Martin Gardner and His Influence on Recreational Math

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Abstract

Recreational mathematics is a relatively new field in the world of mathematics. While sometimes overlooked as frivolous since those who practice it need no advanced knowledge of the subject, recreational mathematics is a perfect transition for people to experience the joy in logically establishing a solution. Martin Gardner recognized that this pattern of proving solutions to questions is how mathematics progresses. From his childhood on, Gardner greatly influenced the mathematical world. Although not a mathematician, he inspired many to pursue careers and make advancements in mathematics during his 25-year career with *Scientific American*. He encouraged novices to expand their knowledge, enlightened professionals of computer science developments, and established his own proofs.

Martin Gardner and His Influence on Recreational Math

Introduction

Martin Gardner, a man known for presenting formal proofs and developing concepts in simple ways, never received a formal education in mathematics. He graduated from the University of Chicago in philosophy (Gardner, 2013), and although he was not viewed as an expert in mathematics, is believed to have done more to increase modern interest in recreational mathematics than anyone else before him through his writings (Pickover, 2001). Through his personal development, studies, and interactions with mathematicians, he became skilled in the subject, sharing the topics with the world through *Scientific American* articles, various books, and stimulating puzzles. He saw that students, both young and old, became interested in, and learned more about, mathematics when it was approached “in a spirit of play” (Mulcahy, 2014, para. 3). This approach to mathematics through games, puzzles, or magic illustrated both the complexity and surprising accessibility of the subject. Intrigued by the theory and beauty within math, he wrote not only on subjects developed by others but also on proofs which he personally produced.

For many people, mathematics begins as an exciting subject, but quickly turns arduous when answers come slowly. When problems lose their simplicity and become abstract, students doubt the benefit of developing a deep understanding of the subject, pushing it off as unnecessary, boring, and impractical (Boaler, 2016). Throughout his career, Gardner sought to disrupt that type of thinking by reintroducing the use of puzzles and word games to display how mathematics is about noticing patterns in the world then developing new knowledge from these observations (Boaler, 2016). Through this type of presentation, he helped his readers develop a better sense of logical intuition. In each of his articles, Gardner presented both emerging and established mathematical concepts, encouraging his readers to work through the theories presented to better understand the ideas. His engaging and simple writing style motivated young and old, experienced and inexperienced, to create their own flexagon, discover snarks, and explore computer applications (O'Connor & Robertson, 2010).

The Beginning

An enthusiast from a young age, Gardner credited his interest in mathematics and its puzzles to his father, who gifted him a copy of *Cyclopedia of Puzzles* by Sam Loyd (O'Connor & Robertson, 2010). The exciting puzzles presented kept Gardner intrigued for hours as a kid. They taught him logical thinking and provided the groundwork which would be built upon throughout his career. Mathematics and physics, his best subjects throughout high school, shaped his deductive skills and fostered his passion for mathematics (O'Connor & Robertson, 2010). Although planning to study physics in college, Gardner decided to major in philosophy after becoming fascinated with the subject (O'Connor & Robertson, 2010). He was extremely interested in philosophy, but jobs in the field were difficult to find during the depression in 1936. Instead, he took several different jobs, eventually becoming a reporter for the local newspaper before serving in the Navy during WWII. After the war, Gardner realized a career in writing would prove more beneficial than one in philosophy. He became a writer for various magazines, eventually writing his own book in 1956 titled *Mathematics, Magic and Mystery* (O'Connor & Robertson, 2010), which demonstrated how mathematics serves as the basis of magic, and how an understanding of one creates a deeper appreciation for the other.

Gardner's career with *Scientific American* began while he was working for a children's magazine in New York City. He attended weekly magic shows, which introduced him to the young and innovative genre of recreational math. At one particular show in 1956, the magician Royal Vale Heath (Gardner, 2001) showed Gardner a contraption made from strips of paper. These strips were marked with equilateral triangles, then folded in a particular way to create what appears to be a hexagon. The hexagon can be *flexed* so different colors are presented when the flexagon is folded (Gardner, 2001). This introduction to flexagons ignited in Gardner a passion to learn more about the topic and the inventors, leading him to study the mathematical reasoning which allowed for such a creation. He proceeded to find and interview the creators of the original hexaflexagon, which culminated in an article published on the subject by *Scientific American* (Gardner, 1956), beginning a quarter-of-a-decade-long career with the magazine.

Flexagons

Although flexagons were not popularized until the late 1950s, the interactive folded-paper toy was created in 1939 by four graduate students at Princeton. After trimming the excess paper from American paper to fit in his English binder, Arthur Stone began to fold the strip diagonally in three spots, creating a hexagon. With further tampering, Stone discovered some interesting qualities about his creation (Gardner, 1956). When the hexagon was manipulated to push two triangles together, a new face of the hexagon would appear. The initial design with three faces was extended to six, then later to even more, faces (Gardner, 1956).

