PREDICTION FOR CONTROL

Julio E. Normey-Rico *,[1] **Eduardo F. Camacho** **

* *Dpto. Automação e Sistemas, Univ. Fed. de Sta Catarina, Brasil*
** *Depto. Ing. Sist. y Automática, Univ. de Sevilla, España.*
E-mails: julio@cartuja.us.es, eduardo@cartuja.us.es

Abstract: This paper shows that "optimal" controllers based on "optimal" predictor structures are not "optimal" in their closed loop behaviour and that predictors should be designed taking into account closed-loop considerations. This is first illustrated with a first order plant with delay. The ISE index is computed for two typical optimal controllers (minimum variance controller and generalized predictive controller) when a stochastic disturbance is considered. The results are compared to those obtained by the use of a non optimal PI controller that uses a non optimal Smith predictor and performs better than the optimal controllers for the illustrative example. A general structure for predictors is proposed. In order to illustrate the results, some simulation examples are shown. *Copyright © 1998 IFAC*

Résumé: Ce papier montre que des lois de commandes "optimales" basées sur des structures prédictives "optimales" ne sont pas "optimales" dans leur comportement en boucle fermée et que la synthèse de prédicteurs devrait prendre en compte des considérations de boucle fermée. Cela est d'abord illustré avec un système du premier ordre à retard. l'index ISE est calculé pour deux lois de commandes optimales typiques (loi de commande à variance minimale et loi de commande prédictive généralisée), quand une perturbation stochastique est considérée. Les résultats sont comparés à ceux obtenus avec un régulateur PI non optimal basé sur un prédicteur de Smith non optimal et sont, pour l'exemple illustratif, meilleurs que ceux obtenus avec un régulateur optimal. Une structure générale de prédicteur est proposée. Pour illustrer les résultats, des exemples de simulations sont montrés.

Keywords: predictors, robustness, dead time systems.

1. INTRODUCTION

Predictor based control structures have been used in many control applications Smith (1958), Clarke et al. (1987), Camacho and Bordons (1995). The performance of the closed loop system can be improved by the use of a predictor structure in two main cases: (i) when the process has a significant dead time and (ii) when the future reference is known. In the first case the main objective of the predictor in the closed-loop system is to eliminate the effect of the dead time. In the second case the predictive controller allows the "anticipation" of the control action. In both cases the predictive strategy includes a model of the process in the structure of the controller.

[1] Work supported in part by CICYT-Spain Contract TAP and CAPES-BRASIL Contract BEXO448/95-6

The Smith predictor (SP) was the first structure of predictive control presented at the end of the 50's Smith (1958), and it was used to improve the performance of classical controllers (PI or PID controllers) for plants with time delay. Later, numerous extensions and modifications of the SP have been proposed: to use it with unstable plants Watanabe and Ito (1981), Matausek and Micic (1996), Furukawa and Shimemura (1983); to improve its disturbance rejection properties Palmor and Powers (1985), Palmor (1996); to extend it to the multivariable case Ogunnaike and Ray (1979), Bhaya and Desoer (1985); to study or improve the robustness Palmor and Halevi (1983), Normey-Rico et al. (1997). Optimal predictors (OP) Goodwin and Sin (1984) were introduced in the model based predictive controller (MBPC) context. While SPs are used to compensate pure dead time, OPs are usually employed to predict the future behaviour of the plant in a multi-step ahead receding horizon. OPs do not explicitly appear in the resulting MBPC structure, although it has been shown in Camacho (1993) that the MBPC structure is equivalent to an OP plus a primary controller. A further difference is that OPs are developed taking into account the stochastic nature of the disturbances and better behaviour should therefore be expected if OPs are used as a dead time compensator when the plant is affected by stochastic disturbances.

During the last decade several strategies based on OP have been proposed: GMV (General Minimum Variance) Clarke and Gawthrop (1979), GPC (generalized predictive controller) Clarke et al. (1987), EPSAC (Extended Prediction Self Adaptative Control) Keyser and Cuawenberghe (1985), EHAC (Extended Horizon Adaptive Control) Ydstie (1984). The use of the OP in these controllers is based on its optimal properties, that is, the OP can generate the "best" prediction of the output of the plant in an open loop configuration and considering deterministic and random disturbances. Several studies of the performance and robustness of these control strategies have been presented in recent years Clarke and Mothadi (1989), Robinson and Clarke (1991), Yoon and Clarke (1995), etc, but the influence of the predictor structure on robustness is only discussed in Normey-Rico and Camacho (1996).

