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Optimizing the Number and Location of Warehouses in Logistics Networks Considering the Optimal Delivery Routes and Set Level of Reserve Stock

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SUMMARY

Forming the optimal structure of a warehouse network is one of the main strategic tasks in arranging an efficient logistics network. The suggested mathematical model and iterative method allow an optimal number and location of warehouses (in the area served by the logistics network) to be defined – with the minimum of total logistics costs for shipping of goods from suppliers through warehouses to customers. The focus is on optimal delivery routes and the optimal level to set for reserve stock contained in the storage network.

Keywords: logistics network, warehouse, optimal warehouse location, logistics costs, optimaldelivery route.
Journal of Economic Literature (JEL) code: C21

INTRODUCTION

The globalization of business requires efficient management in the movement of goods between manufacturers, resellers, and consumers, which can only be ensured by the establishment of large multi-format logistics networks. At the heart of setting up any logistics network (dealing with distribution of material flows in the field of supply and distribution logistics), there is the complex task of arranging the optimal structure for its most important subsystem – the warehouse network, which includes defining the number of warehouses and their geographical location in the service area. The solution of this task requires a large amount of initial data: the scope of supply and demand for goods, locations of suppliers and consumers, estimated capacity of warehouses, features of transportation services, logistics costs, etc., which requires the use of economic-mathematical methods and models.

The urgency of the task is due to the fact that the number of warehouses and their locations in the logistics network (with extensive transport infrastructure) have a significant effect on costs involved in the delivery of goods to consumers, thus affecting the final cost of sold products. Given that transport and storage costs show different responses to a change in the number of warehouses and their locations (with regards to producers and consumers), the dependence between total costs for the operation of a logistics network and the number of warehouses is parabolic, i.e., the task can be formulated as a search for an optimal or sub-optimal (close to optimal) solution.

ANALYSIS OF EXISTENT METHODS

The complexity and multi-factor character of this complex task have led to (in the majority of cases) separate solutions for its two components.

The first task – defining the number of warehouses – is solved by the method of economic compromises, regarding the totality of all costs associated with their maintenance. This task is a basis for the establishment of centralized and decentralized distribution of goods. The best-known approach to its solution is based on the qualitative dependence between various logistics costs and numbers of warehouses, which assumes the selection of their optimal number – in the absence of formulas and quantitative characteristics, based on logic and common sense (Coyle et al., 1985). The analytical method of solving this task assumes selecting the number of warehouses with use of a linear programming model, which optimizes the general costs (including the costs for construction of warehouses, costs for shipping of goods to consumers, and costs for warehouse handling) (Dybskay, 2008).

The second task (choosing the warehouse location) was first reviewed within the theory of labor forces siting (production companies) – by German scientist A. Weber, who formulated it as mathematical problem of choosing the lowest transportation costs for shipping of goods between warehouse and a group of spatially distributed consumers (while considering the distance factor only) (Weber, 1929).

With development of logistics as an economic science, there was an increase in the importance of tasks aiming to define the location of warehouses (when forming a spatially distributed logistics network). While formulating the three basic strategies of positioning the distribution warehouses (depending on the principle of their arrangement: near markets, near production sites, or in an intermediate location), well-known "golden rules" of sales logistics still require some mathematical calculations (to determine the specific areas of their placement) (Hoover, 1948).

Later, the methodology of solving a task on defining the location of warehouses (distribution centers) in logistics networks developed towards a combination of economic-mathematical models – due to more complete accounting of influencing factors and economization of its optimizing criteria.

A conceptual vision of the task (spatial location of warehouses in logistics network) and the main approaches to its solution (regarding the possible restrictions, optimizing and multi-purpose nature of the task) are presented in Tansel et al. (1983a, 1983b). A typology of tasks (placing the objects in a logistics network depending on management objectives), parameters affecting their solution, and performance indicators were reviewed in Brandeau and Chiu (1989).

Depending on the nature of variation, they distinguish the methods of solving a continuous and discrete task (defining the location of warehouses). In one case, the warehouse may be located anywhere (in the area under review); in the second case it should be at pre-defined locations, which is more realistic (given the actual transportation services of logistics networks).

