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On new chaotic and hyperchaotic systems: A literature survey

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Abstract. This paper provides a thorough survey of new chaotic and hyperchaotic systems. An analysis of the dynamic behavior of these complex systems is presented by pointing out their originality and elementary characteristics. Recently, such systems have been increasingly used in various fields such as secure communication, encryption and finance and so on. In practice, each field requires specific performances with peculiar complexity. A particular classification is then proposed in this paper based on the Lyapunov exponent, the equilibriums points and the attractor forms.

Keywords: chaotic systems, hyperchaotic systems, Lyapunov exponent, attractor form, equilibrium.

1 Introduction

Recently, the chaotification [55] of dynamical systems, which appoints the task of creating chaos, has attracted the scientific community. Chaos has revalorized the evolution of technology as soon as its appearance in 1963. This is justified by their unpredictable dynamic behaviors and their extremely sensitivity to initial conditions. It has been proven that an infinitesimal variation in initial conditions grows exponentially with time and so gives unexpected results [25]. This sensitive dependence is among the basic ideas of chaos and the most visible signature of its behavior. This phenomenon is ancient but was highlighted by Lorenz using the butterfly effect [36], which lies in the fact that a flap of butterfly's wings will impact after some time the atmosphere behaviors and bring back it to the chaos.

Chaotic system is defined as a nonlinear system with only one Lyapunov exponent [61] where hyperchaotic system is defined as a chaotic system with more than one positive Lyapunov exponent. Thus, the hyperchaotic attractor is deployed in several directions contrary to the chaotic attractor, which deployed in only one direction.

Recently, generation of deliberate chaotic and hyperchaotic signals has been described as the key issue in several highly engineering and technological applications such as

neural networks [59], finance [57], encryption [50], nonlinear circuits [19], secure communication [35], and so on. In practice, each new system imposes peculiar technical requirements. This variety involves the absolute necessity to know the system performances in order to know, which one will be used. Actually, certain applications are more critical than others such as the control of cardiac rhythm [16]. Thus, comes the idea to classify the recently chaotic and hyperchaotic systems according to their performances since certain system characteristics can restrict their exploitation in nontraditional applications. Based on the analysis of the dynamic behaviors of these new systems, three classification criteria are extracted. Each criterion influences the system performances. Moreover, this classification will make the possibility to choose which performances will be optimized.

The present survey is organized as follows. Section 2 presents the basic chaotic and hyperchaotic systems in the field of the chaos theory by describing their mathematical models as well as their main characteristics. In Section 3, new chaotic and hyperchaotic systems are introduced and analyzed by focusing on their originality. Section 4, proposes a particular classification of these complex systems mainly based on the Lyapunov exponent, the equilibriums points and the attractor forms. Finally, Section 5 provides the possible challenges for such new systems. For our analysis, we consider throughout our study $(a, b, c, d, e, f, k, h, r)$ as the system constant parameters and (x, y, z, w) as the state variables.

2 Basic chaotic and hyperchaotic systems

In this section, we describe the mathematical models of basic chaotic and hyperchaotic systems. This description is, purposely, very brief in order to use these models only in the analysis of new chaotic and hyperchaotic systems.

2.1 Basic chaotic systems

The generalized Lorenz system family is described as follows [10]:

$$\begin{aligned} \dot{x} &= \gamma x + \beta y, \\ \dot{y} &= \alpha x + \theta y - xz, \\ \dot{z} &= xy - bz, \end{aligned} \tag{1}$$

where (γ, β, α, θ) are the constant parameters of system (1). If $(\gamma, \beta, \alpha, \theta)$ is equal to $(-a, a, c, -1)$, we find the Lorenz system [36] described as

$$\begin{aligned} \dot{x} &= a(y - x), \\ \dot{y} &= cx - y - xz, \\ \dot{z} &= xy - bz. \end{aligned} \tag{2}$$

Its first Lyapunov exponent is equal to 0.898 and the sign of $\beta\alpha$ is positive. Besides, this system has been extensively used in secure communication [9].

If $(\gamma, \beta, \alpha, \theta) = (-a, a, c-a, c)$, we find the Chen attractor written as [4]

$$\begin{aligned} \dot{x} &= a(y-x), \\ \dot{y} &= (c-a)x + cy - xz, \\ \dot{z} &= xy - bz. \end{aligned} \tag{3}$$

If $(\gamma, \beta, \alpha, \theta) = (-a, a, 0, c)$, we find the Lü system [37] described as

$$\begin{aligned} \dot{x} &= a(y-x), \\ \dot{y} &= cy - xz, \\ \dot{z} &= xy - bz. \end{aligned} \tag{4}$$

Rössler system, discovered later the Lorenz system, is described as follows [44]:

$$\begin{aligned} \dot{x} &= -(y+z), \\ \dot{y} &= x + ay, \\ \dot{z} &= zx + b. \end{aligned} \tag{5}$$

System (5) is the simplest chaotic system with unsymmetrical property.