In spite of the open loop definition of the predictors, the performance and robustness of the complete control structure should be analysed. Making a comparative analysis of these two problems Normey-Rico and Camacho (1996) showed that, for plants that can be modelled by a first or second order transfer function plus a delay, the Smith predictor (a classical non stochastic predictor) has similar closed loop performance to and better robustness than the optimal predictor in the presence of disturbances and parameter uncertainties.

This result is somehow equivalent to the one obtained in the LQG/LTR problem (or in the identification/adaptation problem Gevers (1991)) where, in general, the best closed loop robust behaviour cannot be obtained using an optimal state estimator (an optimal identifier) and the Kalman filter (the identifier) has to be detuned in order to increase robustness. In this case the Smith predictor allows better closed loop robust conditions although the open loop output prediction is not optimal.

Furthermore in the class of optimal controllers (GPC, minimum variance controllers, etc) optimization is made in two steps. First the prediction of the output of the plant is computed using an open loop model of the plant. Then using the obtained predictor structure the control law is computed by the optimization of a defined cost function. As will be shown in this paper, this type of procedure does not allow an optimal closed loop performance.

The paper is organized as follows. In section 2 we show that an optimal predictor plus an optimal controller do not give the best closed loop performance in the presence of noise. Section 3 presents a general structure for prediction and defines a method to design the controller in order to attempt some performance and robustness specifications. Some simulation results are shown in section 4 and finally the conclusions and perspectives of the work are presented in section 5.

2. OPTIMAL PREDICTORS: CLOSED-LOOP ANALYSIS

As has been mentioned several optimal controllers use the optimal predictors in their structure. GPC is a model based predictive controller based on an optimal prediction of the output of the plant when ARIMA disturbances are considered Clarke et al. (1987). The control sequence is computed in order minimize a multistage cost function of the form

$$J(N_1,N_2) = \sum_{j=N_1}^{N_2} \delta(j)[\hat{y}(t+j \mid t) - w(t+j)]^2$$

$$+ \sum_{j=1}^{N_2-d} \lambda(j)[\Delta u(t+j-1)]^2 \qquad (1)$$

where N_1 and N_2 are the minimum and maximum costing horizons, $\delta(j)$ and $\lambda(j)$ are weighting sequences, $w(t+j)$ is a future set-point or reference sequence and $\hat{y}(t+j \mid t)$ is the j-step ahead optimal prediction of the system output on data up to time t.

If the control weighting factor is set to zero and $\delta(j) = 1$ the minimization of the cost function should give a closed loop output with minimum $\sum_1^N [\hat{y}(t+j \mid t) - w(t+j)]^2$.

In the particular case when the horizon of prediction is set to one, the GPC is equivalent to the minimum variance control MVC Goodwin and Sin (1984).

However, to evaluate the performance of the closed-loop the real error between the reference and the output must be analysed. It is presumed that the prediction $\hat{y}(t+j \mid t)$ is computed using an optimal predictor which considers the stochastic properties of the perturbations.

One should expect that using this type of control the ISE of the error will be minimum, because the quadratic error is minimized at each sample.

However, it will be shown in the following counterexample that this is not true.

Counterexample:

The model of the plant is given by the discretization of a first order system:

$$P(z) = G(z^{-1})z^{-d} = \frac{bz^{-1}}{1 - az^{-1}}z^{-d}$$

where the nominal values of the parameters are: $d_n = 10, a_n = .9$ and $b_n = 0.1$. The GPC controller is computed first using $N = 1$, $\lambda = 0$ and $\delta = 1$.

The performance of this control system will be compared to a structure composed by a Smith predictor Smith (1958) and a PI controller given by:

$$C(z) = k_1 + \frac{k_2}{1 - z^{-1}} \quad (2)$$

where $k_1 = 9$ and $k_2 = 10$.

