

Electron-electron attraction in an engineered electromechanical system

Gábor Széchenyi,¹ András Pályi,^{1,2,3} and Matthias Droth^{2,*}

¹*Institute of Physics, Eötvös University, 1518 Budapest, Hungary*

²*Department of Physics, Budapest University of Technology and Economics, 1111 Budapest, Hungary*

³*MTA-BME Condensed Matter Research Group, Budapest University of Technology and Economics, 1111 Budapest, Hungary*

(Received 21 April 2017; revised manuscript received 10 October 2017; published 8 December 2017)

Two electrons in a quantum dot repel each other: their interaction can be characterized by a positive interaction energy. From the theory of superconductivity, we also know that mechanical vibrations of the crystal lattice can make the electron-electron interaction attractive. Analogously, if a quantum dot interacts with a mechanical degree of freedom, the effective interaction energy can be negative; that is, the electron-electron interaction might be attractive. In this work, we propose and theoretically study an engineered electromechanical system that exhibits electron-electron attraction: a quantum dot suspended on a nonlinear mechanical resonator, tuned by a bottom and a top gate electrode. We focus on the example of a dot embedded in a suspended graphene ribbon, for which we identify conditions for electron-electron attraction. Our results suggest the possibility of electronic transport via tunneling of packets of multiple electrons in such devices, similar to that in superconducting nanostructures, but without the use of any superconducting elements.

DOI: [10.1103/PhysRevB.96.245302](https://doi.org/10.1103/PhysRevB.96.245302)

I. INTRODUCTION

Two electrons usually repel each other due to the Coulomb force. However, mechanical vibrations of a crystal lattice can mediate an effective attractive interaction between the delocalized electrons, leading to the formation of Cooper pairs and the emergence of superconductivity [1,2]. Moreover, recent experiments have demonstrated attractive interaction in the absence of superconductivity, between electrons confined in engineered nanostructures: in a carbon nanotube double quantum dot [3], where the attraction was induced by capacitive coupling to a nearby auxiliary quantum-dot system [4], and in a sketched quantum dot at the SrTiO₃/LaAlO₃ interface [5,6], where the mechanism of attraction has not been revealed. A recent proposal [7] describes how to engineer electron-electron attraction in an artificial nanostructure using a careful design of orbital and tunneling energies.

The possibility of vibration-mediated attractive interaction among confined electrons has been discussed, e.g., in the context of amorphous semiconductors [8], vacancies in silicon [9,10], fullerenes [11], and molecular junctions [12–15]. Figure 1(a) depicts the simplest model capturing the basic ingredients of the effect: it involves (i) a single vibrational mode (phonon), characterized by a mass m and frequency ω , (ii) a single electronic orbital that can be occupied by one or two electrons, i.e., the occupation number is $N \in \{0, 1, 2\}$; this orbital is characterized by an on-site energy ϵ and a repulsive Coulomb energy $U > 0$, (iii) the coupling between the phonon and the confined charge, characterized by the force λ encoding the coupling strength, and (iv) a (zero-temperature) electron reservoir, with Fermi energy $\mu = 0$, which can supply electrons to the orbital. We refer to this as the Anderson-Holstein model [15]. (See Methods for more details.) The presence of the electron-phonon coupling leads to an effective Coulomb energy $U_{\text{eff}} = U - \frac{\lambda^2}{m\omega^2}$, which becomes negative if the coupling λ is strong enough, i.e., if $\lambda > \omega\sqrt{mU}$. That is, a

strong enough electron-phonon coupling implies an attractive electron-electron interaction.

In a system with tunable on-site energy and tunable electron-phonon coupling strength, this attractive electron-electron interaction could lead to remarkable equilibrium and transport properties, as indicated in Figs. 2(a) and 2(b). Figure 2(a) shows the *charge stability diagram*, that is, the number N_{eq} of electrons occupying the orbital in equilibrium at zero temperature, as the function of the two tunable parameters ϵ and λ . For weak electron-phonon coupling $\lambda < \omega\sqrt{mU}$, the filling sequence of the orbital is regular: for example, at $\lambda = 0$, as ϵ is decreased, the occupation of the orbital increases by one at $\epsilon = 0$ and again by one at $\epsilon = -U$. In contrast, for strong electron-phonon coupling $\lambda > \omega\sqrt{mU}$, the occupation is increased abruptly by two as the 0/2 boundary, i.e., the boundary between the $N_{\text{eq}} = 0$ and $N_{\text{eq}} = 2$ regions, is crossed. When tuned to the 0/2 boundary, such a system is expected to show an exotic transport effect when embedded between a source and a drain electrode, reminiscent of Cooper-pair transport in a normal-superconductor junction [16]: current is carried by tunneling of electron pairs [12,15]. Furthermore, the current-bias voltage curve exhibits a smooth “Coulomb hill” instead of a sharp Coulomb plateau [12]; shot noise [15] (Fano factor) increases compared to its value for single-electron tunneling, corresponding to an increased granularity of the charge quanta carrying the current; and unconventional Coulomb-blockade features are induced also in the regimes of single-electron tunneling [17].

The steady progress in the fabrication of molecular junctions allows for electrical control of the orbital energies [18,19]; however, tuning the strength of the electron-phonon coupling in these systems is very challenging. Accordingly, to our knowledge, the effects discussed above have not been observed in molecular junctions.

In this work, we propose an engineered nanostructure to observe electron-electron attraction, electron-pair tunneling, and the associated interesting phenomenology discussed above. The structure we suggest is a suspended quantum dot [20–23], see Fig. 1(b), with a top and a bottom gate electrode. This

