MODEL OF THE OPTIMAL ALLOCATION OF HETEROGENEOUS RESOURCES IN A VERTICALLY INTEGRATED COMPANY

Sysoiev Volodymyr
PhD, Professor, Faculty of Economics, National technical University "Kharkiv Polytechnic Institute", Ukraine
e-mail: sysoevvv@ukr.net

SUMMARY
One of the main problems of management in a vertically integrated company – the allocation of heterogeneous resources between its organizational units in conditions a limited budget is considered. Mathematical models of the optimal allocation of heterogeneous resources are proposed, taking into account the importance of different types of resources and priorities of organizational units of the company that take into account the types of tasks they perform and the types of activity.

1. INTRODUCTION

Vertically integrated companies have a hierarchical structure, which stipulates the centralized management of the resource support of subordinate organizational units (departments, branches, subsidiaries). One of the main problems of resource management in vertically integrated companies is the allocation of heterogeneous resources between their organizational units in a limited budget, taking into account the priority of organizational units and the importance of different types of resources.

The activities of the company's organizational units are considered from the point of view of implementing a set of activities in the performance of tasks assigned to them. At the beginning of each planning period, organizational units form and submit to the central office of the company their primary needs with reference to tasks and activities and detailing to a specific resource type. The central office of the company in the process of resource planning establishes limits on the costs for all types of resources in the implementation of each type of activities to perform the tasks. The amount of the cost limit for individual organizational units should not exceed the funding limit established by the central office for the planning period.

In conditions of limited funding, it is advisable to establish the needs of the organizational units of the company in each type of resources at the minimum and maximum levels, which ensures the performance of the minimum necessary and full scope of tasks assigned to them, respectively.

The problem of optimal allocation of limited heterogeneous resources is the most in demand in various spheres of management and is considered in the framework of the theory of operations research (Jensen & Bard, 2003; Taha, 2007). Gurin et al. (1968) proposed methods and models for the optimal allocation of resources in one- and two-index problems. Barkalov et al. (2002) used the dual network model to solve the problem of optimal allocation of resources taking into account the time of their displacement. Prilutsky & Vlasov (2004) proposed to use in the tasks of calendar and volume-calendar planning for optimal allocation of resources, approximate algorithms, called "front" algorithms of limited search. Grigoriev et al. (2005) developed a mathematical model for the optimal allocation of heterogeneous resources in production by the criterion of minimum deviation from the norms. Romanenkov et al. (2016) proposed an interval variant of the matrix model for assessing the relative efficiency level of hierarchic system of business processes of the company, which allowed to reduce the problem of optimal resource allocation to the linear programming problem with

interval given target function. Gorbaneva (2006), Sacks & Harel (2006), Korgin (2012) used game-theoretic models to solve resource allocation problems.
However, in these studies, the distribution models for heterogeneous resources, taking into account the importance of resources and the priorities of their consumers, were not considered.

2. MATHEMATICAL MODEL

Let’s review the following basic parameters of the model:
C^0 – funds allocated for purchase of resources for all organizational units of the company;
K – number of organizational units of the company;
N – number of types of tasks that are performed by organizational units of the company;
M – number of types of activities that are carried out by organizational units of the company in the performance of their assigned tasks;
S – number of resource types required for the activities of the company's organizational units;
$a_{kmns}^{\min}, k=\overline{1,K}; m=\overline{1,M}; n=\overline{1,N}; s=\overline{1,S}$ – minimum demand of the k-th organizational unit in resources of type s, necessary for the implementation activity of type m when performing the task of type n;
$a_{kmns}^{\max}, k=\overline{1,K}; m=\overline{1,M}; n=\overline{1,N}; s=\overline{1,S}$ – maximum demand of the k-th organizational unit in resources of type s, necessary for the implementation activity of type m when performing the task of type n;
$u_k, k=\overline{1,K}$ – priority coefficient of the k-th organizational unit;
$v_{ks}, k=\overline{1,K}; s=\overline{1,S}$ – importance coefficient of resources of type s, which are used to provide activities of the k-th organizational unit;
Variables of the model are as follows:
$x_k, k=\overline{1,K}$ – required amount of all types of resources (in value terms) allocated to the k-th organizational unit;
$y_{ks}, k=\overline{1,K}; s=\overline{1,S}$ – required amount of financial resources allocated for purchase resources of type s for the k-th organizational unit.
It’s obvious that:

