Application of the generalized Kudryashov method to the Eckhaus equation

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Abstract. In this paper, the generalized Kudryashov method is presented to seek exact solutions of the Eckhaus equation. From these solutions we can derive solitary wave solutions as a special case. The proposed method is direct, effective and convenient and can be applied to many nonlinear evolution equations in mathematical physics.

Keywords: nonlinear evolution equations, exact solutions, Eckhaus equation, generalized Kudryashov method.

1 Introduction

It is well known that nonlinear evolution equations (NLEEs) are widely used to describe physical phenomena in various scientific and engineering fields, such as fluid mechanics, plasma physics, optical fibers, biology, solid state physics, etc. [1–7, 10, 12–15, 20, 21, 23–29, 31–36]. In order to understand the mechanisms of those physical phenomena, it is necessary to explore their solutions and properties. Solutions for the NLEEs can not only describe the designated problems, but also give more insights on the physical aspects of the problems in the related fields. In recent years, various powerful methods have been presented for finding exact solutions of the NLEEs in mathematical physics, such as $\tanh$-function method [28], extended $\tanh$-function method [10, 29], sine-cosine method [27], Jacobi elliptic function method [13, 32], F-expansion method [1, 31], $\exp$-function method [14], (G'/G)-expansion method [26], Q-function method [20] and so on.

The Q-function method, which is a direct and effective algebraic method for computing exact travelling wave solutions, was first proposed by Kudryashov [16]. The Q-function method that is known as the Kudryashov method is one of the most effective methods

for finding the exact solution of high order NLEEs [17]. The most complete description of this method was given in [19]. The successful application of this method to NLEEs was performed in works [8,18,22]. In the present work, we apply the generalized Kudryashov method [8] to the Eckhaus equation, which has the following form:

$$i\psi_t + \psi_{xx} + 2\bigl(|\psi|^2\bigr)_x \psi + |\psi|^4 \psi = 0,$$

where $\psi = \psi(x,t)$, $\psi : \mathbb{R}^2 \to \mathbb{C}$ is a complex-valued function of two real variables x, t. This equation is a nonlinear Schrödinger-type equation (NLSE) that can be linearized to the free linear Schrödinger equation, which was found in [9] as an asymptotic multiscale reduction of certain classes of nonlinear partial differential equations. In [11], many of the properties of the Eckhaus equation were investigated, including the linearization, soliton solutions etc.

This NLSE is another nonlinear evolution equation that is available in the literature. In fact, this is a dissipative equation. Therefore, this is not studied in the context of fiber optics. The first term is the linear evolution term, while the second term is accounted for dispersion that is commonly referred to as group velocity dispersion (GVD). The third term is a dissipative terms that is responsible for the damping of the soliton solution. Finally, the last term is the nonlinear term.

2 The generalized Kudryashov method

Suppose that we have a nonlinear evolution equation in the form

$$F(u, u_t, u_x, u_{xx}, u_{xt}, \dots) = 0, \tag{1}$$

where $u = u(x,t)$ is an unknown function, F is a polynomial in u and its various partial derivatives u_t, u_x with respect to t, x respectively, in which the highest order derivatives and nonlinear terms are involved.

Step 1. Using the traveling wave transformation

$$u(x,t) = u(\xi), \quad \xi = k(x - ct) + \xi_0, \tag{2}$$

where ξ_0 is an arbitrary constant and k, c are constant to be determined later. Then Eq. (1) is reduced to a nonlinear ordinary differential equation (NODE) of the form

$$P(u, u_\xi, u_{\xi\xi}, \dots) = 0. \tag{3}$$

Step 2. Suppose that the solution of Eq. (3) has the following form:

$$u(\xi) = \frac{\sum_{i=0}^{N} a_i Q^i(\xi)}{\sum_{j=0}^{M} b_j Q^j(\xi)} = \frac{A(Q(\xi))}{B(Q(\xi))}, \tag{4}$$

where a_i $(i = 0, 1, \dots, N)$ and b_j $(j = 0, 1, \dots, M)$ are constants to be determined such that $a_N \neq 0$, $b_M \neq 0$ and

$$Q(\xi) = \frac{1}{1 + \kappa e^\xi} \tag{5}$$

is the solution of the equation

$$Q_\xi = Q^2 - Q, \tag{6}$$

where κ is an arbitrary constant.