Methods of solving the continuous task have included the center of gravity method for cargo flows (Bowersox, 1974) and,,based upon it, the method of the equilibrium system center, which accounts for transportation costs (serving as weight factors) (Coyle et al., 1985; Mirotin, 2002), as well as the method of searching for minimum transport work (Sergueyev, 1997). In methods based on the "mass center of a physical body", distances (even average-weighted) are determined by coordinate axes (considered in straight lines), which requires the imposition of a coordinate grid on the map of potential warehouse locations and is effective only if the area under review is provided with a developed network of transportation services. The fundamental difference of a method searching for minimum transport work lies in defining the distance between objects as a "hypotenuse", as well as in applying the iterative algorithm of combined search, which allows the detection of optimal warehouse location (through successive evaluation of options).

The majority of methods for solving a discrete task (defining the warehouse location) are not optimizing methods. The method of factor-rating systems lies in a point-based assessment of factors affecting the choice of warehouse location; despite the possibility of accounting for qualitative indicators, this method has the significant drawback of all expert methods – subjectivity in defining the point scale and in assessment of factors (their weight ratios) (Wild, 1995). The autoregressive method, which enables more strict definition of the most important features (during selection of warehouse location), requires a large scope of statistical material and does not allow the dependence of variables (multicollinearity of factors) (Chase et al., 1998). In some scientific papers, the solution of a discrete problem is viewed using a method of defining the zones of influence (on consumers), which are used in marketing: the method of isochronous lines (Engel, 1995), method of potential sales areas (Tjapuhin, 2001), method of identifying and segmenting the trade zones (Yager, 1982). However, all of these are characterized by a high degree of subjectivity and do not ensure an optimal solution.

The majority of the above-mentioned methods can solve either the task of defining the number of warehouses or the task of defining their location; at that, the warehouse is usually viewed only as a source of material flow, which does not reflect the specifics of arranging the logistics networks, where the warehouse is a link between suppliers and consumers (both in the field of supply and sales).

In this view, particular interest is caused by methods for solving a complex task which consider the interaction of warehouses with all members of a logistics network. As a rule, such methods assume the shared use of optimization models and heuristic methods, which adjust the obtained theoretical solutions based on actual network infrastructure, transportation services, features of vehicles, possible variations in demand, level of reserve stock, time restrictions, etc.

There are several approaches to the formulation of a complex task, which differ in optimization criteria: the first one focuses on distance characteristics (Wilson, 1974); the second on cost characteristics, while using the full cost of the storage network (Giddings et al., 2000) or total transport work (Lukinskiy (Ed.), 2007) or total transport and storage costs as indicators (Khumawala and Kelly, 1974); the third one considers time characteristics (O'Kelly, 1986); and the fourth (complex) one simultaneously considers multiple characteristics (Cheong et al., 2007; Salihov, 2007).

Among the main drawbacks of actual approaches to solving a complex task, we should distinguish among one-criterion character of models, two-stage optimization of the delivery process (before/after the warehouses), and a snap-to-coordinate grid, which, apart from additional heuristic procedures required to specify the location of warehouses, also cannot consider the diversity of actual transportation services (in the area served by the logistics network) or define the optimal routes for door-to-door delivery of goods (from suppliers, through warehouses, and to consumers).

The purpose of this paper is to develop a model and method for optimizing the number and location of warehouses – regarding optimal routes of cargo delivery in the logistics network, storage costs, andthe set scope of reserve stock of goods at warehouses.

MODEL

Let us review the logistics network comprising of suppliers I and consumers J (for a certain kind of goods) and having geographical pointsMpossible locationsof warehouses (for their storage and handling).