$$a_{ks}^{\min} = \sum_{m=1}^{M}\sum_{n=1}^{N} a_{kmns}^{\min}, k=\overline{1,K}; s=\overline{1,S}, \tag{1}$$

$$a_{ks}^{\max} = \sum_{m=1}^{M}\sum_{n=1}^{N} a_{kmns}^{\max}, k=\overline{1,K}; s=\overline{1,S} \tag{2}$$

represent respectively the minimum and maximum demands of the k-th organizational unit in resources of type s, and

$$a_{k}^{\min} = \sum_{s=1}^{S} a_{ks}^{\min}, k=\overline{1,K}, \tag{3}$$

$$a_{k}^{\max} = \sum_{s=1}^{S} a_{ks}^{\max}, k=\overline{1,K} \tag{4}$$

are respectively the minimum and maximum demands of the k-th organizational unit in all types of resources.

Note that if the k-th organizational unit does not participate in the performance of the n-th task, then $a_{kmns} = 0,\ m = \overline{1,M};\ s = \overline{1,S}$, if an activity of type m is not realized in this case, then $a_{kmns} = 0,\ s = \overline{1,S}$, if the same does not require resources of type s, then $a_{kmns} = 0$.
At the first stage of the solution of the problem, we present estimates of heterogeneous resources in value units, which is due to the following reasons:

- constructing a mathematical model for the optimal allocation of heterogeneous resources directly through resource variables (for example, $x_{kmns},\ k = \overline{1,K};\ m = \overline{1,M};\ n = \overline{1,N};\ s = \overline{1,S}$) leads to a complex four-index optimization problem;
- accounting simultaneously the priority of organizational units and the importance of heterogeneous resources for each of the types of tasks performed and the types of activities implemented leads to a labor-intensive system of restrictions;
- use of the solution obtained $X = \left\| x_{kmns} \right\|_{K,M,N,S}$ can cause difficulties in changing environmental factors due to excessive regulation of the control.

The strategy for allocating the limited budget of a vertically integrated company to the purchase of resources should be defined in such a way, so that the demands of the organizational units that have the highest priority, taking into account the limitations on their minimum and maximum provision, were most satisfied.
On this basis, we formulate the first optimization problem: it is required to determine the optimal plan for allocating budgetary funds by the criterion of the maximum of the total value of the financial resources allocated to ensure the solution of the tasks that the organizational units of the company face in view of their priorities.
The mathematical model of this problem is as follows:

$$F(x) = \sum_{k=1}^{K} u_k x_k \to \max; \tag{5}$$

$$\sum_{k=1}^{K} x_k \le C^0;$$

$$a_k^{\min} \le x_k \le a_k^{\max},\ k = \overline{1,K};$$

$$u_k \ge 0,\ k = \overline{1,K};$$

$$\sum_{k=1}^{K} u_k = 1.$$

The problem (5) is a linear programming problem and can be solved by standard procedures of the simplex method.
To determine the priority coefficients of organizational units of the company, it is proposed to use the hierarchy analysis method, whose essence consists in decomposition of the problem into simpler components and subsequent processing of the sequence of judgments based on paired comparisons, which allows expressing the relative degree of interaction of elements in the hierarchy (Saaty, 1988).
Consider in general form the decomposition of the problem of determining the priority coefficients of the organizational units of the company (Fig. 1). At the first level – the goal: “Determination of the priority coefficients (weights) of organizational units”. At the second

level are the criteria specifying the goal, i.e. the tasks that are performed by organizational units: T-1, T-2,..., T-*N*. At the third level, there are sub-criteria that reflect the types of activity of the organizational units that provide the tasks: A-1, A-2,..., A-*M*. At the fourth level are the objects being compared, i.e. organizational units: OU-1, OU-2,..., OU-*K*, the priority coefficients (weights) of which are to be determined.

