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PRE-SERVICE TEACHERS' CONCEPTIONS OF MATHEMATICAL ARGUMENTATION

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Drawing on a situated perspective on learning, we analyzed written, open-ended journals of 52 pre-service teachers (PSTs) concurrently enrolled in mathematics and pedagogy with field experience courses for elementary education majors. Our study provides insights into PSTs' conceptualizations of mathematical argumentation in terms of its meanings. The data reveals how PSTs perceive teacher actions, teaching strategies, classroom expectations, mathematics content, and tasks that facilitate student engagement in mathematical argumentation. It also shows what instructional benefits of enacting mathematical argumentation in the elementary mathematics classroom they perceive.

Keywords: Teacher Education-Preservice, Teacher Beliefs, Reasoning and Proof

Background

For more than two decades, standards documents (e.g., National Council of Teachers of Mathematics, 2000; National Governors Association Center for Best Practices & Council of Chief State School Officers, 2010) continue placing a great deal of emphasis on the practice of mathematical argumentation in the mathematics classrooms across all grade levels. Previous studies have shown that by engaging in this practice, students develop their mathematical understanding and improve their mathematics achievement (e.g., Cross, 2009; Francisco, 2013). Mathematical argumentation is an important aspect of developing mathematically proficient students, but teachers often view curricular expectations about engaging students in mathematical argumentation as challenging. Graham and Lesseig (2018) noted that "teachers—both novice and experienced—have difficulty incorporating argumentation in the classroom" (p. 173).

Research-based understanding of elementary practicing and pre-service teachers' (PSTs') interpretations of mathematical argumentation is limited. The existing studies on mathematical argumentation have focused predominantly on teachers' perceptions of mathematical argumentation from the perspective of proof (e.g., Martin & Harel, 1989; Stylianides & Stylianides, 2009), teachers' classroom discourse practices in argumentation (e.g., Brown, 2017; Yackel, 2002), or teachers' evaluations of student arguments (e.g., Morris, 2007; Shinno, Yanaginomo, & Uno, 2017). Our work adds to this body of research. We provide a window into PSTs' conceptions of mathematical argumentation by answering the following research question: *How do PSTs conceptualize mathematical argumentation as a pedagogical practice in the context of elementary mathematics classrooms*? Our study builds a foundation for professional development efforts that aim to help PSTs meet the challenges of teaching elementary mathematics with a focus on mathematical argumentation.

Conceptual Framework

This research is grounded in a situated perspective on learning (Lave & Wenger, 1991; Peressini, Borko, Romagnano, Knuth, & Willis, 2004). Using the situated perspective to frame

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our work allowed us to explore PSTs' pedagogical conceptions of mathematical argumentation in elementary school mathematics as they positioned themselves as future teachers. Consistent with this perspective, we believe that PSTs build their views about mathematical argumentation by negotiating and renegotiating its meaning for themselves as they participate and reflect on their experiences with mathematical argumentation within and across different contexts. In our analysis then, we considered multiple contexts (i.e., mathematics and teacher preparation courses, field experiences) to include PSTs' experiences as both learners and apprentice-teachers.

Methods

Our study draws on data from a larger project conducted in a midwestern university in the United States. The overarching project was designed to explore K-8 PSTs' knowledge development about mathematical argumentation and proof in a teacher preparation program. The data were collected in two different semesters. Participants were two cohorts of PSTs (n = 52) concurrently enrolled in two courses for elementary education majors: a mathematics content course and a first of two mathematics-oriented pedagogy with field experience courses. Curricula of both courses were coordinated and addressed fundamental to elementary school mathematics topics and their teaching. For this paper, we purposefully selected PSTs' written responses to open-ended reflective journals which they completed throughout the semester (see Table 1), and in which they shared their views on mathematical argumentation. Using the qualitative content analysis and constant comparative methods (Glaser & Strauss, 1976; Mayring, 2014) we analyzed 380 responses in total (some participants did not consistently respond to all prompts).

Table 1: Journal Information

Timeline	Journal Prompts & Number of Responses
4th and 9th week of the semester	Thinking about yourself as an elementary school teacher define the term *mathematical argumentation*. How would you explain its meaning to a parent, for example? (J1, P1, 48 responses; J6, P1, 46 responses)
4th, 9th and 14th week of the semester	Describe the practices that characterize an elementary mathematics classroom in which a teacher engages students in mathematical argumentation. What practices could a visitor (e.g., a parent) see observing that teacher? How these practices can support students' argumentation skills. (J1, P2, 45 responses; J6, P2, 45 responses; J10, P2, 48 responses)
5th week of the semester	Are there any areas or topics of study in elementary mathematics that you view as more or less suitable for engaging students in mathematical argumentation? If so which one. Why? (J2, P1, 50 responses)
5th week of the semester	Describe characteristics of mathematical tasks that have high potential to engage students in mathematical argumentation. How are the tasks you described different from tasks that do not encourage mathematical argumentation? (J2, P2, 50 responses)
14th week of the semester	Describe how your experiences this semester influenced your ideas about teaching elementary mathematics with a focus on mathematical argumentation. (J10, P1, 48 responses)

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Results and Discussion

Table 2 gives a summary of outcome space that describes PSTs' pedagogical views on mathematical argumentation. Given the space limitation, we only discuss selected results.