In his inaugural article, Gardner taught readers how to create a hexaflexagon. After making individualized flexagons, his readers would write him of their joy in flexing, frustrations of certain creations, and ideas for other flexagons. This article launched Gardner's influence on "generations of readers to the pleasures and uses of logical thinking" (Pickover, 2001, p. 88). Giving a great introduction to mathematics for novices and challenging advanced students, flexagons taught both the importance of finding patterns and the concepts of transformation and symmetry.

Gardner's first article also spurred development in flexagon theory which had previously been stalled. The initial theory behind flexagons was explored in 1940, shortly after the creation of the toy. Bryant Tuckerman, one of the original four students, developed the *Tuckerman traverse*, a simple procedure which displays all six faces of a hexaflexagon with the shortest cycle of flexes (Gardner, 1956). The traverse occurs when the same corner is flexed until it is unable to do so, resulting in 12 flexes where three faces appear three times as often as the other three faces. A Feynman Diagram, created by another of the students, Richard Feynman, can be used to illustrate the sequence of faces of the Tuckerman traverse. The group of creators, who called themselves the Flexagon Committee, eventually found that even-numbered flexagons have two distinct sides, while odd-numbered ones have a single side (Gardner, 1956). Thoroughly enjoying their new findings, they constructed a theory for constructing flexagons of various sizes and types. Because of the war, however, the four creators never published their findings. Gardner's article, though, inspired many to publish papers expanding on these initial theories, extending hexaflexagons to edge and point flexagons.

Because the article on flexagons garnered such appreciation, the magazine's publisher, Gerry Piel, decided to hire Gardner to write a column regularly on recreational mathematical topics, entitled "Mathematical Games." From his almost thirty years as a writer on the subject, the reader can see his development from elementary to more advanced levels of both concepts and understanding (Yam, 2010). These articles, which allowed Gardner to develop his personal mathematical skills, also inspired countless others to view mathematics as more than a difficult and useless subject. In fact, one reader developed such a passion for mathematics that, while pursuing her graduate degree in the subject, gave a seminar talk on the topic Gardner introduced to the world—hexaflexagons—which Arthur Stone attended (Goldstine, n.d.). Throughout Gardner's career with *Scientific American*, many readers would have similar responses after learning about a variety of new concepts.

Writing His Column

Gardner's "Mathematical Games" spanned many topics. He began by writing on recreational math, introducing his readers to new puzzles and teaching them ways to find the solution. His first articles were inspired by Sam Loyd, whose book began Gardner's love for mathematics and puzzles. Through reading Loyd's books, Gardner recognized the importance that pictures, along with simple explanations, had on a person who was learning the subject. He realized that mathematics was more than just a subject of memorizing formulas, and that the beauty of mathematics appeared through the patterns observed and logic developed. With every article, Gardner left a puzzle for the reader to solve. Sometimes, solutions were known and published in the following article. Other times, these answers were unknown even to the experts Gardner contacted for the topic, and rewards were offered for correct answers. In presenting the puzzles, Gardner exemplified what Conrad Wolfram states are the four stages of mathematics: (1) posing a question; (2) going from the real world to a mathematical model; (3) performing a calculation; and (4) going from the model back to the real world to answer the question (Boaler, 2016). Though many were unaware, Gardner's readers were learning to comprehend the underlying mathematical principles, forming a strong basis for future studies. His column inspired expert mathematicians to further improve featured topics and encouraged younger readers, many of whom would later pursue careers in mathematics, to ask questions.

Along with Loyd, Lewis Carroll, the pseudonym of Oxford University professor Charles Dodgson, amazed and inspired Gardner. When Carroll wrote his fanciful *Alice in Wonderland*, he included many mathematically inspired elements, which he believed gave readers a better sense of analysis (Abeles, 2015). Carroll's wordplays sometimes referenced questioning characteristics of mathematics like quaternions, such as the Hatter's unsolvable riddle "Why is a raven like a writing-desk?" This question was asked during the tea party, thought to represent William Hamilton's work in quaternions, an unsuccessful study until time was included in the system (Devlin, 2010). Whether Carroll's account was a form of satire of new developments in abstract algebra (Devlin, 2010) or simply an imaginative account, the story encouraged his readers to think logically. Gardner recognized this and wrote a book based on the messages, chess moves, and caricatures in Carroll's books to help people better understand the references (Yam, 2010). He explained how the chess match in *Through the Looking Glass* could be accomplished and provided instructions for each move (Carroll, Gardner, & Tenniel, 2000). He also gave background information on certain words, so modern readers could appreciate the book more. These explanations in his most successful book *The Annotated Alice* introduced "many readers to the pleasures and complexities of critical reading" (Susina, 2016, para. 3).