*matthias.droth@mail.bme.hu

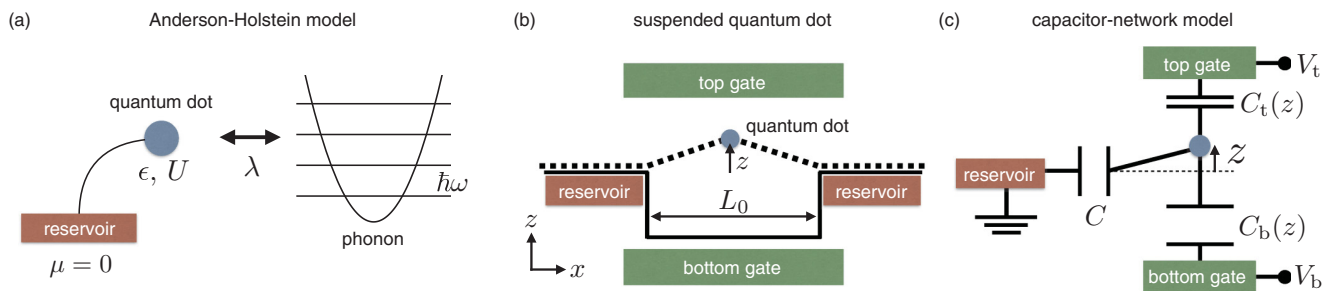


FIG. 1. Electromechanical systems showing mechanically assisted electron-electron attraction. (a) Electron-electron attraction arises in the Anderson-Holstein model, where the charge on a single electronic orbital (quantum dot) interacts with a single vibrational mode (phonon). (b) Electron-electron attraction can also be engineered in a suspended quantum dot (blue spot), which is located on a nonlinear nanomechanical resonator (dashed line). The dot can be displaced along the z direction. The equilibrium occupation and displacement of the dot can be tuned by the top and bottom gate voltages, V_t and V_b , respectively. (c) Capacitor-network model of the suspended dot, which is coupled to a grounded charge reservoir via capacitance C , and to top and bottom gates via z -dependent gate capacitances $C_t(z)$ and $C_b(z)$.

combination allows for independent control of the orbital energy and the electron-phonon coupling strength: in short, the average gate voltage defines the former, whereas the gate-voltage difference defines the latter.

Importantly, in this setup the electron-phonon coupling is of extrinsic origin [20–23]; i.e., it arises due to the external electric field created by the gates and not due to intrinsic mechanisms (e.g., deformation potential, bond-length change). Utilizing this extrinsic mechanism brings two advantages: the electron-phonon coupling is tunable via the gate voltages, and the corresponding extrinsic force can well exceed those arising from the intrinsic mechanisms (see Methods). We focus on the example of a dot embedded in a suspended graphene ribbon [23], for which we identify conditions for electron-electron attraction. Furthermore, our results reveal the possibility of electronic transport via tunneling of packets of multiple electrons in such devices, similar to that in superconducting nanostructures, but without the use of any superconducting elements.

II. RESULTS

A. Setup

We consider a quantum dot embedded in a mechanical resonator, as shown in Fig. 1(b). For concreteness, we formulate a model for the case when the resonator is a graphene nanoribbon, suspended over a trench [24–27], as shown in Fig. 1(b). The system is controlled by voltages V_t and V_b applied on the top and bottom gate electrodes, respectively. The geometry of the resonator is characterized by the width W and the length L_0 of the suspended part of the ribbon. The ribbon might be stretched even if the gates are inactive; characterized by the residual strain (prestrain) $u_{xx,0} = (L_0 - L_u)/L_u$, where L_u is the unstretched length of the suspended part of the ribbon.

The dot, indicated by the blue spot in Fig. 1(b), is located on the suspended part of the resonator. The dot interacts with an electron reservoir; the capacitive part of this interaction is characterized by the capacitance C . In addition, electrons can also tunnel between the reservoir and the dot. The dot is coupled to the top and bottom gates via the displacement-dependent capacitances $C_t(z)$ and $C_b(z)$, respectively. As indicated in Fig. 1(c), the displacement dependence of the capacitances

arises since the displacement of the dot changes the distance between the capacitor plates. To keep the number of parameters to the minimum, we assume that the three capacitances are equal at $z = 0$, and that the displacement dependencies are that of a planar capacitor $C_t(z) = C_b(-z) = Cd/(d - z)$, where d is the distance between the plates. First, we consider the case of an ‘n-type semiconducting’ dot, by which we mean that the number N of excess electrons in the dot at zero gate voltages is zero, and at finite gate voltages it can only be non-negative, $N \geq 0$.

B. Charge stability diagram

Our primary goal is to determine the charge stability diagram of the dot; that is, to determine how the number N_{eq} of electrons in the dot at zero-temperature equilibrium depends on the gate voltages V_t and V_b . Motivated by the result of the Anderson-Holstein model [Fig. 2(a)], we look for the 0/2 boundary that separates regions of the empty dot ($N_{\text{eq}} = 0$) and the doubly occupied dot ($N_{\text{eq}} = 2$). To this end, we express the total energy $E(N, z)$ of the system (see Methods) which depends on the parameters W , L_0 , $u_{xx,0}$, C , d , V_t , and V_b , as well as the dot occupation N and the dot displacement z . We take into account the geometrical nonlinearity of the resonator by keeping the mechanical energy term that is of fourth order in the displacement z ; this is required to avoid the apparent charge instability arising in the case of a purely harmonic oscillator [13]. Then, we minimize the total energy $E(N, z)$ with respect to N and z , to obtain the zero-temperature equilibrium occupation N_{eq} and displacement z_{eq} .

The charge stability diagram is shown in Fig. 2(c), for a certain realistic parameter set (see caption). To be able to compare the diagram with that of the Anderson-Holstein model [Fig. 2(a)], we plot N_{eq} as the function of the average gate voltage $\bar{V} = (V_t + V_b)/2$ and the gate-voltage difference $\delta V = (V_t - V_b)/2$: intuitively, \bar{V} controls the on-site energy of the dot, whereas δV controls the electric field that acts on the dot and hence controls the coupling strength between the dot charge and the resonator.

