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Modeling the effect of deforestation caused by human population pressure on wildlife species

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Abstract. The increase in human population has posed several problems for the ecosystem. One of these problems is the decrease in forestry resources, which leads to decline in forest area and thus threaten the survival of wildlife species as the intraspecific competition among the wildlife species increases. Moreover, these wildlife species can also be apprehended easily by poachers and smugglers. This affects the biodiversity across the globe. In this paper, we have proposed and analyzed a nonlinear mathematical model to see the effect of deforestation caused by population and its pressure on wildlife species. The analysis of proposed model reveals that, as the parameters in respect to the increase in population pressure increase, wildlife species decrease. To support analytical findings, we have done numerical simulation.

Keywords: mathematical model, forestry resources, wildlife species, human population, stability.

1 Introduction

Deforestation is the consequence of large growth in human population. The basic needs, like houses, roads, agricultural land for food production, etc. have increased with the increase in population size. To fulfil these needs, the forestry resources are overexploited by population to clear forest land. This has resulted to deforestation at a large scale across the globe and threatened the survival of wildlife species, which are wholly dependent on forestry resources [6, 9, 16]. As the forest land reduces, the wildlife species lose their natural habitat and easily reach to the hands of poachers and animal smugglers [14, 15]. Thus, the overgrowth in population has negative effect on the wildlife species. According to Adebayo [1], the over growth in population leads to increase in population pressure and poverty, which are the prime reasons for mowing the forest resources and

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convert the forest land for agricultural purposes. Jhoom cultivation is an example for the encroachment of forest region by population. In this cultivation, people slash and burn forestry resources for clearing the land to be used for agriculture, and when soil loses its fertility, they move to another forest region and apply the same technique [10, 26].

In the past few decades, forestry resources and wildlife species are under stress due to increase in population and its associated pressure. Other reasons behind the deforestation are the increase in industrialization and pollution, which are also linked to human population [3, 11, 13, 17, 25, 28, 32]. Some researchers have shown their concern for the depletion of forestry resources as well as wildlife species and made suggestions for their conservation. In this respect, Shukla and Dubey [27] and Shukla et al. [29] have presented nonlinear mathematical models to see the effect of habitat destruction and reduction in resources on the survival of species. They have shown that as habitat shrinks and resources decrease, the species which are wholly dependent upon them may become extinct. Keeping in mind the necessity of the use of forestry resources by human population, Shukla et al. [31] have suggested that the application of technological efforts, such as genetically engineered plants, are helpful in conserving forestry resources along with fulfilling the need of human population. In addition to this, Misra and Lata [20] have shown that these technological efforts are beneficial to conserve forestry resources when they are implemented within appropriate time. Some studies have been conducted to see the effect of wood based industries on forestry resources, and it is shown that forestry resources decrease as the number of industries increases [2, 24, 30]. Moreover, Lata et al. [17] have shown that the wood and nonwood-based industries continuously emit the pollution into the environment, and due to uptake of this pollution by forestry resources, the metabolism of forestry resources is affected. Further, population pressure is a prime factor behind industrialization, and in this regard, Dubey et al. [12] have studied the effect of population and increase in industrialization due to population pressure on the depletion of forestry resources. It is shown that as population pressure increases, industrialization increases, which leads to deforestation. To reduce the population pressure, in some studies, it is suggested that economic efforts in the form of incentives, like fuel efficient stoves and biogas, subsidies on the products, which are the alternate of forestry resources (tin, fibre, metal, etc.) [5, 22, 23], and alternative resources, like synthetic [4, 7, 8], may be provided to the people. It is found that these efforts are helpful to check deforestation. Moreover, Misra and Lata [21] have studied the conservation of forestry resources by using economic as well as technological efforts. In this study, it is found that economic as well as technological efforts both are beneficial in the conservation of forestry resources only upto a certain level. Recently, Lata and Misra [18] have proposed a mathematical model for the conservation of forestry resources using economic efforts (money), where some part of the money is used to reduce population pressure, whereas the other part is used for plantation. The study clearly demonstrates when more money should be spent on plantation to increase the forestry resources and when the money should be spent to reduce the population pressure.

From the above it may be pointed out that the increase in population and its associated pressure are the major factors behind deforestation and thus threatens the survival of wildlife species. Motivated from prior discussion, in this paper, we study the effect of

deforestation: (i) directly caused by human population and (ii) indirectly by the clearance of forest to fulfill the land's requirement for agriculture and housing complexes on the wildlife species.

2 Mathematical model

In this section, we present a mathematical model dealing with the subject of depletion of forestry resources and its effect on wildlife species. It is assumed that wildlife species wholly depend upon forestry resources for food and shelter. As the encroachment of human population increases in the forest region due to the demand of agricultural land for food production, wood, space for housing complexes and economic position, the wildlife species are also affected. Therefore, we consider a forest habitat, where forestry resources and wildlife species both are under endangered situation due to enlargement in human population and their demand (known as population pressure). Let $R(t)$ and $H(t)$ represent the density of forestry resources and human population, respectively, in the region under consideration at any time t . Further, let $P(t)$ and $S(t)$ denote the intensity of population pressure and density of wildlife species, respectively, at time t in the same region.

We propose the following nonlinear mathematical model:

$$\begin{aligned}\frac{dR}{dt} &= sR\left(1 - \frac{R}{L}\right) - \alpha_1RH - \alpha_2R^2P - \phi RS, \\ \frac{dH}{dt} &= rH\left(1 - \frac{H}{K}\right) + \pi_1\alpha_1RH, \\ \frac{dP}{dt} &= \lambda H - \lambda_0P, \\ \frac{dS}{dt} &= \theta\phi RS - \delta_0S^2 - \theta_1PS - \delta_1PS^2 - \theta_0S,\end{aligned}\tag{1}$$

where $R(0), H(0), P(0), S(0) \geq 0$. Here all the parameters are assumed to be positive.