Initial data for the task:

$W_i\left(i=\overline{1,I}\right)$ – scope of supplying the goods from the i-th supplier;

$V_j\left(j=\overline{1,J}\right)$ – scope of demand in goods of the j-th consumer, while $\sum_{i=1}^{I} W_i \geq \sum_{j=1}^{J} V_j$;

$\mu_i\left(i=\overline{1,I}\right)$, $\beta_j\left(j=\overline{1,J}\right)$ – weight factors of the i-th supplier and j-th consumer, respectively (reflecting the additional factors that affect the plan on optimal attachment of consumers to suppliers – e.g., impossibility of direct transit (warehouse) supplies or their priority with regards to other supplies);

$d_{ij}\left(i,j=\overline{1,M}\right)$ – distance between all points of the logistics network (linked by relevant transportation services);

Z – scope of total stock reserve for certain type of goods, which must always be maintained in warehouses of the logistics network under review.

It is necessary to determine the optimal number of warehouses and their locations in potential points of logistics network at the smallest possible total of logistics (transportation and storage) costs. Cost indicatorsare shown inconventional monetary units (CMU).

Regarding the minimizing nature of optimality criterion, it would be appropriate to convert and normalize the weight factors:

$$\mu_i' = \frac{1}{\mu_i}; i=\overline{1,I}; \qquad \beta_j' = \frac{1}{\beta_j}; j=\overline{1,J};$$

$$\bar{\mu}_i = \frac{\mu_i'}{\sum_{i=1}^{I}\mu_i'}; \qquad \bar{\beta}_j = \frac{\beta_j'}{\sum_{j=1}^{J}\beta_j'}.$$

A mathematical model of the task (defining the number and location of warehouses in the logistics network, based on actual transportation services and the need to maintain reserve stocks in the warehouse network) looks like:

$$B(X,Y) = \left[\sum_{i=1}^{I}\sum_{k=1}^{N} d_{ik}\bar{\mu}_i x_{ik} + \sum_{k=1}^{N}\sum_{j=1}^{J} d_{kj}\bar{\beta}_j y_{kj}\right]T_0 + \varphi\left[G_{k_1}^{(N)}; G_{k_2}^{(N)}; ...; G_{k_N}^{(N)}\right] \rightarrow \min,$$

$$\sum_{k=1}^{N} x_{ik} \leq W_i; i=\overline{1,I};$$

$$\sum_{i=1}^{I} x_{ik} \leq G_k; k=\overline{1,N}; \qquad (1)$$

$$\sum_{i=1}^{I} x_{ik} - \sum_{j=1}^{J} y_{kj} \geq \frac{Z}{N}; k=\overline{1,N};$$

$$\sum_{k=1}^{N} y_{kj} = V_j; j=\overline{1,J};$$

$$x_{ik} \geq 0; i=\overline{1,I}; k=\overline{1,N};$$

$$y_{kj} \geq 0; k=\overline{1,N}; j=\overline{1,J},$$

where B(X,Y) – total logistics costs for transportation of goods from suppliers to warehouses and from warehouses to suppliers (including the storage costs);

N – required number of warehouses in points of the logistics network;

d_{ik}, d_{kj} – distances between suppliers and warehouses, warehouses and consumers (accordingly);

$X = \|x_{ik}\|_{I,N}$ – matrix – scopes of goods transported from suppliers to warehouses;

$Y = \|y_{kj}\|_{N,J}$ – matrix – scopes of goods transported from warehouses to suppliers;

T_0 – freight rate, CMU/t·km;

$\varphi[G_{k_1}^{(N)}; G_{k_2}^{(N)}; ...; G_{k_N}^{(N)}]$ – function of storage costs (depends on number and capacity of warehouses);

$G_k^{(N)}; k=\overline{1,N}$ – capacity of the k-th warehouse (at N-th number of warehouses).

Ratios – equations and inequations of the model include:

- limitations on the total export of goods from each supplier (to warehouses);
- limitations on the total import of goods from suppliers (to each warehouse);
- limitations on the size of reserve stock (at each warehouse);
- limitationson the needsof consumers andnon-negativityconditionson thequantity ofthe goods transported.

Task (1) assumes mathematical programming; its solution would cause no trouble if the location and number of warehouses (N) were known. For large logistics networks, determining a solution via sorting the possible number of warehouses and their locations is unacceptable (due to the inevitably large number of solution options).

METHOD

We suggest an iterative method of solving the task (the algorithm is shown in Fig. 1).