The key features in Fig. 2(c) are as follows. (i) Overall, the diagram shows strong qualitative similarities with that of the Anderson-Holstein model [Fig. 2(a)]. (ii) Similarly to the Anderson-Holstein case [Fig. 2(a)], Fig. 2(c) also shows a

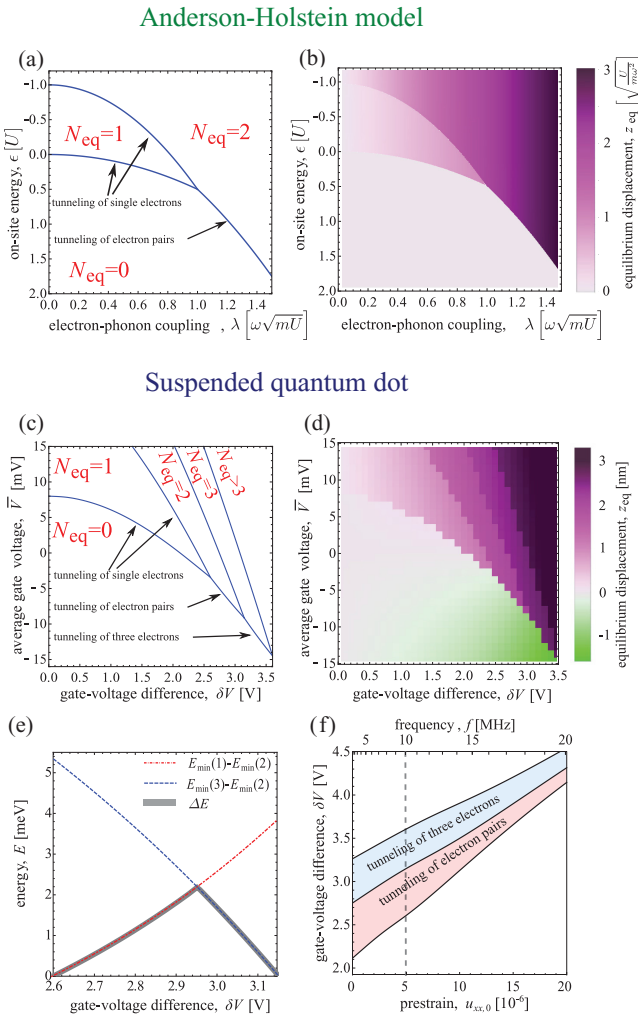


FIG. 2. Characteristics of mechanically assisted electron-electron attraction. Charge stability diagram (a) and displacement stability diagram (b) of the Anderson-Holstein model. Analogous results for the suspended quantum dot are shown in (c) and (d), respectively [28]. (e) Charge excitation gap ΔE (thick gray line) along the 0/2 boundary shown in (c) (labeled as “tunneling of electron pairs”). (f) Gate-voltage range of the 0/2 (pink) and 0/3 (blue) boundaries, as a function of prestrain. The upper horizontal axis corresponds to the frequency f associated to the resonator (see Methods). Parameters: ribbon length $L_0 = 1.5 \mu\text{m}$, ribbon width $W = 0.4 \mu\text{m}$, ribbon-gate distance $d = 150 \text{ nm}$, capacitance $C = 5 \text{ aF}$. The prestrain in (c)–(e) is $u_{x,x,0} = 5 \times 10^{-6}$.

triple point between the $N_{\text{eq}} = 0, 1, 2$ regions. The coordinates of this triple point are $\delta V_2 \approx 2.6 \text{ V}$ and $\bar{V}_2 \approx -3.5 \text{ mV}$. (iii) In addition to the Anderson-Holstein result, the figure also shows a triple point between the $N_{\text{eq}} = 0, 2, 3$ regions, at $\delta V_3 \approx 3.1 \text{ V}$ and $\bar{V}_3 \approx -9 \text{ mV}$, and a triple point between the $N_{\text{eq}} = 0, 3, 4$ regions, at $\delta V_4 \approx 3.6 \text{ V}$ and $\bar{V}_4 \approx -14.5 \text{ mV}$. (iv) Similarly to the Anderson-Holstein case, a 0/2 boundary is observed in Fig. 2(c), labeled as “tunneling of electron pairs.” This 0/2 boundary connects the triple points at $(\delta V_2, \bar{V}_2)$ and $(\delta V_3, \bar{V}_3)$. (v) Importantly, Fig. 2(c) shows that the 0/2 boundary arises at a gate-voltage difference of a few volts, and an average gate voltage of a few millivolts, which suggests that the

experimental observation of this charge stability diagram, and the pair-tunneling transport effects it implies [12], is feasible. (vi) In addition to the 0/2 boundary, Fig. 2(c) also shows 0/ N_{eq} boundaries with $N_{\text{eq}} > 2$; e.g., the 0/3 boundary between the triple points $(\delta V_3, \bar{V}_3)$ and $(\delta V_4, \bar{V}_4)$, labeled as “tunneling of three electrons.”

C. Displacement stability diagram

Besides the charge stability diagram, it is instructive, and, for the description of transport effects, is crucial to describe how the equilibrium displacement z_{eq} varies as the gate voltages are tuned. We will refer to this function as the *displacement stability diagram*. Figure 2(b) shows the displacement stability diagram of the Anderson-Holstein model, whereas Fig. 2(d) shows the diagram for our suspended quantum dot model, for a specific set of parameter values listed in the caption. [The pixel structure in Fig. 2(d) is due to data being obtained on a δV - \bar{V} grid of size 36×30 .] Naturally, the equilibrium displacement varies smoothly within the regions belonging to a certain N_{eq} , and jumps abruptly along the boundaries between those regions. Note that for the suspended quantum dot, the characteristic scale of these jumps is nanometer.

The Anderson-Holstein displacement stability diagram [Fig. 2(b)] shows zero displacement $z_{\text{eq}} = 0$ for an uncharged dot, i.e., in the $N_{\text{eq}} = 0$ region. However, the uncharged suspended quantum dot can be displaced toward the bottom gate, as indicated by the green region of Fig. 2(d). This effect arises due to the capacitive coupling to the reservoir, as shown by the following argument. First, consider a specific setting: the nonequilibrium situation when $N = 0$, $z = 0$, the top gate is grounded, $V_t = 0$, and the bottom gate voltage is negative, $V_b < 0$. That corresponds to $\delta V > 0$ and $\bar{V} < 0$, i.e., the region where the negative z_{eq} is observed in Fig. 2(d). In this case, a charge q_r is accumulated on the plate of the reservoir, and the charges accumulated on the three plates r, t, b associated to the dot [see Fig. 1(c)] are $-q_r, -q_r$, and $2q_r$, respectively. This means that the Coulomb attraction between the plates of the bottom capacitor is four times stronger than for the top capacitor, implying that the dot is pulled toward the bottom gate. In a more general case, when the gate voltages are not specified, but $\delta V > 0$ and $\bar{V} < 0$ still hold, we can say that a finite charge will accumulate on the plate facing the reservoir, and therefore the sum of the charges on the plates facing the top and bottom gates does not vanish. In turn, that sum determines the force acting on the dot, hence we conclude that that force is nonzero, and therefore the dot is displaced.