The first equation for forestry resources accounts for its logistic growth with intrinsic growth rate s and carrying capacity L . As a result of use of forestry resources by the human population, the forestry resources undergo a loss in their growth rate expressed by the term α_1RH [31]. This use of forestry resources mainly covers the cutting of trees for wood, medicinal requirement, grazing, etc. In addition to the direct use of forestry resources by human population, the forest land is also cleared for the agricultural production, housing complexes, etc. to fulfill the increasing demand of ever increasing human population. This reduces the forest land in the region under consideration. We have explicitly modeled this aspect by the third term α_2R^2P in the first equation of model system (1). This is due to the fact that once forest land cleared for the above purposes cannot be used for the regeneration of forest and thus adversely affected the carrying capacity of forestry resources. Finally, the last term expresses the predation rate of the wildlife species modeled using bilinear interaction, denoting possible feeding satiation, with predation rate ϕ .

The dynamics of human population is described by the second equation. It is assumed that human population also follow the logistic growth with intrinsic growth rate r and carrying capacity K and is subject to further growth due to use of forestry resources for their livelihood $\pi_1\alpha_1RH$, where π_1 is a proportionality constant [31].

As the human population increases, the demand of population for food, housing complexes, etc. increases. To fulfil this demand, the forest land is cleared for agriculture, making houses, roads, etc. Thus, the growth rate in the demand of population is assumed to be proportional to the human population (i.e., λH) and is modeled in the third equation of model system (1). However, this demand of population reduces on applying some other efforts (e.g., economic) by making more agricultural production using fertilizers, constructing multi-storey buildings, etc. and is modeled by the second term in this equation.

The wildlife species, as described in fourth equation, is assumed to be wholly dependent on forestry resources for food, shelter, etc. thereby decreasing the forestry resources; it is subject to mortality due to crowding at a rate δ_0 . As a result of population demand (population pressure), the forest land is cleared, which reduces the forest area, and thus, the intraspecific competition among wildlife species increases. Moreover, the wildlife species can also be easily captured by the poachers and smugglers. This aspect has been explicitly modeled by the third and fourth terms in this equation (i.e., θ_1PS and δ_1PS^2), where θ_1 represents declination in the wildlife species due to capture by poachers and smugglers, whereas δ_1 accounts for the increment in the intraspecific competition among the wildlife species due to the reduction in the forest area on account of population pressure. Furthermore, as time passes, the wildlife species migrate from the forest region and also depleted naturally at a rate θ_0 .

From first equation of model system (1),

$$\begin{aligned}\frac{dR}{dt} &= sR - \frac{sR^2}{L} - \alpha_1RH - \alpha_2R^2P - \phi RS \\ &= (s - \alpha_1H - \phi S)R - R^2\left(\frac{s}{L} + \alpha_2P\right).\end{aligned}$$

From the above equation it may be easily noted that in presence of human population and wildlife species, intrinsic growth rate of forestry resources at any time $t > 0$ is $s - \alpha_1H - \phi S$. To avoid the extinction of forestry resources and thus the feasibility of model system (1), $s - \alpha_1H - \phi S$ must be positive for all time $t > 0$. Similarly, from the last equation of the above model system (1), to avoid the extinction of wildlife species and the feasibility of model system (1), $\theta\phi R - \theta_1P - \theta_0$ must be positive for all time $t > 0$. Therefore, in the rest of the paper, we assume that $s - \alpha_1H - \phi S$ and $\theta\phi R - \theta_1P - \theta_0$ are positive.

3 Boundedness of solutions

Boundedness is necessary to analyze the model system (1). For this, we require the following lemma, which ensures boundedness of the model system (1).

Lemma. *The set $\Omega = \{(R, H, P, S): 0 \leq R \leq L; 0 \leq H \leq H_m; 0 \leq P \leq P_m; 0 \leq S \leq (\phi\theta L - \theta_0)/\delta_0\}$, where $H_m = (K/r)(r + \pi_1\alpha_1L)$ and $P_m = (\lambda/\lambda_0)H_m$, is the region of attraction for model (1) and attracts all solutions initiating in the interior of the positive orthant. Here it is assumed that $\phi\theta L - \theta_0 > 0$.*

Proof of above lemma is given in Appendix A.

4 Equilibrium analysis

Since the proposed model is nonlinear, we analyze it qualitatively using the stability theory of differential equations. For this, we first obtain the equilibrium solutions of the model by setting the growth rate of all the dynamical variables to zero. It is found that only six nonnegative equilibria are feasible, which are given as follows:

- (i) $F_0(0, 0, 0, 0)$, (ii) $F_1(L, 0, 0, 0)$, (iii) $F_2\left(0, K, \frac{\lambda K}{\lambda_0}, 0\right)$,
 (iv) $F_3\left(\frac{s + \phi\theta_0/\delta_0}{s/L + \theta\phi^2/\delta_0}, 0, 0, \frac{s(\phi\theta L - \theta_0)}{\delta_0 L(s/L + \theta\phi^2/\delta_0)}\right)$, provided $\phi\theta L - \theta_0 > 0$,
 (v) $F_4(R_4, H_4, P_4, 0)$, and (vi) $F^*(R^*, H^*, P^*, S^*)$.

The feasibility of equilibria F_0, F_1, F_2, F_3 is obvious, hence omitted, and in the following, we show the feasibility of equilibria F_4 and F^* .