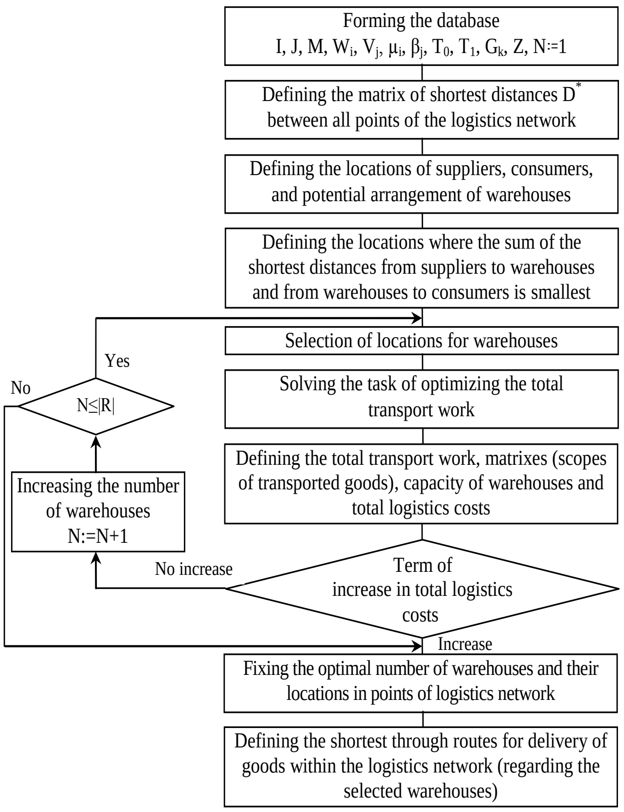

Figure 1. Algorithm of optimizing the number and location of warehouses in logistics network

Let us review in detail the main stages of the suggested task-solving procedure:

First stage. Let us define the matrix of distances between points of logistics network $D = \|d_{ij}\|_{M,M}$ as follows:

d_{ij}, if points i and j are directly linked to each other;
$D_{ij}= \infty$, if points i and j are not directly linked;
0, if i = j.

Second stage. Let us define the matrix of shortest distances between all points of the logistics network $D^* = \|d^*_{ij}\|_{M,M}$, while using a Bellman–Shimbel algorithm of search for the shortest path (Golstein, 1966):

$$d_{ij}^{2r} = \min_{1\le\lambda\le M}[d_{i\lambda}^{r} + d_{\lambda j}^{r}],\ r = 1,2,... \quad (2)$$

For convenience of recording the expressions, let us assume that suppliers are located in points: 1,2,…,I and consumers – in points: M-J+1,M-J+2,…,M.

Let us define the set R as a subset of logistics network points {I+1,I+2,…,M-J} (possible locations of candidate warehouses) regarding the various constraints: geographic, economic, technical, social, etc. Thus, the warehouses can be placed in locations: $R \subseteq \{I+1, I+2,..., M-J\}$.

Third stage. Let us define the points of logistics network $k_s \in R; s = \overline{1,|R|}$, where the sum of the shortest distances (from suppliers to warehouses and from warehouses to consumers) is minimal:

$$k_1 = \arg\min_{k_s\in R}\Big[\sum_{i=1}^{I} d^*_{ik_s} + \sum_{j=M-J+1}^{M} d^*_{k_s j}\Big];$$

$$k_2 = \arg\min_{\substack{k_s\in R\\ k_s\neq k_1}}\Big[\sum_{i=1}^{I} d^*_{ik_s} + \sum_{j=M-J+1}^{M} d^*_{k_s j}\Big];$$

$$\vdots \quad (3)$$

$$k_{|R|-1} = \arg\min_{\substack{k_s\in R\\ k_s\neq k_1; k_s\neq k_2,...,\\ k_s\neq k_{|R|-1}}}\Big[\sum_{i=1}^{I} d^*_{ik_s} + \sum_{j=M-J+1}^{M} d^*_{k_s j}\Big];$$