It is expected that transport, e.g., electron-pair tunneling, through such a suspended quantum dot will be sensitive to the size of the jumps on the displacement stability diagram: the larger the displacement jump, the lower the current flowing through the device. This effect is known as the Franck-Condon blockade [12, 15, 29–31] and arises for the following reason. In a transport situation, the system is voltage-tuned to a point along one of the instability lines, e.g., the 0/2 boundary. Then, electrons can hop between the leads and the dot, and hence the occupation of the dot can fluctuate between 0 and 2. As argued above, the ground-state displacement for the two occupancies is different. As a consequence, the overlap

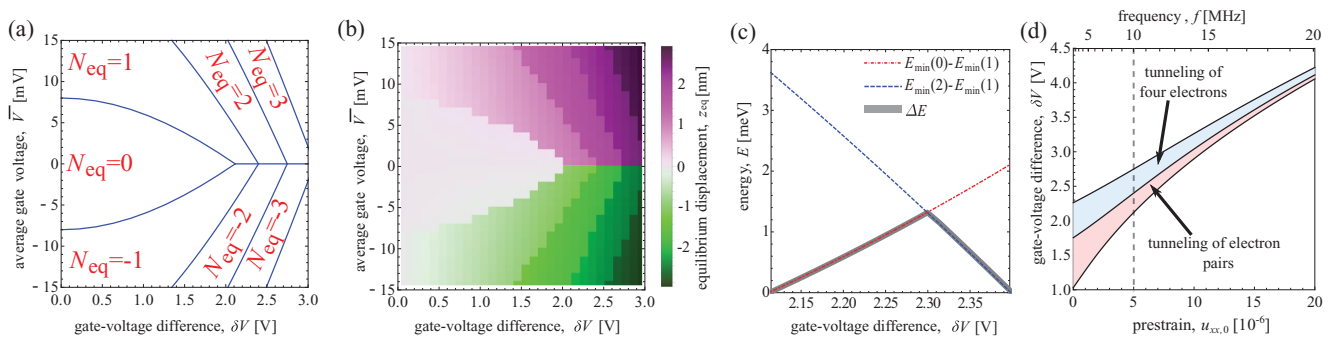


FIG. 3. Results for a suspended metallic quantum dot. (a) Charge stability diagram. (b) Displacement stability diagram [28]. (c) Charge excitation gap ΔE (thick gray line) along the 1/-1 boundary shown in (a). Parameters: see caption of Fig. 2. (d) Gate-voltage ranges of the 1/-1 and 2/-2 boundaries, as a function prestrain. The upper horizontal axis corresponds to the frequency f associated to the resonator (see Methods).

between the resonator's quantum states corresponding to the two charge occupancies can become much smaller than unity; the larger the displacement jump at the selected point of the 0/2 boundary, the stronger the suppression of that overlap. In turn, that overlap controls the electron's tunnel rates between the leads and the dot, and hence these tunnel rates and thereby the current through the device are also suppressed. The quantitative characterization of these effects in the presence of mechanical nonlinearities, multiple mechanical modes, finite temperature, and arbitrary electronic occupation is an important future theory task.

D. Results for a metallic suspended quantum dot

So far, we presented and discussed the results corresponding to an n -type semiconducting quantum dot, where the number of excess electrons is restricted to nonnegative integers, $N \geq 0$. However, depending on the specifics of the experimental setup, it can happen that the quantum dot is occupied by a large number N_0 of, say, conduction-band electrons, when the top and bottom gate voltages are set to zero. In that case, which we refer to as a “metallic” dot [13], the number of these electrons can not only be increased, but also decreased by tuning the system via the gate voltages. To characterize this scenario, in Fig. 3 we show the results corresponding to the metallic dot. To obtain the results in Fig. 3, we simply extended the energy minimization procedure (described in Methods) to include negative occupancies, $N \in \{-4, -3, \dots, 3, 4\}$.

There are two major differences between the metallic (Fig. 3) and semiconducting (Fig. 2) results. (i) Naturally, the stability diagrams of the metallic dot show perfect antisymmetry with respect to the average gate voltage \bar{V} ; this antisymmetry is absent in the semiconducting case. Note that it is expected that the $\bar{V} > 0$ part of the semiconducting and metallic stability diagrams are identical, and that expectation is confirmed by comparing Fig. 2(c) with Fig. 3(a) and Fig. 2(d) with Fig. 3(b). (ii) The charge stability diagram of the metallic dot [Fig. 3(a)] shows boundaries of the type $N - N$, along the $\bar{V} = 0$ line. In principle, tuning the system to such a boundary could imply transport via tunneling of electron packets of size $2N$.

III. DISCUSSION

Steady advances in nanofabrication now allow the tailoring of electron-phonon interaction in suspended quantum dots [22]. For graphene-based mechanical resonators, the tension and hence the nonlinearity can be tuned *in situ* [32] and the real-space mode shape can be visualized [33]. Suspended nanostructures with top and bottom gates have been fabricated and studied in various experiments [26,31,34]. In addition, quantum dots on suspended graphene ribbons have been created [26], and voltage-controlled charge-phonon coupling in such devices have been demonstrated [23]. Such devices bear the promise of combining few-electron transport with the outstanding mechanical characteristics, e.g., high Q-factor and low mass density, of this material [25]. Similar structures could be fabricated using members of the recently discovered family of two-dimensional materials [35]. These developments suggest that the fabrication of an engineered device as proposed here is within reach.

A conceptually simple experiment to observe the charge stability diagram in Fig. 2(c) could be based on charge sensing; that is, N_{eq} could be measured via the current flowing through a mesoscopic conductor that is capacitively coupled to the suspended dot and therefore sensitive to N_{eq} . Without the charge sensor, the boundaries of the diagram could also be mapped by sending a current through the suspended quantum dot, e.g., by utilizing the left and right reservoir depicted in Fig. 1(b) as a source and drain contact, respectively. For a fixed value of the gate-voltage difference below the $N_{\text{eq}} = 0, 1, 2$ triple point of Fig. 2(c), that is, for $\delta V < \delta V_2 \approx 2.6$ V, the current $I(V_{\text{sd}}, \bar{V})$ at finite bias voltage V_{sd} would show standard Coulomb-blockade features. However, in the range $\delta V_2 < \delta V < \delta V_3$, the characteristics of electron-pair tunneling [12,15] discussed in the Introduction are expected to appear when $I(V_{\text{sd}}, \bar{V})$ is measured.