Snarks and Boojums

Beyond just his *Alice* books, Carroll wrote a poem titled "The Hunting of the Snark," which left a lasting impression on Gardner. The poem centers around a group of men looking for a rare creature called a Snark, marked by five characteristics. A special breed of Snarks, known as Boojums, is even rarer. The poem enthralled Gardner so much that he used the term *Snark* to describe a specific type of graph. As a result of Gardner's designation, a graph with a chromatic index of three that is simple, connected, and bridgeless is now known as a Snark. In one column, Gardner (1976) introduced this new name for what is classified as nontrivial uncolorable trivalent graphs, beating out his initial title of NUT. Gardner continued to allude to Carroll's poem by classifying planar Snarks as Boojums. These graphs are just like the creatures in Carroll's ballad, Snarks are not easily found and a Boojum never discovered.

Created from nodes and edges connecting the nodes, graphs can be classified as directed or undirected, cyclic, or weighted. As evidenced by the different classifications, many terms are used to explain a particular graph. Nontrivial, uncolorable, and trivalent are the three specific terms which classify a Snark. By nontrivial, we mean that the graph contains no bridge. A bridge is an edge which connects two nodes such that if the edge is removed, the graph is divided into two graphs. A graph is uncolorable if every edge cannot be colored with three colors with every color meeting at all nodes (Gardner, 1976). The last attribute of a snark is that every node has three edges which connect to it, creating a trivalent graph.

The Boojum is important because if such a graph is found, the four-color theorem could be successfully disproved. The four-color theorem asserts that any planar map can be colored with a maximum of four colors such that no two neighboring areas are the same color (Gardner, 1976). Since maps can be diagrammed as graphs, where the nodes represent the region and edges connect neighboring regions, the coloring pattern of the map can then be converted to coloring the edges of a graph, which has one less color than the map. With this, Boojums can be used to authenticate the theorem. The four-color theorem was considered false by many mathematicians until 1976, when Kenneth Appel and Wolfgang Haken established a computer-assisted proof (Appel & Haken, 1976). Before that, Peter Tait proved in 1880 that the four-color theorem is equivalent to the statement that no Snark is planar, meaning edges of the graph intersect only at the nodes (as cited in Gardner, 1976). Tait also proved that any planar trivalent graph whose map has four-colored regions will have a graph with three-colored edges. Tait, ignoring nonplanar and graphs with bridges, assumed all trivalent graphs are three-colorable, implying that any graph with three edges meeting at every node cannot possibly be uncolorable and thus Snarks could never exist (Gardner, 1976). But Gardner wondered what would happen if nonplanar graphs were considered. Thus he distinguished between planar Boojums and nonplanar Snarks. While a planar trivalent map is three-colorable, nothing can be assumed about a nonplanar trivalent map. Thus, the hunt for Snarks and Boojums may not be completely futile.

The smallest and first Snark discovered is the Petersen graph, which consists of 10 nodes and 15 edges. The discovery of this graph in 1891 preceded the discovery of a second snark, having almost twice as many nodes, by 50 years. Rufus Isaacs, a notable graph theorist, made further developments on Snark theory in the mid-1970s. In addition to finding two infinite sets of Snarks, he also asserts that a Petersen graph, although not a subgraph of every Snark, is contained in every known Snark, thereby preventing any Snark from potentially masquerading as a Boojum (Gardner, 1976). If this assertion is expanded and proved such that all possible Snarks have Petersen graphs, then the four-color theorem is proved true since no Boojums could exist where the nonplanar Petersen graph in a Snark is contained in a planar Boojum (Gardner, 1976).

As in all his articles, Gardner did not simply offer the information on Snarks as facts that his readers should accept without proving the facts to themselves. Rather, he provided many examples of Snarks and three-colorable trivalent graphs so that readers could know what constituted graphs as Snarks. Since one of the easiest ways to dismiss a graph as a Snark is to show it is three-colorable, he offered useful techniques to color a graph by taking a node, coloring its edges, traveling to the next node, then label the colors of those edges as dependent on the previous choice. When an issue arises, Gardner recommends going one step back to find which choices can be changed to possibly obtain a colorable map. Through eight repeatable steps, Gardner taught those new to the subject how to effectively prove whether a graph is colorable or uncolorable (Gardner, 1976). Because the trivalent and nontrivial aspects of a graph are easily noticed, the given process helps readers conclude whether a graph is a Snark. Readers were able to develop their skills in coloring by following the procedure to color known trivalent graphs, then were encouraged to prove that the Petersen graph and Boojums are uncolorable. Concluding his article, Gardner challenged his readers to find their own Snarks by creating and coloring their own trivalent graphs.

