

DYNAMIC MODELLING OF THE BEHAVIOUR OF THE OUARKISS EARTHEN DAM UNDER SEISMIC LOADS

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ABSTRACT

The dynamic modelling of the behaviour of the Ouarkiss earthen dam under seismic loads was performed using the finite elements method (FEM), with an approach in effective stresses. The soil behaviour is described by the Mohr-Coulomb criterion. A numerical method and a procedure of analysis are presented in this work. The seismic response of an earthen dam was evaluated. Particular emphasis is placed on the calculation of stresses, displacements, deformations and interstitial overpressures recorded during the seismic solicitation. It has been shown that numerical simulation is able to highlight the fundamental aspects of the displacements and deformations processes experienced by the structure of dam and to produce preliminary results for the evaluation of the seismic behaviour of the structure taking into account the physical non-linearity of the materials constituting the body of the dam and the effect of the rigidity of the different zones of the dam and the foundation.

Keywords: Displacement; Earthen dam; Mohr-Coulomb criterion; Seismic analysis, Stress

1 INTRODUCTION

The study presented here revolves around the stability of earthen dams under large-scale seismic load. Indeed, it is necessary to carry out this study in order to assess the probable seismic impact on these dams which could lead to a total rupture of these structures. Particular attention will be paid to the use of a finite element calculation model of displacements, deformations induced by the earthquake that affected the site.

The emergence of the finite element method as well as increasingly powerful and fast computers have allowed the development of increasingly elaborate calculation codes. However, the effectiveness of these codes is largely related to the validity of the law of behaviour used. In recent years many models have been proposed in the literature and the user can sometimes be confused by this multiplicity. This diversity is explained by the fact that, after dreaming of the universal model that gives answers to everything, specialists seem to agree on the definition of large classes of models, each one being well adapted to the resolution of certain types of problems. It is therefore essential to define clearly the field of validity of each model and the degree of complexity of its identification.

Earthquakes always present hazards to dams that engineers must take in consideration during the safety analysis of these structures [1]. Nowadays, there are no zones that can be considered as non-seismic. Water resources security is directly linked to the mobilization of these resources; this often leads us to build dams in areas with high seismicity, hence the need for thorough seismic analysis to prevent the eventuality of dam failure during earthquakes.

The seismic computation of dams [2] has always been carried out using pseudo-static methods with horizontal accelerations of the ground equal to 0.1g. Whereas nowadays, the earthquakes [3] induce much higher accelerations than this value and in both directions, horizontal and vertical and that the methods to be used must effectively reproduce the real behaviour of the structures, in order to make a good decision on their seismic safety.

A considerable development has been recorded in the field of dynamic analysis in the case of linear elasticity with the introduction of the computer tool, but in the non-linear field the research have undergone a net evolution through in particular, the improvement of constitutive laws and the improvement of numerical methods (finite elements or finite difference) to implement the tools and models enabling the engineer to better describe seismic behaviour and propose suitable solutions.

In this study, the seismic behaviour of the Ouarkiss earthen dam will be analysed taking into account the physical non-linearity of the materials constituting the body of the dam and the effect of the rigidity of the different zones of the dam and of the foundation [4, 5].

2 DYNAMIC MODELLING OF THE BEHAVIOUR OF THE EARTHEN DAM

The objective of this study is to improve the understanding of dynamic phenomena in earthen dams, particularly amplification, resonance, etc. and to evaluate the constitutive and numerical formulations best suited to dynamic analysis by putting them on seismic data obtained on the Ouarkiss dam.

The many models proposed for this purpose are the models adapted each to the resolution of non-linear problems, complex posed nowadays to the engineers. The models therefore, commonly used for the seismic stability analysis of earth dams, range from the simplest equilibrium analysis to highly sophisticated numerical modelling techniques. They include:

- Pseudo Static Analysis, Newark's Approach, Seed-Lee-Idriss Analysis, etc.
- Numerical modelling techniques (total stress and effective stress).

As well as the soil behaviour model of Mohr-Coulomb used in this case. This model takes into account the different phases of the life of these works, from their construction, until they are filled and their resistance to a major seismic load. The numerical finite element method [6], with an effective stress approach is therefore used in this case.

Dynamic Soil-Structure Interaction

Ground-structure interaction refers to movement modification of the ground (or of the structure) during an earthquake due to the presence of the other component (structure or soil). This interaction is of course more or less important depending on the nature of the ground, the characteristics of the structure and its foundation mode. For some structures, superficially founded, the need to study the seismic response of a structure, not considering it in isolation but as an integral part of an ensemble comprising the ground and surrounding structures, makes ground-structure interaction analyses imperative for embankment dams. It is useful to formulate in a general way the problem of soil-structure interaction. This formulation is oriented towards a finite element treatment of the problem such that the use of numerical methods is almost inevitable. The motion equations are obtained by reference to Figure 1 which schematizes a floor-structure assembly.

In general, the equation of motion is written

$$[[M]]\{\ddot{U}\} + [[C]]\{\dot{U}\} + [[K]]\{U\} = \{Q_f\} \quad (1)$$

The total displacement of the structure is defined by $\{U\} = \{U_f\} + \{U_i\}$

The problem is broken down into two sub-problems:

Problem of soil response in free field

$$[M_f]\{\ddot{U}_f\} + [C_f]\{\dot{U}_f\} + [K_f]\{U_f\} = \{Q_f\} \quad (2)$$

Source problem

$$[M_i]\{\ddot{U}_i\} + [C_i]\{\dot{U}_i\} + [K_i]\{U_i\} = \{Q_i\} \quad (3)$$

Such as

$$\{Q_i\} = ([M] - [M_f])\{\ddot{U}_i\} + ([C] - [C_f])\{\dot{U}_i\} + ([K] - [K_f])\{U_i\} \quad (4)$$

From this last equation, we conclude that there is interaction as soon as there is a difference in mass or stiffness between the ground and the structure.

Figure 1 illustrates linear and non-linear approaches to soil-structure interaction. Figure 1a corresponds to the global methods whose solution is obtained by direct resolution of equation (1). They do not involve any notion of superimposition and are therefore theoretically adapted to non-linear problems. Alternatively, sub-structure methods rely on the decomposition of Figures 1b and 1c to solve the problem in stages. These methods are, of course, applicable only to linear problems, justifiable for superimposition.

To take into account the effect of the soil-structure interaction on the seismic response of structures in general and dams in particular, several methods have been developed among which impedance function, global method, sub-method structure and hybrid method.

