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A STOCHASTIC PREDICTIVE CONTROL APPROACH TO PROJECT RISK MANAGEMENT [1]

Ascensión Zafra-Cabeza, Miguel A. Ridao
Eduardo F. Camacho

Escuela Superior de Ingenieros. Universidad de Sevilla
Camino de los Descubrimientos, s/n
41092 Seville (SPAIN)
email:{asun, ridao, eduardo}@cartuja.us.es

Abstract: This work shows a control policy based on MPC and applied to project risk management. MPC has been applied due the properties that presents such as the easy constraint treatment or the extension to multivariable case. The control actions are the mitigation actions to execute in order to reduce the risk exposure. Stochastic variables have been introduced to model the uncertainties of risk impacts. Integer variables are involved in the optimization problem modelling the mitigation actions. *Copyright ©2005 IFAC*

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1. INTRODUCTION

Project risk management is a very extended field in economic systems due to the accomplishments that can be reached. The limited knowledge about the process, the economic system's complexity and the uncertainty in strong points, have played a decisive role. Nowadays, methods and disciplines that help to face challenges are being highly accepted in companies. Organizations which better understand the nature of the risks and can manage them more effectively can not only avoid unforeseen disasters but can work with tighter margins and less contingency (Chapman and Ward, 2000). Several previous studies have aimed to develop methodologies or formalizations about risk management (Crouhy *et al.*, 2000),(Jaafari, 2001). In (Zafra-Cabeza *et al.*, 2004) an optimal scheduling and risk assessment of projects is carried out through static modelling.

This paper studies control policies applied to project scheduling. The objective of the paper is to maintain the cost of the project within budget, according to a given reference and taking into account risk management. The manipulated variables are the mitigation actions to undertake in order to reduce risk exposure and the controlled variable is the cost of the system. A dynamic model of the process is proposed where there are explicit constraints imposed by the system.

Model predictive control (MPC) is an optimal control strategy based in the explicit use of a dynamic model to predict the process output at future time instants (Camacho and Bordons, 2004). MPC disciplines are widely applied in industry (Richalet, 1993) and economic systems (Herbert and Bell, 2001).

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The control methodology applied to this study has been MPC. Some of the reasons in which this decision has been based are the easy treatment of the constraints, the extension to multivariable case and the main role of the model.

This paper develops a methodology to reduce the risk exposure. The control policies are based upon a model that predicts the policy targets. The control variables (actions) can be integer; therefore, the optimization problem is stated as a mixed integer problem. Constraints are explicitly introduced upon the control variables so that they may be limited to economically realistic values.

The impacts caused by risks can be modelled as deterministic or stochastic variables. In that case, a special kind of constraints called *chance constraints* are introduced requiring that constraints should be held with a probability exceeding α. The addition of these variables gives rise to an stochastic optimization problem and it will be treated in the paper.

This work is organized as follows. Section 2 describes the system, providing the dynamic model and the risk modelling. Section 3 presents the optimal control problem based on MPC and the constraint description where stochastic variables are involved. A case study is depicted in Section 4. The experiments have been realised on a true project. Some concluding remarks are described in Section 5.

2. SYSTEM DESCRIPTION

The considered system corresponds to the cost estimation of projects. Assuming that the scheduling of the tasks that comprise the project and the set of risks that can affect to these tasks are known, the objective of this process is to minimize the cost of the whole project through actions that reduce the impacts of the identified risks.

The cost of the project until time instant t has been considered as:

$$y(t) = y(t-1) + T(t) + R(u,t) \quad (1)$$

where

$y(t)$ **and** $y(t-1)$ is the cost of the project until t and $(t-1)$ instant times, respectively.
$T(t)$ is the nominal cost contribution of the tasks that are being executed at instant time t, and
$R(u,t)$ is the term that contains the risk management at time t

As it can be observed, the control actions affect term $R(u,t)$. The following subsection describes how risk mitigation has been modelled in this work.