Feasibility of equilibrium F_4

$$s\left(1 - \frac{R}{L}\right) - \alpha_1 H - \alpha_2 R P = 0, \quad (2)$$

$$r\left(1 - \frac{H}{K}\right) + \pi_1 \alpha_1 R = 0, \quad (3)$$

$$\lambda H - \lambda_0 P = 0. \quad (4)$$

From equations (3) and (4) we get the values of H and P as

$$H = \frac{K}{r}(r + \pi_1 \alpha_1 R), \quad P = \frac{\lambda H}{\lambda_0}.$$

Now, putting the above values of H and P in equation (2), we get the following quadratic equation in R :

$$\frac{\pi_1 \alpha_1 \alpha_2 \lambda K}{\lambda_0 r} R^2 + \left(\frac{s}{L} + \frac{\pi_1 \alpha_1^2 K}{r} + \frac{\alpha_2 \lambda K}{\lambda_0}\right) R - (s - \alpha_1 K) = 0.$$

Above equation possesses a unique positive root, which is given as follows:

$$R = \frac{\lambda_0 r}{2\pi_1 \alpha_1 \alpha_2 \lambda K} \left(-\left(\frac{s}{L} + \frac{\pi_1 \alpha_1^2 K}{r} + \frac{\alpha_2 \lambda K}{\lambda_0}\right) + A\right),$$

where $A = \sqrt{(s/L + \pi_1\alpha_1^2K/r + \alpha_2\lambda K/\lambda_0)^2 + 4\pi_1\alpha_1\alpha_2\lambda K(s - \alpha_1K)/(\lambda_0r)} > 0$, provided $s - \alpha_1K > 0$. Now using this positive value of $R = R_4$ (say), the positive values of H_4 and P_4 can be obtained. Thus, the equilibrium F_4 is feasible without any condition.

Feasibility of equilibrium F^ .* On solving the following algebraic equations, we get the values of R^* , H^* , P^* , and S^* :

$$s\left(1 - \frac{R}{L}\right) - \alpha_1H - \alpha_2RP - \phi S = 0, \quad (5)$$

$$r\left(1 - \frac{H}{K}\right) + \pi_1\alpha_1R = 0, \quad (6)$$

$$\lambda H - \lambda_0P = 0, \quad (7)$$

$$\theta\phi R - \delta_0S - \theta_1P - \delta_1PS - \theta_0 = 0. \quad (8)$$

From equation (6) we get

$$H = \frac{K}{r}(r + \pi_1\alpha_1R) = f(R) \quad (\text{say}). \quad (9)$$

Further, from equation (7) we have

$$P = \frac{\lambda}{\lambda_0}f(R) = g(R) \quad (\text{say}). \quad (10)$$

Again, from equation (8) we get

$$S = \frac{\phi\theta R - \theta_1g(R) - \theta_0}{\delta_0 + \delta_1g(R)} = h(R) \quad (\text{say}). \quad (11)$$

Now, using the values of H , P , and S , we obtain the following cubic equation in R :

$$p_0R^3 + p_1R^2 + p_2R + p_3 = 0, \quad (12)$$

where

$$p_0 = \frac{1}{r^2\lambda_0^2}\pi_1^2\alpha_1^2\alpha_2\lambda^2\delta_1K^2,$$

$$p_1 = \frac{1}{\lambda_0^2r^2L}[\lambda_0\delta_1\lambda L\pi_1^2\alpha_1^3K^2 + 2r\pi_1\alpha_1\delta_1\alpha_2\lambda^2K^2L + sr\lambda_0\pi_1\alpha_1\delta_1\lambda K + r\pi_1\lambda_0\delta_0\lambda\alpha_1\alpha_2KL],$$

$$p_2 = \frac{1}{\lambda_0^2rL}[-\pi_1\lambda_0\lambda\delta_1\alpha_1sKL - \theta\phi\lambda\lambda_0\pi_1\alpha_1KL + \delta_0\lambda_0^2rs + \lambda_0\lambda\delta_1srK + 2\pi_1\alpha_1^2\lambda_0\lambda\delta_1K^2L + \theta\phi^2\lambda_0^2rL + \pi_1\delta_0\lambda_0^2\alpha_1^2KL + r\lambda\lambda_0\delta_0\alpha_2KL + r\delta_1\lambda^2\alpha_2K^2L],$$

$$p_3 = -\left((s - \alpha_1 K)\left(\delta_0 + \frac{\lambda \delta_1 K}{\lambda_0}\right) + \phi \theta_0 + \frac{\phi \theta_1 \lambda K}{\lambda_0}\right).$$

Here it is noted that p_0 and p_1 are positive, whereas p_3 is negative, provided $s - \alpha_1 K > 0$. Hence, by Descartes' rule of signs equation (12) has unique positive root. Further, this positive root can be obtained by Cardan's method and is given as $R = (A_3 - A_2/A_3 - p_1)/3p_0 = R^*$ (say), where $A_1 = 2p_1^3 - 9p_0p_1p_2 + 27p_0^2p_3$, $A_2 = 3p_0p_2 - p_1^2$, and A_3 is the real positive value of the root $(-A_1 + \sqrt{A_1^2 + 4A_2^3}/2)^{1/3}$. After knowing the positive value of $R = R^*$, the positive values of $H = H^*$, $P = P^*$, and $S = S^*$ can be obtained from equations (9), (10), and (11), respectively.

5 Stability analysis

5.1 Local stability analysis

The local stability behavior of equilibria F_0, F_1, F_2, F_3, F_4 , and F^* is discussed in this section. The local stability means that if we perturb the system in the neighborhood of that equilibrium, then system approaches to its equilibrium. The local stability behavior of equilibria F_0, F_1, F_2, F_3, F_4 , and F^* are described in the following theorem.