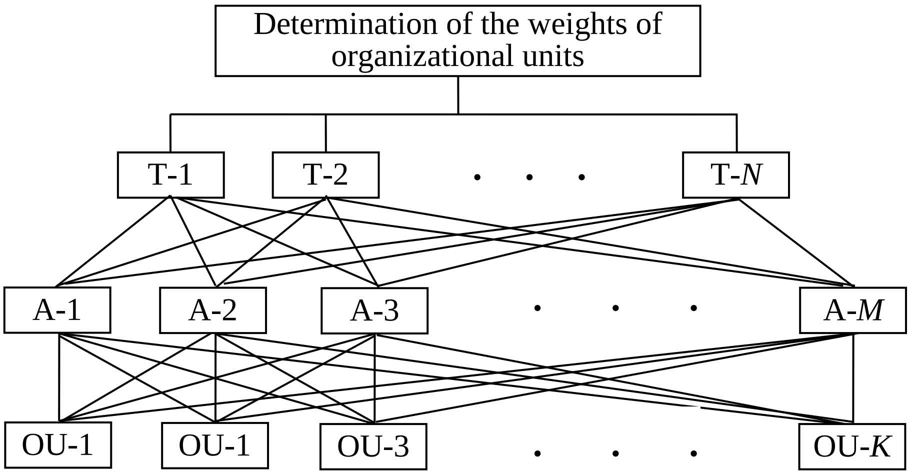

Figure 1. Four-level hierarchical model for determining the priority coefficients (weights) of organizational units of the company

Source: own creation

According to the algorithm of the hierarchy analysis method, at the first step we determine the weighting coefficients of the problems – the vector $\eta = [\eta_1, \eta_2, \ldots, \eta_N]$. At the second step, the local weighting coefficients of the activities are determined – the matrix $\Psi = \left\| \psi_{mn} \right\|_{M,N}$, and then the vector of the weighting coefficients of the activities – $\psi = [\Psi\eta]$. In the third step, the matrix of local weights of organizational units is determined similarly –$U = \left\| u_{km} \right\|_{K,M}$, and then the vector of global weights of organizational units – $u = [U\psi]$. All steps of the algorithm are accompanied by calculations of the indices and assessments of the consistency of expert judgment.

As a result of solving the problem (5), the optimal budget allocation plan was obtained from the point of view of the selected efficiency criterion:

$$x^* = [x_1^*, x_2^*, \ldots, x_K^*]. \tag{6}$$

In accordance with the plan (6), company resource management services strive to purchase on the funds allocated to each organizational unit such necessary resources that would best meet their demands, taking into account the importance of resource types and restrictions on minimum and maximum provision for each type.

Thus, in the second stage, the problem of the optimal allocation of financial resources allocated to each organizational unit – $x_k^* (k = \overline{1,K})$ for the purchase of dissimilar resources that they need is solved. As a criterion for optimization, we will select the criterion of the maximum of the total value of financial resources allocated for the purchase of heterogeneous resources, taking into account their importance for the activities of organizational units.

The mathematical model of this problem has the form:

$$G_k(y) = \sum_{s=1}^{S} v_{ks} y_{ks} \to \max ; \tag{7}$$

$$\sum_{s=1}^{S} y_{ks} \le x_k^* ;$$

$$a_{ks}^{\min} \le y_{ks} \le a_{ks}^{\max},\ k = \overline{1,K}; s = \overline{1,S} ;$$

$$v_{ks} \ge 0, k = \overline{1,K}; s = \overline{1,S} ;$$

$$\sum_{s=1}^{S} v_{ks} = 1, k = \overline{1,K} .$$

The importance coefficients of types of resources for each organizational unit are determined using the following four-level hierarchy (Fig. 2). At the first level – the goal: "Determination the importance coefficients (weights) of types resources for the *k*-th organizational unit". At the second level are the tasks performed by this organizational unit: T_k-1, T_k-2,..., T_k-*N*. At the third level – its types of activity: A_k-1, A_k-2, A_k-3,..., A_k-*M*. At the fourth level, At the fourth level, the types of resources needed for this organizational unit: R_k-1, R_k-2, R_k-3,..., R_k-*S*.