In theory, global methods are likely to understand non-linear behaviours due to the law of behaviour of materials or to the soil-structure interfaces. These are the most direct calculation time. Soil and structure are modelled by finite elements. We use the rock accelerogram directly and we get the free field accelerogram, and the accelerogram at the base of the structure, as well as the complete response of the soil deposit and the structure. The problem is to be solved often using more complex methods, especially because they will be defined by the equation (1).

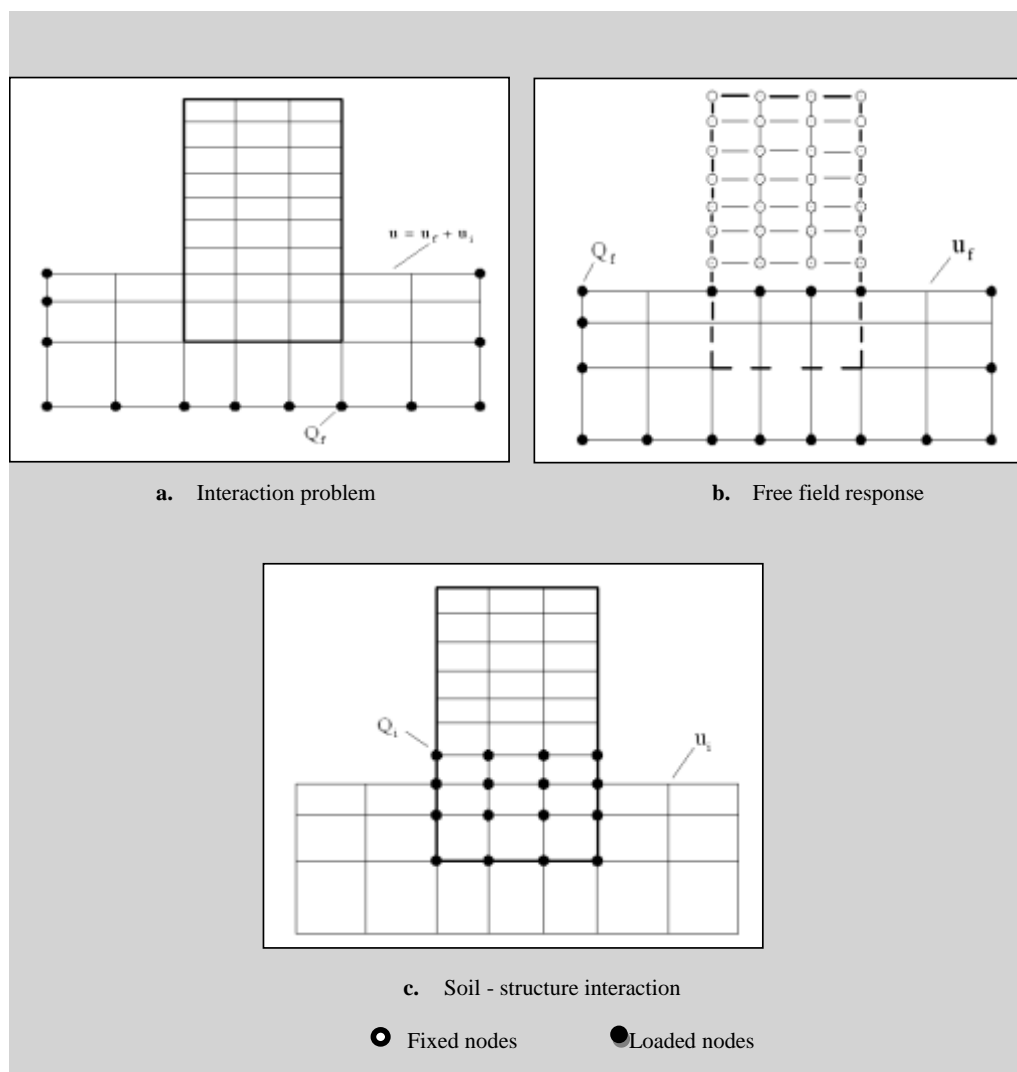


Figure 1. Superposition theorem for soil-structure interaction

Fluid-structure interaction

Seismic response analysis of an earthen dam requires the consideration of fluid-structure interaction effects [7, 8]. These effects can introduce substantial changes in the modal characteristics of the dam, such as frequencies and proper modes of vibrations. The problem of determining hydrodynamic pressure in an earthquake is an essential part of ensuring the seismic stability of dams, and the first approach to this problem was addressed by Westergaard [9], who calculated the repartition of pressures on a vertical screen limiting a semi-infinite reservoir of constant depth under the assumption of a harmonic horizontal movement of the screen of a period T .

The problem of water compressibility was taken into consideration by the formulation of Westergaard [9], and from these results, he noticed that for a face height limited to 100 m, the pressure increase does not exceed 5%, so compressibility can be neglected.

The effect of the non-compressible fluid can simply be taken into account by adding an appropriate mass to each dam-fluid contact node of the upstream face, thus depending on the structure. The exact solution given by Westergaard is expressed as a series development of sinusoidal functions:

$$C_n = \sqrt{1 - \frac{16\gamma_w H_1^2}{n^2 g K T^2}} = \left[1 - \frac{1}{n^2} \left(\frac{T_0}{T}\right)^2\right]^{1/2} \text{ et } T_0 = \frac{4H}{c} \quad (5)$$

$$P(Z) = \frac{8\alpha H}{\pi^2} \gamma_w \sum_{1,3,5} \frac{1}{n^2 C_n} \sin\left(\frac{n\pi}{2H} Z\right) \quad (6)$$

Westergaard's distribution

$$P(Z) = \frac{7}{8} \alpha \gamma_w \sqrt{HZ} \quad (7)$$

The system to be studied consists of a core-type earthen dam, supported by a horizontal surface of a flat elastic medium, the latter being limited in depth by the horizontal rocky base. The acceleration is applied to the base of the foundation.

The dam is subdivided into two sub-structures. The dam is composed of a core, a downstream and upstream facing, represented by a mesh of triangular finite elements with six nodes which have a better accuracy. This offers the advantage of determining the dam dynamic response, with a very good accuracy on the one hand, and significantly reduces machine time on the other hand, especially in non-linear dynamic analysis. The material constituting the dam is considered homogeneous, elastic and isotropic in the case of linear analysis, and in the case of non-linear analysis is considered elastic perfectly plastic. The foundation, limited by a rigid horizontal surface, was also modelled in finite elements of the same type as the dam. For the action of the fluid is taken into account by using the Westergaard theory.