Influence on Computer Science

Always looking for new concepts to write about and receiving topics from many mathematicians, Gardner became very involved in computer science, an emerging discipline in the late 1950s. Having written previous articles on graph theory, he applied his gained knowledge to computer applications. He read articles by leading experts in the field and discussed the fascinating results with his readers. Occasionally, the same experts who established innovative solutions for computer applications would write to Gardner, asking him to promote support of research for a specific topic. Gardner built on the knowledge of those before him, which inspired readers to research further what the intellectuals began (Berlekamp, 2014).

Crossing Numbers

Along with his introduction of Snarks in conjunction with the four-color theorem, Gardner studied and wrote on many other developments in graph theory, integrating the subject with computer science. Because graphs are a useful way to model networking, scheduling, and other computer applications (Gardner, 1973), the study has and continues to be extremely relevant to growing developments in computer programming and networking. One important aspect of networking involves the crossing number. When edges of a graph meet at a point other than a node, the graph is said to have a crossing. The minimal number of crossings which occur in a good drawing, where no more than two edges go through any particular crossing, is known as the graph's crossing number (Gardner, 1973). As mentioned earlier, the graph is classified as planar when the crossing number is zero. A complete graph occurs when every node is connected by one edge, such that for n nodes, we have $\Sigma(n-1)$ edges. With complete graphs, the crossing number is easily seen, and can be used to prove that graphs with more than four nodes are nonplanar since adding a fifth node, either inside or outside a four-point graph, will always create a crossing point, as noted by the dotted line in Figure 1. Although it proves that five regions in a map cannot share a boundary and not the four-color theorem, this fact is useful in other graph theory applications.

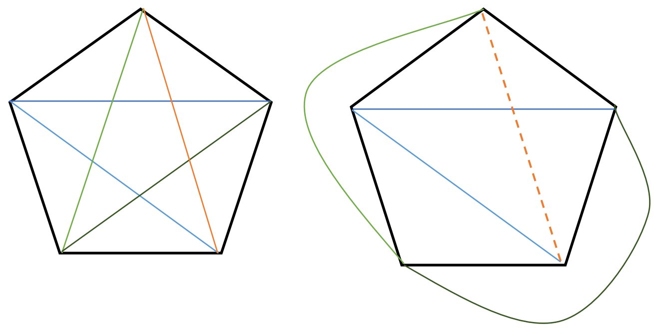

Figure 1. Five-point complete graph. Notice the graph is not planar since the dotted edge in the right graph cannot be drawn without crossing another edge.

For instance, crossing numbers appear to correlate with the chromatic number of a graph. The crossing number of a complete graph can be found using a formula proposed by Richard Guy (Gardner, 1973), and these values have been used to bound the chromatic index above by the one-fourth power of the crossing number (Zhu, 2015). One challenge which Gardner offers his readers is to determine a formula which gives the max number of edges which, as part of a complete graph, have no crossings.

Although Gardner never mentions the fact, crossing numbers are useful in designing integrated circuits for very large-scale integrating, or VSLI, chips. A low crossing number is preferred when creating a chip so that the wires can perform efficiently. Thus, an important part of creating chips is finding the number of edges which gives a zero-crossing. If a zero-crossing cannot be established, a minimal crossing number is instead found. The size and cost of chips are decreased when the number is minimized, so establishing an upper bound was beneficial in the field (Zhu, 2015). While Gardner may not have personally inspired these theorems on VSLI chips, his article on the crossing numbers opened others' eyes to the complexity and excitement of graph theory.

RSA Cryptography

In 1977, Gardner would continue to write on computer science with an article about the new development by Ron Rivest, Adi Shamir, and Leonard Adleman after the three sent Gardner a letter explaining their findings. Rivest, Shamir, and Adleman recognized the importance of electronic communication in the future, and like many mathematicians before them, sought to create a cipher which would keep communication safe by encrypting messages with a cipher that was theoretically unbreakable. The created system further implemented the Diffie-Hellman cryptosystem, which is itself a trapdoor one-way function. Their cipher was the first of its kind, allowing senders to use a public function to encode a message for the recipient. The recipients had the inverse function to decode the received message (Gardner, 1977). The significance was that although everyone could access the public key, the private key was virtually impossible to uncover. Using prime numbers for the public key, Rivest, Shamir, and Adleman developed an advanced way to secure messages. This improvement of the Diffie-Hellman for public-key encryption is now commonly referred to as RSA cryptography.