Several simulation tests were performed using these two controllers with an ARIMA model of the noise $n(t)$:

$$n(t) = \frac{T(z^{-1})}{D(z^{-1})} = \frac{1 + 0.7z^{-1}}{(1 + 0.4z^{-1})(1 - z^{-1})}$$

- case 1: considering $w(t) = 0$, no deterministic perturbations and an exact model of the plant and disturbances.
- case 2: considering a change in the set-point from 0 to 1 at $t = 0$, a 10% step perturbation at the output of the plant at $t = 60$ and with an exact model of the plant and disturbances.
- case 3: considering the same simulation conditions as in case 2 but using a real disturbance different from the model:

$$D_r(z^{-1}) = (1{+}0.3z^{-1})(1{-}z^{-1}) \quad T_r(z^{-1}) = 1{+}0.8z^{-1}$$

- case 4: considering the same simulation conditions as in case 2 but using a real plant with denominator $1 - 0.92z^{-1}$.

In all the cases the ISE index was computed and the following results were obtained:

- case 1: ISEgpc= 0.64 ISEsp= 0.94
- case 2: ISEgpc= 12.07 ISEsp= 11.87
- case 3: ISEgpc= 11.50 ISEsp= 11.23
- case 4: ISEgpc= 1667 ISEsp= 11.01

Note that in the ideal case where no changes in the set-point or load perturbations are considered the GPC performs better than the SP, but in all the other real cases, even when the model of the plant and disturbances are the same as the real plant and disturbances, the SP performs better than the GPC. From this example we can conclude that:

- the procedure used in the GPC (and also in other optimal controllers) which considers two optimal designs in separate steps (optimal predictor plus optimal controller) is not optimal.
- as the solution of the problem gives a minimum of the predicted quadratic error at each sample but not a minimum of the real ISE index, the use of an optimal predictor in the closed-loop configuration is not appropriate.
- the use of the correct polynomials in the optimal predictor does not guarantee the minimal ISE in the closed-loop.
- the performance of the GPC is less robust than the SP. Note that for case 4 where a small error in the pole of the plant is considered, the GPC becomes unstable while the SP has a good performance.

The behaviour of the closed loop system is shown in figure 1 for case 1, in figure 2 for case 2 and in figure 3 for case 4. Case 3 is not shown because the results are similar to case 2.

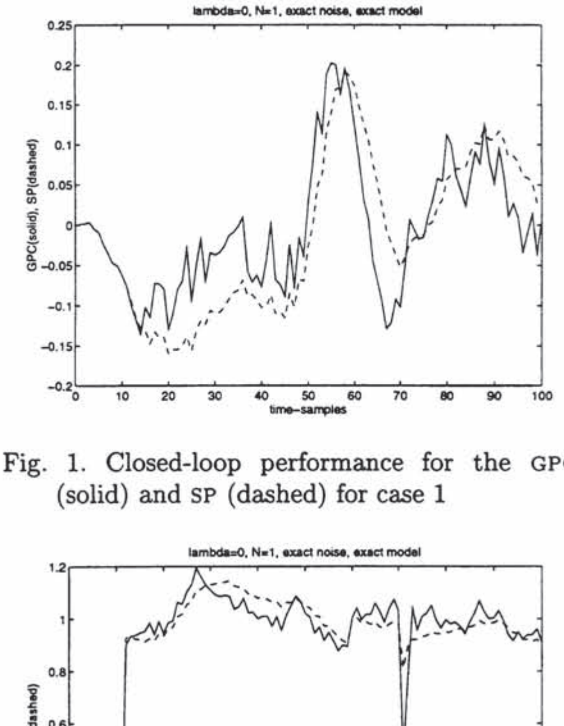


Fig. 1. Closed-loop performance for the GPC (solid) and SP (dashed) for case 1

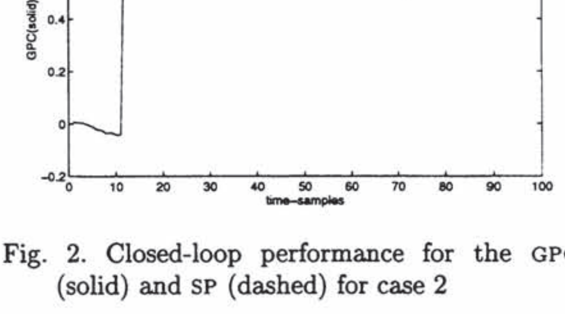


Fig. 2. Closed-loop performance for the GPC (solid) and SP (dashed) for case 2

The previous results are valid not only for the case of $\lambda = 0$ and $N = 1$. Using $N = 10$ and $\lambda = 1$ the behaviour of the GPC is compared to the SP using a PI with $k_p = 2.75$ and $k_i = 0.82$. The noise was generated with the same model as in the previous example. Again in this counterexample the error in the SP has lower ISE than in the GPC: When the model of the disturbances in the optimal predictor