Theorem 1. *The equilibria F_0, F_1, F_2, F_3 , and F_4 are always unstable. The interior equilibrium F^* is locally asymptotically stable if the following condition holds:*

$$\frac{16}{9} \frac{r\lambda_0^2}{\pi_1\lambda^2K} > \max\left\{\frac{\alpha_2^2 R^{*2}}{\left(\frac{s}{L} + \alpha_2 P^*\right)}, \frac{(\delta_1 S^* + \theta_1)^2}{\theta(\delta_0 + \delta_1 P^*)}\right\}. \quad (13)$$

For the proof of above theorem, see Appendix B

5.2 Global stability analysis

In the present section, we discuss the global stability behavior of interior equilibrium F^* , and for this, we state the following theorem.

Theorem 2. *The interior equilibrium F^* is globally asymptotically stable inside the region of attraction Ω , provided the following inequality holds:*

$$S_1 = \frac{16r\lambda_0^2}{9\pi_1\lambda^2K} - \max\left\{\frac{\alpha_2^2 L^2}{\left(\frac{s}{L} + \alpha_2 P^*\right)}, \frac{(\delta_1(\phi\theta L - \theta_0) + \theta_1\delta_0)^2}{\theta\delta_0^2(\delta_0 + \delta_1 P^*)}\right\} > 0. \quad (14)$$

For the proof of above theorem, see Appendix C.

Remark 1. It may be pointed out that for small values of α_2, δ_1, ϕ , and λ , the stability conditions (13) and (14) will be easily satisfied. This indicates that the large values for the depletion in carrying capacity of forestry resources and wildlife population due to population pressure, use of forestry resources by wildlife species and the growth in population pressure due to population, respectively, destabilize the system (see Fig. 1).

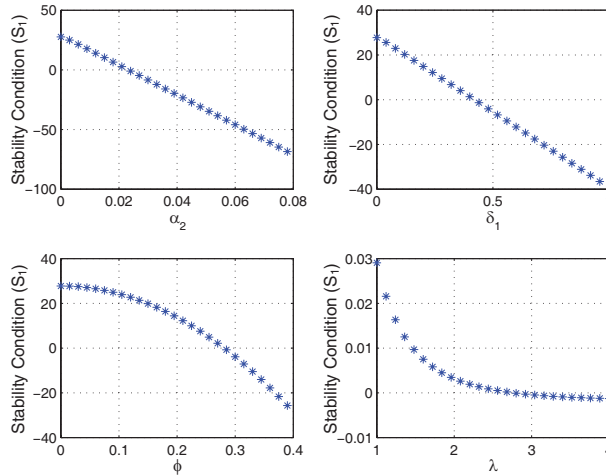


Figure 1. The destabilizing effect of parameters α_2 , δ_1 , ϕ , and λ on the coexistence equilibrium. The remaining parameters are same as given in (16).

6 Persistence

The permanence or uniform persistence plays a significant role to understand the survival of biological species. It ensures that all the species will be present in future if they are initially present in the system. Mathematically, system (1) is said to be uniformly persistence if there exists positive constants L_1 and L_2 such that each positive solution $(R(t), H(t), P(t), S(t))$ of the system with positive initial condition satisfies

$$L_1 \leq \liminf_{t \rightarrow \infty} V(t) \leq \limsup_{t \rightarrow \infty} V(t) \leq L_2, \tag{15}$$

where $V(t) = (R(t), H(t), P(t), S(t))$.

The following theorem gives the criteria for the uniform persistence of the model system (1).

Theorem 3. *The model system (1) is uniformly persistent if conditions (26) and (28) hold.*

For the proof of this theorem, see Appendix D.

7 Numerical simulation

In the present section, we simulate model system (1) by choosing the following set of parameter values, which are given as follows:

$$\begin{aligned} K &= 100, & L &= 100, & r &= 0.5, & s &= 0.8, \\ \alpha_1 &= 0.003, & \alpha_2 &= 0.00004, & \delta_0 &= 0.08, & \delta_1 &= 0.006, \\ \phi &= 0.021, & \lambda &= 0.04, & \lambda_0 &= 0.5, & \pi_1 &= 0.05, \\ \theta &= 0.8, & \theta_0 &= 0.00002, & \theta_1 &= 0.0001. \end{aligned} \tag{16}$$

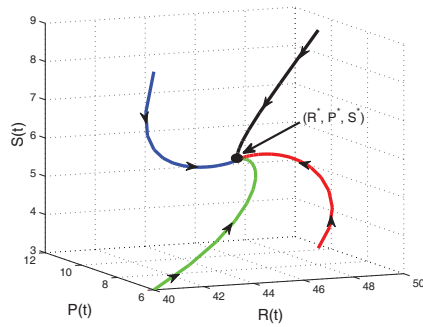


Figure 2. Global stability of interior equilibrium F^* in R, P, S -space. All parameters are same as given in (16).

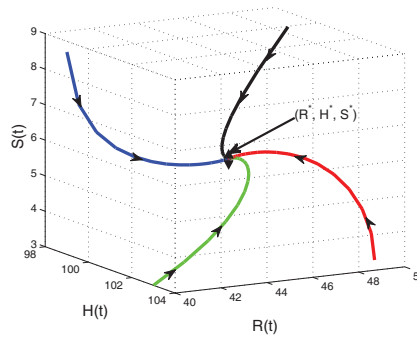


Figure 3. Global stability of interior equilibrium F^* in R, H, S -space. All parameters are same as given in (16).