To establish a foundation for understanding the system, Gardner first introduced his readers to Fermat's Little Theorem, which provides a quick way to determine if a number is prime. This theorem is an extension of Euler's Theorem, and is used often in number theory studies. If p is prime, and a is a positive integer less than p, then Fermat's Little Theorem tells us that $a^{p-1} \equiv 1(mod\ p)$. The encryption with the RSA algorithm is reliant on having large prime numbers, so Fermat's Theorem allows for a number's primality to be quickly verified. This allows the creator to easily confirm that their choice of factors is, in fact, prime. Since no other information about the number, such as its prime factors, are given in RSA, the hacker cannot easily work backwards to determine the encryption values. These facts assured the creators that their system, based on primes, would cause decryption without the cipher to be very demanding.

To begin, RSA cryptography takes two primes both normally over 40 digits, p and q, which are kept secret. However, the values $n = pq$ and *s*, which is relatively prime to Euler's Totient $\phi(n) = (p-1)\ (q-1)$, are made public. The encryption begins by first changing the letters of the message to numerical values. These values are raised to the power *s*, then the result is reduced modulo *n*. This results in the coded message, which is delivered and decoded using the inverse algorithm. The decryption algorithm is dependent on the hidden values of *p* and *q*, which are difficult to discover, and uses the multiplicative inverse of *s*, labeled *t*. Since *s* is coprime to $\phi(n)$, we know that *t* can only be found if the distinct primes *p* and *q* are provided since $t \equiv s^{-1} mod\ \phi(n)$ (Gardner, 1977). When the encrypted message is received, the receiver has the private key and decrypts the message to find the original by raising the entire code to the power *t*, then reducing that value modulo *n*.

For example, suppose we choose $p = 7$ and $q = 11$ and wish to encrypt the message "Math is fun." Then $n = 77$, $\phi(n) = 60$, and choose *s* to be coprime to $\phi(n)$ such that $s = 17$, which means $t = 53$ since $1 \equiv 17 \times 53\ (mod\ 77)$. The numeric representation of the phrase is 120019070818052013, which is divided up into smaller groups of two integers so that each segment is less than *n*. The first segment of the message is encoded: $12^{17}(mod\ 77) = 45$, then the remaining segments are similarly encrypted. After transforming all two-number segments, the resulting value is 45002428572034862. This is sent to the receiver, who will take two-number segments, raise them to the 53rd power, and find the total modulo 77. Taking the first two numbers, notice that $45^{53} \equiv 12 \ (mod\ 77)$, which corresponds to the original letter "M" of the message. Continuing this process, the original message will be received.

Gardner wrote this article after Rivest mailed him what the group had found and developed. In fact, this article introduced the world to RSA cryptography (Greenemeier, 2015). As in all his articles, Gardner challenges his readers to take part in personally learning how the cipher works, which is the reason he explained Fermat's Theorem and how to select the numbers which the system is based upon. Along with telling Gardner how the algorithm worked, Rivest, Shamir, and Adleman included an encrypted message and the public key for the readers, offering a prize to whomever could decipher the code first. Gardner sees this cipher as so powerful, that millions of years could pass before the cipher is broken (Gardner, 1977). Yet he underestimated human's ingenuity, and the help computers would offer, because in 1991, within less than twenty years, the code was successfully decrypted. Gardner recognized that cryptography developments were very likely to occur, but his gross overestimation of the resiliency of RSA causes one to wonder if he viewed the subject as unprofitable. The study of cryptanalysis, though, is as, if not more, important now than when Gardner presented this article in 1977.

Conway's Game of Life

With the growing use of computers in the latter half of the twentieth century, research of recreational mathematics as an independent subject area increased more than many imagined. Some of those developments occurred after Gardner (1970) wrote an article on John Conway's Game of Life. This article introduced many of his readers, which included skilled mathematicians, to the subject and various applications which the game provided. For this article, he interviewed John Conway, the creator of the game and a close friend of Gardner.

Gardner and Conway often wrote to each other with their discoveries and developments of new puzzles. In one meeting, while he discussed his many developments since their last meeting together, Conway introduced Gardner to his new creation, the Game of Life (Izhikevich, Conway, & Seth, 2015). Gardner correctly predicted that this new game would influence the world of recreational math, especially with the growing prominence of computers. Not only did his inexperienced readers enjoy learning about this new simulation board game, but so did his experienced readers who were able to use computers to play the game.

A misnomer, the Game of Life does not require any players to be functioning when a computer is used as the creator intended. Life is a computer program composed of an infinite grid with cells that are either alive or dead. The Game is classified as a cellular automaton, meaning the rules which Conway established are applicable only to the cells and their neighbors within the grid. The rules Conway established determine whether each cell will be "on" or "off," making the population's behavior unpredictable so the system is analogous to living organisms and indeterminate chaos (Caballero, Hodge, & Hernandez, 2016). To begin, a person, or an algorithm, determines the initial set of counters, or living organisms, which will be affected by the existential rules. The three rules are (1) that every counter with two or three neighboring counters survives; (2) a counter dies if it has four or more neighboring counters (from overpopulation) or if it has less than one neighboring counter (from isolation); (3) each empty cell adjacent to precisely three neighbors has a counter the following move (Gardner, 1970). One can tell that these rules create a system entirely dependent on the initial arrangement of counters. Arrangements may result in a stable pattern, an oscillator, a glider, or a train. Like any good mathematician, Conway wanted to determine if and what type of a pattern would emerge dependent on the initial counters. He noticed that patterns immediately emerged when stability or blinkers developed. Other times, Conway would watch the computer screen and marvel at his creation of a non-repeating pattern (Gardner, 1970).