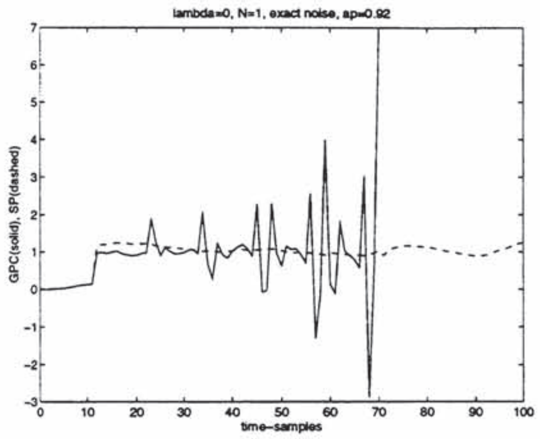


Fig. 3. Closed-loop performance for the GPC (solid) and SP (dashed) for case 4

is exact: ISEgpc= 13.32 and ISEsp= 13.28 and when plant model uncertainties are considered (error of 2 samples in the delay): ISEgpc= 29.07 and ISEsp= 20.91.

It is a well known result that the robustness of the GPC can be improved by the use of filters in the definition of the prediction-model Clarke and Mothadi (1989), Yoon and Clarke (1995), P.Ansay and Wertz (1997). In this approach, first the controller is computed using $T = 1$ and $D' = 1$ and then the filter T is modified to increase the robustness, that is, the polynomial T is not related to the characteristics of the noise, and so the prediction is not optimal. Also the choice of an appropriate filter to increase the robustness of the GPC results in complex controllers.

These counterexamples suggest that it is necessary to analyse the closed loop of the predictor based structures in a more general way. In this new approach the design of the predictor and the primary controller must be made in one step, considering the effects of the choice of the predictor structure and the control law on the closed loop system. The guide-lines of this new approach called "Prediction for Control" will be addressed in the next section.

3. DESIGNING PREDICTOR BASED CONTROL SYSTEMS

All the predictors proposed in the literature compute the prediction of the output of the plant using the actual and previous values of the output and input of the plant, giving a block diagram like the one shown in figure 4 Normey-Rico et al. (1996).

In figure 4 blocks R and Q represent the predictor structure and blocks C and W represent the control algorithm. In Normey-Rico et al. (1996) the expressions of R and Q are obtained for the Smith, analytical and optimal predictors.

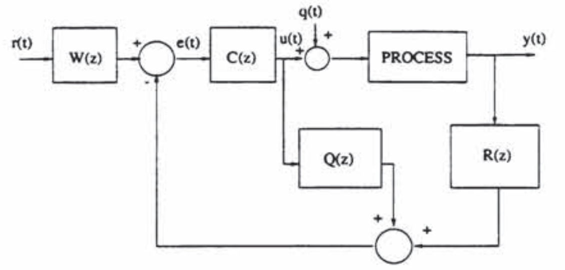


Fig. 4. General Predictor Based Control Structure

Using the block diagram of figure 4 the closed loop transfer functions from the reference to the output ($L(z)$) and from the disturbance to the output ($H(z)$) are given by:

$$L(z) = \frac{W(z)C(z)P(z)}{1 + C(z)(P(z)R(z) + Q(z))}$$

$$H(z) = \frac{P(z)(1 + C(z)Q(z))}{1 + C(z)(P(z)R(z) + Q(z))} \quad (3)$$

and the norm-bound uncertainty region for δP ($P = P_n + \delta P$) to maintain closed-loop stability is Morari and Zafiriou (1989):

$$\triangle P(j\omega) = \frac{\mid 1 + C(P_n R + Q) \mid}{\mid CR \mid}(j\omega) \quad \omega \in [0, \pi] \quad (4)$$

Thus, the following parametrization of the predictor structure is proposed: define two rational functions in z, R and X such that:

$$Q = X - P_n R \quad (5)$$

so the nominal closed loop transfer functions and the norm bound uncertainty region are given by:

$$L_n = \frac{CWP_n}{1 + CX} \quad H_n = \frac{P_n(1 + CQ)}{1 + CX} \quad \triangle P = \frac{\mid 1 + CX \mid}{\mid CR \mid} \quad (6)$$

In this paper we give the guide-lines for the design of C, W, R and X.

