Pre-posterior analysis of inspections incorporating degradation of concrete structures

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1 Abstract

The framework of pre-posterior decision analysis has a large potential as a decision support tool in structural engineering. It seems ideally suited to tackle problems related to determining the value of Structural Health Monitoring and is commonly applied in inspection and maintenance planning. However, the application of this methodology for integrated life-cycle cost decision making related to monitoring of time-dependent and spatial degradation phenomena in concrete structures, needs further investigation. In this work, the time-dependent and spatial degradation phenomena will be coupled to the pre-posterior decision making approach and applied on concrete beams under bending, subjected to corrosion of the reinforcement. A framework is set up to determine the value of information of inspections enabling adequate decision-making. The methodology incorporates Bayesian updating based on the uncertain inspection outcomes. The framework will be illustrated by application on a simply supported reinforced concrete beam.

Keywords: Concrete, degradation, Bayesian updating, pre-posterior analysis, decision making, inspection

2 Introduction

Deterioration processes such as reinforcement corrosion are subjected to large uncertainties, reducing the reliability of reinforced concrete structures. Measuring the structure's condition state can reduce this uncertainty. Since inspections and maintenance represent a significant part of the total life cycle cost of a structure, pre-posterior analyses should be conducted to determine their cost-effectiveness, by quantification of the value of information (VoI) of an inspection strategy [1]. The fundamental decision whether to consider additional, however inherently uncertain and a priori unknown, information is expressed by this VoI, as given by equation (1) [2].

$$VoI = C_{prior} - \left[\int_{Y} f_{Y}(\boldsymbol{y}) \min_{a} \left[\sum_{i=1}^{m} c_{E_i}(a) \Pr(E_i | \boldsymbol{Y} = \boldsymbol{y}) \right] d\boldsymbol{y} \right] \quad (1)$$

Here, C_{prior} is the prior expected cost, $f_{Y}(\boldsymbol{y})$ the joint PDF of the monitoring outcomes $\boldsymbol{Y}$, $c_{Ei}(a)$ the expected cost of an action a and $Pr(E_i|\boldsymbol{Y}=\boldsymbol{y})$ the probability of an event or damage state E_i, given the inspection outcomes $\boldsymbol{Y}=\boldsymbol{y}$.

To account for the spatial distribution of corrosion, the framework provided by Schneider et al.[3] can be used. When inspection results are used to update the reliability of the structure, it is important to consider the spatial correlation of corrosion. Hence, measurements at one location also give indirect information on the corrosion state at another location. For this purpose, the structure is divided in different elements, with a length smaller than the correlation length of the relevant parameters. As such, the influencing factors within each element are strongly correlated and the deterioration state at time *t* is constant within the elements. These elements are grouped into zones, where one zone has common exposure conditions and material characteristics. Between different zones, there is no interdependence of the deterioration state. Within a zone, the statistical interdependence between the elements can be modelled by random hyperparameters. Realizations of the hyperparameters are identical for all elements in a zone. The method described by Schneider et al. [3] is very promising, however, it still has to be implemented in a pre-posterior decision making framework.

In this work, a framework is set up that allows evaluation of the VoI of inspections, accounting for the time-variant and spatial character of degradation. The result is a pre-posterior expected value of the lifetime costs, making it possible to determine in advance whether inspections will be worth their costs.

3 Set-up of the framework

The structure is spatially discretized and a deterioration model is assigned. At each time step, the failure probability and total costs are evaluated. The VoI is calculated at the final time step considered (e.g. the expected service life).

3.1 Time-dependent degradation

Because of corrosion, concrete structures lose resistance over time, among others due to a reduction in rebar diameter. The total expected reinforcement area at time *t* is given by equation (2) and (3) [4].

$$A(t-T_i) = A_0 - \alpha V_{corr}(t-T_i)\sqrt{n_r \pi A_0} + \frac{n_r\pi}{4}\alpha^2 V_{corr}^2 (t-T_i)^2 \tag{2}$$

$$A(t) = A(t-T_i)F_{T_i}(t) + A_0\left(1-F_{T_i}(t)\right) \tag{3}$$

Here, A_0 is the initial reinforcement area in the element considered and α is the pitting factor. V_{corr} is the corrosion rate, written as $V_{corr,a} \cdot ToW$, which equals the mean corrosion rate while corrosion is active, multiplied by the time of wetness; n_r is the number of bars corroding, T_i the initiation period and F_{Ti} the Cumulative Distribution Function (CDF) of T_i.