In this work, the dam is treated as a two-dimensional system in which we consider a planar behaviour, the dam and the foundation ground are considered as two sub-structures of the combined system, represented by a mesh of two-dimensional finite elements, each free node of the finite element considered has two degrees of freedom (u_x, u_y). To simplify the treatment of lateral end nodes, in the case of an earthquake of horizontal direction, we assume that these points are free to move in the horizontal direction, but not in the vertical direction.

3 DYNAMIC ANALYSIS

General formulation of motion equations by the finite element method

A deformable solid is in equilibrium if the virtual work of the external forces is less than the virtual work of the internal forces, the expression of the principle of virtual works is obtained by applying the variational (integral) formulation to the equations of motion using the finite elements method.

$$- \int_{\Omega} \delta \varepsilon^t \sigma d\Omega + \int_{\Gamma} \delta U(-\rho) \vec{n}^t d\Gamma + \int_{\Omega} \delta U \rho r \ddot{u}_g d\Omega - \int_{\Omega} \delta U \rho \ddot{u} d\Omega = 0 \quad (8)$$

Hence the fact that

$-\int_{\Omega} \delta \varepsilon^t \sigma d\Omega$	Internal virtual work
$\int_{\Gamma} \delta U(-\rho) \vec{n}^t d\Gamma$	Virtual work of surface forces
$\int_{\Omega} \delta U \rho r \ddot{u}_g d\Omega$	Virtual work of volume forces (seismic loading)
$\int_{\Omega} \delta U \rho \ddot{u} d\Omega$	Virtual work of inertial forces

Dynamic characteristics of the system

The stiffness matrices for the two sub-structures, dam and foundation, are given by

$$[K_f] = \begin{bmatrix} 0 & 0 & 0 \\ 0 & K_{ii} & K_{if} \\ 0 & K_{fi} & K_{ff} \end{bmatrix} \quad [K_b] = \begin{bmatrix} K_{bb} & K_{bi} & 0 \\ K_{ib} & K_{ii} & 0 \\ 0 & 0 & 0 \end{bmatrix} \quad (9)$$

$$[K] = \begin{bmatrix} K_{bb} & K_{bi} & 0 & 0 \\ K_{ib} & K_{ii} & 0 & 0 \\ 0 & 0 & K_{ii} & K_{if} \\ 0 & 0 & K_{fi} & K_{ff} \end{bmatrix} \quad (10)$$

And the mass matrices

$$[M_f] = \begin{bmatrix} 0 & 0 & 0 \\ 0 & m_{ii} & m_{if} \\ 0 & m_{fi} & m_{ff} \end{bmatrix} \quad (11)$$

$$[M_b] = \begin{bmatrix} m_{bb} & m_{bi} & 0 \\ m_{ib} & m_{ii} & 0 \\ 0 & 0 & 0 \end{bmatrix} \quad (12)$$

$$[M] = [M_f] + [M_b] \quad (13)$$

$$[M] = \begin{bmatrix} Km_{bb} & m_{bi} & 0 & 0 \\ Km_{ib} & m_{ii} & 0 & 0 \\ 0 & 0 & m_{ii} & m_{if} \\ 0 & 0 & m_{fi} & m_{ff} \end{bmatrix} \quad (14)$$

The damping matrices for the two sub-structures dam-foundation are calculated by the relationship

$$\text{Foundation}[C_f] = \alpha_{Rf}[M_f] + \beta_{Rf}[K_f] \quad (15)$$

α_R, β_R are coefficients satisfying the condition of orthogonality of C in the modal base. They can be determined from the modal analysis of the structure by taking two proper pulsations of two distinct modes.

$$[C_f] = \begin{bmatrix} 0 & 0 & 0 \\ 0 & C_{ii} & C_{if} \\ 0 & C_{fi} & C_{ff} \end{bmatrix} \quad (16)$$

$$\text{Dam} \quad [C_b] = \alpha_{Rb}[M_b] + \beta_{Rb}[K_b] \quad (17)$$

$$[C_b] = \begin{bmatrix} C_{bb} & C_{bi} & 0 \\ C_{ib} & C_{ii} & 0 \\ 0 & 0 & 0 \end{bmatrix} \quad (18)$$

Then the overall damping matrix is obtained by assembling the damping matrices corresponding to each sub-structure

$$[C] = \begin{bmatrix} C_{bb} & C_{bi} & 0 & 0 \\ C_{ib} & C_{ii} & 0 & 0 \\ 0 & 0 & C_{ii} & C_{if} \\ 0 & 0 & C_{fi} & C_{ff} \end{bmatrix} \quad (19)$$

The most appropriate or effective method of resolution depends on the behaviour of the structure to be studied (linear or non-linear) and the way in which the applied load is defined [10]. It should be realized that in most practical applications, obtaining an analytical solution is inaccessible and the use of numerical methods is essential.

The calculations were carried out by the PLAXIS 8.2 software, which uses the Mohr-Coulomb elastic-plastic behaviour law, which is one of the most widely used models. Indeed, it requires few input parameters and allows to easily model the behaviour of soil under various loading conditions. Its behaviour can be described by two linear phases, the first representing the elasticity of the material, which is based on Hooke's law in one dimension linking stresses to deformations, and the second represented by a constant value line, modelling the plastic behaviour of the material after it has passed its breaking point [11].

Behavioural laws in PLAXIS

Materials whose behaviour is elastoplastic are characterised by the appearance of elastic reversible deformations and irreversible plastic deformations. On the loading surface, two behaviour cases are possible: in the case of the Mohr-Coulomb model, the loading surface does not evolve, we then speak of a perfectly plastic elastic law; when the loading surface evolves during the loading, we are talking about an elastoplastic model with work hardening.