After Gardner published his article on the subject, Conway's game obtained instant fame, inspiring mathematicians and scientists to develop variations and find valuable inferences from the game (Izhikevick et al., 2015). Life was so addictive with its complexity that it is believed that mathematicians in the government wasted many hours, and millions of dollars, watching Life evolve (Roberts, 2015). This article may not have provided the most responses to a column, but there is no doubt that the widely circulated article left many expanding the usefulness of Life. Readers appreciated that three simple rules can govern a complex program, and some scientists applied the automaton of Life to the epigenetic principle, which states that genetic action and developmental processes are inseparable dimensions of a single biological system. Through the counter's state, the development of a person or organism, based on successes or failures from the past, is represented. Caballero et al. (2016) noticed that the program is beneficial in replicating the complex system of epigenetics and researched how it correlated to well-known models created by other mathematicians, Alan Turing and Alan Edelman. They found that using Life to simulate the epigenetic landscape performed very well in establishing theories for biological science. Gardner knew that recreational mathematics had connections which extended further than just pure math, which was why he introduced the applications, such as biology, in which Life could be utilized.

Readers were inspired not only to use Life for real-world applications, but also to develop other personal discoveries based on the game. Gardner invited his readers to prove or disprove a question that Conway himself proposed: that no pattern of a finite amount of counters can grow without limit (Gardner, 1970). A group from MIT disproved this conjecture with the discovery of the glider gun. R. William Gosper, Jr., the leading developer of the group, constructed the arrangement of counters now known as the glider gun. Every 30 moves, or generations, the gun creates a new glider, or pattern that travels across the grid. The eight gliders which result repeatedly populate 36 cells, thus establishing an initial arrangement of counters which have unbounded growth (Dewdney, 1985). Gosper's gun was the smallest gun in terms of the period and grid-size discovered for 45 years, until a gun with a larger period but fewer live cells was discovered in 2015 (Scorbie, 2015). Shortly after that first discovery, though, Conway could prove that the Game was equivalent to a universal Turing Machine because the initial pattern can be used to simulate any Turing machine (Izhikevick et al., 2015). Although Gardner did not invent the Game of Life, he certainly impacted the effect the game had by introducing the Game to the world. The many readers of *Scientific American*, as interested in the topic as Gardner, enjoyed making new developments in the emerging field of computer science through simulating interactions with Life.

Personal Developments in Math

As Gardner deepened his knowledge about mathematics through interviews with formally educated mathematicians, he began expanding his skills from writing on mathematical topics to developing proofs. Two topics that especially fascinated him were serial isogons and the length of Steiner trees on a checkerboard. From his time writing "Mathematical Games," Gardner was able to work with experts in the two fields to create proofs on the topics. He thoroughly enjoyed the process of writing and publishing the proofs, recognizing the beauty of pure mathematics that comes with composing a proof. In his first work, he paired with three men to create a proof that all 90-degree serial isogons must have a total number of sides summing to a multiple of eight, while his later work on Steiner trees was expounded on by prominent mathematicians.

Serial Isogons of 90 Degrees

Serial isogons are a closed path created of segments with lengths that increase in sequential order with a turn of 90 degrees at the end of each segment (Sallows, Gardner, Guy & Knuth, 1991). Different turn-angles for serial isogons could be considered, but Gardner and his colleagues focused solely on 90-degree angles. The paths can intersect, touch, or overlap, but they must be closed. Gardner, along with Lee Sallows, Richard Guy, and Donald Knuth, found that the smallest isogon of 90-degrees has order eight. Of notable importance is that the created tile also satisfies the Conway criterion, meaning the tile's boundary can be partitioned into six parts with the first and fourth partitions equal, and every other fragment has rotational symmetry with the others (Sallows et al., 1991). The 8-sided polyomino was the first serial-sided tile found which satisfies the Conway criterion, allowing the polyomino to tile a plane.