Equation (4) gives an expression for the initiation period T_i [5], [6].

$$T_i = \frac{1}{4D}\frac{c^2}{\left(erf^{-1}\left(1-\frac{C_{cr}}{C_s}\right)\right)^2} \tag{4}$$

Here, *D* is the diffusion coefficient of the concrete, C_{cr} the critical chloride concentration, C_s the concentration of chlorides at the surface and *c* the concrete cover. The distributions used are given in Table 1.

3.2 Spatial character

The parameters of the corrosion model are not only uncertain; they also are spatially correlated. To account for this spatial aspect, the structure is subdivided into different zones according to Schneider et al. [3], which are characterized by the same hyperparameters. In this work there is assumed that the mean values of C_s and *D* are the same for all elements in a zone, hence these are modelled as hyperparameters (Table 1). On the other hand, to model dependency as a function of the geometrical location, random fields might be more appropriate. One random field then overlaps with one zone.

As indicated in Table 1, a random field is assigned to the concrete cover to account for its spatial variation. The element length of the random field should be taken smaller than the correlation length l_c. The correlation lengths typically used for the concrete properties are 1 m, 2 m and 3.5 m [7]. In this work, a correlation length of 1 m is used. The correlation between the elements of the random field is given by equation (5), which represents an exponential correlation function, with x and x' two points in the random field.

$$\rho_X(x,x') = \exp\left(-\frac{||x-x'||}{l_c}\right) \quad (5)$$

Table 1. Distributions for corrosion parameters

Variable	Mean	Standard deviation	Distribution	Reference
α [-]	2	-	Det.	[6]
$V_{corr,a}$ [mm/yr.]	0.03	0.02	Weibull	[8]
ToW [-]	0.75	0.2	Normal	[8]
C_s [wt.-%/c]	μ_{Cs}	0.9	Lognormal	[1]
D [mm²/yr.]	μ_D	5	Lognormal	[1]
μ_{Cs} [wt.-%/c]	1.5	0.15	Lognormal	[1]
μ_D [mm²/yr.]	20	2	Lognormal	[1]
C_{cr} [wt.-%/c]	0.6	0.15	Lognormal	[5]
c [mm]	30	2.1	Gaussian Random Field	[9]

3.3 Bayesian updating

Since the inspection results are not known on beforehand, they are simulated with Latin Hypercube Sampling (LHS). Different possible test results are sampled corresponding to different branches in the decision tree.

The updating is performed using Markov Chain Monte Carlo (MCMC) simulations. The prior distributions are updated to posterior distributions, accounting for the available measurement information. Test results y_j can generally be written as $M(\boldsymbol{\vartheta})$ minus a measurement uncertainty. Here, M is a model representing the response to the input parameters $\boldsymbol{\vartheta}$. If N test results y_j are available, the likelihood function is given by equation (6).

$$L(\boldsymbol{\vartheta}|y) \sim \prod_{j=1}^{N} \frac{1}{\sigma} exp\left(-\frac{1}{2}\frac{\left(y_j - M(\boldsymbol{\vartheta})\right)^2}{\sigma^2}\right) \quad (6)$$

This equation is based on the assumptions of a Gaussian measurement uncertainty with standard deviation σ and independent measurements.

By applying MCMC, the parameters of the inspected element, including the hyperparameters, are updated. By updating the hyperparameters, information is also provided on the non-inspected elements. Furthermore, based on the MCMC, the mean values of the random field are altered for the inspected elements. Using this knowledge, the total updated random field is given by equations (7) and (8) [10].

$$\boldsymbol{\mu}_{F|Y} = \boldsymbol{\mu}_F + \boldsymbol{\Sigma}_F \boldsymbol{R}_Y^T \left(\boldsymbol{R}_Y \boldsymbol{\Sigma}_F \boldsymbol{R}_Y^T + \boldsymbol{\Sigma}_\epsilon\right)^{-1} (\boldsymbol{y} - \boldsymbol{R}_Y \boldsymbol{\mu}_F) \quad (7)$$

$$\boldsymbol{\Sigma}_{F|Y} = \boldsymbol{\Sigma}_F - \boldsymbol{\Sigma}_F \boldsymbol{R}_Y^T \left(\boldsymbol{R}_Y \boldsymbol{\Sigma}_F \boldsymbol{R}_Y^T + \boldsymbol{\Sigma}_\epsilon\right)^{-1} \boldsymbol{R}_Y \boldsymbol{\Sigma}_F^T \quad (8)$$