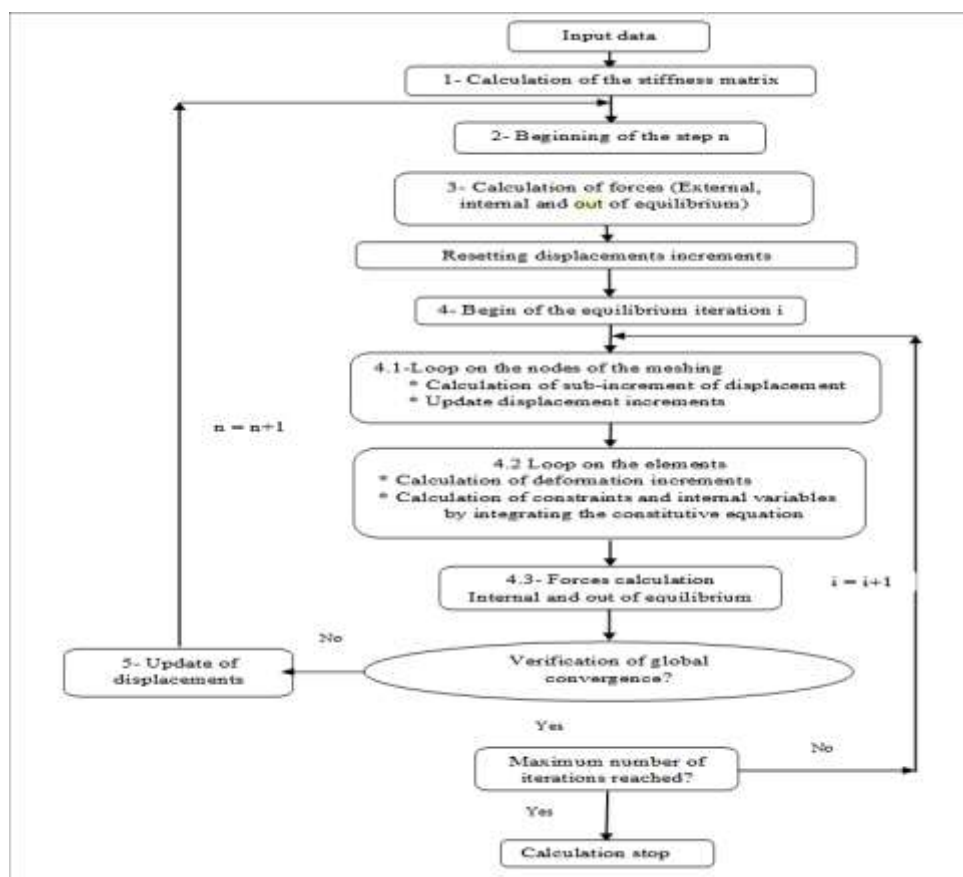


Figure 2. General Resolution algorithm of the Plaxis code

4 MODELLING THE DYNAMIC BEHAVIOUR OF THE OUARKISS EARTHEN DAM

The Ouarkiss dam

The Ouarkiss dam is located in the wilaya of Oum El-Bouaghi, about 25km south of the town of Ain Fakroune, downstream of the confluence of the Oued El-Kebir and Ouarkiss, in a gorge, through Djebel Oumkeriche, along the road that connects the localities of Ain Fakroune and Bougrhara. The Ouarkiss dam is a balancing dam, its main role is storing water from the Beniharoun dam, transiting by the dam of Oued El-Athmania (w) of Mila. The latter is an earthen dam with a clay core of a capacity of 65 Hm³ and a height of 40 m.

Seismic risk

The seismic analysis of the Constantine region is undoubtedly an important element in the assessment of the seismic hazard of this region. Some earthquakes are widely described and studied. These include the destructive earthquakes of 1908, 1947, the most recent of 1985. The historical seismicity analysis of the study area gave the distribution of maximum earthquake intensities and summarized in Figure 3.

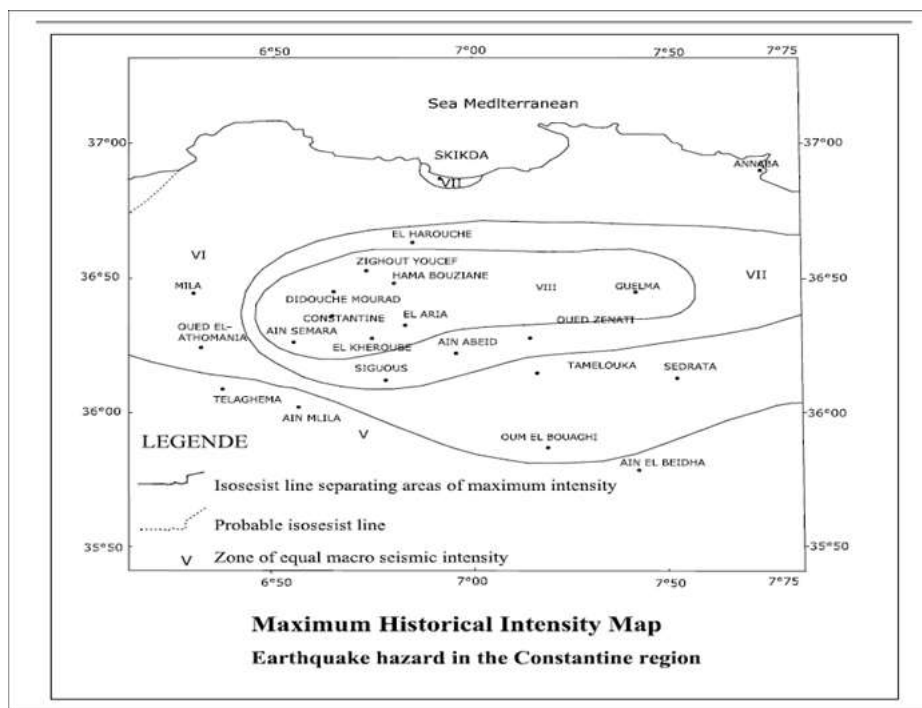


Figure 3 . Maximum intensity observed in the wilayas of eastern Algeria

The seismic source model of the Ain Fakroune region is obtained through a probabilistic analysis that allows us to assess the level of seismic hazard by the intensity, also considering seismic source areas modelled on the seismic distribution and the geological nature of the affected site. The minimum level of seismic protection accorded to a structure of its purpose and its importance vis-à-vis the objectives of protection. For the calculation of dam structures, the acceleration to be considered is normally that which would correspond to a minimum return period of 1000 years. It is of the order of 0.15g.

Table 1. Peak Acceleration for Earthquakes

Return period (years)	100	200	500	1000
Peak acceleration in (g)	0.06	0.09	0.10	0.15

Soil Parameters

In the PLAXIS 8.2 analysis, the following soil properties were used – see Table 2.

Table 2. Soil properties for using PLAXIS 8.2 analysis

Mohr-Coulomb		Foundation	Transition layer	Core	Filter and Drain	Upstream facing	Downstream facing	Protection
γ_{unsat}	[kN/m ³]	26.5	18	19.3	26.5	19	19	19
γ_{sat}	[kN/m ³]	27.5	21	22.3	27.5	20.3	20.3	21
K x	m/s	1E -09	1E -05	1E -09	1E -09	1E -05	1E -05	1E -07
K y	m/s	1E -09	1E -05	1E -09	1E -09	1E -05	1E -05	1E -07
ν	[-]	0.31	0.3	0.35	0.31	0.28	0.28	0.3
E _{ref}	[kN/m ²]	450 000	30 000	643140	450 000	800 000	800 000	40 000
C _{ref}	[kN/m ²]	8	1	10	8	5	5	10
ϕ	[°]	35	34	20	35	30	30	36
ψ	[°]	34	33	10	34	30	30	31
R _{inter.}	[-]	1	0.67	0.5	0.67	1	1	1

Meshing

The analysis of the dynamic behaviour of the dam is done using the finite element method (FEM) using triangular elements (06 nodes) with two degrees of freedom (u_x , u_y).