2.1 Risk Modelling

In this paper, risks are characterized by a probability of occurrence (P_i) and initial impacts (II_i) which may affect cost of the project, if risks become facts and if no actions are taken. The link between tasks, risks and actions is provided in a Risk-Based Structure (RBS). An example of (RBS) is depicted in Figure 1. Every task may have some risks (R_i) associated as a result of the risk assessment performed. Actions (A_i) can be taken to manage risks and their consequences. Several actions may reduce the same risk and one action may reduce more than one risk. The same risk can be associated to different tasks. Tasks, risks and actions are depicted by rectangles, triangles and circles, respectively.

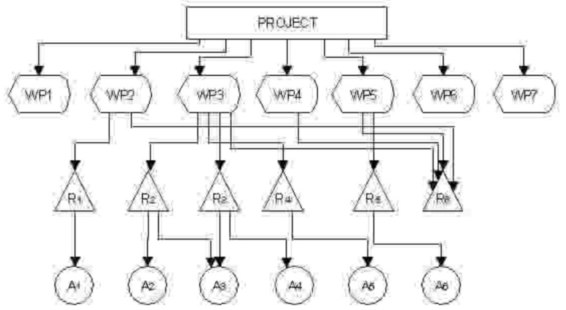

Fig. 1. Risk-based structure of a project

Four types of actions are usually considered in the risk management context:

- Mitigate: Reduce the impact of a source of risk.
- Prevent: Change the probability of occurrence.
- Avoid: Plan to avoid specified sources of risk
- Accept: Accept risk exposure, but do nothing about it.

Only mitigation and accept actions have been implemented in this paper. Mitigation actions will reduce the initial impact of a risk on a task, but the project will be charged with additional cost not included in the initial scheduling. Notice that the impact is probabilistic (only if the risk occurs), but the cost of the action is a fact. Examples of mitigation actions are the contracts of new workers or the purchasing of new machinery to prevent delays in a task. Insurance contracts are also an example and, perhaps, the most common practice to mitigate risks. In fact, insurance companies have an increasing interest in improving risk estimates to encourage mitigation through scientific modelling (Kleindorfer and Kunreuther, 1999). Every mitigation action is described by a set of three elements:

$$A_i = \{u_i, f_i, g_i\} \quad i = 1...p$$

where p is the number of mitigation actions and the decision variable for the action (A_i) is denoted by u_i. $f_i : \Re \rightarrow \Re$ is a function that determine the impact reduction as a function of u_i in each unit time; thus, f_i is the cost reduction of initial impact when the action (A_i) is applied. The cost of executing an action in an unit time is modelled by functions $g_i(u_i) : \Re \rightarrow \Re$. f and g functions can be linked to an additional parameter to state the period time considered for the reduction.
In previous works (Zafra-Cabeza *et al.*, 2002) it was seen that the decision about a mitigation action is not usually a execute/don't-execute decision. The intensity of the action has to be taken into account when deciding how to execute the action. The impact and the cost of the action depend on the number of workers to be contracted, or the amount insured; that is, the decision will be taken depending on the value of the mitigation action control variable u_i. Thereby, $u_i \in \Re$ or $u_i \in \aleph$.

Therefore, the term $R(u,t)$ presented in equation (1) can be described as follows:

$$R(u,t) = \sum_{i=1}^{m} risk_t(i,t) RE_i(u,t) \qquad (2)$$

where the number of risks is denoted by m. Terms $RE_i(u,t)$ models the effect of the risk R_i at time t. This term is called "Risk Exposure" and it is defined as:

$$RE_i(u,t) = P_i(II_i - \sum_{j=1}^{p} f_j(u_j)) + \sum_{j=1}^{p} risk_a(i,j) g_j(u) \qquad (3)$$

where P_i is the probability of the risk R_i and II_i denotes the initial impact of the risk R_i affecting the cost. The sum of functions f means the reduction of the initial impact by executing actions. $risk_t(i,t) = 1$ indicates that the risk R_i could take place at time t according to the risk identification and otherwise $risk_t(i,t) = 0$. $risk_a(i,j) = 1$ if the risk R_i is mitigated by action j. $g_j(u_j)$ is the cost of the mitigation action A_j.