Here, $\boldsymbol{\mu}_F$ is the initial mean vector of the random field, $\boldsymbol{\Sigma}_F$ the initial covariance matrix and $\boldsymbol{\Sigma}_\epsilon$ the covariance matrix of the measurement uncertainties. $\boldsymbol{R}_y$ indicates the measurement scheme and contains rows of length equal to the number of elements with all zeros. When an element is inspected, the corresponding element in $\boldsymbol{R}_y$ is one instead of zero. For example, when the third and fifth element of a 1D random field consisting of eight elements are inspected, $\boldsymbol{R}_y$ looks as given by equation (9).

$$\boldsymbol{R}_Y = \begin{bmatrix} 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 \end{bmatrix} \quad (9)$$

3.4 Inspection strategies

Different types of measurements can be done. Two examples will be given below and discussed further for a simply supported beam.

3.4.1 Tests on concrete samples

Core samples from the concrete structure could be taken and, for example, compression tests could be executed on them, directly updating the strength of the concrete.

Chloride concentration could be measured in test samples as well, updating the distribution of the chlorides at the inspected location. The mean of the chloride concentration is modelled by a hyperparameter μ_{Cs} and is hence uncertain itself. When measuring the chloride concentration in an element, the distribution of C_s at that element is updated together with the distribution of the hyperparameter μ_{Cs}. This updated distribution of the hyperparameter μ_{Cs} will also be used to evaluate the deterioration state in the non-inspected elements.

3.4.2 Half-cell potential measurements or visual inspections

A visual inspection or half-cell potential measurement gives an indication on whether or not corrosion has initiated. This information updates the distribution of the initiation period T_i. When there is an indication that corrosion has initiated at some time before the inspection, the updated PDF of the initiation period is given by equation (10).

$$f''_{Ti,ind}(t) = \begin{cases} \dfrac{f'_{T_i}(t)}{F'_{T_i}(t_{insp})} & t < t_{insp} \\ 0 & t > t_{insp} \end{cases} \quad (10)$$

Here t_{insp} indicates the time at which the inspection is performed, ' marks the prior and " the posterior distributions.

When there is no indication that corrosion has started some time before the inspection, the updated PDF of the initiation period is given by equation (11).

$$f''_{Ti,no\ ind}(t) = \begin{cases} 0 & t < t_{insp} \\ \dfrac{f'_{T_i}(t)}{1 - F'_{T_i}(t_{insp})} & t > t_{insp} \end{cases} \quad (11)$$

Updating T_i will also give posterior distributions for μ_{Cs}, μ_D, C_{cr} and c. In the case of C_{cr}, this is done according to equation (12).

$$f(C_{cr}|(no)indication) = \sum_{T_i} f(C_{cr}|T_i) f''(T_i) \Delta T_i \quad (12)$$

A similar procedure is used for the other parameters.

3.5 Actions and cost model

Different actions can be considered, e.g. doing nothing, repairing a structure when a certain safety level or damage threshold is crossed, demolishing the structure and rebuilding it…. The prior expected cost is the minimum over the actions, as each action corresponds to a particular final lifetime cost. For a specific test outcome, the corresponding posterior cost is also the minimum over the possible actions.

The costs are calculated at the end of a predefined service life T_{SL}, including the costs of the actions (C_R at t_{rep}) and inspections (C_{insp} at t_{insp}), in addition to the expected failure cost. The total costs C_T at time t are calculated according to equation (13). The final cost at T_{SL} correspond to a summation of all inspections and all repairs executed before T_{SL} and of the failure costs over the whole service life.

$$\begin{aligned} C_T(t) &= C_T(t-1) + \frac{C_{insp}\left(1 - P_f(t_{insp})\right)}{(1+r)^{t_{insp}}} \delta_{t,t_{insp}} \\ &+ \frac{C_R\left(1 - P_f(t_{rep})\right)}{(1+r)^{t_{rep}}} \delta_{t,t_{rep}} \\ &+ C_F \Delta P_f(t) \frac{1 - P_f(t)}{(1+r)^t} \end{aligned} \quad (13)$$

$$with\ \Delta P_f(t) = \frac{P_f(t) - P_f(t-1)}{\Delta t\left(1 - P_f(t-1)\right)}$$

In equation (13), r is the interest rate, $\Delta P_f(t)$ the annual probability of failure in year t, given no failure before t and C_F is the expected cost associated to failure of the structure. The Kronecker deltas $\delta_{t,t_{insp}}$ and $\delta_{t,t_{rep}}$ equal one if $t = t_{insp}$ or $t = t_{rep}$ respectively, and zero for all other t.