Then, the Chua's circuit [7] was proposed. It is written as follows [8, 21, 39, 45]:

$$\begin{aligned} \frac{\mathrm{d}v_1}{\mathrm{d}t} &= \frac{G(v_2 - v_1) - f(v_1)}{C_1}, \\ \frac{\mathrm{d}v_2}{\mathrm{d}t} &= \frac{G(v_1 - v_2) + i_3}{C_2}, \\ \frac{\mathrm{d}i_L}{\mathrm{d}t} &= \frac{-v_2 - R_0 i_3}{L}, \end{aligned} \tag{6}$$

where v_1 and v_2 are the voltage cross the capacitors C_1 and C_2. i_L is the current through the inductance L. $G = 1/R$ and $R_0 i_3$ reflects the small resistance of the inductor. $f(v_1)$ is a piecewise linear function of the Chua's diode given by

$$f(v_1) = G_b v_1 + 0.5(G_a - G_b)\big(|v_1 + E| - |v_1 - E|\big), \tag{7}$$

where $G_a G_b < 0$ and E is the breakpoint voltage of the Chua's diode.

The Liu system, characterized by three equilibriums, is described as [33]

$$\begin{aligned} \dot{x} &= a(y-x), \\ \dot{y} &= bx - kxz, \\ \dot{z} &= -cz + hx^2. \end{aligned} \tag{8}$$

Finally, the Qi system, characterized by five equilibriums, is written as [42]

$$\begin{aligned} \dot{x} &= a(y-x) + yz, \\ \dot{y} &= cx - y - xz, \\ \dot{z} &= xy - bz. \end{aligned} \tag{9}$$

2.2 Basic hyperchaotic systems

The first hyperchaotic system was discovered by Rössler [44]. It is written as

$$\begin{aligned} \dot{x} &= -(y+z), \\ \dot{y} &= x + ay + w, \\ \dot{z} &= xz + b, \\ \dot{w} &= -cz + dw. \end{aligned} \tag{10}$$

Then, the hyperchaotic Chua's system was discovered [1, 24, 48]. It is described as

$$\begin{aligned} \dot{x} &= a\big(y - x - f(x)\big), \\ \dot{y} &= x - y + z + k(v - y), \\ \dot{z} &= -by, \\ \dot{u} &= a\big(v - u - f(u)\big), \\ \dot{v} &= u - v + w, \\ \dot{w} &= -bv, \end{aligned} \tag{11}$$

where $f(x)$ and $f(u)$ are the piecewise linear functions of the Chua's diode.

In 2005, Li and Chen discovered a new hyperchaotic system [31], described as

$$\begin{aligned} \dot{x} &= a(y - x) + w, \\ \dot{y} &= dx + cy - xz, \\ \dot{z} &= xy - bz, \\ \dot{w} &= yz + rw. \end{aligned} \tag{12}$$

Later, other Chen systems [18, 32] were discovered. The new system [18], invariant to the transformation $(x, y, z, w, k) \mid (-x, -y, -z, -w, -k)$, is written as

$$\begin{aligned} \dot{x} &= a(y - x), \\ \dot{y} &= dx - xz + cy - w, \\ \dot{z} &= xy - bz, \\ \dot{w} &= x + k. \end{aligned} \tag{13}$$

In 2006, Wang and Liu generated a hyperchaotic system [15] described as

$$\begin{aligned} \dot{x} &= a(y - x), \\ \dot{y} &= bx - kxz + w, \\ \dot{z} &= -cz + hx^2, \\ \dot{w} &= -dx. \end{aligned} \tag{14}$$

According to d, this system pivots between the chaotic, hyperchaotic and periodic attractor. Its electric circuit was realized for security applications [54].

Then, the hyperchaotic Lü system was described by the following differential equations [3]:

$$\begin{aligned} \dot{x} &= a(y-x)+w, \\ \dot{y} &= cy-xz, \\ \dot{z} &= xy-bz, \\ \dot{w} &= xz+dw. \end{aligned} \tag{15}$$

In 2007, Gao et al. built the hyperchaotic system, based on system (2), by adding a nonlinear feedback controller. The novel system is described as [17]

$$\begin{aligned} \dot{x} &= a(y-x), \\ \dot{y} &= cx+y-xz-w, \\ \dot{z} &= xy-bz, \\ \dot{w} &= kyz. \end{aligned} \tag{16}$$

This system has been implemented for secure communication and encryption [63]. In the same year, Jia discovered another hyperchaotic system written as [23]

$$\begin{aligned} \dot{x} &= a(y-x)+w, \\ \dot{y} &= cx-y-xz, \\ \dot{z} &= xy-bz, \\ \dot{w} &= -xz+rw. \end{aligned} \tag{17}$$

Later, Chen et al. discovered a hyperchaotic system, described as [6]

$$\begin{aligned} \dot{x} &= a(y-x)+eyz, \\ \dot{y} &= cx+y-dxz+w, \\ \dot{z} &= xy-bz, \\ \dot{w} &= -ky. \end{aligned} \tag{18}$$

This system has one equilibrium and its first Lyapunov exponent is equal to 4.409.