Our results indicate that such a device would allow for the exploration of transport via multi-electron tunneling as well. For example, when voltage-tuned to the 0/3 boundary of Fig. 2(c), current could be carried by the simultaneous tunneling of three electrons. To our knowledge, it is an open challenge for both theory and experiment to characterize such exotic scenarios.

So far, our discussion focused on the case of zero temperature. To observe a sharp $0/2$ boundary of the charge stability diagram shown in Fig. 2(c), the thermal energy scale $k_B T$ should be much lower than the charge excitation gap ΔE along the $0/2$ boundary. Along this boundary, the charge excitation gap is the difference between the energy of the lowest-energy excited charge configuration, being the lower of $E_{\min}(N=1)$ and $E_{\min}(N=3)$, and that of the twofold degenerate ground state $E_{\min}(N=0) = E_{\min}(N=2)$. Here, $E_{\min}(N) = \min_z E(N, z)$. We plot the charge excitation gap in Fig. 2(e), for the parameter set listed in the caption. The plot shows ΔE (thick gray line) as the two gate voltages are varied simultaneously such that we move along the $0/2$ boundary. For this example, the charge excitation gap is tuned between zero, at the triple points, to a maximum of ≈ 2.2 meV, reached around the center of the considered gate-voltage range, at $\delta V \approx 2.95$ V. Regarding the fact that temperatures of the order of 100 mK corresponding to an energy scale of $k_B T \approx 10 \mu\text{eV}$ are available, reaching the above condition $k_B T \ll \Delta E$ seems experimentally feasible. Similar conclusion can be reached in the case of the metallic dot; see Fig. 3(c).

Figure 2(c) proves the existence of the $0/2$ and $0/3$ boundaries in the charge stability diagram for a specific parameter set, in case of the semiconducting dot. We demonstrate the robustness of these boundaries with respect to parameter variations by revealing how they change as the prestrain $u_{xx,0}$ of the ribbon is varied. In Fig. 2(f), we plot the prestrain dependence of the gate-voltage-difference coordinates characterizing the three triple points of Fig. 2(c), that is, $\delta V_2, \delta V_3, \delta V_4$. Note that the prestrain value $u_{xx,0} = 5 \times 10^{-6}$ corresponds to Figs. 2(c)–2(e). Figure 2(f) indicates that the gate-voltage intervals $[\delta V_2, \delta V_3]$ and $[\delta V_3, \delta V_4]$ of the $0/2$ and $0/3$ boundaries shrink as the prestrain is increased. However, in the considered range of prestrain, the order of magnitude of the required gate-voltage difference remains to be a few volts, and the order of magnitude of the widths $\delta V_3 - \delta V_2$ and $\delta V_4 - \delta V_3$ remains to be a few hundred millivolts. The corresponding result in the case of the metallic dot are shown in Fig. 3(d).

As pointed out above, it is expected that transport through a suspended quantum dot can take the form of sequential tunneling of electron pairs, in a fashion reminiscent of certain electronic circuits containing superconducting leads [17]. This possibility naturally leads to the question: are there any electronic arrangements, where the functionality of a superconducting lead can be mimicked by a suspended quantum dot? In Fig. 4, we present such a setup. Figure 4(a) shows the sketch of a Cooper-pair splitter circuit [36–38], where a superconducting source electrode serves as a source of spin-singlet Cooper pairs, and the two quantum dots transfer the electrons to the left and right normal leads such that the entanglement of their spins is maintained. Hence, this device supplies a stream of spatially separated, spin-entangled pairs of electrons. It has been suggested that this functionality can be achieved even if the superconducting source is replaced by a normal-metal electrode, and a third (conventional) quantum dot [39]. Here, we suggest to utilize the mechanical degree of freedom of the suspended dot for the same purpose, in the setup shown in Fig. 4(b). The suspended quantum dot should be gate-voltage-tuned to the $0/2$ boundary, which could guarantee that only spin-singlet pairs of electrons can tunnel from the source

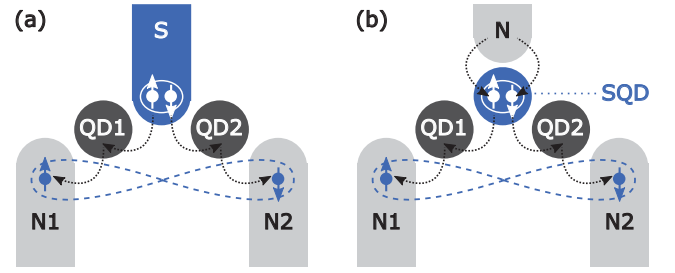


FIG. 4. Mechanically assisted electron-electron attraction for creating a stream of spatially separated spin-entangled electrons. (a) A Cooper-pair splitter circuit, based on the controlled emission of Cooper pairs from a superconducting source electrode (S), through two quantum dots (QD1 and QD2), to two different normal drain electrodes (N1 and N2). (b) A similar functionality is offered if the superconducting electrode is replaced by a normal electrode (N) and a suspended quantum dot (SQD) that is tuned to the $0/2$ boundary of its charge stability diagram.

to the triple-dot system, and thereby allow for the creation of a stream of spatially separated spin-entangled electrons.

IV. METHODS

A. Anderson-Holstein model: Attractive electron-electron interaction and stability diagrams

In the Introduction, we discussed that an effectively attractive electron-electron interaction can arise [8] in the Anderson-Holstein model depicted in Fig. 1(a). Here, we summarize how that conclusion is reached, and outline how the charge and displacement stability diagrams shown in Figs. 2(a) and 2(b) are obtained.

As discussed in the Introduction, the energy of the Anderson-Holstein model is the sum of three contributions, $E_{\text{AH}} = E_{\text{AH,m}} + E_{\text{AH,o}} + E_{\text{AH,int}}$. The mechanical energy is $E_{\text{AH,m}} = \frac{1}{2} m \omega^2 z^2$, the energy of the electrons occupying the orbital is $E_{\text{AH,o}} = \epsilon N + \frac{U}{2} N(N-1)$, whereas the electron-phonon interaction energy is $E_{\text{AH,int}} = \lambda z N$.