To calculate the expected posterior cost, the inspection outcomes y_i and their corresponding probabilities $P(y_i)$ are considered by applying equation (14).

$$E[C_{posterior}] = \sum_{i=1}^{NoS} P(y_i) \min_a (C_{F,a,y_i}(T_{SL})) \quad (14)$$

Here, NoS is the number of sets of test outcomes considered.

Given the prior and posterior costs, equation (15) gives the expected VoI.

$$E[VoI] = C_{prior} - E[C_{posterior}] \quad (15)$$

When this is larger than zero, an inspection strategy is worth its cost.

4 Simply supported beam

As an example, a simply supported beam of length 4 m subjected to corrosion is considered. Uniform exposure is assumed for this beam, hence it represents one zone according to the framework of Schneider et al. [3]. The beam is subdivided in eight elements as visualized in Figure 1. Based on a convergence study for the probability of failure under bending, an element length of 0.5 m is chosen.

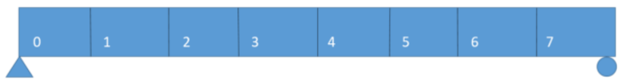

Figure 1. Simply supported beam subdivided in eight elements

The spatial aspect is modelled by the hyperparameters μ_{Cs} and μ_D and by the random field assigned to the concrete cover.

4.1 Time-dependent reliability

The limit state for bending is given by equation (16).

$$g = K_R A_s(t) f_y \left(h - c - 0{,}5 \frac{A_s(t) f_y}{b f_c} \right) - K_E(G + Q) \quad (16)$$

Here, K_R is the resistance model uncertainty, h the height of the beam, b the width of the beam, f_y the yield strength of the reinforcement, f_c the concrete strength, K_E the load model uncertainty and G and Q are the bending moments due to permanent and variable loads respectively. The distributions used are given in Table 2. The limit state is evaluated at each time step by application of a FORM analysis as explained underneath.

The beam is subdivided in elements and modelled as a vector-valued series system: it fails when one of its eight elements fails. It is assumed that the moments originate from a uniformly distributed load, with maximum moments given in Table 2.

Table 2. Distributions of variables in the limit state under bending

Variable	Mean	Standard deviation	Distribution	Reference
K_R [-]	1	0.05	Lognormal	[9]
f_y [MPa]	550	11	Normal	[9]
h [mm]	500	10	Normal	[9]
b [mm]	300	-	Det.	
f_c [MPa]	38.8	4.7	Lognormal	[9]
K_E [-]	1	0.10	Lognormal	[9]
G [kNm]	50	2.5	Normal	[9]
Q [kNm]	17.5	10.5	Gumbel	[9]
A_0 [mm²]	785.4	-	Det.	

The failure probability can be calculated according to equation (17).

$$P_{f,s}(t) = 1 - \Phi_m(\boldsymbol{\beta}(t), \boldsymbol{\rho}(t)) \quad (17)$$

Here, $\boldsymbol{\beta}(t)$ is the vector of element reliabilities at time t, Φ_m the multivariate normal CDF of degree m (here m= 8) and $\boldsymbol{\rho}(t)$ the correlation matrix, containing the correlations between the element reliabilities, calculated based on the sensitivity factors resulting from the FORM analysis according to equation (18).

$$\rho_{ij}(t) = \boldsymbol{\alpha}_i^T(t)\boldsymbol{\alpha}_j(t) \quad (18)$$

Since hyperparameters are involved, predicative reliability indexes will be used for the element reliabilities (equation (19)).

$$\tilde{\beta} \cong \frac{\mu_B}{\sqrt{1 + \sigma_B^2}} \quad (19)$$

Here, μ_B is the reliability index based on a FORM analysis with mean values of the hyperparameter. The variance of the reliability index is given by equation (20).

$$\sigma_B^2 \cong \sum_{i=1}^{l} \frac{\alpha_i^2}{\sigma_i^2} \sigma_{M_i}^2 \quad (20)$$

Here, σ_{Mi} is the standard deviation of the hyperparameter representing the mean of the variable i, α_i is the sensitivity factor of the variable i

in the FORM analysis performed to determine μ_B and σ_i is the standard deviation of variable i. For lognormal distributions, σ_i is replaced by σ_{lni}.