3 New chaotic and hyperchaotic systems

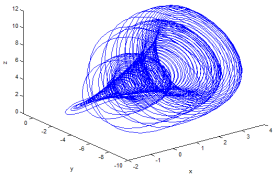
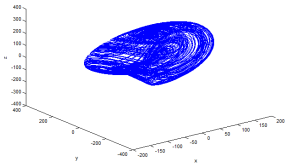
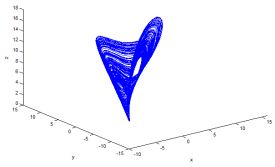
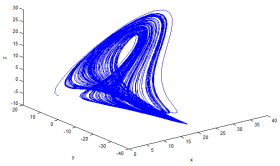
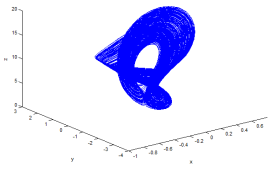
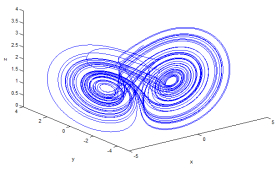
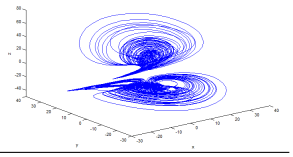
3.1 New chaotic systems

In order to follow the evolution of the technology, various new chaotic systems have emerged. For example, in 2005, Qi et al. generated a new four dimensional autonomous chaotic system [43] characterized by 65 equilibriums. It is written as

$$\begin{aligned} \dot{x} &= a(y-x)+yzw, \\ \dot{y} &= d(x-y)+xzw, \\ \dot{z} &= -cz+xyz, \\ \dot{w} &= -dw+xyz. \end{aligned} \tag{19}$$

To illustrate the main characteristics of new systems emerging in the last five years, Table 1 is established. Seven novel systems are selected. An analysis of these systems will be then given in order to understand their complex dynamic behaviors.

Table 1. Description of new chaotic systems: Lyapunov exponents (LEs), equilibriums and phase diagram (figures attractors simulated in matcont).

Source	Equilibriums	LEs	Attractors
Dong et al. [14]	1, 2 or 3	$\mathrm{LE}_1 = 0.067$ $\mathrm{LE}_2 = 0.003$ $\mathrm{LE}_3 = -0.013$	
Pan et al. [40]	3	$\mathrm{LE}_1 = 0.697$ $\mathrm{LE}_2 \simeq 0$ $\mathrm{LE}_3 = -1.481$	
Li and Ou [29]	1 or 3	$\mathrm{LE}_1 = 0.426$ $\mathrm{LE}_2 \simeq 0$ $\mathrm{LE}_3 = -7.426$	
Liu and Zang [34]	1, 3 or 5	A largest LE spectrum	
Li et al. [26]	3	$\mathrm{LE}_1 = 1.868$ $\mathrm{LE}_2 \simeq 0$ $\mathrm{LE}_3 = -17.736$	
Cang et al. [2]	2	$\mathrm{LE}_1 = 0.296$ $\mathrm{LE}_2 \simeq 0$ $\mathrm{LE}_3 = -1.295$	
Guan et al. [20]	3, 4 or 5	A largest LE spectrum	

As described in Table 1, the system suggested by Dong et al. was discovered in 2009. This chaotic system is described as [14]

$$\begin{aligned} \dot{x} &= ax - bx^2 - c(y+z), \\ \dot{y} &= -dy - ez + kx - hx(x-z), \\ \dot{z} &= -r(z-f)(x-g), \end{aligned} \tag{20}$$

where $(a, b, c, d, e, f, g, k, h, r)$ are the constant parameters of system (20). The originality of this system lies in its attractor form. Indeed, the chaotic system generates a stranger attractor with multi-layers. The number of these layers varies by varying the value of Lyapunov dimension, which is calculated from the sum of Lyapunov exponents. Four forms of attractors are obtained namely three-layer, four-layer, five-layer and multi-layer attractor. If the parameter d varies, the number of equilibriums changes. In fact, if $d < 0$, system (20) has only one equilibrium. If $d = 0$, it has two equilibriums and if $d > 0$, it has three equilibriums.

One year later, Pan et al. discovered a new chaotic system derived from the Chen system (3) via chaos controlling from its continuous time TS fuzzy model and with the non-resonant parametric perturbation approach [40]. This system has three equilibriums and is symmetric about the z axis. It is described as

$$\begin{aligned} \dot{x} &= a(y-x) + dxz, \\ \dot{y} &= (f-a)x - xz + fy, \\ \dot{z} &= -ex^2 + xy + cz. \end{aligned} \tag{21}$$

This system has a quadratic term in the third equation and a cross-product term in each equation. It generates a three-scroll attractor as highlighted in Table 1. System (21) is characterized by a first Lyapunov exponent equal to 0.6971. Also, the third Lyapunov exponent of this system is equal to -1.4812 and it is very low compared to the chaotic Chen system, which is equal to -12.0003. So, the new chaotic system has a lower degree of orbital disorder and randomness because this system has a slower contracting rate phase space than the Chen system.