Using this set of parameter values, we have obtained the components of the interior equilibrium F^* , which are given as

$$R^* = 44.8281, \quad H^* = 101.3448, \quad P^* = 8.1075, \quad S^* = 5.8477.$$

Further, we have obtained the eigenvalues of Jacobian matrix evaluated at F^* of the model system (1) and are given as

$$-0.5568 + 0.2443i, \quad -0.5568 - 0.2443i, \quad -0.5511, \quad \text{and} \quad -0.4672.$$

Here it is noted that the eigenvalues of J_{F^*} are either negative or with negative real part. Therefore, the negativity of the sign ensures the local stability of interior equilibrium F^* for the above set of parameter values. We have plotted Figs. 2 and 3 to show the global stability behavior of equilibrium F^* . In these figures, it is apparent that all solution trajectories initiating inside the region of attraction are approaching towards the equilibrium values, showing the global stability behavior. To see the influence of some crucial parameters, like λ_0 , δ_1 , ϕ , λ , and α_2 , on the system dynamics, we have also plotted Figs. 4–7. First, we have shown the effect of ϕ (i.e., the depletion rate coefficient of forestry resources R due to wildlife species) on the variables R and P with respect to time t in Fig. 4. In this figure, it is manifested that the equilibrium levels of forestry resources and population pressure decrease as ϕ increases. Further, the variation of forestry resources R and wildlife species S for different values of λ by taking the value of $\phi = 0.0021$ and the remaining parameters as in (16) is demonstrated in Fig. 5. From this figure it is apparent that the equilibrium level of forestry resources as well as wildlife species decreases as the growth rate coefficient of population pressure due to population increases. This means that, due to pressure, population is compelled to clear more forestry resources and forest land to fulfil the demand of more food and space. Again, in Fig. 6, the variation of forestry resources R and wildlife species S with respect to time for the different values of λ_0 by taking the value of $\phi = 0.0021$ and the remaining parameters as in (16) has been shown. From this figure it is clear that as λ_0 (i.e., the declination rate coefficient due to natural and use of some economic efforts) increases, the equilibrium

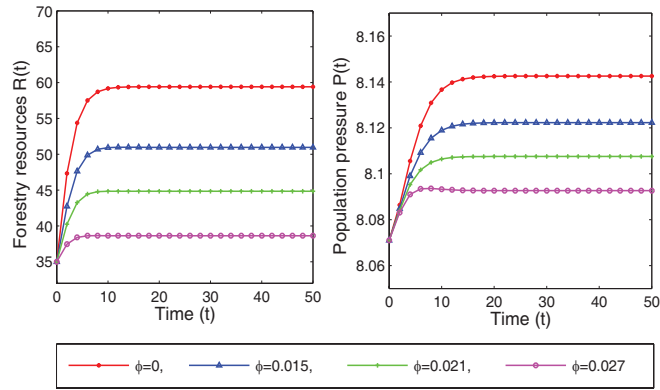


Figure 4. Effect of different values of ϕ on the forestry resources and population pressure with time. The remaining parameters are same as given in (16).

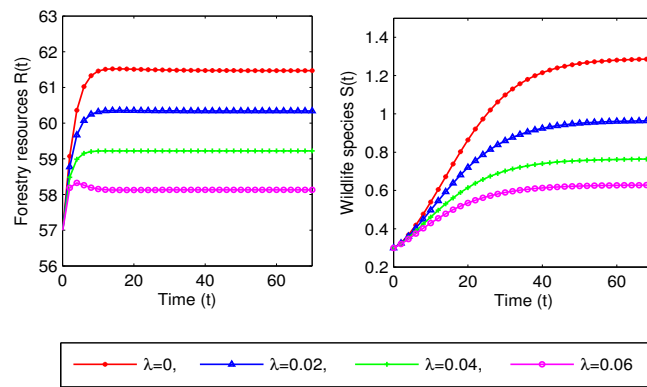


Figure 5. Effect of different values of forestry resources and wildlife species with time for different values of λ by taking $\phi = 0.0021$. The remaining parameters are same as given in (16).

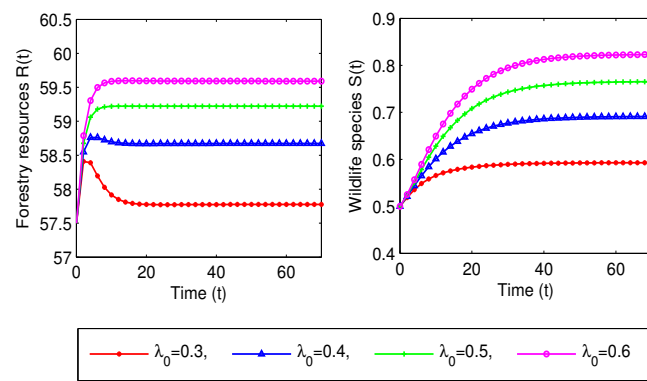


Figure 6. Variation of forestry resources and wildlife species with time for different values of λ_0 by taking $\phi = 0.0021$. The remaining parameters are same as given in (16).

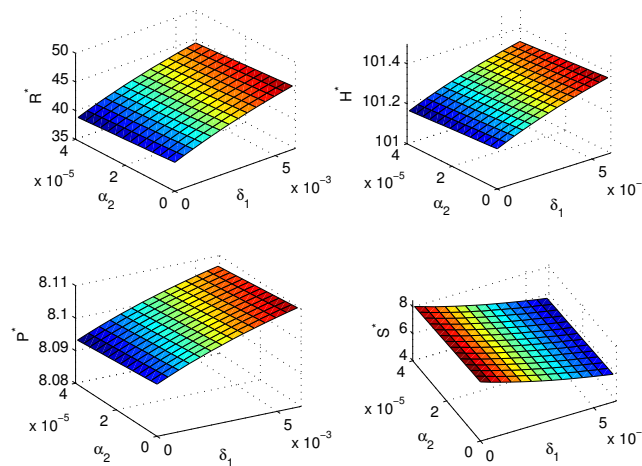


Figure 7. Effect of the different values of α_2 and δ_1 on the equilibrium level of R^* , H^* , P^* , and S^* , respectively. The remaining parameters are same as given in (16).

level of forestry resources and wildlife species both increase. In Fig. 7, we have shown the cumulative effect of the parameters α_2 and δ_1 on the equilibrium values R^* , H^* , P^* , and S^* . From this figure it is manifested that when α_2 and δ_1 both are zero, all the equilibrium values of interior equilibria F^* are at their maximum level. Moreover, for the value of α_2 to be zero, on increasing the value of δ_1 , the equilibrium levels of forestry resources, population, population pressure increase, whereas the equilibrium level of wildlife species decrease. Further, δ_1 to be zero, on increasing the value of α_2 , the equilibrium level of forestry resources, population, population pressure, and wildlife species decreases. Here it is interesting to note that as the values of α_2 and δ_1 increase simultaneously, the equilibrium levels of all variables decrease. Because of population pressure, peoples clear the forestry resources and land, therefore, forestry resources and wildlife species both decrease. Hence, from the prior discussion it may be noted that large population size is the major driver behind the deforestation and is also responsible for the endangered situation of wildlife species.