4.2 Actions and cost model

The actions in this example are either doing nothing or repairing the elements of which the reliability index drops below 3.8. It is considered that repair stops the degradation of the repaired element.

For the results presented in the following, the cost related parameters in Table 3 are used. Here, C_{insp} is subdivided into different components, where $C_{insp,i}$ is the initial cost for an inspection strategy, $C_{insp,el}$ is the extra cost per inspected element and $C_{indinsp}$ is the cost per measurement performed at an element. It should be pointed out that all these costs are relative costs.

Table 3. Cost parameters

Parameter	Value	Parameter	Value
C_F	1	$C_{indinsp}$	10^{-6}
$C_{insp,i}$	10^{-3}	C_R	10^{-2}
$C_{insp,el}$	10^{-4}	r	0.02

4.3 Updating based on inspections

4.3.1 Tests on samples

When a low chloride content (0.85 wt.-%/c) is measured at element 4 at t= 15 years, the updated distribution of μ_{Cs} is given in Figure 2. The mean of μ_{Cs} is shifted towards lower values and the standard deviation is lower. Hence, the uncertainty on this parameter is reduced. The standard deviation of C_s is only updated for the inspected element, as visible in Figure 3. When determining the final costs according to equation (14) different possible test results are sampled from the distribution of C_s and the corresponding final costs are weighed according to their probabilities.

The VoI depends on the time and location of the inspections. Hence, by varying these, the VoI can be optimized and the best inspection strategy can be chosen. For example, when measuring the chloride content at element 3 at 5 years, the VoI according to equation (15) is -0.0001949, corresponding to a posterior cost of 0.001073 and a prior cost of 0.0008781. On the other hand, when the measurement would be executed later (but at the same element), the posterior costs are only 0.0009307, corresponding to a VoI of -0.0000526. Hence, there might be an optimal time of inspection, leading to the lowest final costs and the largest VoI.

For this example, the VoI is negative and hence the inspections are not economic. It must however be pointed out that this outcome is case dependent.

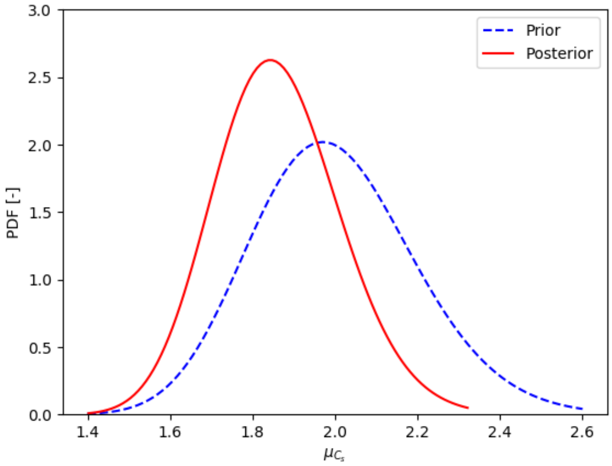

Figure 2. PDF of μ_{Cs} when measuring low chloride content (0.85 wt.-%/c) at element 4

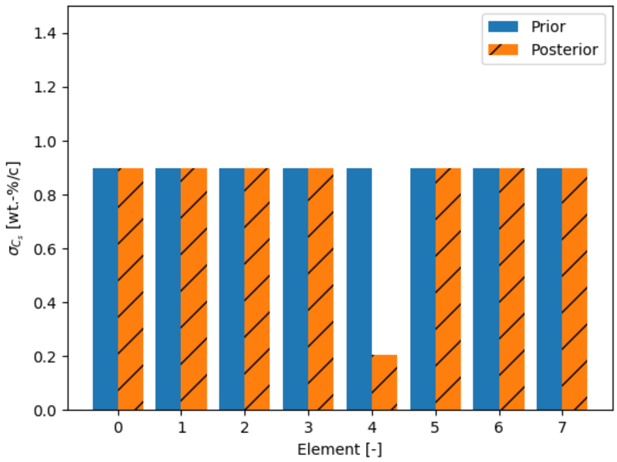

Figure 3. Prior and posterior standard deviation of C_s along the length of the beam (measuring low chloride content (0.85 wt.-%/c) at element 4)