For further extension of serial isogons, the authors used various techniques to prove that for any 90-degree serial isogon, the path of order N must be a multiple of eight. One direction was taken to prove that, treating a move east and north as positive, and a move west and south as negative, the sum of a closed circuit equals zero only when N is of the form $8z$ for some z in the integers (Sallows et al., 1991). To begin, notice that to be closed, N must be at least a multiple of four, as can be shown with a rook moving along a checkerboard. Thus, let $N = 4k$ for an integer k. Consider the north-south direction, which form the sequence of even integers. Then the total for both directions is $(2 + 4 + \cdots + 4k) = 2\sum_{j=1}^{2k} j = 2k(2k+1)$, where half are northward and half southward. Since the distance north and south must be a multiple of 2 to have a closed isogon, the sum will never be zero (Sallows et al., 1991). Now if *k* is an odd integer, we get that the length of travel in one direction is odd since total north-travel is $k(2k + 1) \Rightarrow odd \times odd = odd$. But since this value is equivalent to the summation of even integers, we reach a contradiction and conclude that *k* cannot be odd. So, $k = 2j$ for some *j* in the integers, and thus $N = 4(2j) = 8j$. So the authors established one technical proof for the number of sides in a 90-degree serial isogon. Gardner, expanding this technical proof, used a coloring pattern to prove that *N* must be a multiple of eight.

Gardner created a unique grid of white and black squares to construct a bicolor proof showing that $N \equiv 0 \; mod \; 8$. The grid is composed of black squares separated above by three white rows and beside by one white row. Starting the serial isogon on a black square and then first moving in a horizontal direction, the end of each segment appears on the repeated sequence of colors: *WWWWWWBB*, regardless of the direction that each succeeding segment turns (Sallows et al., 1991). By the grid's coloring, the segment will only reach a black tile with moves that are a multiple of 7 or 8. Because the last segment is only perpendicular to the first segment, thus resulting in a closed path, on even turns, the possibility of the moves being a multiple of seven is removed. This result gave Gardner such pleasure, he listed it as one of four times in which he discovered the beauty in thinking through and writing an elegant proof (Gardner, 2006). Gardner did exactly what he inspired his readers to do—ask a question then use mathematical logic to prove your answer. He successfully completed this task and would continue asking and proving questions throughout his life.

Minimal Steiner Tree on Checkerboard

Other than his work on proving the order of a right-angled serial isogon, Gardner further developed research on minimum Steiner trees. Further exemplifying how he had grown from a writer on mathematics to a fellow mathematician, Gardner worked with two experts, Fan Chung and Ron Graham, in 1989 to find whether a minimal network of Steiner trees existed which joined every corner of the chessboard. Although he was not able to prove the network he found was of minimal length, he did conjecture the Steiner trees for large square lattices (Chung et al., 1989). The proof of their conjectures was completed by other scholars only four years after their paper was published.

A Steiner tree on a given set of points in a metric space is defined as the shortest network connecting all of the points, differing slightly from a minimum spanning tree because extra points, called Steiner points, may be added for minimal distance. Giving costs to the distance between the points allows Steiner trees to be used in many applications, such as finding the minimal time of data transmission or modeling wire routing within VLSI circuits. The difference between minimum spanning trees and Steiner trees are evident when finding the length between the three nodes of an equilateral triangle with side length one. The minimum spanning tree connecting the nodes will have a length of two, but if a point is added to the middle and the three nodes are connected via the added point, a Steiner tree results with total length of $\sqrt{3} \approx 1.73$. So a three-point tree has its length reduced by 13% when constructed as a Steiner tree rather than a spanning tree, as illustrated in figure 2. Sometimes more than one point is added, as in the created Steiner network connecting the four edges of a square, reducing the length from 3 to 2.73 (Chung, Gardner, & Graham, 1989).

Figure 2. Steiner tree (left) with the added Steiner point and Spanning tree (right) connecting the vertices of an equilateral triangle.

A nondeterministic polynomial, or NP, problem does not have solutions produced in polynomial time, but given solutions can be verified with an algorithm in polynomial time. A specific problem which any NP problem can be solved with, or reduced to, is classified as NP-complete. One such NP-complete problem is that of finding the path of a Steiner tree for a random set of points (Chung et al., 1989). Algorithms exist for finding the minimum Steiner tree that connects less than twenty nodes, providing simple ways to validate answers for certain problems. But for an arbitrary set of points relating to more complex problems, many mathematicians agree that no efficient algorithm exists to create a minimum Steiner tree (Chung et al., 1989). Gardner, Chung, and Graham questioned the effect of arranging the points in a square lattice for creating such an algorithm. Chung and Graham (1978), who previously established minimum Steiner trees for $2 \times n$ rectangular arrays, or ladders, of points, worked with Gardner to expand the research, developing conjectures and results for rectangular and square arrays.