8 Conclusion

Millions of wildlife species are living in the forest habitat and depend upon forestry resources for food and shelter. In the past few decades, forestry resources have been remarkably reduced due to overgrowth in population and its pressure and thus threatened the survival of wildlife species. Therefore, to study this problem, we have considered that forestry resources and human population follow logistic growth in their natural habitats, and wildlife species wholly depend upon forestry resources for their survival. Further, it is considered that, due to increase in population, population pressure increases, which depletes the forestry resources and increases the intraspecific competition among wildlife species and thus also decreases the wildlife species. The stability theory of differential

equations is used to make the qualitative analysis of the proposed model. It is found that for the proposed model, only six equilibria are feasible. The components in each equilibria are obtained by setting the growth rate of dynamical variables to zero. The stability behavior of all the equilibria is performed and found that only coexistence equilibrium is stable (locally as well as globally), provided condition (14) is satisfied. It is noted that the coefficients related to the increase in population pressure (λ) decrease in the forestry resources due to the use of wildlife species (ϕ) and decrease in the growth rate of wildlife species and forestry resources due to population pressure (δ_1 and α_2), all of them destabilize the system. To see the survival of wildlife species, the uniform persistence is also performed. It is found that wildlife species persist if the coefficients related to the depletion of forestry resources and wildlife species, due to population pressure, are small. The analysis suggests that the reduction in population pressure by applying some efforts (e.g., economic efforts) is helpful to conserve forestry resources and wildlife species, and it also stabilizes the system. Therefore, for sustainable development and retaining the biodiversity of wildlife species, it is important that the population pressure be reduced by making some additional efforts, like increasing the agricultural production using fertilizers, constructing multi-storey buildings for housing complexes, etc.

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Appendix A: Proof of the Lemma

Now, here only the outline of the proof is given. For detail, see [19].

From the first equation of model system (1) we get

$$\frac{dR}{dt} \leq sR \left(1 - \frac{R}{L}\right) \implies \limsup_{t \rightarrow \infty} R(t) \leq L.$$

Again, through the second equation of model system (1) we obtain

$$\begin{aligned} \frac{dH}{dt} &\leq rH \left(1 - \frac{H}{K}\right) + \pi_1 \alpha_1 LH \\ \implies \limsup_{t \rightarrow \infty} H(t) &\leq \frac{K}{r} (r + \pi_1 \alpha_1 L) = H_m \quad (\text{say}). \end{aligned}$$

Further, from the third equation of model system (1) we get

$$\frac{dP}{dt} \leq \lambda H_m - \lambda_0 P.$$

Therefore,

$$\limsup_{t \rightarrow \infty} P(t) \leq \frac{\lambda K}{r \lambda_0} (r + \pi_1 \alpha_1 L) = P_m \quad (\text{say}).$$

Now, from the fourth equation of model system (1) we obtain

$$\frac{dS}{dt} \leq \phi\theta LS - \delta_0 S^2 - \theta_0 S \implies \limsup_{t \rightarrow \infty} S(t) \leq \frac{\phi\theta L - \theta_0}{\delta_0}.$$

Hence the proof.

Appendix B: Proof of Theorem 1

For the model system (1), the Jacobian matrix B is given as follows:

$$B = \begin{pmatrix} a_{11} & -\alpha_1 R & -\alpha_2 R^2 & -\phi R \\ \pi_1 \alpha_1 H & a_{22} & 0 & 0 \\ 0 & \lambda & -\lambda_0 & 0 \\ \theta \phi S & 0 & -\delta_1 S^2 - \theta_1 S & a_{44} \end{pmatrix},$$

where

$$\begin{aligned} a_{11} &= s \left(1 - \frac{2R}{L} \right) - \alpha_1 H - 2\alpha_2 R P - \phi S, \\ a_{22} &= r \left(1 - \frac{2H}{K} \right) + \pi_1 \alpha_1 R, \\ a_{44} &= \theta \phi R - 2\delta_0 S - 2\delta_1 P S - \theta_0 - \theta_1 P. \end{aligned}$$

From the Jacobian matrix B (evaluated at F_0) it is apparent that two eigenvalues, i.e., s and r are positive. Thus, equilibrium F_0 is always unstable in R - H -plane.

From the Jacobian matrix B (evaluated at F_1) it is apparent that two eigenvalues, i.e., $(r + \pi_1 \alpha_1 L)$ and $(\phi\theta L - \theta_0)$ are positive. Thus, equilibrium F_1 is always unstable in H - S -plane.

Again, from Jacobian matrix B (evaluated at F_2) it is straightforward that one eigenvalue, (i.e., $s - \alpha_1 K$) is positive. Hence, the equilibrium F_2 is always unstable in R -direction.

Similarly, from Jacobian matrix B (evaluated at F_3) it is apparent that one eigenvalue (i.e., $r + \pi_1 \alpha_1 R_3$) is positive. Thus, the equilibrium F_3 is always unstable in H -direction.