Table 2: Results Summary

Major Category	Sub-Category	Number of PSTs
1. Notion of the term mathematical argumentation	Individual perspective	38
	Social perspective	10
2. Benefits of the use of mathematical argumentation in teaching mathematics	For students	26
	For teachers	22
3. Teacher actions that support argumentation	Teacher questioning	33
	Teacher encouragement	13
4. Teaching strategies that promote argumentation	Discussion	38
	Concrete manipulatives or visual representations	19
5. Mathematics content where argumentation can be implemented	Selective topics	30
	All topics	14
6. Tasks that can be used with argumentation	Call for justifications	30
	Open to multiple solution strategies	21
7. Classroom expectations	Student actions	31
	Classroom environment	13

Note. Some of the participants shared more than one view.

While defining mathematical argumentation, the vast majority of our PSTs discussed argumentation from the perspective of an *individual.* They focused on a person's ability to explain and justify the thinking and reasoning used to solve a problem. We illustrate this perspective with an excerpt from PST A38's journal: "Mathematical argumentation is the ability for a student to reach mathematical conclusions through logical reasoning" (J6, P1). A much less prevalent interpretation of mathematical argumentation stemmed from perceiving argumentation as a *social* activity. PSTs with the *social* perspective conveyed the view of mathematical argumentation as a process of communicating mathematical ideas to others to justify, convince, or to provide a challenge. We illustrate this view using PST A19's response:

> Mathematical argumentation is the process of explaining and justifying to others clearly how you got an answer to a particular mathematical problem or question…When questions from others arise, one must be able to answer those questions and must also be able to answer questions of others based on their work if they are unsure about how someone else goes about their answer. (J6, P1)

Across the analyzed journals, 36 PSTs discussed teacher actions which they viewed as essential for engaging students in mathematical argumentation. Most frequently, they attended to teacher *questioning* and teacher efforts of *encouraging* students to participate in argumentation. With a focus on teacher questioning, PSTs often shared that teachers who regularly ask the "*how*" and "*why*" questions engage students in mathematical argumentation by having them to explain and justify their thinking. PST A1's journal entry exemplifies this view:

> A classroom that fosters mathematical argumentation should contain a teacher who is constantly asking his/her students to explain how they reached their answers and why they think it [the answers] make sense… Teachers should be asking their students questions like "how did you come to this answer?," "why does that make sense?," and "how did you know where to start in this problem?" (J1, P2)

PSTs who focused on teacher encouragement discussed that teachers foster mathematical argumentation by prompting students to present their thinking, inviting students to critique each other's reasoning, or correcting misconceptions. PST A29, for instance, wrote:

> A teacher who fosters mathematical argumentation should…encourage students to share their methods of thinking through a problem with the class. [This] promote[s] mathematical argumentation within the classroom and will help students to build their skills in math by getting them to talk to one another and figure out what methods do and do not work for solving math problems. (J1, P2)

Summary Discussion and Conclusions

Our PSTs' largely individual-focused perceptions of mathematical argumentation was clearly visible when PSTs discussed teacher actions in support of argumentation. In their descriptions, only a few PSTs considered how a teacher might support *collective* efforts in which students jointly build on each other's ideas and collectively establish a mathematical claim. The vast majority of our PSTs concentrated on how teachers might encourage individual students to explain and justify their thinking for themselves, to other students, or to teachers. Even while discussing how a teacher might support a group of students, our PSTs painted pictures of individual students developing their own arguments drawing on ideas from others.

We hypothesize that PSTs' experiences with mathematical argumentation in their mathematics content and pedagogy courses could possibly contribute to their largely individual-focused views on mathematical argumentation. Even though in their mathematics content and pedagogy courses instructors frequently engaged PSTs in sharing, analyzing, critiquing, and building arguments collectively, the social aspects of argumentation or any instructional decisions in support of collective argumentation were not explicitly discussed. While mathematical argumentation was also a focal aspect of PSTs' field experiences, culminating activities in which PSTs engaged consisted of one-on-one interactions with students. It might be, then, that in their field experience classrooms most PSTs saw mathematical argumentation from the perspective of individual students focusing on each student's ability to generate arguments.

Drawing on past research which established the relationship between teachers' beliefs and their instructional practices (e.g., Stipek, Givvin, Salmon, & MacGyvers, 2001; Thompson, 1984), it appears reasonable to expect that PSTs with mostly individual-focused views of mathematical argumentation might less likely use argumentation as a pedagogical tool for constructing meaning of mathematics collectively. That is, they might not routinely consider engaging their students in collective examination of assertions and provide them with opportunities to build on and critique each other's ideas. To help our PSTs develop a richer perspective on mathematical argumentation we are now more explicitly draw PSTs' attention to both individual and social aspects of mathematical argumentation in both courses (i.e., mathematics content and pedagogy). Research needs to further examine PSTs' views on mathematical argumentation in relationship to their experiences with mathematical

argumentation to provide directions for learning activities that can help PSTs develop richer perspectives on mathematical argumentation.

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