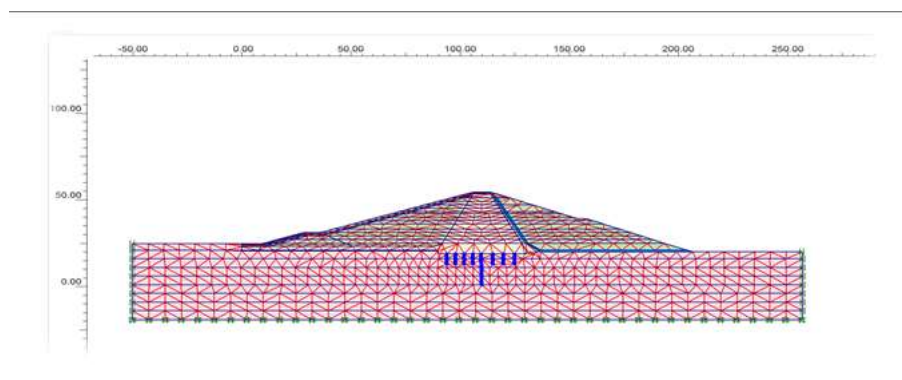


Figure 4. Meshing of the structure at 06 nodes

Analysis of the dynamic behaviour of the dam under seismic solicitation

a) Characteristics of the supposed earthquake

Figure 5 shows the characteristics of the earthquake in which it is assumed that the dam is under its solicitation, such as the magnitude, epicentre distance, peak acceleration.

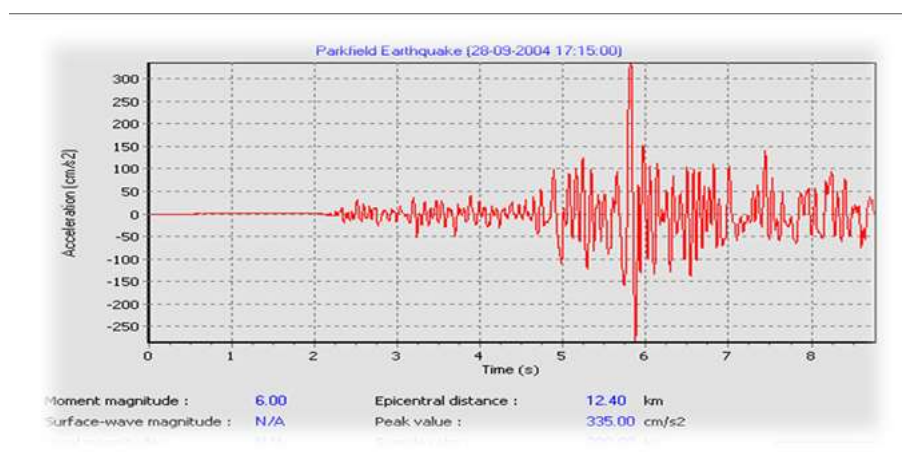


Figure 5. Characteristics of the supposed earthquake

5 RESULTS OF THE CALCULATIONS UNDER THE EFFECT OF THE SEISMIC LOAD

The displacements

Analyzing Figure 6, we notice that the total displacement is maximum at the downstream face and reaches a value of 3.88 m, which indicates that in this region there is a high risk of instability. The horizontal displacement is represented according to the cutting (A -A') in Figure 7, it reaches a maximum value of 2.45 m, and according to the vertical cutting (B -B') in Figure 8, it is of the order of $12.00 \cdot 10^{-3}$ m.

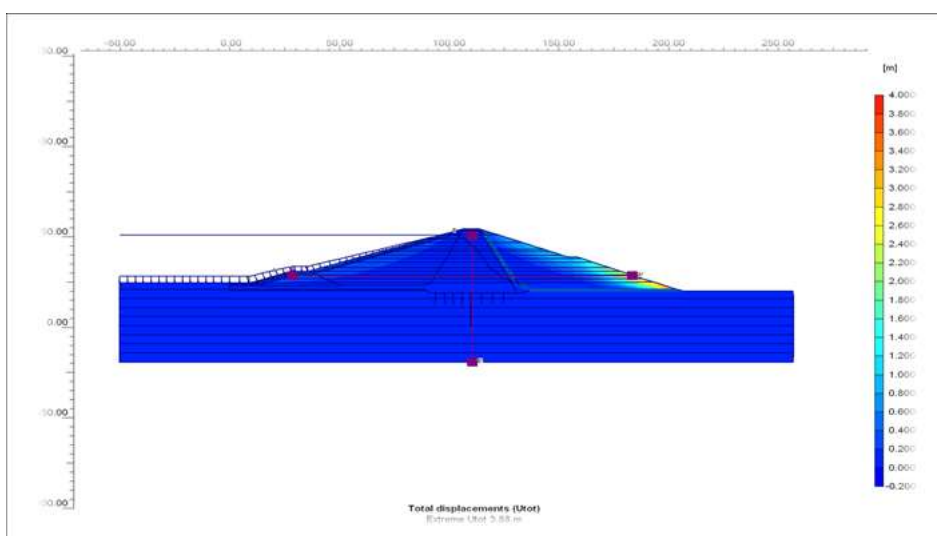


Figure 6. The total displacements (Extreme U tot 3.88m)

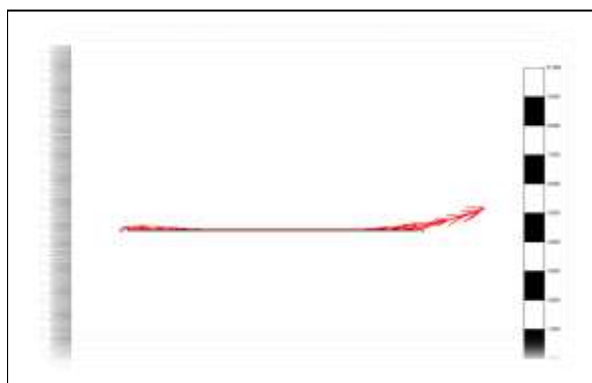


Figure 7. Diagram of displacements along the horizontal section (cutting horizontal A-A')