Further, from Jacobian matrix B (evaluated at F_4) it is easy to note that one eigenvalue (i.e., $\theta\phi R_4 - \theta_0 - \theta_1 P_4$) is positive. Hence, equilibrium F_4 is always unstable in S -direction.

Now, to determine the nature of eigenvalues of Jacobian matrix B (evaluated at F^*) is difficult by using Routh–Herwitz criterion, so we use Lyapunov’s stability theory. By using the Taylor’s series expansion, the linearized system of model system (1) about $F^*(R^*, H^*, P^*, S^*)$ can be written as follows:

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \\ \dot{x}_4 \end{bmatrix} = \begin{bmatrix} -(\frac{s}{L} + \alpha_2 P^*)R^* & -\alpha_1 R^* & -\alpha_2 R^{*2} & -\phi R^* \\ \pi_1 \alpha_1 H^* & -\frac{rH^*}{K} & 0 & 0 \\ 0 & \lambda & -\lambda_0 & 0 \\ \theta \phi S^* & 0 & -(\delta_1 S^* + \theta_1)S^* & -(\delta_0 + \delta_1 P^*)S^* \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix}$$

where “.” denote $d/(dt)$ and x_1, x_2, x_3, x_4 are small perturbations around the equilibrium F^* , i.e., $R = R^* + x_1, H = H^* + x_2, P = P^* + x_3, S = S^* + x_4$.

Now consider the positive definite function which is given as follows:

$$X = \frac{1}{2} \left(\frac{x_1^2}{R^*} + k_1 \frac{x_2^2}{H^*} + k_2 x_3^2 + k_3 \frac{x_4^2}{S^*} \right), \quad (17)$$

where k_1, k_2 , and k_3 are some positive constants to be chosen appropriately.

The differentiation of equation (17) along the solutions of linearized system of (1) with respect to t we get

$$\begin{aligned} \frac{dX}{dt} = & - \left(\frac{s}{L} + \alpha_2 P^* \right) x_1^2 - k_1 \frac{r}{K} x_2^2 - k_2 \lambda_0 x_3^2 - k_3 (\delta_0 + \delta_1 P^*) x_4^2 \\ & - \alpha_1 (1 - k_1 \pi_1) x_1 x_2 - \alpha_2 R^* x_1 x_3 - \phi (1 - k_3 \theta) x_1 x_4 \\ & + k_2 \lambda x_2 x_3 - k_3 (\delta_1 S^* + \theta_1) x_3 x_4. \end{aligned}$$

Choosing $k_1 = 1/\pi_1$ and $k_3 = 1/\theta$, dX/dt is simplified as

$$\begin{aligned} \frac{dX}{dt} = & - \left(\frac{s}{L} + \alpha_2 P^* \right) x_1^2 - \frac{r}{\pi_1 K} x_2^2 - k_2 \lambda_0 x_3^2 - \frac{1}{\theta} (\delta_0 + \delta_1 P^*) x_4^2 \\ & - \alpha_2 R^* x_1 x_3 + k_2 \lambda x_2 x_3 - \frac{1}{\theta} (\delta_1 S^* + \theta_1) x_3 x_4. \end{aligned}$$

Now, if the following inequalities hold:

$$\alpha_2^2 R^{*2} < \frac{4}{3} k_2 \lambda_0 \left(\frac{s}{L} + \alpha_2 P^* \right), \quad (18)$$

$$k_2 \lambda^2 < \frac{4}{3} \frac{r}{\pi_1 K} \lambda_0, \quad (19)$$

$$\frac{1}{\theta} (\delta_1 S^* + \theta_1)^2 < \frac{4}{3} k_2 \lambda_0 (\delta_0 + \delta_1 P^*), \quad (20)$$

then dX/dt will be negative definite.

From inequalities (18)–(20) we may easily choose the positive value of k_2 if

$$\max \left\{ \frac{3}{4} \frac{\alpha_2^2 R^{*2}}{\lambda_0 \left(\frac{s}{L} + \alpha_2 P^* \right)}, \frac{3}{4} \frac{(\delta_1 S^* + \theta_1)^2}{\lambda_0 \theta (\delta_0 + \delta_1 P^*)} \right\} < k_2 < \frac{4}{3} \frac{r \lambda_0}{\pi_1 K \lambda^2}. \quad (21)$$

From inequality (21) we assert that dX/dt is negative definite under condition (13), proving the theorem.

Appendix C: Proof of Theorem 2

To prove the global stability of equilibrium F^* , we assume the following Lyapunov function, about equilibrium F^* :

$$Y(R, H, P, S) = \left(R - R^* - R^* \ln \frac{R}{R^*} \right) + z_1 \left(H - H^* - H^* \ln \frac{H}{H^*} \right) + \frac{z_2}{2} (P - P^*)^2 + z_3 \left(S - S^* - S^* \ln \frac{S}{S^*} \right),$$

where $z_1, z_2, z_3 > 0$ and will be chosen appropriately. It can be easily checked that the function Y is zero at the equilibrium $F^*(R^*, H^*, P^*, S^*)$ and positive for all other positive values of R, H, P , and S .