In 2011, Li and Ou discovered a new three dimensional chaotic system [29], based on the chaotic Lorenz system (22). The novel system is written as

$$\begin{aligned} \dot{x} &= a(y-x), \\ \dot{y} &= cx - xz, \\ \dot{z} &= xy - bz + ex^2. \end{aligned} \tag{22}$$

This system is not topologically equivalent to the Lorenz (2) or the Chen system (3), but when $e = 0$, it was equivalent to the Lü system (4). Hence, this system represents also the transition between the Lorenz and the Chen system called the new Lorenz-like system. So, it has a complex dynamic behavior with a variable equilibrium number. In fact, when the product bd is negative, the system has a unique equilibrium at the origin O. When the product bd is positive, besides O, system (22) has still two non-zero equilibriums. Thus,

this system has one or three equilibriums depending on the sign of the product bd. The new system (22) generates a two scroll attractor and has a first Lyapunov exponent equal to 0.4265.

A new system, discovered by Liu and Zhang in 2013 [34], was described as

$$\begin{aligned} \dot{x} &= ax + dyz + ky^2, \\ \dot{y} &= by + exz + hz, \\ \dot{z} &= cz + fxy. \end{aligned} \tag{23}$$

When $(a, b, c, d, e, k) = (-3, 5, -10, 1, 1, -1, 1)$ and $h \in (-50, 50]$, system (23) generates a strange attractor, which has a double scroll form. For calculating the equilibriums, we solve these equations: $\dot{x} = 0$, $\dot{y} = 0$ and $\dot{z} = 0$. First, we have the origin O as one of the equilibria. Next, by combining the last three equations, we obtain the discriminant Δ equal to $h^2 f^2 + 4bcef$. If $\Delta < 0$, system (23) has only one equilibrium at the origin. If $\Delta = 0$, it has three equilibriums and if $\Delta > 0$, it has five equilibriums including the origin respectively. Thus, this system can generates 1, 3 or 5 equilibriums.

In the same years, Li et al. proposed a new chaotic system [26] written as

$$\begin{aligned} \dot{x} &= -ax + fyz, \\ \dot{y} &= cy - dxz, \\ \dot{z} &= ey^2 - bz. \end{aligned} \tag{24}$$

System (24) has three equilibriums and generates a chaotic attractor. Its nonlinearity lies in a two quadratic cross product and a square term. Then, the novel system displays complicated dynamic behavior characterized by an invariable Lyapunov exponent spectrum when f, e and d vary. Such as the parameters d and f are the global parameters of nonlinear amplitude adjuster. Further, the parameter e is a local parameter of nonlinear amplitude adjuster.

In 2014, Cang et al. proposed a new Lorenz-like system [2], described as

$$\begin{aligned} \dot{x} &= -ay - xz, \\ \dot{y} &= -dx + xy, \\ \dot{z} &= -r - xy. \end{aligned} \tag{25}$$

Not equivalent to the Chen system, system (25) contains three cross-product terms and one constant term. This system is characterized by two symmetrical equilibriums with respect to the z axis as highlighted in Table 1. By varying value parameters, system (25) converges to two fixed points or generates a periodic orbit or a chaotic attractor with a first Lyapunov exponent equal to 0.2963.

In the same year, Guan et al. discovered a novel chaotic system [20]. This system has three cross-product terms and one constant term described as

$$\begin{cases} \dot{x} &= ax - yz - y + k, \\ \dot{y} &= -by + xz, \\ \dot{z} &= -cz + xy. \end{cases} \tag{26}$$

System (26) is very sensitive to the initial conditions. Indeed, this system generates multiple coexisting attractors namely multiple chaotic attractor, multiple periodic attractor or multiple periodic and chaotic attractors. For example, when $(a, b, c, k) = (4, 10, 5, 0)$, system (26) can generate three coexisting chaotic attractors whose Lyapunov exponents are equal to 0.699, 0.457 and 0.520. Also, this system is characterized by a variable equilibriums number by varying the parameter c. If $c < 16bk^2/(4ab + 1)^2$, system (26) has three equilibriums. If $c = 16bk^2/(4ab + 1)^2$, it has four equilibriums and if $c > 16bk^2/(4ab+1)^2$, it has five equilibriums. Thus, this system has 3, 4 or 5 equilibriums according to the parameters a, b, c and k.

Recently, Luo et al. have proposed a new chaotic system, which generates various grid multi-wings butterfly chaotic attractor [38], described as

$$\begin{aligned} \dot{x} &= a(y - x), \\ \dot{y} &= by - xz, \\ \dot{z} &= G(y) - c, \end{aligned} \tag{27}$$

where $G(y) = y^2$. By replacing $G(y)$ with a multi-piecewise square function with adjustable parameters, system (27) generates a complex grid multi-wing chaotic attractor. Thus, the number of grid multi-wing attractors depends on the values of the adjustable parameters. Also, these parameters can control the number, the sizes and the relative positions of the attractor wings.