3. PREDICTIVE CONTROL APPROACH

The control objective is to maintain the total cost of the project according to a given reference in each step. The manipulated variables are the actions to undertake in order to reduce risk exposure.
Model predictive Control (MPC) is an optimal control strategy based on the explicit use of a dynamic model to predict the process output at future time instants (Camacho and Bordons, 2004). The future time interval considered in the optimization is called *prediction horizon* (N). The set of future control signals is calculated by optimizing a determined criterion or objective function that usually is quadratic. The predicted outputs depend on the known past inputs and outputs values up to instant t and on the future control signals. Only the control signal calculated for instant t is sent to the process whilst the next control signals are rejected. Therefore an optimization problem is solved at each time instant. Note that the receding horizon concept is applied.
The objective function uses to include the control effort and the error between the predicted output and the reference:

$$J_N = \sum_{j=1}^{N} \delta(j)[\hat{y}(t+j|t) - w(t+j)]^2 + \lambda(j) \sum_{j=1}^{N} [\triangle u(t+j-1)]^2 \qquad (4)$$

MPC disciplines are being widely accepted by the academic world and by industry (Richalet, 1993). Some advantages that MPC presents over other methods are the easiness to implement the control law, the extension to multivariable case or the addition of constraints in the optimization. However, note the main role that the model of the process takes place.
The previous statements have been decisive to select this control strategy for this work. Equations (1),(2) and (3) can be rearranged and the cost can be expressed as the following 1-output, n-input model:

$$A(z^{-1})y(t) = B(z^{-1})u(t-1) + d(t) \qquad (5)$$

where $A(z^{-1}) = 1 - (z^{-1})$, $B(z^{-1})$ is a $n \times 1$ polynomial vector and $d(t)$ is an offset term including the nominal cost of the project and the mean cost of impacts for the corresponding time period, not depending or control actions. $B(z^{-1})$ is time-varying and is calculated in each step; the model process changes at each time instant as consequence of the risk occurrence and their probabilities. In order to simplify the instant to execute the mitigation actions, the start day of the tasks has been selected. The sample time has been considered one day.

3.1 Constrainst description

MPC presents a great advantage over other methods in the treatment of constraints. This optimization problem is usually subject to constraints on the control u which can be expressed as:

$$R_u u(t) \leq u \leq r_u(t)$$

When the risk identification is carried out, the accurate value of the impacts can be unknown at that time. This work comprises the case where impacts can be modelled as deterministic or stochastic variables, according the available information about them. In the second case, II_j has been supposed as a stochastic variable with normal distribution ($II_j \sim N(\mu_j, \sigma_j^2)$).
Chance constrained optimization is a stochastic method that attempts to reconcile optimization over uncertain constraints. The constraints, which contains stochastic parameters, are guaranteed to be satisfied with a certain probability at the optimum found (Kall and Wallace, 1994). Chance constraints can be added to the optimization problem under the format:

$$Pr\{RE_{cj}(\xi, u) \leq K_{cj}\} \geq \alpha_{cj} \quad (6)$$

Taking into account that

$$Pr\{II_{cj}(\xi) \leq h(u)\} = F(h(u)) \quad (7)$$

with F the value of the cumulative distribution function of a standard normal distribution, it can be stated that $F(h(u)) \geq \alpha_{cj}$. Therefore, the equation (6) can be transformed to:

$$h(u) \geq F^{-1}(\alpha_{cj}) \quad (8)$$

The chance constraints are convex $\forall\alpha \in [0, 1]$(Kall and Wallace, 1994).