Now the derivative of Y with respect to t along the trajectories of model system (1) is given by

$$\begin{aligned} \frac{dY}{dt} = & - \left(\frac{s}{L} + \alpha_2 P^* \right) (R - R^*)^2 - \frac{r z_1}{K} (H - H^*)^2 - z_2 \lambda_0 (P - P^*)^2 \\ & - z_3 (\delta_0 + \delta_1 P^*) (S - S^*)^2 - \alpha_1 [1 - z_1 \pi_1] (R - R^*) (H - H^*) \\ & + z_2 \lambda (P - P^*) (H - H^*) - \alpha_2 R (R - R^*) (P - P^*) \\ & - \phi (1 - z_3 \theta) (R - R^*) (S - S^*) - z_3 (\delta_1 S + \theta_1) (P - P^*) (S - S^*). \end{aligned}$$

Choosing $z_1 = 1/\pi_1$ and $z_3 = 1/\theta$, we have

$$\begin{aligned} \frac{dY}{dt} = & - \left(\frac{s}{L} + \alpha_2 P^* \right) (R - R^*)^2 - \frac{r}{\pi_1 K} (H - H^*)^2 - z_2 \lambda_0 (P - P^*)^2 \\ & - (\delta_0 + \delta_1 P^*) \frac{1}{\theta} (S - S^*)^2 - \alpha_2 R (R - R^*) (P - P^*) \\ & + z_2 \lambda (P - P^*) (H - H^*) - \frac{1}{\theta} (\delta_1 S + \theta_1) (P - P^*) (S - S^*). \end{aligned} \tag{22}$$

Now dY/dt will be negative definite inside the region of attraction Ω , provided the following inequalities:

$$\alpha_2^2 L^2 < \frac{4}{3} z_2 \lambda_0 \left(\frac{s}{L} + \lambda_2 P^* \right) \tag{23}$$

$$z_2 \lambda^2 < \frac{4}{3} \frac{r}{\pi_1 K} \lambda_0 \tag{24}$$

$$\frac{1}{\theta} \left(\frac{\delta_1}{\delta_0} (\theta \phi L - \theta_0) + \theta_1 \right)^2 < \frac{4}{3} z_2 \lambda_0 (\delta_0 + \delta_1 P^*). \tag{25}$$

From inequalities (23)–(25) we may choose positive z_2 if inequality (14) is satisfied. Thus, dY/dt is negative definite under condition (14).

Appendix D: Proof of Theorem 3

First, from the lemma we define

$$L_2 = \max \left\{ L, H_m, P_m, \frac{1}{\delta_0}(\phi\theta L - \theta_0) \right\}.$$

Hence, it implies that $\limsup_{t \rightarrow \infty} V(t) \leq L_2$.

This also shows that for any adequately small $\epsilon_i > 0$, there exists a $T_i > 0$ such that $R(t) < L + \epsilon_1$ for all $t \geq T_1$, $H(t) < H_m + \epsilon_2$ for all $t \geq T_2$, $P(t) < P_m + \epsilon_3$ for all $t \geq T_3$, and $S(t) < (\phi\theta L - \theta_0)/\delta_0 + \epsilon_4$ for all $t \geq T_4$. Taking $\epsilon = \max \epsilon_i$, $T = \max T_i$, $i = 1, \dots, 4$, one can say that for $\epsilon > 0$, there exists a $T > 0$ such that for all $t \geq T$, the following holds: $R(t) < L + \epsilon$, $H(t) < H_m + \epsilon$, $P(t) < P_m + \epsilon$, $S(t) < (\phi\theta L - \theta_0)/\delta_0 + \epsilon$.

Now from the first equation of model system (1), for all $t \geq T$, we can write

$$\begin{aligned} \frac{dR}{dt} &\geq sR - \frac{sR^2}{L} - \alpha_1(H_m + \epsilon)R - \alpha_2(P_m + \epsilon)R^2 - \phi \left(\frac{1}{\delta_0}(\phi\theta L - \theta_0) + \epsilon \right) R, \\ &= \left(s - \alpha_1(H_m + \epsilon) - \phi \left(\frac{1}{\delta_0}(\phi\theta L - \theta_0) + \epsilon \right) \right) R - \left(\frac{s}{L} + \alpha_2(P_m + \epsilon) \right) R^2. \end{aligned}$$

It gives

$$\liminf_{t \rightarrow \infty} R(t) \geq \frac{(s - \alpha_1(H_m + \epsilon) - \phi((\phi\theta L - \theta_0)/\delta_0 + \epsilon))}{s/L + \alpha_2(P_m + \epsilon)},$$

which is true for every $\epsilon > 0$, thus,

$$\liminf_{t \rightarrow \infty} R(t) \geq \frac{s - \alpha_1 H_m - (\phi/\delta_0)(\phi\theta L - \theta_0)}{s/L + \alpha_2 P_m} = R_a \quad (\text{say}),$$

here R_a is positive, provided

$$s - \alpha_1 H_m - \frac{\phi}{\delta_0}(\phi\theta L - \theta_0) > 0. \quad (26)$$

Again, from the second equation of model system (1) we have

$$\frac{dH}{dt} \geq rH - \frac{rH^2}{K} + \pi_1 \alpha_1 R_a H = (r + \pi_1 \alpha_1 R_a)H - \frac{rH^2}{K}, \quad (27)$$

it follows that $\liminf_{t \rightarrow \infty} H(t) \geq (K/r)(r + \pi_1 \alpha_1 R_a) = H_a$ (say), which is true for every $\epsilon > 0$.

Further, from the third equation of model system (1) we get

$$\frac{dP}{dt} \geq \lambda H_a - \lambda_0 P,$$

this implies that $\liminf_{t \rightarrow \infty} P(t) \geq \lambda H_a / \lambda_0 = P_a$ (say).

Similarly, through the fourth equation of model system (1) we get

$$\frac{dS}{dt} \geq \phi\theta R_a S - \delta_0 S^2 - \delta_1 (P_m + \epsilon) S^2 - \theta_0 S - \theta_1 (P_m + \epsilon) S,$$

and so $\liminf_{t \rightarrow \infty} R(t) \geq (\phi\theta R_a - \theta_0 - \theta_1 P_m) / (\delta_0 + \delta_1 P_m) = S_a$ (say), which is true for every $\epsilon > 0$ if

$$\phi\theta R_a - \theta_0 - \theta_1 P_m > 0, \quad (28)$$

and hence, taking $L_1 = \min(R_a, H_a, P_a, S_a)$, the theorem follows.

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