Step 3. Determine the positive integer numbers N and M in Eq. (4) by using the homogeneous balance between the highest order derivatives and the nonlinear terms in Eq. (3) after substituting Eq. (6) and the necessary derivatives of u, which have the form

$$u_\xi = \left(Q^2 - Q\right)\left(\frac{A'B - AB'}{B^2}\right), \tag{7}$$

$$u_{\xi\xi} = \frac{(Q^2 - Q)^2}{B^3}\left\{B(BA'' - AB'') - 2B'(A'B - AB')\right\}$$
$$+ (2Q - 1)(Q^2 - Q)\left(\frac{A'B - AB'}{B^2}\right), \tag{8}$$

where the prime $'$ denotes the derivative $\mathrm{d}/\mathrm{d}Q$.

Step 4. Substitute Eqs. (4), (7) and (8) into Eq. (3). As a result of this substitution, we get a polynomial of Q. In this polynomial we gather all terms of same powers and equating them to be zero, we obtain a system of algebraic equations, which can be solved by the Maple or Mathematica to get the unknown parameters a_i $(i = 0, 1, \ldots, N)$, b_j $(j = 0, 1, \ldots, M)$, k, c. Consequently, we obtain the exact solutions of Eq. (1).

3 The Eckhaus equation

The Eckhaus equation is a nonlinear Schrödinger-type equation, which can be written as

$$\mathrm{i}\psi_t + \psi_{xx} + 2\left(|\psi|^2\right)_x\psi + |\psi|^4\psi = 0. \tag{9}$$

This equation has been solved by using the (G'/G)-expansion method [30]. Let us now solve Eq. (9) by using the generalized Kudryashov method. To this end, we use the following wave transformation:

$$\psi(x, t) = u(\xi)\mathrm{e}^{\mathrm{i}(\alpha x + \beta t)}, \quad \xi = k(x - 2\alpha t) + \xi_0, \tag{10}$$

where k, α, β are constants to be determined later and ξ_0 is an arbitrary constant. Now Eq. (9) is reduced to the following NODE:

$$k^2 u_{\xi\xi} - \left(\beta + \alpha^2\right)u + 4ku_\xi u^2 + u^5 = 0. \tag{11}$$

Suppose that

$$u = v^{1/2}. \tag{12}$$

Then Eq. (11) can be reduced to the following NODE:

$$2k^2 vv_{\xi\xi} - k^2 v_\xi^2 - 4\left(\beta + \alpha^2\right)v^2 + 8kv^2 v_\xi + 4v^4 = 0. \tag{13}$$

Balancing $v v_{\xi\xi}$ with v^4 in Eq. (13), we get the formula $N = M + 1$. If we choose $M = 1$ and $N = 2$, then

$$v = \frac{A(Q)}{B(Q)} = \frac{a_0 + a_1 Q + a_2 Q^2}{b_0 + b_1 Q}. \tag{14}$$

Substituting v and it's their necessary derivatives into (13) and equating all the coefficients of Q to zero, we obtain

$$3a_2^2 b_1^2 k^2 + 8a_2^3 b_1 k + 4a_2^4 = 0, \tag{15}$$

$$\begin{aligned}
&-4a_2^2 b_1^2 k^2 + 8a_2^2 b_0 b_1 k^2 + 4a_1 a_2 b_1^2 k^2 + 16a_2^3 b_0 k - 8a_2^3 b_1 k + 16a_1 a_2^2 b_1 k \\
&\quad + 16a_1 a_2^3 = 0, \tag{16}
\end{aligned}$$