3.2 New hyperchaotic systems

Our study of new hyperchaotic systems will be essentially focused on systems that have appeared during these last five years. We analyze nine new hyperchaotic systems. In Table 2, the main characteristics relative to each one will be indicated and followed after by a detailed analysis.

In 2010, two new hyperchaotic systems based on the chaotic Qi system (9) appeared. In fact, the first one was proposed by Yujun et al. [58] by introducing a dynamic feedback controller to the chaotic system. It is described as

$$\begin{aligned} \dot{x} &= a(y - x) + yz, \\ \dot{y} &= -cx - y - xz + w, \\ \dot{z} &= -xy - bz, \\ \dot{w} &= -xz + rw. \end{aligned} \tag{28}$$

By varying the parameters (a, b, c), system (28) generates different attractors namely chaotic, hyperchaotic and some periodic attractor. For the hyperchaotic attractor, its first Lypunov exponent is equal to 1.4106.

At the same time, Wang et al. have discovered a new hyperchaotic system by adding nonlinear controller to the Qi system (9). Contrary to the previous system, this system contains five cross-product terms, which generates more complex dynamic behavior as

described in Table 2. It is described as fllows [51, 52]

$$\begin{aligned} \dot{x} &= a(y-x) + yz + bw, \\ \dot{y} &= cx - y - xz, \\ \dot{z} &= xy - bz + yw, \\ \dot{w} &= -yz + kw. \end{aligned} \tag{29}$$

System (29) can also generate chaotic attractor, hyperchaotic attractor and a periodic orbit by varying parameter k. However, the first Lypunov exponent corresponding to the hyperchaotic attractor is relatively larger than the first system. It is equal to 2.285.

After one year, Pang and Liu derived a new system based on Lü system (4) by adding a linear feedback controller. This system is written as [41]

$$\begin{aligned} \dot{x} &= a(y-x), \\ \dot{y} &= cy - xz + w, \\ \dot{z} &= xy - bz, \\ \dot{w} &= kx - hy. \end{aligned} \tag{30}$$

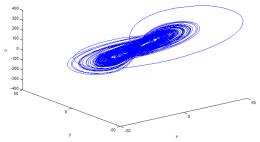
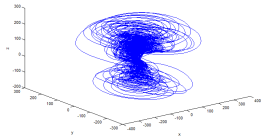
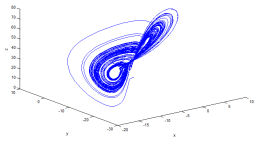
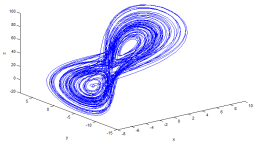
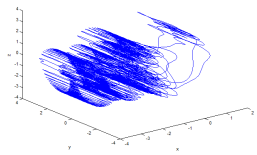
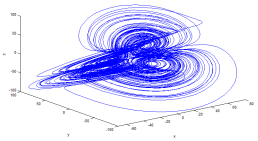
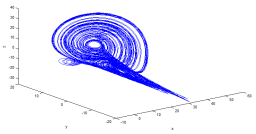
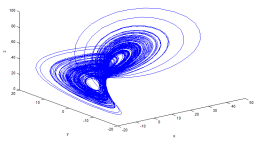
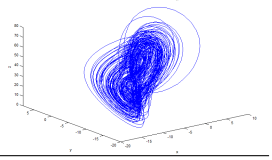
System (30) exhibits chaotic attractor, hyperchaotic attractor and periodic orbits by varying parameters k and h and fixing the others. When $(a, b, c, k, h) = (36, 3, 20, 2, 2)$, a hyperchaotic attractor was obtained of which the first Lyapunov exponent is equal to 1.4106 as shown in Table 2. Its electrical implementation has been verified. Furthermore, the proposed system exists over an extremely broad high magnitude bandwidth, which is very sought in secure communication.

In 2011, Li et al. proposed a novel hyperchaotic system based on Lorenz system by adding a simple state feedback controller. The new system is described as [30]

$$\begin{aligned} \dot{x} &= a(y-x), \\ \dot{y} &= -xz + bx - cy + w, \\ \dot{z} &= xy - dz, \\ \dot{w} &= -ky - rw. \end{aligned} \tag{31}$$

By setting (a, b, c, d, r) to $(12, 23, 1, 2.1, 0.2)$ and varying the parameter k, four types of attractors are obtained. When $2.34 \leqslant k < 8.04, 12.64 \leqslant k < 12.69$ and $12.79 \leqslant k < 15.70$, this system generates a hyperchaotic attractor. If $0 \leqslant k < 2.34, 8.04 \leqslant k < 8.12, 12.69 \leqslant k < 12.79$ and $15.70 \leqslant k < 16.11$ a chaotic attractor is obtained. Finally, when $8.12 \leqslant k < 12.40$ and $16.11 \leqslant k \leqslant 20$ or $12.40 \leqslant k < 12.64$, the system generates a regular attractor, periodic orbit and quasi-periodic attractor respectively. Thus, the proposed system commutes between various types of dynamic behavior. Also, the number of equilibriums is variable according to the report k/r as described in Table 2. In fact, if $k/r < 22$, it has three equilibriums and if $k/r > 22$, it has only one equilibrium at the origin.