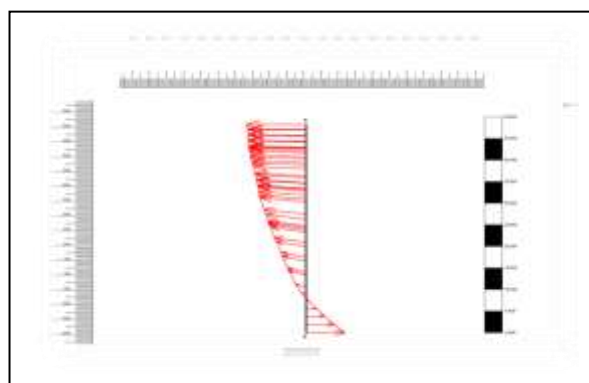


Figure 8. Diagram of displacements along the vertical section (cutting vertical B-B')

Accelerations distributions in the different zones of the dam

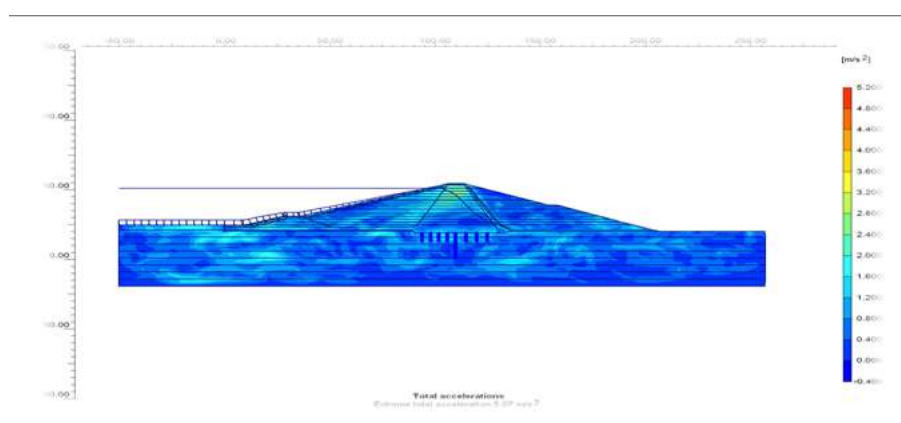


Figure 9. Total accelerations (Extreme total acceleration 5.07 m / s²)

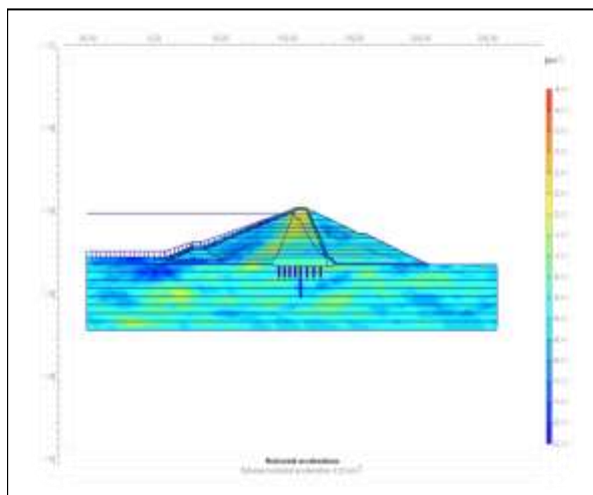


Figure 10. Horizontal accelerations - Extreme horizontal acceleration 4.32 m/s^2

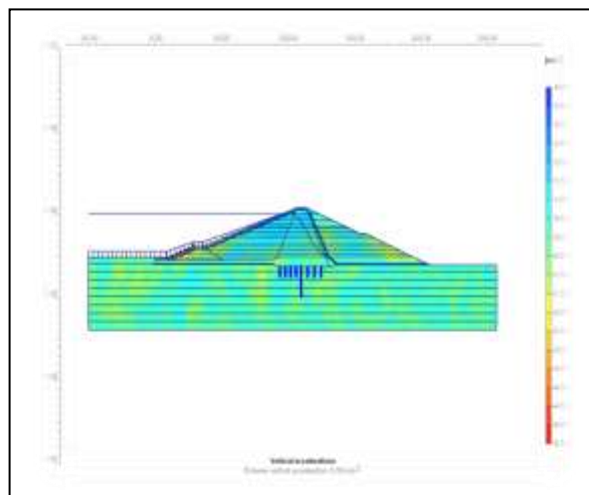


Figure 11. Vertical accelerations - Extreme vertical acceleration -5.06 m/s^2

Stresses

The maximum values of stresses are located at the interface between the dam and the foundation under the waterproof mat upstream of the dam. Horizontal, vertical and shear stresses fade away as they move away from the interface dam-foundation.

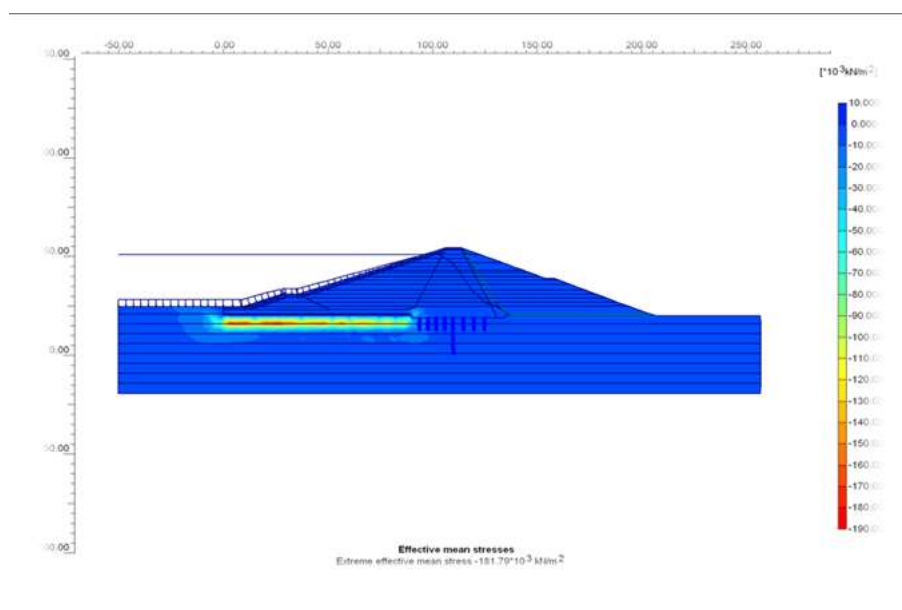


Figure 12. Effective mean stresses (Extreme effective mean stress $-181.79 \cdot 10^3 \text{ KN/m}^2$)