4. EXPERIMENTS

In order to illustrate the proposed technique, a true research project has been taken as an example: the AESOP [2] (Assessment of Energy Saving in Oil Pipelines) project. The main objective of the project was to study and develop techniques for the use of flow improvers or drag reducers (DRA) in pipelines to reduce the energy consumption and to increase the transport capabilities of oil pipeline networks. Table 1 describes the tasks of the project.

Initial costs assigned to the different tasks, without considering risks, are shown in Table 2 (these data are not true in order to keep the confidentiality of the project). After the initial scheduling of the project, risk assessment identifies the main risks associated to each task. The RBS that was identified is shown in Figure 1. Table 3 shows the risks that are going to be considered in this example and their consequence on the project.

[2] AESOP project (ENK6-CT2000-00096) is a research and technological development project partially supported by the Energy, Environment and Sustainable Development Programme of the European Union Fifth Framework Programme (http://www.esi2.us.es/aesop/)

Table 1. Tasks of AESOP project.

Task	Description
$WP1$	Kick-off meeting and starting of the project
$WP2$	Assessment of effect on fuel performance for high additive concentrations
$WP3$	Experimental Field Studies of flow improvers in Pipelines
$WP4$	Effectivity model development
$WP5$	Methodology for the use of long-chain polymers in Pipelines
$WP6$	Project management
$WP7$	End of the project. Last meeting

These risks have to be modelled, determining the probability of occurrence, the impacts and the set of mitigation actions that can be executed. The impacts in terms of cost and the proposed mitigation actions are described in Table 4.

Table 2. Task description

Task	Time	Start month	End Month	Cost
$WP1$	0.1	0	0	9
$WP2$	8	0	7	430
$WP3$	10	0	9	625
$WP4$	5	10	14	195
$WP5$	21	15	36	510
$WP6$	36	0	36	54
$WP7$	0.1	36	36	9

Table 3. Risk description

Risk	Description
R_1	Conclusive results about the impossibility of using DRA in pipelines because of the fuel performance
R_2	Difficulty in calibration of measurement equipment because of the DRAs
R_3	Not enough or lack of quality in the collected data
R_4	Adverse work conditions situation in the test pipeline
R_5	Overrun
R_6	Failure of a partner

The set of possible mitigation actions are $\{A_i\}$ with $i = 1...6$. Therefore, there are six control actions, $u_1, ..., u_6$. Variables u_2 and u_6 are boolean and the rest of them are real. The considered process is a first-order linear system without dead time and therefore $B(z^{-1}) = B_0(t)$. $B_0(t)$ and $d(t)$ are defined as follows

$$B_0(t) = \begin{bmatrix} (-P_1 f_1 + g_1) risk_t(1,t) \\ (-P_2 f_2 + g_2) risk_t(2,t) \\ (-P_2 f_3 - P_3 f_3 + g_3) max(risk_t(2,t), risk_t(3,t)) \\ (-P_3 f_4 + g_4) risk_t(4,t) \\ (-P_4 f_5 + g_5) risk_t(5,t) \\ (-P_5 f_6 + g_6) risk_t(6,t) \end{bmatrix}$$

$$d(t) = \sum_{i=1}^{nrisks} P_i II_i risk_t(i,t) + \sum_{i=1}^{ntasks} task_t(i,t)$$

$task_t(i,t) = 1$ if the task WP_i is being executed at time t. In other case, $task_t(i,t) = 0$.