Table 2. Description of new hyperchaotic systems.

Source	Equilibriums	LEs	Attractors
Niu et al. [58]	5	$\mathrm{LE}_1 = 1.416$ $\mathrm{LE}_2 = 0.531$ $\mathrm{LE}_3 \simeq 0$ $\mathrm{LE}_4 = -39.101$	
Wang et al. [52]	5	$\mathrm{LE}_1 = 2.285$ $\mathrm{LE}_2 = 0.864$ $\mathrm{LE}_3 \simeq 0$ $\mathrm{LE}_4 = -19.504$	
Pang and Liu [41]	1	$\mathrm{LE}_1 = 1.410$ $\mathrm{LE}_2 = 0.123$ $\mathrm{LE}_3 \simeq 0$ $\mathrm{LE}_4 = -20.533$	
Li et al. [30]	1 or 3	$\mathrm{LE}_1 = 1.4106$ $\mathrm{LE}_2 = 0.1232$ $\mathrm{LE}_3 \simeq 0$ $\mathrm{LE}_4 = -20.533$	
Wang et al. [53]	0	$\mathrm{LE}_1 = 0.87$ $\mathrm{LE}_2 = 0.03$ $\mathrm{LE}_3 \simeq 0$ $\mathrm{LE}_4 = -1.01$	
Dadras et al. [11]	1	$\mathrm{LE}_1 = 1.844$ $\mathrm{LE}_2 = 0.500$ $\mathrm{LE}_3 \simeq 0$ $\mathrm{LE}_4 = -49.212$	
Jia et al. [22]		This class contains 6 different systems	
Wan et al. [49]	1	$\mathrm{LE}_1 = 2.108$ $\mathrm{LE}_2 = 2.108$ $\mathrm{LE}_3 \simeq 0$ $\mathrm{LE}_4 = -19.123$	
Li et al. [27]	1	$\mathrm{LE}_1 = 1.230$ $\mathrm{LE}_2 = 0.094$ $\mathrm{LE}_3 \simeq 0$ $\mathrm{LE}_4 = -33.816$	
Zhou et al. [65]	∞	$\mathrm{LE}_1 = 0.240$ $\mathrm{LE}_2 = 0.086$ $\mathrm{LE}_3 \simeq 0$ $\mathrm{LE}_4 = -25.826$	

In 2012, Wang et al. proposed a new hyperchaotic system. It is written as [53]

$$\begin{aligned} \dot{x} &= y, \\ \dot{y} &= -x + yz + axzw, \\ \dot{z} &= 1 - y^2, \\ \dot{w} &= z + bxz + cxyz. \end{aligned} \tag{32}$$

Such a system has no equilibrium points. When $(a, b, c) = (8, 2.5, 30)$, this system generates the hyperchaotic attractor whose first Lyapunov exponent is equal to 0.87. As shown in Table 2, Its attractor form indicates a complex dynamic behavior different from the other hyperchaotic systems with one or three equilibriums.

In the same year, Dadras et al. proposed a new system with one equilibrium [11], described as

$$\begin{aligned} \dot{x} &= ax - yz + w, \\ \dot{y} &= xz - by, \\ \dot{z} &= xy - cz + xw, \\ \dot{w} &= -y. \end{aligned} \tag{33}$$

System (33) is dissipative and symmetrical about axis z. The only equilibrium point of this system is $O\ (0, 0, 0, 0)$. By analyzing its dynamic behavior, system (33) can exhibit chaotic and hyperchaotic attractors over a large range of parameters. Indeed, the parameters (a, b, c) are equal to $(8, 40, 14.9)$, the proposed system generates a four-wing hyperchaotic attractor, as shown in Table 2. The first Lyapunov exponent relative to this attractor is equal to 1.844.

In 2013, Jia and Wang discovered a novel class of hyperchaotic systems [22]. This class is obtained by adding a state feedback to system (2). It is written as

$$\begin{aligned} \dot{x} &= a(y - x), \\ \dot{y} &= bx - 10xz + y + w, \\ \dot{z} &= -cz + T(x, y), \\ \dot{w} &= -dQ(x, y, z, w) + R(x, y, z, w), \end{aligned} \tag{34}$$

where $T(x, y)$ and $R(x, y, z, w)$ are nonlinear continuous functions and $Q(x, y, z, w)$ is a linear continuous function. By varying T, R and Q, the system can exhibit six several hyperchaotic systems. These six systems create hyperchaotic attractors, where $a = 20$, $c = 3$, $d = 10$ and $b \in (20, 60)$.