- to increase robustness R must have low pass characteristics (for instance $R(z) = (\frac{1-\beta}{1-\beta z^{-1}})^{\nu}$, $0 < \beta < 1$ and $\nu \geq 1$)
- to maintain internal stability X must be chosen with the unstable poles of P in order to obtain a stable Q. If the plant is stable it is possible to choose $Q = P_n(z^d - R)$. Using this expression for Q, R can be chosen independently from the nominal performance.
- C and W must be computed considering the closed loop performance as in a classical two-degree-of-freedom controller.

As can be seen this design procedure is not optimal but gives better results than the "optimal" controllers based on "optimal predictors" and "optimal primary control laws".

4. EXAMPLES

Two examples illustrate the advantages of the proposed control algorithm.

Example 1: Consider an oscilatory stable plant with a dead time:

$$P(s) = \frac{1}{1 + m_1 s + m_2 s^2} e^{-4s}$$

with a nominal pole $p = -2.5 + 0.7j$. The uncertainties are defined as 10% in the delay, 5% in the static gain and 10% in the poles of the plant.

The controller must be computed in order to obtain (for the nominal case) a set point step response with small overshoot and to maintain stability for all plants in the family. For this case we use $X = G_n$,

$$C = \frac{-9.407z^2 + 14.45z - 5.646}{(z-1)(z+.07)},$$

$$W = \frac{0.6}{-9.407z^2 + 14.45z - 5.646}$$

and the sampling time $T = 0.25$. To attempt the robustness conditions the filter R is chosen as $R(z) = (\frac{0.3}{z-0.7})$.

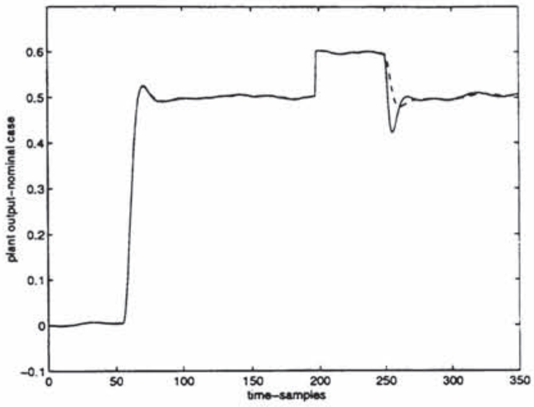
The closed loop behaviour of the closed loop system is compared to a GPC that is computed in order to obtain the same performance as the proposed controller (see figure 5.a). The closed loop behaviour when parameter uncertainties are considered (the static gain= 1.05, the pole $p = -2.3+0.55j$ and the delay= 4.3) is shown in figure 5.b. At $t = 0$ a 0.5 step change in the reference is performed and at $t = 200$ a 10% step disturbance is added at the output of the plant. The ISE index has been computed for both controllers in the nominal case obtaining: ISEgpc= 14.86 and ISEpc= 14.84, that is, they have the "same" nominal performance. The noise polynomial of the GPC is $T = 1$ and the perturbations are generated using the same polynomial. Note that a different T could be used in order to stabilize the time response of figure 5.b, but in this case the disturbance rejection will be slower than the one obtained with the proposed controller P.Ansay and Wertz (1997). That is, if both controllers are computed to obtain similar nominal performance, then the GPC is less robust than the proposed controller.

Example 2: Consider here $P(z) = \frac{z^{-2}}{1-1.1z^{-1}}$ and suppose that the time delay can vary between 1 and 2 samples. It is desirable to have zero steady state for step references and an overshoot of less than 5%. The obtained controller must guarantee robust stability. In this case we compute:

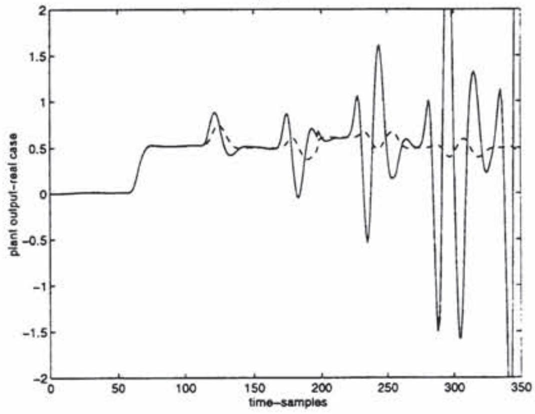
$$R(z) = \frac{0.2z}{z-0.8} \qquad C(z) = \frac{0.28(1-.9z^{-1})}{1-z^{-1}}$$

$$W = \frac{0.1}{1-0.9z^{-1}} \qquad Q(z) = \frac{-3.33(1-z^{-1})}{1-0.8z^{-1}}$$