4.3.2 Half-cell potential measurements or visual inspection

Half-cell potential measurements or visual inspections update the knowledge on the initiation period and the parameters on which it depends

according to equation (4). The updated distributions of μ_{Cs}, μ_D and C_{cr}, are visualized in Figure 4 to Figure 6 when inspection of element 2 reveals that corrosion has initiated at some time before t= 15 years. The updated mean and covariance matrix of the random field for c are shown in Figure 7 and Figure 9 respectively. Figure 8 shows the prior covariance matrix of c.
To determine the VoI, both cases of indication and no indication should be considered, together with their corresponding probabilities.

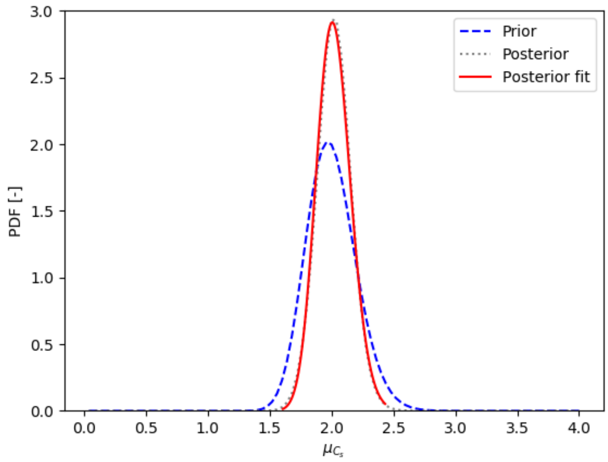

Figure 4. PDF of μ_{Cs} after indication of corrosion initiation at t= 15 years at element 2

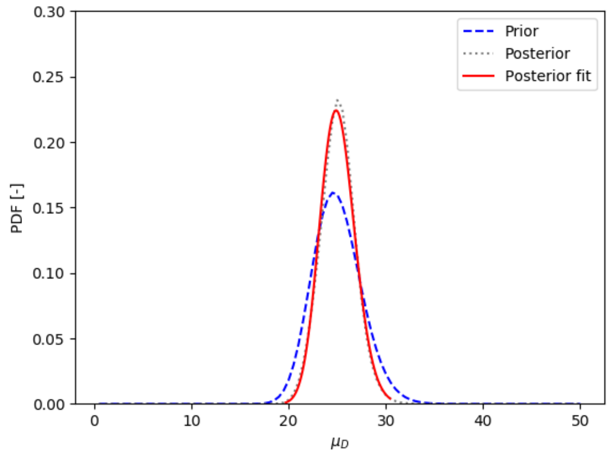

Figure 5. PDF of μ_D after indication of corrosion initiation at t= 15 years at element 2

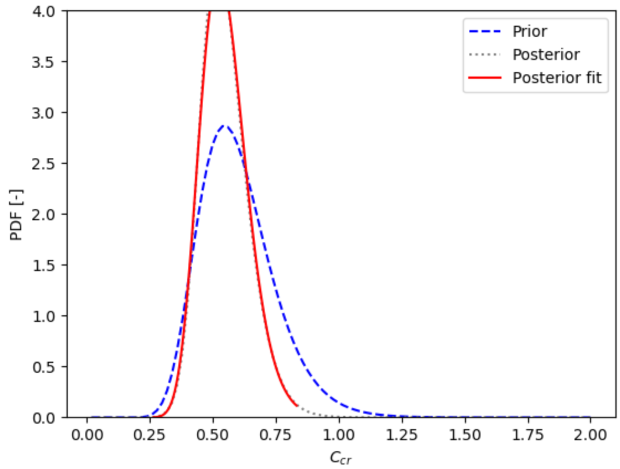

Figure 6. PDF of C_{cr} after indication of corrosion initiation at t= 15 years at element 2