$$\begin{aligned}
&-4\alpha^2 a_2^2 b_1^2 - 4a_2^2 \beta b_1^2 + 8a_2^2 b_0^2 k^2 + a_2^2 b_1^2 k^2 - 10a_2^2 b_0 b_1 k^2 + 6a_0 a_2 b_1^2 k^2 \\
&\quad - 6a_1 a_2 b_1^2 k^2 + 10a_1 a_2 b_0 b_1 k^2 - 16a_2^3 b_0 k + 40a_1 a_2^2 b_0 k + 8a_0 a_2^2 b_1 k \\
&\quad - 16a_1 a_2^2 b_1 k + 8a_1^2 a_2 b_1 k + 16a_0 a_2^3 + 24a_1^2 a_2^2 = 0, \tag{17}
\end{aligned}$$

$$\begin{aligned}
&- 8\alpha^2 a_2 a_1 b_1^2 - 8\alpha^2 a_2^2 b_0 b_1 - 8a_2 a_1 \beta b_1^2 - 8a_2^2 \beta b_0 b_1 + 12a_2 a_1 b_0^2 k^2 \\
&\quad + 2a_2 a_1 b_1^2 k^2 - 12a_2 a_1 b_0 b_1 k^2 - 12a_2^2 b_0^2 k^2 - 12a_0 a_2 b_1^2 k^2 + 2a_2^2 b_0 b_1 k^2 \\
&\quad + 12a_0 a_2 b_0 b_1 k^2 + 32a_2 a_1^2 b_0 k - 8a_2 a_1^2 b_1 k - 40a_2^2 a_1 b_0 k + 32a_0 a_2^2 b_0 k \\
&\quad - 8a_0 a_2^2 b_1 k + 16a_2 a_1^3 + 48a_0 a_2^2 a_1 = 0, \tag{18}
\end{aligned}$$

$$\begin{aligned}
&- 4\alpha^2 a_1^2 b_1^2 - 16\alpha^2 a_2 a_1 b_0 b_1 - 4\alpha^2 a_2^2 b_0^2 - 8\alpha^2 a_0 a_2 b_1^2 - 4a_1^2 \beta b_1^2 - 16a_2 a_1 \beta b_0 b_1 \\
&\quad - 4a_2^2 \beta b_0^2 - 8a_0 a_2 \beta b_1^2 + 3a_1^2 b_0^2 k^2 + 2a_1^2 b_0 b_1 k^2 - 18a_2 a_1 b_0^2 k^2 - 2a_0 a_1 b_1^2 k^2 \\
&\quad - 2a_0 a_1 b_0 b_1 k^2 + 2a_2 a_1 b_0 b_1 k^2 + 4a_2^2 b_0^2 k^2 + 12a_0 a_2 b_0^2 k^2 - a_0^2 b_1^2 k^2 + 6a_0 a_2 b_1^2 k^2 \\
&\quad - 20a_0 a_2 b_0 b_1 k^2 + 8a_1^3 b_0 k - 32a_2 a_1^2 b_0 k - 8a_0 a_1^2 b_1 k + 48a_0 a_2 a_1 b_0 k \\
&\quad - 32a_0 a_2^2 b_0 k - 8a_0^2 a_2 b_1 k + 4a_1^4 + 48a_0 a_2 a_1^2 + 24a_0^2 a_2^2 = 0, \tag{19}
\end{aligned}$$

$$\begin{aligned}
&-8\alpha^2 a_1^2 b_0 b_1 - 8\alpha^2 a_2 a_1 b_0^2 - 8\alpha^2 a_0 a_1 b_1^2 - 16\alpha^2 a_0 a_2 b_0 b_1 - 8a_1^2 \beta b_0 b_1 \\
&\quad - 8a_2 a_1 \beta b_0^2 - 8a_0 a_1 \beta b_1^2 - 16a_0 a_2 \beta b_0 b_1 - 4a_1^2 b_0^2 k^2 - 2a_1^2 b_0 b_1 k^2 + 4a_0 a_1 b_0^2 k^2 \\
&\quad + 6a_2 a_1 b_0^2 k^2 + 2a_0 a_1 b_1^2 k^2 + 4a_0 a_1 b_0 b_1 k^2 - 20a_0 a_2 b_0^2 k^2 - 4a_0^2 b_0 b_1 k^2 \\
&\quad + 8a_0 a_2 b_0 b_1 k^2 - 8a_1^3 b_0 k + 16a_0 a_1^2 b_0 k + 8a_0 a_1^2 b_1 k - 48a_0 a_2 a_1 b_0 k \\
&\quad - 16a_0^2 a_1 b_1 k + 16a_0^2 a_2 b_0 k + 8a_0^2 a_2 b_1 k + 16a_0 a_1^3 + 48a_0^2 a_2 a_1 = 0, \tag{20}
\end{aligned}$$