Table 4. Mitigation actions description

Risk	Cost	Actions	Description	Cost Re.	Cost Function
R_1	175	A_1	Insurance contract 1 (Real)	$f_{11} = 100u_1$	$g_1 = u_1$
R_2	242	A_2	Auxiliary measurement equipment purchasing (Boolean)	$f_{12} = 225u_2$	$g_2 = 45.6u_2$
		A_3	Subcontracting the measurement in the injection points (Real)	$f_{13} = 6.5u_3$	$g_3 = u_3$
R_3	205	A_3		$f_{13} = 6.5u_3$	$g_3 = u_3$
		A_4	Preliminary and exhaustive analysis on experimental activities (Real)	$f_{14} = 8au_4$	$g_4 = u_4$
R_4	93	A_5	Insurance contract 2 (Real)	$f_{15} = 60u_5$	$g_5 = u_5$
R_5	0.1CT	A_6	Contract more qualified staff (Boolean)	$f_{16} = 76.5u_6$	$g_6 = 23.7u_6$
R_6	183	-			

The horizon, N has been established to 10 and the control effort, ($\lambda = 0.01$) to allow high changes in the control. The constraints that have been considered are the following:

$$\begin{aligned} u_i \geq 0 \quad \text{with} \quad i = 1,3,4,5 \\ f_2(u_2) + f_3(u_3) \leq max(II_2, II_3) \\ f_3(u_3) + f_4(u_4) \leq max(II_2, II_3) \\ f_5(u_5) \leq 4.9II_4 \\ f_6(u_6) \leq 7II_5 \\ f_1(u_1) \leq II_1 \\ \sum g_i \leq 0.2 * CT \end{aligned} \tag{9}$$

First constraint states that variable u_1, u_3, u_4 and u_5 are real but they can not be negative. The next constraints do not allow the reduction of the cost for the risks to be higher than an amount of the initial impacts. The additional cost of the mitigation actions can not be higher than the twenty percent of the total cost of the project (CT); it is stated in the last constraint. CT is the total cost of the project when risks are not considered.

The results of the optimization problem have been obtained using a solver developed in Matlab (Bemporad, 2002) that can be used for the mixed integer programming. These algorithms use branch and bound methods.

The reference trajectory $w(k)$ can be chosen to indicate the desired execution rate of the process and may be linked to the financial policy of the company for a particular project. In figure 2 the reference has been established to 0 in order to minimize the cost. The thin solid line ($ynom$) means the cost considering risks, but no mitigation actions. The cost for the non risk case is depicted in dashed-dotted line ($ynor$). The upper bold solid lines represent the proposed solutions under the constraints described in equation (9). If some of the initial constraints are removed, it is possible to observe how the cost decreases. It is stated in the lower bold solid line when constraint ($f_1(u_1) \leq II_1$) is removed. Figure 3 shows the control actions. They are always executed at the beginning of the actions. In fact, notice that $ynom$ is highly increased in the points where tasks begin. The risk probabilities have been taken randomly. Note how variables u_2, u_6 only take values in $\{0, 1\}$.

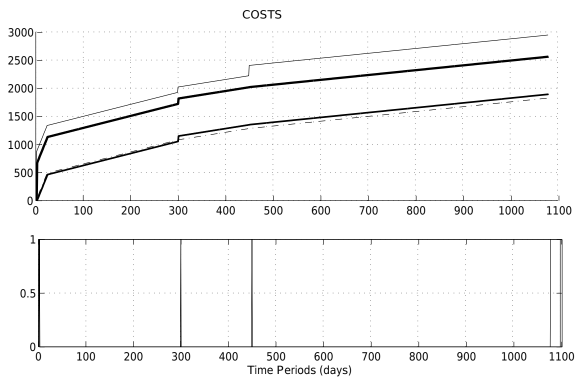

Fig. 2. Cost for the non risk case (dashed-dotted line), without mitigation actions (thin solid line) and the proposed solution (bold solid line).