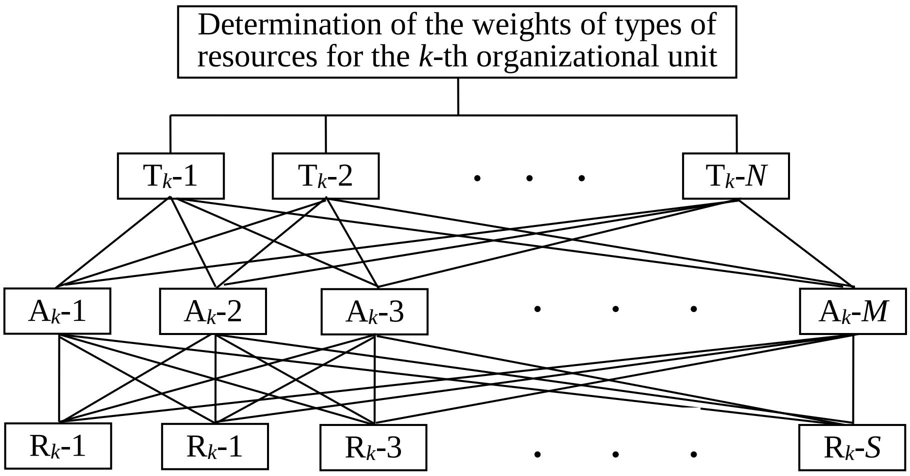

Figure 2. Four-level hierarchical model for determining the importance coefficients of types of resources for the *k*-th organizational unit
Source: own creation

The algorithm for determining the importance coefficients of different types of resources is similar to the algorithm for determining the priority coefficients of organizational units described above.
The problem (7) is also a linear programming problem and is solved by standard procedures of the simplex method. The matrix of optimal allocation plans for all organizational units of a vertically integrated company:

$$Y^* = \left\| y_{ks}^* \right\|_{K,S} \tag{8}$$

is used to purchase heterogeneous resources, based on the unit cost of resources of each type c_{ks}^{o}:

$$x_{ks}^{*} = \frac{y_{ks}^{*}}{c_{ks}^{o}},\ k = \overline{1,K}; s = \overline{1,S}. \qquad (9)$$

3. CONCLUSIONS

The problem of allocation heterogeneous resources of different importance between the multi-priority organizational units with a limited budget is very important for vertically integrated companies in countries with transformational economies.

The proposed mathematical models for solving this problem allow to optimally allocate the heterogeneous resources of the company among its various organizational units in conditions a limited budget, taking into account their minimum and maximum demands, the importance of different types of resources and the priorities of organizational units that take into account the types of tasks they perform and the types of their activities.

In order to create a logically complete mathematical apparatus for the optimal allocation of heterogeneous resources in vertically integrated companies, further research should be directed toward solving the problem of assessing the effect of the allocation plan of different types of resources on the efficiency of its multi-priority organizational units.

REFERENCES

Barkalov P.S., Burkova I.V., Glagolev A.V. & Kolpachev V.N. (2002): *Tasks of resource allocation in project management*, Moscow: ICS RAS.

Gorbaneva O.I. (2006): Cooperative-game simulation of resource allocation. News of higher educational institutions. The North Caucasus region. Natural Sciences. Appendix, 2, pp. 10-16.

Grigoryev V.P., Kalyuta V.N. & Kiselev K.A. (2005): Model of the optimal distribution of resources in production. *Proceedings of the Tomsk Polytechnic University*, 308 (5), pp. 179-181.

Gurin L.S., Dymarsky Ya.S. & Merkulov A.D. (1968): *Tasks and methods for optimal allocation of resources*, Moscow: Soviet Radio.

Jensen P.A., Bard J.F. (2003): *Operations research models and methods*, New York: John Wiley & Sons Incorporated.

Korgin N.A. (2012): Presentation of the mechanism of sequential resource allocation as a non-manipulable mechanism of multi-criteria active expertise. *Management of large systems: a collection of proceedings*, ICS RAS, 36, pp. 186-208.

Prilutskii M.Kh., Vlasov S.E. (2004): Optimal allocation of resources in the tasks of calendar and volume-calendar planning. *Proceedings of Nizhny Novgorod State Technical University. Information processing and management systems*, 11, pp. 31-36.

Romanenkov Yu.A., Vartanyan V.M. & Zeyniev T.G. (2016): Interval optimization model of resource allocation in the business process system of the company. *Mathematical modeling and computer modeling. A series of technical sciences*, 13, pp. 126-138.

Saaty T.L. (1988): *Multicriteria decision making : the analytic hierarchy process; planning, priority setting, resource allocation*. 2nd ed., New York: McGraw-Hill.

Sacks R. & Harel M. (2006): An economic game theory model of subcontractor resource allocation behavior. *Construction Management and Economics*, 24 (8), pp. 869-881.

Taha H.A. (2007): *Operations Research: An Introduction*. 8th ed., New Jersey: Pearson Prentice Hall, Upper Saddle River.