$$\begin{aligned}
&-4\alpha^2 a_0^2 b_1^2 - 8\alpha^2 a_2 a_0 b_0^2 - 16\alpha^2 a_1 a_0 b_0 b_1 - 4\alpha^2 a_1^2 b_0^2 - 4a_0^2 \beta b_1^2 - 8a_2 a_0 \beta b_0^2 \\
&\quad - 16a_1 a_0 \beta b_0 b_1 - 4a_1^2 \beta b_0^2 + a_0^2 b_1^2 k^2 + 6a_0^2 b_0 b_1 k^2 - 6a_1 a_0 b_0^2 k^2 + 8a_2 a_0 b_0^2 k^2 \\
&\quad - 2a_1 a_0 b_0 b_1 k^2 + a_1^2 b_0^2 k^2 - 8a_0^3 b_1 k + 8a_1 a_0^2 b_0 k - 16a_2 a_0^2 b_0 k + 16a_1 a_0^2 b_1 k \\
&\quad - 16a_1^2 a_0 b_0 k + 16a_2 a_0^3 + 24a_1^2 a_0^2 = 0, \tag{21}
\end{aligned}$$

$$\begin{aligned}
&-8\alpha^2 a_0^2 b_0 b_1 - 8\alpha^2 a_1 a_0 b_0^2 - 8a_0^2 \beta b_0 b_1 - 8a_1 a_0 \beta b_0^2 - 2a_0^2 b_0 b_1 k^2 + 2a_1 a_0 b_0^2 k^2 \\
&\quad + 8a_0^3 b_1 k - 8a_1 a_0^2 b_0 k + 16a_1 a_0^3 = 0, \tag{22}
\end{aligned}$$

$$-4\alpha^2 a_0^2 b_0^2 - 4a_0^2 \beta b_0^2 + 4a_0^4 = 0. \tag{23}$$

Solving system (15)–(23) with the aid of Mathematica, we obtain the following results.

Result 1.

$$a_0 = 0, \quad a_1 = \frac{-kb_0}{2}, \quad a_2 = \frac{-kb_1}{2}, \quad k = \pm 2\sqrt{\beta + \alpha^2}. \tag{24}$$

Substituting (24) into (14) with (5), (10) and (12), we obtain the following solutions of Eq. (9):

$$\psi_{1,2}(x,t) = \pm \left\{ \sqrt{\beta + \alpha^2} \frac{1}{1 + \kappa e^{\pm 2\sqrt{\beta + \alpha^2}(x - 2\alpha t) + \xi_0}} \right\}^{1/2} e^{i(\alpha x + \beta t)}, \tag{25}$$

$$\psi_{3,4}(x,t) = \pm i \left\{ \sqrt{\beta + \alpha^2} \frac{1}{1 + \kappa e^{\pm 2\sqrt{\beta + \alpha^2}(x - 2\alpha t) + \xi_0}} \right\}^{1/2} e^{i(\alpha x + \beta t)}. \tag{26}$$