$$k_{|R|}.$$

Fourth stage.The total transportation work can be applied as the objective function instead of the total logistics costs in order to avoid the nonlinearityof the Task (1). The total transportation work is the essential characteristic of the carriage of goods process from suppliers to consumers via warehouses as well. Thus, the solution of the Task (1) can be similarly provided as a solution of the linear programming task:

$$F_1(X,Y) = \sum_{i=1}^{I}\sum_{k=1}^{N} d_{ik}\bar{\mu}_i x_{ik} + \sum_{k=1}^{N}\sum_{j=1}^{J} d_{kj}\bar{\beta}_j y_{kj} \to \min,$$

first, at N=1 ($k = k_1$), let us assume that the initial value of storage capacity is large enough (e.g. $G_{k_1} = \sum_{i=1}^{I} W_i$) and determine (for this case) the total transportation work $F_1^{(1)}$, matrixes (scopes of goods transported from suppliers to warehouse $X^{(1)} = \| x_{ik}^{(1)} \|_{I,1}$ and from warehouse to consumers) $Y^{(1)} = \| y_{kj}^{(1)} \|_{1,J}$, as well as capacity of the warehouse: $G_{k_1}^{(1)} = \sum_{i=1}^{I} x_{ik_1}^{(1)}$.

In this case, the minimum total (average-weighted) logistics costs (storage and transportation) will be:

$$B^{(1)} = [\sum_{i=1}^{I} \overline{\mu}_i L_{ik_1}^{(1)} d_{ik_1}^* + \sum_{j=1}^{J} \overline{\beta}_j L_{k_1j}^{(1)} d_{k_1j}^*] T_1 + \varphi(G_{k_1}^{(1)}) \quad (4)$$

where $L_{ik_1}^{(1)}, L_{k_1j}^{(1)}$ are the number of cargo trips from the i-th supplier to the warehouse and from the warehouse to the j-th consumer, respectively, defined as:

$$L_{ik_1}^{(1)} = [\frac{x_{ik_1}^{(1)}}{q\gamma}]; \quad L_{k_1j}^{(1)} = [\frac{y_{k_1j}^{(1)}}{q\gamma}],$$

where q – rated carrying capacity of vehicle used for transportation;

γ – factor of utilizing the carrying capacity of vehicle;

T_1 – freight rate, CMU/km;

$\varphi(G_{k_1}^{(1)})$ – function of storage costs at one warehouse.

Then, while solving Task (1) for its objective function (total transportation work), where N=2 ($k = k_1, k = k_2$), let us define the total transportationwork $F_1^{(2)}$, matrixes (scopes of goods delivered from suppliers to two warehouses: $X^{(2)} = \| x_{ik}^{(2)} \|_{I,2}$ and from warehouses to consumers: $Y^{(2)} = \| y_{kj}^{(2)} \|_{2,J}$), as well as capacities of both warehouses: $G_{k_1}^{(2)} = \sum_{i=1}^{I} x_{ik_1}^{(2)}$, $G_{k_2}^{(2)} = \sum_{i=1}^{I} x_{ik_2}^{(2)}$.

In the case of two warehouses, the minimum total (average-weighted) logistic costs (storage and transportation) will be:

$$B^{(2)} = [\sum_{i=1}^{I} \sum_{s=1}^{2} \overline{\mu}_i L_{ik_s}^{(2)} d_{ik_s}^* + \sum_{s=1}^{2} \sum_{j=1}^{J} \overline{\beta}_j L_{k_sj}^{(2)} d_{k_sj}^*] T_1 + \phi[G_{k_1}^{(2)}; G_{k_2}^{(2)}] \quad (5)$$

where $L_{ik_s}^{(2)}, L_{k_sj}^{(2)}; s = \overline{1,N}$ are the number of cargo trips from the i-th supplier to the two warehouses and from the two warehouses to the j-th user, respectively, defined as:

$$L_{ik_s}^{(2)} = [\frac{x_{ik_s}^{(2)}}{q\gamma}]; s = \overline{1,N}; \quad L_{k_sj}^{(2)} = [\frac{y_{k_sj}^{(2)}}{q\gamma}]; s = \overline{1,N};$$

$\varphi[G_{k_1}^{(2)}; G_{k_2}^{(2)}]$ is thefunction of storage costs at the two warehouses.