. Using R, X and C the norm-bound uncertainty



(a)



(b)

Fig. 5. Behaviour of the GPC (dashed) and proposed (solid) for the nominal case (a) and with plant uncertainties (b)

is compared to the unmodelled dynamics in figure 6. As can be seen the controller is robust.

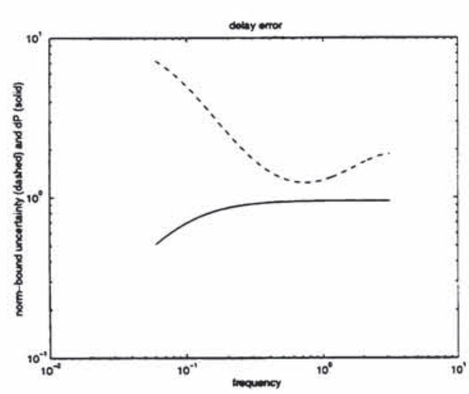


Fig. 6. Norm-bound uncertainty (dashed) and δP (solid)

The final control law is implemented using the block diagram of figure 4. The closed loop performance is analysed in figure 7. In order to make a comparative analysis with a predictive optimal controller a GPC is computed so as to obtain the same nominal performance as the proposed controller. The output of the plant and the control action are shown in figure 7.a for the nominal case and in 7.b for an error of 1 sample in the delay. At

$t = 0$ a step change in the reference is applied to the system and at $t = 50$ a 10% step disturbance is introduced. Note that the proposed controller preserves the stability but the GPC does not. As was analysed in P.Ansay and Wertz (1997) if a filter is used in order to stabilize the system based on the GPC the nominal disturbance rejection will be deteriorated.

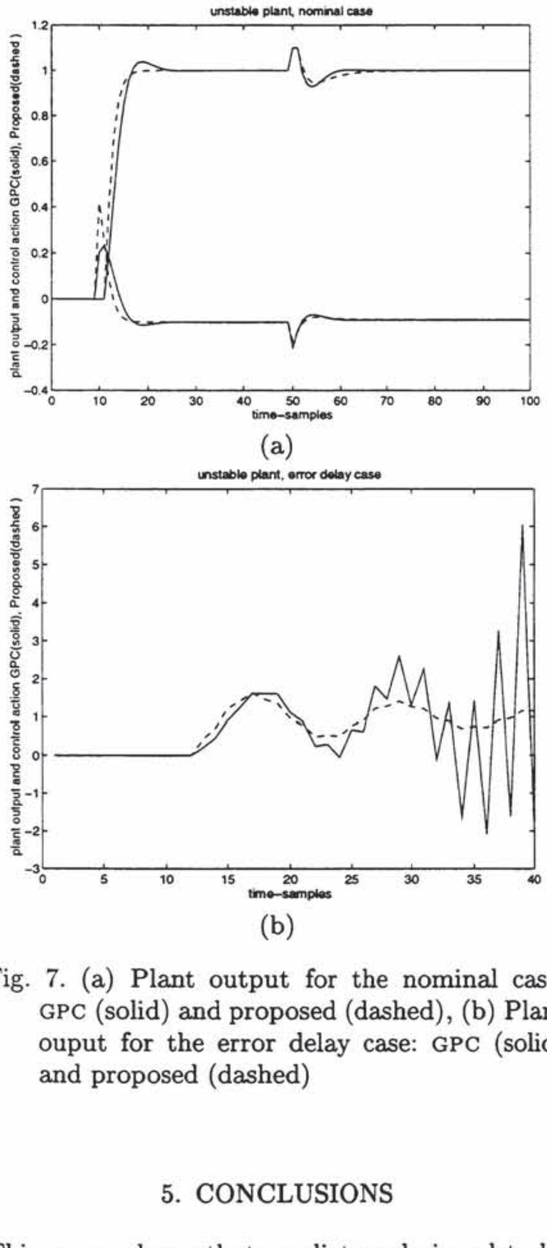


Fig. 7. (a) Plant output for the nominal case: GPC (solid) and proposed (dashed), (b) Plant ouput for the error delay case: GPC (solid) and proposed (dashed)