If we set $\kappa = 1$, we obtain

$$\psi(x,t) = \pm \left\{ \frac{\sqrt{\beta + \alpha^2}}{2} \left(1 - \tanh\left(\pm\sqrt{\beta + \alpha^2}(x - 2\alpha t) + \xi_0\right)\right) \right\}^{1/2} e^{i(\alpha x + \beta t)}, \tag{27}$$

$$\psi(x,t) = \pm i \left\{ \frac{\sqrt{\beta + \alpha^2}}{2} \left(1 - \tanh\left(\pm\sqrt{\beta + \alpha^2}(x - 2\alpha t) + \xi_0\right)\right) \right\}^{1/2} e^{i(\alpha x + \beta t)}. \tag{28}$$

If we set $\kappa = -1$, we obtain

$$\psi(x,t) = \pm \left\{ \frac{\sqrt{\beta + \alpha^2}}{2} \left(1 - \coth\left(\pm\sqrt{\beta + \alpha^2}(x - 2\alpha t) + \xi_0\right)\right) \right\}^{1/2} e^{i(\alpha x + \beta t)}, \tag{29}$$

$$\psi(x,t) = \pm i \left\{ \frac{\sqrt{\beta + \alpha^2}}{2} \left(1 - \coth\left(\pm\sqrt{\beta + \alpha^2}(x - 2\alpha t) + \xi_0\right)\right) \right\}^{1/2} e^{i(\alpha x + \beta t)}. \tag{30}$$

Result 2.

$$a_0 = \frac{kb_0}{2}, \quad a_1 = \frac{k}{2}(b_1 - b_0), \quad a_2 = \frac{-kb_1}{2}, \quad k = \pm 2\sqrt{\beta + \alpha^2}. \tag{31}$$

Substituting (31) into (14) with (5), (10) and (12), we obtain the following solutions of Eq. (9):

$$\psi_{5,6}(x,t) = \pm \left\{ \sqrt{\beta + \alpha^2} \left(1 - \frac{1}{1 + \kappa e^{\pm 2\sqrt{\beta + \alpha^2}(x - 2\alpha t) + \xi_0}}\right) \right\}^{1/2} e^{i(\alpha x + \beta t)}, \tag{32}$$

$$\psi_{7,8}(x,t) = \pm i \left\{ \sqrt{\beta + \alpha^2} \left(1 - \frac{1}{1 + \kappa e^{\pm 2\sqrt{\beta + \alpha^2}(x - 2\alpha t) + \xi_0}}\right) \right\}^{1/2} e^{i(\alpha x + \beta t)}. \tag{33}$$

If we set $\kappa = 1$, we obtain

$$\psi(x,t) = \pm \left\{ \frac{\sqrt{\beta + \alpha^2}}{2} \left(1 + \tanh\left(\pm\sqrt{\beta + \alpha^2}(x - 2\alpha t) + \xi_0\right)\right) \right\}^{1/2} e^{i(\alpha x + \beta t)}, \tag{34}$$

$$\psi(x,t) = \pm i \left\{ \frac{\sqrt{\beta + \alpha^2}}{2} \left(1 + \tanh\left(\pm\sqrt{\beta + \alpha^2}(x - 2\alpha t) + \xi_0\right)\right) \right\}^{1/2} e^{i(\alpha x + \beta t)}. \tag{35}$$

If we set $\kappa = -1$, we obtain

$$\psi(x,t) = \pm\left\{\frac{\sqrt{\beta+\alpha^2}}{2}\big(1 + \coth\big(\pm\sqrt{\beta+\alpha^2}(x-2\alpha t) + \xi_0\big)\big)\right\}^{1/2} \mathrm{e}^{\mathrm{i}(\alpha x+\beta t)}, \quad (36)$$

$$\psi(x,t) = \pm\mathrm{i}\left\{\frac{\sqrt{\beta+\alpha^2}}{2}\big(1 + \coth\big(\pm\sqrt{\beta+\alpha^2}(x-2\alpha t) + \xi_0\big)\big)\right\}^{1/2} \mathrm{e}^{\mathrm{i}(\alpha x+\beta t)}. \quad (37)$$

Result 3.

$$a_0 = \frac{3kb_0}{4}, \quad a_1 = -2kb_0, \quad a_2 = kb_0, \quad b_1 = -2b_0, \quad k = \pm 2\sqrt{\frac{2(\beta+\alpha^2)}{5}}. \quad (38)$$