The energy of the Anderson-Holstein model can be rewritten [8, 12, 13], using the definitions $z_0 = \lambda / (m \omega^2)$, $\epsilon_{\text{eff}} = \epsilon - \lambda^2 / (2m \omega^2)$, and $U_{\text{eff}} = U - \lambda^2 / (m \omega^2)$, as

$$E_{\text{AH}} = \frac{1}{2} m \omega^2 (z + N z_0)^2 + \epsilon_{\text{eff}} N + \frac{1}{2} U_{\text{eff}} N(N-1). \quad (1)$$

This energy function is the same as that of a system where the harmonic oscillator has an occupation-dependent equilibrium position at $-N z_0$, and the electronic orbital is characterized by the on-site energy ϵ_{eff} and the electron-electron interaction energy U_{eff} . As mentioned in the main text, U_{eff} becomes negative, and hence can be interpreted as an attractive electron-electron interaction, for sufficiently strong electron-phonon coupling strengths $\lambda > \omega \sqrt{m U}$.

The charge and displacement stability diagrams in Figs. 2(a) and 2(b) characterize the lowest-energy state of the electron-phonon system composed with a zero-temperature electron reservoir with Fermi energy $\mu = 0$ [Fig. 1(a)]. The occupation N_{eq} and displacement z_{eq} of the lowest-energy states are plotted in Figs. 2(a) and 2(b). These are analytical results, obtained by minimizing the energy function $E_{\text{AH}}(N, z)$ with respect to N and z .

B. Charge and displacement stability diagrams of the suspended quantum dot

The stability diagrams in Figs. 2(c) and 2(d) are obtained by minimizing the total energy $E(N, z)$ of the suspended quantum dot depicted in Fig. 1(b). This energy $E = E_m + E_{em} + W$ includes a purely mechanical (E_m), an electromechanical (E_{em}), and a purely electronic (W) contribution. Here we describe how these contributions are estimated, and how the energy function is used to obtain the stability diagrams.

First, we describe the purely mechanical contribution, associated to the deformation of the graphene ribbon. We assume that most of the excess charge of the quantum dot is localized in a narrow region around the center of the suspended part of the ribbon. Then, the gate-induced forces stretch the ribbon in the way shown in Fig. 1(b), and the deformation-induced elongation of the ribbon is assumed to be homogeneous for simplicity. The mechanical energy arising from this stretching deformation can be expressed as a function of the ribbon's parameters and the displacement z of the dot:

$$E_m(z) = \frac{1}{2} Y W L_u u_{xx}^2(z). \quad (2)$$

Here, $Y = 340 \text{ N/m}$ is the Young modulus of graphene [40], and $u_{xx}(z) = (L(z) - L_u)/L_u$ is the relative elongation (strain) of the ribbon. Simple geometrical considerations imply

$$u_{xx}(z) = \sqrt{(1 + u_{xx,0})^2 + 4(z/L_u)^2} - 1. \quad (3)$$

Substituting this into Eq. (2) and expanding the latter up to fourth order in z , we find, up to a constant,

$$E_m(z) \approx \alpha_2 z^2 + \alpha_4 z^4, \quad (4)$$

where $\alpha_2 \approx 2Y u_{xx,0} W/L_0$ and $\alpha_4 \approx 2Y W/L_0^3$; the latter two expressions are accurate up to leading order in the small prestrain $u_{xx,0} \ll 1$.

The electromechanical contribution to the energy is associated to the effective capacitors [41] shown in Fig. 1(c):

$$E_{em}(N, z) = q_r^2/2C + q_t^2/2C_t(z) + q_b^2/2C_b(z), \quad (5)$$

where $q_{r,t,b}$ are the charges accumulated on the reservoir, top gate, and bottom gate, respectively. The relation of these charges to the dot occupancy N and displacement z can be established using that (i) the quantized dot charge $-|e|N$ can be expressed as $-|e|N = -q_r - q_t - q_b$, (ii) the top gate voltage is the sum of the voltages dropping on the reservoir and top capacitors, $V_t = q_t/C_t(z) - q_r/C$, (iii) analogously for the bottom gate voltage, $V_b = q_b/C_b(z) - q_r/C$. This linear set of three equations for $q_{r,t,b}$ is solved, and the solutions are inserted into Eq. (5), to obtain the explicit dependence of the electromechanical energy E_{em} on N and z .

The third, last contribution to the total energy of the system is the work done by the voltage sources [41] $W(N, z) = -q_t V_t - q_b V_b$.

To find the equilibrium occupation N_{eq} and displacement z_{eq} of the suspended quantum dot, we minimize the energy $E(N, z)$ as a function of displacement z and occupation number N , using the following procedure. We focus on the case of small displacements $z \ll d$, hence we Taylor-expand the electromechanical E_m and the electronic W terms up to second order in the variable z around zero. By this expansion, the total energy function becomes a fourth-order polynomial of z ,

which we can minimize numerically with respect to z for the different occupation numbers $N \in \{0, 1, 2, 3, 4\}$, yielding the minimum value $E_{\min}(N) = \min_z E(N, z)$ of the energy and the corresponding displacement $z_{\min}(N)$, assuming dot occupancy N . The equilibrium occupation N_{eq} at zero temperature is then found by minimizing the energy $E_{\min}(N)$, also yielding the equilibrium displacement $z_{eq} = z_{\min}(N_{eq})$. Repeating this procedure for various values of the gate voltages, we obtain the charge and displacement stability diagrams shown in Figs. 2(c) and 2(d), respectively. The same procedure is followed to obtain the results for the metallic dot, Figs. 3(a) and 3(b), with the generalization that we allow for negative occupation numbers as well, $N \in \{-4, -3, \dots, 3, 4\}$.

C. Relation between the Anderson-Holstein model and the suspended quantum dot model

Here, we establish the relation between the Anderson-Holstein model and the semiconducting suspended quantum dot model. Focusing on the regime of large gate-voltage differences, e.g., to the vicinity of the 0/2 boundary where electron pair tunneling is expected, we show that the energy of the suspended quantum dot model incorporates two terms that are absent in the energy of the Anderson-Holstein model.