5. CONCLUSIONS

This paper shows that predictors designed to be optimal in an open loop configuration are not optimal when working in closed loop structures. It is shown that Smith predictor based control structures are most robust and produce similar nominal performance as the ones based on optimal predictors, even when working with the theoretical situations treated by optimal predictors (ARMA and ARIMA processes). This suggests that a new approach ("Prediction for Control"), that takes into account that predictors will work in a closed loop structure, has to be taken. The paper presents a procedure for designing this type of control strategy that considers nominal performance and robustness of the resulting closed loop.

References

A. Bhaya and C. A. Desoer. Controlling plants with delay. *Int. J. Control*, 41:813–830, 1985.

E.F. Camacho. Constrained generalized predictive control. *IEEE-TAC*, 38:327–332, 1993.

E.F. Camacho and C. Bordons. *Model Predictive Control in Process Industry*. Springer Verlag, 1995.

D.W. Clarke and P.J. Gawthrop. Self tuning control. *IEE-D*, 123:633–640, 1979.

D.W. Clarke and C. Mothadi. Properties of generalized predictive control. *Automatica*, 25(6):859–875, 1989.

D.W. Clarke, C. Mothadi, and P.S. Tuffs. Generalized Predictive Control. Part I The Basic Algorithm and Part II Extensions and Interpretations. *Automatica*, 23(2): 137–160, 1987.

T. Furukawa and E. Shimemura. Predictive control for systems with time delay. *Int. J. Control*, 37:399–412, 1983.

M. Gevers. Connecting identification and robust control: a new challenger. In *Plenary lecture, 9th IFAC Symp. on Identification and System Parameter Estimation*, pages 1-10, Budapest, 1991.

G. Goodwin and K. Sin. *Adaptive filtering prediction and control*. Prentice Hall, 1984.

R.M.C. De Keyser and A.R. Cuawenberghe. Extended prediction self adaptive control. In *IFAC Simp. on Ident. and Syst. Parameter Estimation*, pages 1317–1322, York, 1985.

M.R. Matausek and A.D. Micic. A modified smith predictor for controlling a process with a integrator and long dead-time. *IEEE TAC*, 41(8):1199–1203, 1996.

M. Morari and E. Zafiriou. *Robust Process Control*. Prentice Hall, 1989.

J.E. Normey-Rico, C. Bordons, and E.F. Camacho. Improving the robustness of dead-time compensating PI controllers. *Control Engineering Practice*, 5(6):801–810, 1997.

J.E. Normey-Rico and E.F. Camacho. A smith predictor based generalized predictive controller. *Technical Report, University of Seville*, GAR 02-96, 1996.

J.E. Normey-Rico, E.F. Camacho, and C. Bordons. Robustness analysis for generalized predictive controllers for industrial processes. In *Proceedings of Controlo 96*, Porto, 1996.

B.A. Ogunnaike and W.H. Ray. Multivariable controller design for linear systems having multiple time delays. *AIChE journal*, 25:1043–1060, 1979.

Z.J. Palmor. *The Control Handbook: Chapters 10.8-10.10*. CRC Press and IEEE Press, 1996.

Z.J. Palmor and Y. Halevi. On the design properties of multivariable dead time compensators. *Automatica*, 19: 255–264, 1983.

Z.J. Palmor and D.V. Powers. Improved dead time compensator controllers. *AIChE-Journal*, 31:215–221, 1985.

P.Ansay and V. Wertz. Model uncertainties in gpc: a systematic two-step design. *Proc. of the ECC 97, Bruxelles*, 1997.

T. Robinson and D. Clarke. Robustness effects of a prefilter in receding-horizon predictive control. *IEE-D*, 138:2–8, 1991.

M.J.O. Smith. *Feedback Control Systems*. Mc Graw Hill, 1958.

K. Watanabe and M. Ito. A process-model control for linear systems with delay. *IEEE TAC*, 26(6):1261–1269, 1981.

B.E. Ydstie. Extended horizon adaptive control. In *9th IFAC World Congress*, Budapest, Hungary, 1984.

T. Yoon and D. Clarke. Observer design in receding-horizon control. *Int. Journal of Control*, 2:171–191, 1995.