In his June 1986 "Mathematical Games" column, Gardner introduced the topic to his readers by constructing the two trees for square lattices of order 2 and order 9. Before this, he showed his readers how the Steiner tree would appear for a small number of vertices, acting as building blocks for larger arrays. Always aware of different learning styles, Gardner taught his readers a new way to see how Steiner trees arise. To do so, one simply needs to take two sheets of plexiglass, drill holes to represent the nodes of the tree in the glass, then connect the sheets with rods. When the apparatus is immersed and removed from a bubbly soap solution, a film is created. The resulting soap film illustrates a Steiner tree, most often the minimal tree, because the film will always shrink to the smallest area (Gardner, 1986). Giving readers this hands-on approach is why Gardner was loved by his readers. He taught the readers, aiding those who learn through different styles, by encouraging them to visualize the concepts.

Following this example, Gardner next explains how he arrived at a solution for an array of order-9, where the order describes the number of nodes along one edge. He began by expanding the four-point Steiner tree, which he labeled *X* (Gardner, 1986). Since the length of *X* is known to be $1 + \sqrt{3}$, the length of larger arrays of points can easily be calculated by finding the minimum number of *X*'s in constructing the Steiner tree. For the array of order-9, Gardner created a tree with 26 *X*'s and two additional edges which reach opposite endpoints. Thus, his conjectured tree has minimal length $28 + 26\sqrt{3}$. Although all lengths for orders greater than 3 were only conjectured, one commentator, referring to Gardner's paper with Chung and Graham, hopes "a sequence of efforts [to prove the conjectures] will result" (O'Connor & Robertson, 2010). As a result of their paper, new developments to verify the Steiner tree did arise. For the specific case where the rectangular array has an order which is a power of two, it was proven that the construction with only *X*'s will result in the minimum Steiner tree (Brazil et al., 1996). When the lattice is not a $2^k \times 2^k$ array, the use of only *X*'s may not lead to minimal length. Instead of using just *X*'s in the connection of the lattice, Brazil et al. propose adding Steiner points in new locations to connect the grid. The notation used to represent these new trees establishes ways to prove the length of the minimal Steiner tree for a specific lattice. Whereas Chung et al. (1989)

only had four smaller trees to create larger trees, Brazil et al. (1996) expanded to using six smaller trees, allowing for more precise Steiner trees. The minimal lengths of each of the created components were found; then the trees were proved to be minimizing for their respective sizes; finally, a table was created to show the Steiner tree length for lattices of various orders. Using the table, the Steiner tree for order-9 array was found to have length $26 + 26\sqrt{3}$, slightly shorter than what Gardner initially conjectured seven years earlier (Brazil et al., 1996). Although Gardner did not establish a minimal Steiner tree for certain arrays in his paper, he did accomplish the task he envisioned—persuading others to build on what he had begun, as happens in mathematics.

Influence on Recreational Mathematics

Throughout his career, Gardner established himself as an influencer in the world of recreational math. His success was largely due to his use of puzzles to entice readers to think logically and critically, expanding their mathematical skills (Suri, 2015). Gardner's dedication to seeking out the leading experts on the subject, researching as much as possible on the subject, then writing his column in a clear way to be comprehended by the young and old alike also lent to his success in the field (Gardner, 2008). From the beginning, Gardner never settled for second-hand information about a specific subject. With his inaugural piece on hexaflexagons, Gardner interviewed two of the four creators, Bryant Tuckerman and John Tukey. He continued getting first-hand information so that, although untrained in the subject, he could convert highly formal descriptions to simpler explanations, allowing all of his readers to receive and understand the concept accurately. He was always aware that his articles were not just for the uneducated, but those who, like him, were fascinated in math, which is why he always provided a problem to conclude his articles. This insight and enthusiasm in teaching both simple and complex subjects with surprising clarity was why Gardner achieved runner-up in Pickover's list of "Ten Most Influential Mathematicians Alive Today" (Pickover, 2001).

Conclusion

Martin Gardner was not a formally educated mathematician. However, this did not deter him from becoming a very influential person in the world of mathematics. Unlike many current students of math, Gardner recognized from a young age that mathematics is not always about solving problems but about developing the logic which is useful in every aspect of life. With this insight, he was able to write an extremely successful column for *Scientific American*, which influenced a younger generation to the complexities and innovations in math. He inspired many younger people to pursue careers related to mathematics and computer science, along with encouraging older readers to further develop emerging theories such as those in computer science. Through his life, Gardner recognized and appreciated the importance of learning, and his career enabled him to continually learn new concepts while developing proofs of his own. Gardner recognized that his interest in mathematics was encouraged by the puzzles such as those by Sam Loyd, so he always included challenges to close his articles, motivating his readers to further develop their deductive skills. Mathematicians always talk about the beauty of mathematics, but sometimes have difficulty portraying this beauty to those outside its realm. The biggest influence for mathematical beauty came from a man who was not a mathematician, but a simple man who encouraged learning.

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