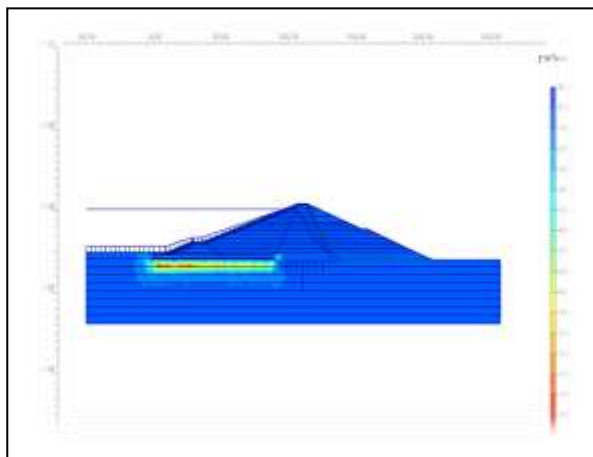


Figure 13. Effective horizontal mean stresses
(Extreme effective horizontal stresses
 $-157.17 * 10^3 \text{ KN/m}^2$)

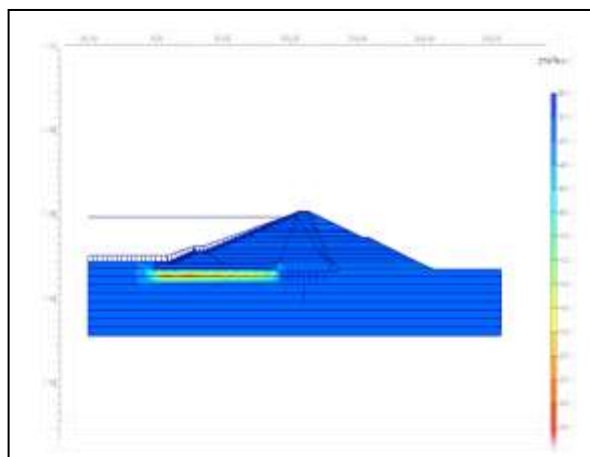


Figure 14. Effective vertical mean stresses
(Extreme effective vertical stress
 $-260.50 * 10^3 \text{ KN / m}^2$)

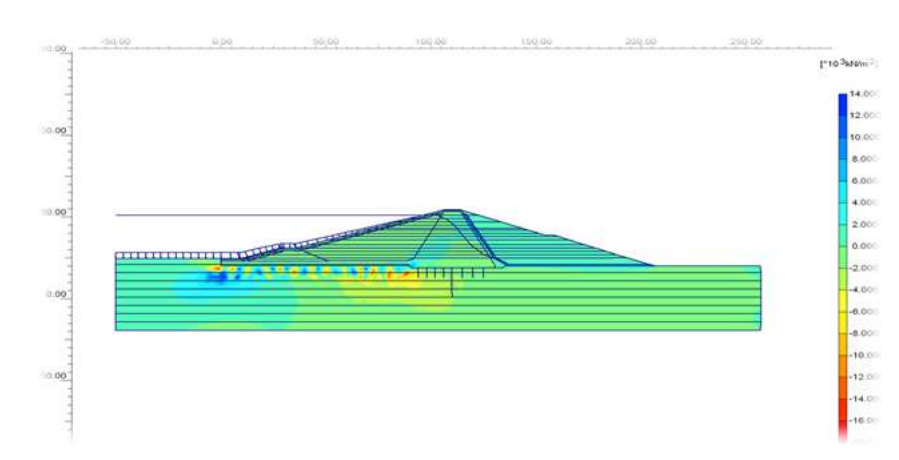


Figure 15. Shear stresses (Extreme shear stress $-16.45 * 10^3 \text{ KN/m}^2$)

Distribution of plasticity in the dam

Zones in the plastic state and zones in a state that does not exceed the boundaries described through the Mohr-Coulomb model.

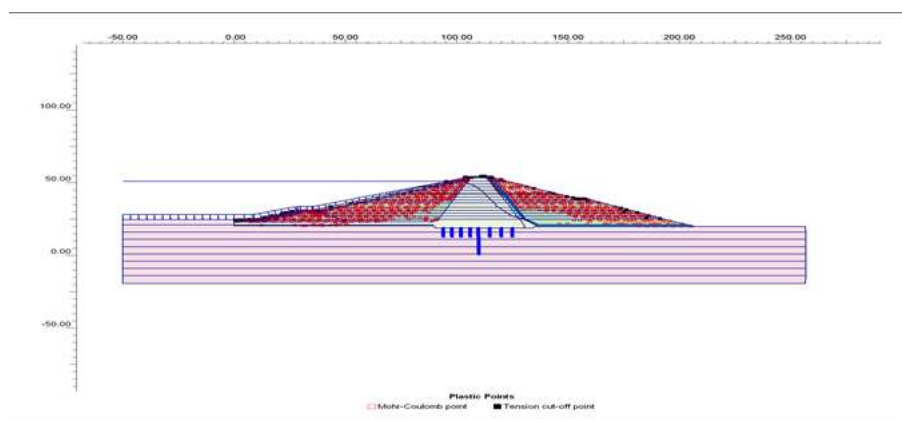


Figure 16. Plastic points Mohr coulomb point ■ Tension cut-off point ■

It is observed that the plasticity is concentrated in the downstream slope and the crest of the dam as well as the foot of the upstream slope, while it is almost zero in the core, which indicates that the seismic loading induces the plasticity in a large part of the face.

6 CONCLUSIONS AND RECOMMENDATIONS

The present work, which dealt with the modelling of the dynamic behaviour of the Ouarkiss earthen dam, relied on the finite element method taking into account the non-linear behaviour of the materials constituting the dam. The study of the dynamic behaviour of earth dams is fundamental because of the stability and sustainability requirements of these structures. The calculation methods used nowadays tend to become more and more sophisticated than those normally used for static loads.

The development of numerical analysis methods has progressed considerably, but more information on dam failure phenomena, particularly earthen dams, is needed before we are able to accurately determine the behaviour of the latter during severe soil quakes. The properties of the foundation materials and the dike body are of primary importance for the stability of earthen dams during earthquakes. Cyclic stresses can lead to high interstitial pressures in saturated non-homogeneous materials. This leads to reduced effective stresses and in the extreme cases to a complete loss of the shear strength of the material. The use of such materials is not desired.