Substituting (38) into (14) with (5), (10) and (12), we obtain the following solutions of Eq. (9):

$$\psi_{9,10}(x,t) = \pm\left\{\frac{1}{2}\sqrt{\frac{2(\beta+\alpha^2)}{5}}\left(3 - \frac{2}{1 + \kappa\mathrm{e}^{\pm 2\sqrt{2(\beta+\alpha^2)/5}(x-2\alpha t)+\xi_0}}\right)\right\}^{1/2}$$
$$\times \mathrm{e}^{\mathrm{i}(\alpha x+\beta t)}, \quad (39)$$

$$\psi_{11,12}(x,t) = \pm\mathrm{i}\left\{\frac{1}{2}\sqrt{\frac{2(\beta+\alpha^2)}{5}}\left(3 - \frac{2}{1 + \kappa\mathrm{e}^{\pm 2\sqrt{2(\beta+\alpha^2)/5}(x-2\alpha t)+\xi_0}}\right)\right\}^{1/2}$$
$$\times \mathrm{e}^{\mathrm{i}(\alpha x+\beta t)}. \quad (40)$$

If we set $\kappa = 1$, we obtain

$$\psi(x,t) = \pm\left\{\frac{1}{2}\sqrt{\frac{2(\beta+\alpha^2)}{5}}\left(2 + \tanh\left(\pm\sqrt{\frac{2(\beta+\alpha^2)}{5}}(x-2\alpha t) + \xi_0\right)\right)\right\}^{1/2}$$
$$\times \mathrm{e}^{\mathrm{i}(\alpha x+\beta t)}, \quad (41)$$

$$\psi(x,t) = \pm\mathrm{i}\left\{\frac{1}{2}\sqrt{\frac{2(\beta+\alpha^2)}{5}}\left(2 + \tanh\left(\pm\sqrt{\frac{2(\beta+\alpha^2)}{5}}(x-2\alpha t) + \xi_0\right)\right)\right\}^{1/2}$$
$$\mathrm{e}^{\mathrm{i}(\alpha x+\beta t)}. \quad (42)$$

If we set $\kappa = -1$, we obtain

$$\psi(x,t) = \pm\left\{\frac{1}{2}\sqrt{\frac{2(\beta+\alpha^2)}{5}}\left(2 + \coth\left(\pm\sqrt{\frac{2(\beta+\alpha^2)}{5}}(x-2\alpha t) + \xi_0\right)\right)\right\}^{1/2}$$
$$\times \mathrm{e}^{\mathrm{i}(\alpha x+\beta t)}, \quad (43)$$

$$\psi(x,t) = \pm\mathrm{i}\left\{\frac{1}{2}\sqrt{\frac{2(\beta+\alpha^2)}{5}}\left(2 + \coth\left(\pm\sqrt{\frac{2(\beta+\alpha^2)}{5}}(x-2\alpha t) + \xi_0\right)\right)\right\}^{1/2}$$
$$\times \mathrm{e}^{\mathrm{i}(\alpha x+\beta t)}. \quad (44)$$

Result 4.

$$a_0 = 0, \quad a_1 = 0, \quad a_2 = kb_0, \quad b_1 = -2b_0, \quad k = \pm\sqrt{\beta + \alpha^2}. \qquad (45)$$

Substituting (45) into (14) with (5), (10) and (12), we obtain the following solutions of Eq. (9):

$$\psi_{13,14}(x,t) = \pm\left\{\sqrt{\beta + \alpha^2}\left(\frac{(1 + \kappa e^{\pm\sqrt{\beta+\alpha^2}(x-2\alpha t)+\xi_0})^{-2}}{1 - 2(1 + \kappa e^{\pm\sqrt{\beta+\alpha^2}(x-2\alpha t)+\xi_0})^{-1}}\right)\right\}^{1/2}$$
$$\times e^{i(\alpha x + \beta t)}, \qquad (46)$$

$$\psi_{15,16}(x,t) = \pm i\left\{\sqrt{\beta + \alpha^2}\left(\frac{(1 + \kappa e^{\pm\sqrt{\beta+\alpha^2}(x-2\alpha t)+\xi_0})^{-2}}{1 - 2(1 + \kappa e^{\pm\sqrt{\beta+\alpha^2}(x-2\alpha t)+\xi_0})^{-1}}\right)\right\}^{1/2}$$
$$\times e^{i(\alpha x + \beta t)}. \qquad (47)$$