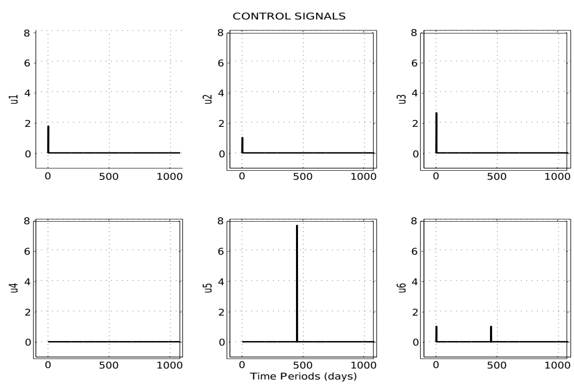

Fig. 3. Control actions

For the next experiment the initial impact of R_1 has been considered as a stochastic variable with normal distribution ($II_1 \sim N(175, 12)$). Hence, two new constraints have been added:

$$\begin{aligned} f_1(u_1) \leq 2II_1(\xi^k) \;\; k = 1, ..., ns \\ Pr\{RE_1(u, \xi^k) \leq 25\} \geq 0.99 \;\; k = 1, ..., ns \end{aligned} \tag{10}$$

ns denotes the number of samples ($ns = 10$). The first constraint states that the reduction for R_1 can not be higher than two times the initial impact $II_1(\xi^k)$. The second inequality is a chance constraint. It states that the *Risk Exposure* should be reduced to 25 with a probability equal to 99%.

Also, the reference has been changed taking into account a possible company policy. $w(k)$ is updated at each time instant, $w(k) = ynor(k) + 0.2 * yrisk(k)$, where $yrisk(k)$ is the additional cost that risks cause. The considered constraints for this experiment have been the first, second, third and forth of equation (9) and additionally, the constraints stated in equation (10). Figure 4 shows the results obtained. There are no important changes in the output; the dotted line is the reference and the bold line is the proposed solution. The main differences are reflected in the control signals. Figure 5 shows the control actions for the deterministic case, without constraints stated in eq. 10 (solid line) and for the stochastic case (dotted line). Note that in the stochastic case, besides all the control signals as in deterministic case, u_1 is maintained an interval with a value not equal to 0 to satisfy the stochastic constraints whilst in the deterministic case only one period time the action A_1 is executed. In the stochastic approach, not only a value of the impact is considered but a discrete distribution as consequence of the uncertainty modelling.

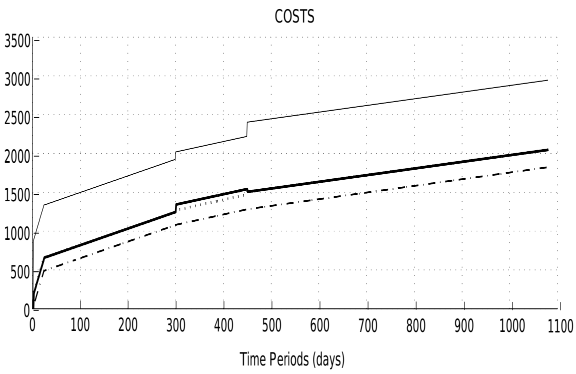

Fig. 4. Costs with probabilistic constraints

5. CONCLUSIONS

This paper describes a control policy that optimizes the cost of a project taking into account risk management. MPC has been the chosen control methodology due to the facility that presents in the treatment of constraints or the extension to multivariable case. The setting of the reference or the control effort can set policies imposed by the company. The introduction of uncertain variables modelled as stochastic variables has given rise to a stochastic optimization.

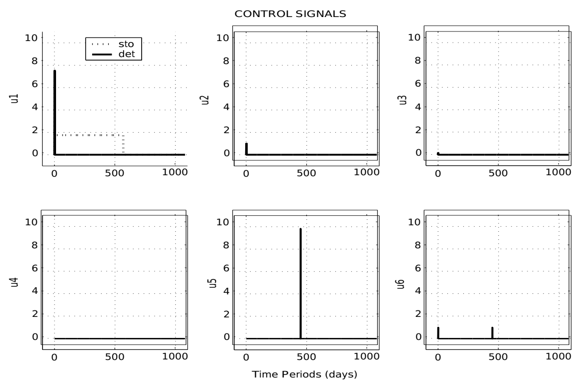

Fig. 5. Control actions for the stochastic case (dotted line) and deterministic case (solid line)

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