In 2014, Wan et al. obtained a hyperchaotic system. This system is constructed without adding any control term to its state equation. It is written as [49]

$$\begin{aligned} \dot{x} &= -ax + yz + y, \\ \dot{y} &= cy - xz + dz - w, \\ \dot{z} &= bxy - hz + yw, \\ \dot{w} &= y. \end{aligned} \tag{35}$$

System (35) has one equilibrium at the origin $(0, 0, 0, 0)$. When the parameters (a, b, d, h) are fixed to $(4, 2, 10, 19)$ and by varying c, this system can display different attractors with variable number of wings. Indeed, several chaotic attractors are obtained such as the two-wing, the three-wing chaotic attractor and the four-wing hyperchaotic attractor. When $c = 5.5$, the four wing hyperchaotic exhibits a first Lapunov exponents equal to 2.108 as illustrated in Table 2. Further, system (35), with one equilibrium, differs from the others because four-wing hyperchaotic attractors are generated by hyperchaotic systems with 5 equilibriums.

In the same year, Li et al. started by creating a new chaotic system in order to lead a novel hyperchaotic system [27]. The proposed chaotic system is constructed by expunging the term $-y$ in the second equation of system (9). It is described by

$$\begin{aligned} \dot{x} &= a(y - z) + yz, \\ \dot{y} &= cx - xz, \\ \dot{z} &= -bz + xy. \end{aligned} \tag{36}$$

This chaotic system have three equilibriums. Now, by adding a state variable to system (36), a new hyperchaotic system is generated. It is written as

$$\begin{aligned} \dot{x} &= a(y - z) + yz, \\ \dot{y} &= cx - xz + kw, \\ \dot{z} &= -bz + xy + hw, \\ \dot{w} &= -dy. \end{aligned} \tag{37}$$

System (37) is characterized by only one equilibrium at the origin $(0, 0, 0, 0)$ and can exhibit several dynamic behaviors in a wide range of parameters. Indeed, by fixing (a, b, c, k, h) to $(30, 2.5, 35, 0.2, 0.5)$ and varying d, the proposed system displays period, quasi-periodic, chaotic and hyperchaotic systems. When $d = 20$, a hyperchaotic attractor is obtained whose first Lyapunov exponent is equal to 1.23062 as shown in Table 2. This system is certainly complex, but its originality lies in its chaotic form (3D), which is distinguished from the classical Qi system.

Recently, Zhou and Yang have discovered a new hyperchaotic system based on the Lü system (4). When $(a, b, c) = (36, 3, 20)$, the Lü system generates a chaotic attractor. Thus, the parameters (a, b) are fixed to $(36, 6)$ in the new hyperchaotic system, and only the parameter c will stay variable. It is described as follows [65]:

$$\begin{aligned} \dot{x} &= 36(y - x) + w, \\ \dot{y} &= cy - xz, \\ \dot{z} &= -3z + xy, \\ \dot{w} &= 18x - 0.5w. \end{aligned} \tag{38}$$

The equilibrium system is $(x, 0, 0, 36x)$, where x is a state variable. Thus, the proposed hyperchaotic system has an infinite number of real equilibria. Moreover, system (38)

can have multiple dynamic behaviors by varying parameter c. When $c \in [13, 16.75]$, system (38) generates the hyperchaotic attractor whose the first Lyapunov exponent is equal to 0.240. When $c \in [4, 13)$ and $(16.75, 33]$, chaotic attractors and periodic orbits are obtained.

In 2015, Chen and Yang propose a novel Lorenz-type hyperchaotic system [5]. It is described as

$$\begin{aligned} \dot{x} &= a(y - x), \\ \dot{y} &= cx - y - xz + w, \\ \dot{z} &= -bz + xy, \\ \dot{w} &= (c - 1)y + w - \frac{x^3}{b}. \end{aligned} \tag{39}$$

System (39) exhibits a high complex dynamical behavior. In fact, it displays coexisting attractors by varying parameters namely chaotic and periodic attractors, coexisting periodic attractors, chaotic attractor and heteroclinc cycles. This system can generate, when $a = 10$, $b = 2$ and $c = 20$, an hyperchaotic attractor with four Lyapunov exponents equal to $\mathrm{LE}_1 = 0.569$, $\mathrm{LE}_2 = 0.045$, $\mathrm{LE}_3 \simeq 0$ and $\mathrm{LE}_4 = -12.607$. Also, system (39) is characterized by a curve of equilibria.

4 Classification of new chaotic and hyperchaotic systems

New systems are proposed to be used in technical applications. Each technology imposes precise performances according to their real needs. Thus, it is important to know if it exists a system, which solves this complexity or if it is necessary to improve certain performances that lead to a new system. A classification of these systems is imperative. Until now, no classification has been made. So, we propose to classify these systems (Tables 1 and 2). Three criteria are considered: the first Lyapunov exponent, the phase space and the number of equilibria.

4.1 According to the first Lyapunov exponent

The Lyapunov exponent LE [13] represents the growth or decline rate of small perturbation along each main axis of the phase space system. It is the principal criteria of chaos. For an autonomous three dimensional system, the LEs are classified by decreasing order ($\mathrm{LE}_1 > \mathrm{LE}_2 > \mathrm{LE}_3$). The chaotic attractor is characterized by a positive first Lyapunov exponent. For these LEs, the sign '$+$' and '$-$' translate, respectively, the expansion and contraction of the strange attractor in the phase space. Besides, the 0 translates the critical nature of this attractor. Thus, if the first Lyapunov exponent (LE_1) increases, then the expansion degree becomes important and will bring more complexity to the dynamic behavior. According to Table 1, we notice that system (24) can display the largest first Lyapunov exponent equal to 1.86852. This system can satisfy the properties of secure communication applications because it has a complex dynamic behavior to conceal the information transmitted. On the other hand, system (20) has the lowest first

Lyapunov exponent equal to 0.067, so, its expansion degree is weak. So, this system is not recommended for fields of security and data transmission.