If we set $\kappa = \pm 1$, we obtain

$$\psi(x,t) = \pm\left\{\frac{\sqrt{\beta + \alpha^2}}{2}\left(1 + \coth\left(\pm\sqrt{\beta + \alpha^2}(x - 2\alpha t) + \xi_0\right)\right)\right\}^{1/2}$$
$$\times e^{i(\alpha x + \beta t)}, \qquad (48)$$

$$\psi(x,t) = \pm i\left\{\frac{\sqrt{\beta + \alpha^2}}{2}\left(1 + \coth\left(\pm\sqrt{\beta + \alpha^2}(x - 2\alpha t) + \xi_0\right)\right)\right\}^{1/2}$$
$$\times e^{i(\alpha x + \beta t)}. \qquad (49)$$

Result 5.

$$a_0 = kb_0, \quad a_1 = -2kb_0, \quad a_2 = kb_0, \quad b_1 = -2b_0, \quad k = \pm\sqrt{\beta + \alpha^2}. \qquad (50)$$

Substituting (50) into (14) with (5), (10) and (12), we obtain the following solutions of Eq. (9):

$$\psi_{17,18}(x,t) = \pm\left\{\sqrt{\beta + \alpha^2}\left(\frac{((1 + \kappa e^{\pm\sqrt{\beta+\alpha^2}(x-2\alpha t)+\xi_0})^{-1} - 1)^2}{1 - 2(1 + \kappa e^{\pm\sqrt{\beta+\alpha^2}(x-2\alpha t)+\xi_0})^{-1}}\right)\right\}^{1/2}$$
$$\times e^{i(\alpha x + \beta t)}, \qquad (51)$$

$$\psi_{19,20}(x,t) = \pm i\left\{\sqrt{\beta + \alpha^2}\left(\frac{((1 + \kappa e^{\pm\sqrt{\beta+\alpha^2}(x-2\alpha t)+\xi_0})^{-1} - 1)^2}{1 - 2(1 + \kappa e^{\pm\sqrt{\beta+\alpha^2}(x-2\alpha t)+\xi_0})^{-1}}\right)\right\}^{1/2}$$
$$\times e^{i(\alpha x + \beta t)}. \qquad (52)$$

If we set $\kappa = \pm 1$, we obtain

$$\psi(x,t) = \pm\left\{\frac{\sqrt{\beta + \alpha^2}}{2}\left(1 + \coth\left(\pm\sqrt{\beta + \alpha^2}(x - 2\alpha t) + \xi_0\right)\right)\right\}^{1/2}$$
$$\times e^{i(\alpha x + \beta t)}, \qquad (53)$$

$$\psi(x,t) = \pm i \left\{ \frac{\sqrt{\beta + \alpha^2}}{2} \left(1 + \coth\left(\pm\sqrt{\beta + \alpha^2}(x - 2\alpha t) + \xi_0\right)\right) \right\}^{1/2}$$
$$\times e^{i(\alpha x + \beta t)}. \tag{54}$$

4 Conclusions

In this paper, we have proposed the generalized Kudryashov method for solving the Enkhaus equation. This work has illustrated that the solutions obtained in [30] are considered as a special case of our obtained solutions and a new results have been obtained using this method. This method is direct, effective and can be extended for solving many systems of nonlinear PDEs. The soliton solutions that are retrievable from this equation are topological and singular soliton solutions only. Being a dissipative model, it is not possible to obtain non-topological soliton solution. Therefore, it makes sense that this model only retrieves singular and topological soliton solutions.

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