As mentioned above, the energy of the suspended quantum dot $E(N, z)$ in the small displacement regime $z \ll d$ can be approximated by its Taylor expansion in the variable z . This yields

$$E(N, z) \approx \alpha_2 z^2 + \frac{e^2 N^2}{6C} - \frac{2}{3} |e| N \bar{V} - \frac{2}{3} |e| N \delta V \frac{z}{d} - \frac{1}{3} C \delta V^2 \left(\frac{z}{d}\right)^2 - \frac{2}{3} C \delta V \bar{V} \frac{z}{d} + \alpha_4 z^4, \quad (6)$$

where terms independent of N and z are neglected. In Eq. (6) we drop further second- and higher-order terms in z , which, in the vicinity of the 0/2 boundary where $|\bar{V}|, |e|/C \ll \delta V$ holds, are much smaller than $C \delta V^2 (z/d)^2$.

We now compare the energy E of the quantum dot model in Eq. (6), with the energy E_{AH} of the Anderson-Holstein model. We claim that the first five terms of E correspond to the four terms of E_{AH} . That is, the former is obtained from the latter via making the substitutions

$$U \mapsto \frac{e^2}{3C}, \quad (7)$$

$$\frac{1}{2} m \omega^2 \mapsto \alpha_2 - \frac{1}{3} C \delta V^2 \frac{1}{d^2}, \quad (8)$$

$$\epsilon \mapsto -\frac{2|e|\bar{V}}{3} + \frac{e^2}{6C}, \quad (9)$$

$$\lambda \mapsto -\frac{2}{3} |e| \delta V \frac{1}{d}. \quad (10)$$

These results confirm the expectations that the average gate voltage \bar{V} controls the on-site energy ϵ [Eq. (9)], and the gate-voltage difference δV controls the electron-phonon coupling strength λ [Eq. (10)]. The second term on the right-hand side of Eq. (8) shows that the oscillator eigenfrequency of the Anderson-Holstein model corresponds to a combination of a mechanical and an electronic contribution in the suspended-dot model. The last two terms in the energy Eq. (6) of the

suspended quantum dot, i.e., the charge-independent term proportional to the displacement and the quartic potential, do not appear in the Anderson-Holstein model.

D. Resonator frequency

The frequency f associated to the resonator in Fig. 2(f) is defined from the quadratic term of the mechanical energy E_m in Eq. (4) via $\alpha_2 z^2 = \frac{1}{2} m (2\pi f)^2 z^2$. Here, $m = WL_0\rho$, which approximates the mass $WL_u\rho$ of the suspended part of the graphene ribbon, with the surface mass density of graphene being $\rho = 7.61 \times 10^{-7}$ kg/m². From these, the frequency is expressed as

$$f = \frac{1}{\pi L_0} \sqrt{\frac{Y u_{xx,0}}{\rho}}. \quad (11)$$

E. Extrinsic forces can be much stronger than intrinsic ones

The force acting on a singly occupied suspended quantum dot due to the extrinsic electron-phonon interaction, i.e., due to the electric field induced by the top and bottom gates, is estimated as $F_e = e(V_t - V_b)/2d$. Inserting the characteristic values $V_t - V_b = 1$ V and $d = 150$ nm, we find $F_e = 3.3$ meV/nm. On the other hand, in equilibrium, the force acting on the quantum dot due to the dominant intrinsic electron-phonon coupling, that is, the deformation potential mechanism, is $F_i = \{\partial_z[\Xi u_{xx}(z)]\}_{z=z_{\text{eq}}} \approx 4\Xi z_{\text{eq}}/L_0^2$. Here, $\Xi = 30$ eV is the in-plane deformation potential of graphene [42,43], we used Eq. (3), and we present the leading-order result in the small quantities z_{eq}/L_0 and $u_{xx,0}$. For the characteristic values of $z_{\text{eq}} = 1$ nm and $L_0 = 1.5$ μ m [see Fig. 2(d)], we find $F_i \approx 0.05$ meV/nm, which indeed fulfills $F_i \ll F_e$. In the suspended quantum dot model, we

have neglected the energy contribution of the intrinsic electron-phonon interaction, and that simplification is justified by these quantitative estimates.

V. CONCLUSIONS

In conclusion, we suggested a way to engineer an electromechanical system that exhibits effective electron-electron attraction. Our study, focused on the example of a suspended quantum dot in a graphene nanoribbon, supports the experimental feasibility of observing the remarkable but so far elusive equilibrium and transport phenomena implied by the attractive nature of the interaction. Furthermore, our work suggests the possibility that certain functionalities of superconducting nanostructures can be achieved by substituting the superconducting elements with appropriately assembled electromechanical systems. These results raise interesting questions regarding, e.g., the feasibility of realizing an electron-based quantum simulator of the attractive Hubbard model, or the design of artificial superconductors based on engineered electromechanical systems as building blocks.

ACKNOWLEDGMENTS

We thank G. Rastelli, P. Simon, and S. Weiss for useful discussions. G.S. acknowledges financial support of the National Research, Development and Innovation Office of Hungary via the National Quantum Technologies Program Grant No. NKP-2017-00001 and the OTKA Grant No. 108676. A.P. acknowledges funding from the EU Marie Curie Career Integration Grant No. CIG-293834 (CarbonQubits), the OTKA Grants No. 105149 and No. PD 100373, and the EU ERC Starting Grant CooPairEnt No. 258789. M.D. acknowledges funding from the Deutsche Forschungsgemeinschaft (DFG, German Research Foundation) within Project No. 317796071.