Regarding the new hyperchaotic systems, four Lyapunov exponents were exhibited. The hyperchaotic attractor is characterized by two positive exponents such as the signs of (LE_1, LE_2, LE_3, LE_4) are equal to $(+,+,0,-)$. A chaotic attractor is generated, when the signs of (LE_1, LE_2, LE_3, LE_4) are equal to $(+,0,-,-)$. Otherwise, one finds the previous cases namely the regular attractors. Since $\mathrm{LE}_1 > \mathrm{LE}_2$, thus the biggest expansion degree will be indicated by the first Lyapunov exponent, which translates the complex dynamic behavior of these systems. According to Table 2, we notice that system (29) can exhibit the largest first Lyapunov exponent equal to 2.2885. However, the second exponent is equal to 0.864. By contrast, system (38) can exhibit two positive Lyapunov exponents equal to 2.1083 ($\mathrm{LE}_1 = \mathrm{LE}_2$). The lowest first Lyapunov exponent was exhibited by system (34), it is equal to 0.24014. The first Lyapunov exponent indicates the dilatation degree of attractors in the phase space. So, we can classify these systems with the largest or the lowest exponent.

4.2 According to the number of equilibriums

In chaos theory, the equilibriums of chaotic and hyperchaotic systems can contribute to analyze their dynamic behaviors specially in the Shil'nikov method [46]. Moreover, these equilibriums are very important for visualizing chaotic and hyperchaotic attractors mainly for showing multiple wings or scrolls attractors.

The number of equilibriums change from one system to another. For most hyperchaotic and chaotic systems namely the basic ones, the number of equilibriums varies from 1 to 3 and sometimes 5 (as shown in Section 2). In these last years, new systems have appeared by innovating this criterion. Indeed, based on Tables 1 and 2, we observe firstly the new systems with a variable number of equilibriums (changes by varying system parameters), but it is equal to the basic systems namely system (22), system (23), (26) and (31). These systems have actually more complexity than the ones of which the number of equilibrium is fixed. Besides, several new systems have a great number of equilibrium but a countable number such as system (19). Finally, there are two other types of systems, without or with infinite equilibrium points. The hyperchaotic system (32) without equilibrium has a complex dynamic behavior compared to systems with one equilibrium, which information is encrypted [53]. As for system (38) with infinite equilibrium points, it may be more recommended for security applications and may not be exploited in chaotic encryption or decryption as indicated in [65].

4.3 According to the attractor form

In this section, we propose to classify the new systems according to their attractor form. Indeed, each form disperses differently in the phase space portraits, which influence their exploitation in technological applications.

Depending on our study, we find the most attractors with symmetrical property like Qi system (9) or with unsymmetrical property like Rössler system (5). Based on Tables 1

and 2, we observe different attractor forms namely wing, scroll or layer ones. We can find attractors with three, four and multi layers namely for system (20). Besides, the number of wings and scrolls can be fixed or varied by changing parameters such as system (23), system (33) and system (35). Thus, this criterion permits to classify chaotic and hyperchaotic systems according to the similarity of their attractor form.

5 Challenges

In this section, we present the possible challenges, which can be envisioned based on our survey. One of the most involved characteristics of the new chaotic and hyperchaotic systems is their complex dynamical behaviors with a high degree of disorder and randomness. Thus, the fractional order calculus becomes an interesting field for the circuit implementation of these systems. It is due to the ability of this calculus to provide greater specific description of several nonlinear phenomena neglected by the integer order calculus. Recently, several applications of fractional chaotic and hyperchaotic systems (continuous or discrete [62]) were proposed [28, 56], but until now, other research results are expected. Chaos control and synchronization [60] are other expanding fields in chaos theory due to their potential application in engineering areas, these last decades. The antisynchronization between two fractional chaotic or hyperchaotic systems is also considered as a challenging problem [47]. This phenomenon is used recently in different applications such as neural networks [64]. Besides, in practice, for analyzing the performances of the fractional chaotic and hyperchaotic systems, several methods exist such as time series. The time series method is based on the numerical calculus of the Lyapunov exponents. By optimizing these exponents, we can obtain more precise and efficient dynamic behaviors [12] of these systems.

6 Conclusions

In many chaotic and hyperchaotic systems, the characteristics of their dynamic behavior vary. However, their implementations for engineering technologies are difficult due to the particularity of each field. In this paper, we first analyzed the most famous chaotic and hyperchaotic systems by indicating their originality namely the chaotic system with coexisting attractors and the hyperchaotic system without equilibrium. Then, we proposed a classification of these systems based on the number of equilibriums, attractor form and the first Lyapunov exponent.

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