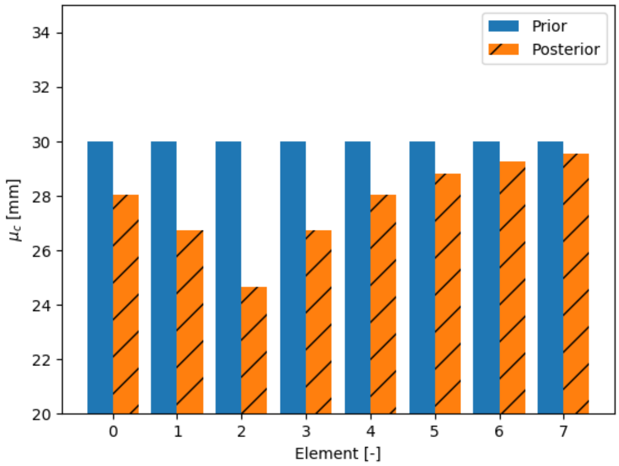

Figure 7. Mean of c after indication of corrosion initiation at t= 15 years at element 2

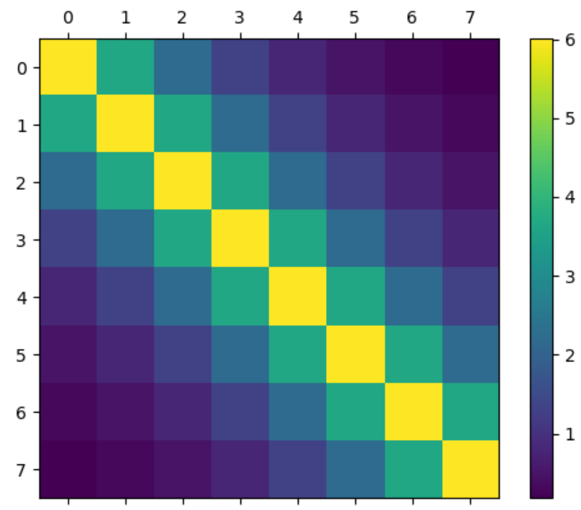

Figure 8. Prior covariance matrix of c

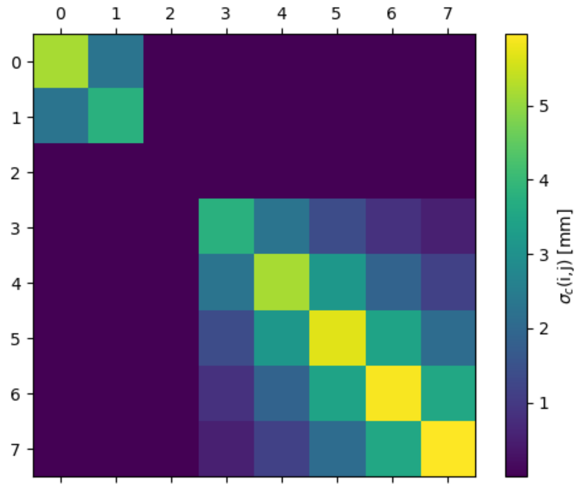

Figure 9. Covariance matrix of c after indication of corrosion initiation at t= 15 years at element 2

5 Discussion and conclusion

In this paper, a framework is set up for a pre-posterior analysis including the time-dependent and spatial behaviour of concrete degradation. The structure is subdivided into zones with common exposure conditions. These zones are subdivided in elements with uniform degradation. Since these elements are correlated, measurements of one element also provide information on other elements in that zone by updating the hyperparameters and the random fields. Due to corrosion, the resistance of a concrete element reduces in time. Hence, at each time step a reliability index and corresponding costs are calculated. Calculation of the VoI is based on the cost at an anticipated service lifetime.

The framework makes it possible to compare different inspection strategies and different repair criteria, accounting for the time and location of the measurements. As such, the maintenance of infrastructure can be economically optimized.

The influence of inspections on the distributions of hyperparameters and random fields is illustrated with two examples. Both types of measurements provide different information and hence induce the updating of other parameters. Whereas measuring the chloride content only influences uncertainty of the non-inspected elements by updating the hyperparameters, half-cell potential measurements also update the random field.

The location and time at which the inspections are executed influence the posterior distribution of the parameters and hence the probability of failure and the corresponding costs. As such, one inspection strategy might be more economical than another, as was shown by varying the inspection time for the chloride measurements.

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