-
- [1] H. K. Onnes, Commun. Phys. Lab. Univ. Leiden **119b** (1911).
 [2] J. Bardeen, L. N. Cooper, and J. R. Schrieffer, *Phys. Rev.* **108**, 1175 (1957).
 [3] A. Hamo, A. Benyamini, I. Shapir, I. Khivrich, J. Waissman, K. Kaasbjerg, Y. Oreg, F. von Oppen, and S. Ilani, *Nature (London)* **535**, 395 (2016).
 [4] W. A. Little, *Phys. Rev.* **134**, A1416 (1964).
 [5] G. Cheng, M. Tomczyk, Sh. Lu, J. P. Veazey, M. Huang, P. Irvin, S. Ryu, H. Lee, C.-B. Eom, C. S. Hellberg, and J. Levy, *Nature (London)* **521**, 196 (2015).
 [6] G. Cheng, M. Tomczyk, A. B. Tacla, H. Lee, Sh. Lu, J. P. Veazey, M. Huang, P. Irvin, S. Ryu, Ch.-B. Eom, A. Daley, D. Pekker, and J. Levy, *Phys. Rev. X* **6**, 041042 (2016).
 [7] M. R. Butler, B. Movaghar, T. J. Marks, and M. A. Ratner, *Nano Lett.* **15**, 1597 (2015).
 [8] P. W. Anderson, *Phys. Rev. Lett.* **34**, 953 (1975).
 [9] G. A. Baraff, E. O. Kane, and M. Schluter, *Phys. Rev. Lett.* **43**, 956 (1979).
 [10] G. A. Baraff, E. O. Kane, and M. Schluter, *Phys. Rev. B* **21**, 5662 (1980).
 [11] M. Lannoo, G. A. Baraff, M. Schluter, and D. Tomanek, *Phys. Rev. B* **44**, 12106 (1991).
 [12] J. Koch, M. E. Raikh, and F. von Oppen, *Phys. Rev. Lett.* **96**, 056803 (2006).
 [13] T. Ojanen, F. C. Gethmann, and F. von Oppen, *Phys. Rev. B* **80**, 195103 (2009).
 [14] J. Koch, E. Sela, Y. Oreg, and F. von Oppen, *Phys. Rev. B* **75**, 195402 (2007).
 [15] M.-J. Hwang, M.-S. Choi, and R. Lopez, *Phys. Rev. B* **76**, 165312 (2007).
 [16] X. Jehl, M. Sanquer, R. Calemczuk, and D. Mailly, *Nature (London)* **405**, 50 (2000).
 [17] T.-F. Fang, S.-F. Zhang, C.-J. Niu, and Q.-f. Sun, *Europhys. Lett.* **105**, 47006 (2014).
 [18] H. Park, J. Park, A. K. L. Lim, E. H. Anderson, A. P. Alivisatos, and P. L. McEuen, *Nature (London)* **407**, 57 (2000).
 [19] S. Kubatkin, A. Danilov, M. Hjort, J. Cornil, J.-L. Bredas, N. Stuhr-Hansen, P. Hedegard, and T. Bjornholm, *Nature (London)* **425**, 698 (2003).
 [20] G. A. Steele, A. K. Huttel, B. Witkamp, M. Poot, H. B. Meerwaldt, L. P. Kouwenhoven, and H. S. J. van der Zant, *Science* **325**, 1103 (2009).
 [21] B. Lassagne, Y. Tarakanov, J. Kinaret, D. Garcia-Sanchez, and A. Bachtold, *Science* **325**, 1107 (2009).

- [22] A. Benyamini, A. Hamo, S. V. Kusminskiy, F. von Oppen, and S. Ilani, *Nat. Phys.* **10**, 151 (2014).
- [23] G. Luo, Z.-Z. Zhang, G.-W. Deng, H. ou Li, G. Cao, M. Xiao, G.-C. Guo, and G.-P. Guo, *Nanoscale* **9**, 5608 (2017).
- [24] J. Scott Bunch, A. M. van der Zande, S. S. Verbridge, I. W. Frank, D. M. Tanenbaum, J. M. Parpia, H. G. Craighead, and P. L. McEuen, *Science* **315**, 490 (2007).
- [25] A. Eichler, J. Moser, J. Chaste, M. Zdrojek, I. Wilson-Rae, and A. Bachtold, *Nat. Nanotechnol.* **6**, 339 (2011).
- [26] M. T. Allen, J. Isert, and A. Yacoby, *Nat. Commun.* **3**, 934 (2012).
- [27] C. Chen, V. V. Deshpande, M. Koshino, S. Lee, A. Gondarenko, A. H. MacDonald, P. Kim, and J. Hone, *Nat. Phys.* **12**, 240 (2016).
- [28] The pixel structure in Figs. 2(d) and 3(b) is due to data being obtained on a δV - \bar{V} grid of size 36×30 .
- [29] S. Braig and K. Flensberg, *Phys. Rev. B* **68**, 205324 (2003).
- [30] J. Koch and F. von Oppen, *Phys. Rev. Lett.* **94**, 206804 (2005).
- [31] R. Leturcq, C. Stampfer, K. Inderbitzin, L. Durrer, C. Hierold, E. Mariani, M. G. Schultz, F. von Oppen, and K. Ensslin, *Nat. Phys.* **5**, 327 (2009).
- [32] R. De Alba, F. Massel, I. R. Storch, T. S. Abhilash, A. Hui, P. L. McEuen, H. G. Craighead, and J. M. Parpia, *Nat. Nanotechnol.* **11**, 741 (2016).
- [33] D. Davidovikj, J. J. Slim, S. J. Cartamil-Bueno, H. S. J. van der Zant, P. G. Steeneken, and W. J. Venstra, *Nano Lett.* **16**, 2768 (2016).
- [34] P. Weber, H. L. Calvo, J. Bohle, K. Go, C. Meyer, M. R. Wegewijs, and C. Stampfer, *Nano Lett.* **15**, 4417 (2015).
- [35] A. Castellanos-Gomez, V. Singh, H. S. J. van der Zant, and G. A. Steele, *Ann. Phys.* **527**, 27 (2015).
- [36] P. Recher, E. V. Sukhorukov, and D. Loss, *Phys. Rev. B* **63**, 165314 (2001).
- [37] L. Hofstetter, S. Csonka, J. Nygard, and C. Schonenberger, *Nature* **461**, 960 (2009).
- [38] L. G. Herrmann, F. Portier, P. Roche, A. L. Yeyati, T. Kontos, and C. Strunk, *Phys. Rev. Lett.* **104**, 026801 (2010).
- [39] D. S. Saraga and D. Loss, *Phys. Rev. Lett.* **90**, 166803 (2003).
- [40] C. Lee, X. Wei, J. W. Kysar, and J. Hone, *Science* **321**, 385 (2008).
- [41] Y. Nazarov and Y. Blanter, *Quantum Transport*, (Cambridge University Press, New York, 2009).
- [42] H. Suzuura and T. Ando, *Physica E* **6**, 864 (2000).
- [43] M. Droth and G. Burkard, *Phys. Status Solidi RRL* **10**, 75 (2016).