Rather exhaustive studies on the seismic phenomenon, its complexity and the harmful consequences on dams of all types have been reported by several authors, as well as the methods of modelling the response of the materials under seismic load. With regards the Ouarkiss earth dam on which the work was carried out, a finite element calculation was carried out and proved to be amply sufficient. Seismic solicitations are assumed to be parallel to the main axes of the structure in question. The calculations were carried out by dividing the structure of the dam into several elements linked together by several nodes. The displacements are introduced as unknown at the different nodes. Their calculations are carried out using the conditions of equilibrium and the behaviour laws of the materials in each node taking into account the criterion of Mohr-coulomb.

The analysis of the results shows that the earthquake induces large lateral displacements. The latter increase on the downstream slope is accompanied by an increase in accelerations and velocities. The density of the upstream and downstream refills has a significant influence on the stresses distribution in the dam body. The decrease in the rigidity of these refills also tends to increase these same stresses.

Seismic loading induces plastic deformations in most of the downstream face, the variation of displacements as a function of horizontal distance shows an increase at the extremities implying a risk of instability in this part of the dam. The decrease in depth leads to a significant increase in plastic deformation in the dam. For the acceleration and velocity, there is considerable variation, especially at the dam crest, and a decrease in stresses from the base to the top of the dam.

The comparison between the response of the elastic and elastoplastic analysis shows that the presence of plastic deformations leads to the attenuation of accelerations in the dam, particularly at the dam crest; this result is attributed to the dissipation of energy by plastic deformation. The results obtained are conclusive in more than one way, compared to the work carried out in the same context [12, 13, 14, 15] and can be used preferentially as a database for subsequent investigation work in the field of modelling the dynamic behaviour of earthen dams.

NOTATIONS

$[[M]]$	Mass matrix	Z	Elevation in the submerged part of the upstream face
$[[C]]$	damping matrix	\ddot{u}_g	Seismic acceleration at the base
$[[K]]$	Stiffness matrix	R	The vector of dynamic coupling
$\{Q_f\}$	The load vector	I	Index of dam-foundation interface nodes
$\{U\}$	Total displacement of the structure	Ω	The considered domain
$\{U_i\}$	Displacement to the node of the structure	Γ	The interface contact
$\{U_f\}$	Displacement to the foundation node	b	Index of dam nodes
P	Pressure on the screen	f	Index of foundation nodes
A	Coefficient of seismic intensity in horizontal direction on site	γ_{unsat}	Unsaturated Soil Density
γ_w	Water density	γ_{sat}	Saturated soil density
H	Water depth	K_x	Permeability of soil according to x direction
K	Water compressibility module	K_y	Permeability of soil according to y direction
T	Period of the screen movement, supposed harmonic	ν	Coefficient of Poisson
T_0	proper period of the reservoir	E_{ref}	The young module of reference
C	Celerity of Compression waves in water C =1440 m/s	C_{ref}	the cohesion of reference
		ϕ	Friction angle
		ψ	Angle of dilatancy

REFERENCES

- [1] SAWADA, Y., H. NAKAZAWA, T. ODA, S. KOBAYASHI, S. SHIBUYA and T. KAWABATA. Seismic performance of small earth dams with sloping core zones and geosynthetic clay liners using full-scale shaking table tests. *Soils and Foundations*. 2018, 58 (3), pp. 519-533. DOI: <https://doi.org/10.1016/j.sandf.2018.01.003>
- [2] GAHLOT, N. P. and A. R. GAJBHIYE. Seismic Analysis for Safety of Dams. *Journal of Mechanical and Civil Engineering*. 2013, 6 (3), pp. 42-47.
- [3] NEWMARK, N.M. Effects of earthquakes on dams and embankments. 5th Rankine lecture. *Geotechnique*. 1965, 15 (2), pp.139-160.
- [4] MOAYED, R. Z. and M. F. RAMZANPOUR. Seismic behavior of zoned core embankment dam. *EJGE*. 2008, 13, Bund. A.
- [5] LYSMER, J. Analytical procedures in Soil dynamics. *ASCE Conference on Earthquake Engineering and Soil Dynamics, Pasadena, CA*. 1978, 3, pp. 1267-1317.
- [6] CHATEAUNEUF, A. Comprendre les éléments finis (Understanding the finite element). *European Journal of Computational Mechanics*. 2012, 16, pp. 1075-1076. DOI: <https://doi.org/10.1080/17797179.2007.9737328>
- [7] NGUYEN-THOI, T., P. PHUNG-VAN, S. NGUYEN-HOANG and Q. LIEU-XUAN. A coupled alpha-FEM for dynamic analyses of 2D fluid-solid interaction problems. *Journal of Computational and Applied Mathematics*. 2014, 271, pp. 130–149. DOI: <https://doi.org/10.1016/j.cam.2014.04.004>
- [8] KÜÇÜKARSLAN, S., S.B. COSKUN and B. TASKIN. Transient analysis of dam-reservoir interaction including the reservoir bottom effects. *Journal of Fluids and Structures*. 2005, 20 (8), pp.1073- 1084.
- [9] WESTERGAARD, H. M. Water Pressures on Dams during Earthquakes. *Transactions of ASCE*. 1933, 98 (2), pp. 418–433.
- [10] CHARATPANGOON, B., J. KIYONO, A. FURUKAWA and C. HANSAPINYO. Dynamic analysis of earth dam damaged by the 2011 Off the Pacific Coast of Tohoku Earthquake. *Soil Dynamics and Earthquake Engineering*. 2014, 64, pp. 50–62.
- [11] GAO, H. S. Modeling Stress Strain Curves for Nonlinear Analysis. *Materials Science Forum*. 2008, 575–578, pp. 539–544. DOI: <https://doi.org/10.4028/www.scientific.net/MSF.575-578.539>
- [12] RAJA, M. A. and B. K. MAHESHWARI. Behaviour of Earth Dam under Seismic Load Considering Nonlinearity of the Soil. *Open Journal of Civil Engineering*. 2016, 6, pp.75-83.
- [13] YANG, X. and S. CHI. Seismic stability of earth-rock dams using finite element limit analysis. *Soil Dynamics and Earthquake Engineering*. 2014, 64, pp.1-10.
- [14] CASCONI, E. and S. RAMPOLLO. Decoupled seismic analysis of an earth dam. *Soil Dynamics and Earthquake Engineering*. 2003, 23, pp. 349–365.
- [15] PAPALOU, A. and J. BIELAK. Nonlinear Seismic Response of Earth Dams with Canyon Interaction. *Journal of Geotechnical and Geoenvironmental Engineering*. 2004, 130 (1), pp. 103–110.