Fifth stage. If total logistics costs for one warehouse are less than total logistics costs for two warehouses:

$$B^{(1)} \leq B^{(2)},$$

then one warehouse should be enough for the logistics network under review.

Otherwise, there is a transition to the fourth stage of the algorithm and Task (1) is solved for its objective function (total transportation work), where N=3 ($k = k_1$, $k = k_2$, $k = k_3$), etc., with relevant checks at the fifth stage, either till meeting the term of increase in total logistics costs or till having tried all items in the logistics network (subsets R from I+1 to M-J), which is practically improbable.

Sixth stage. Let us assume that optimal number of warehouses N and their locations in logistics network: $k_1, k_2, ..., k_N$ have been defined. Following the solution of Task (1) for this set of warehouses, we obtain the optimal values of total transportationwork $F_1^{(N)}$, matrixes (scopes of goods transported to/from the warehouses $X^{(N)} = \| x_{ik}^{(N)} \|_{I,N}$, $Y^{(N)} = \| y_{kj}^{(N)} \|_{N,J}$), and capacity of all warehouses in logistics network:

$$G_{k_1}^{(N)} = \sum_{i=1}^{I} x_{ik_1}^{(N)}, \quad G_{k_2}^{(N)} = \sum_{i=1}^{I} x_{ik_2}^{(N)}, \ldots, \quad G_{k_N}^{(N)} = \sum_{i=1}^{I} x_{ik_N}^{(N)}.$$

In this case, the minimum total (average-weighted) logistic costs (storage and transportation) will be:

$$B^{(N)} = [\sum_{i=1}^{I} \sum_{s=1}^{N} \overline{\mu}_i L_{ik_s}^{(N)} d_{ik_s}^* + \sum_{s=1}^{N} \sum_{j=1}^{J} \overline{\beta}_j L_{k_sj}^{(N)} d_{k_sj}^*] T_1 + \phi[G_{k_1}^{(N)}; G_{k_2}^{(N)}; ...; G_{k_N}^{(N)}] \quad (6)$$

where $L_{ik_s}^{(N)}, L_{k_sj}^{(N)}; s = \overline{1,N}$ are the number of cargo trips from the i-th supplier to N warehouses and from N warehouses to the j-th user, respectively, defined as:

$$L_{ik_s}^{(N)} = [\frac{x_{ik_s}^{(N)}}{q\gamma}]; s = \overline{1,N}; \quad L_{k_sj}^{(N)} = [\frac{y_{k_sj}^{(N)}}{q\gamma}]; s = \overline{1,N};$$

$\varphi[G_{k_1}^{(N)}; G_{k_2}^{(N)}; ...; G_{k_N}^{(N)}]$ – function of storage costs at the N-th number of warehouses.

Seventh stage. Using the values of distance matrix D and shortest distance matrix D^*, let us define the optimal through routes for delivery of goods $\{i, \lambda_1, \lambda_2, ..., \lambda_s, j\}$ (from suppliers, through warehouses, and to consumers):

$$l_{i\lambda_1} = \min_{i \le \lambda \le M} [d_{i\lambda} + d^*_{\lambda j}];$$

$$l_{\lambda_1\lambda_2} = \min_{i \le \lambda \le M} [d_{\lambda_1\lambda} + d^*_{\lambda j}]; \quad (7)$$

$$\vdots$$

$$l_{\lambda_s j} = \min_{i \le \lambda \le M} [d_{\lambda_s\lambda} + d^*_{\lambda j}]$$

for $i = \overline{1, I};\ j = k_1 k_2, ..., k_N$, and then for $i = k_1, k_2, ..., k_N;\ j = \overline{M - I + 1, J}$.

AN EXAMPLE OF APPLICATION OF THE MODEL AND METHOD

Let us imagine the implementation of developed the model and method on the example of logistics network consisting of a six consumers, three suppliers (A, B and C) and four pointspossible location ofthe warehouse (I, II, III and IV) (Fig. 2).

Initial data for modeling the supply processare presented in Table 1.

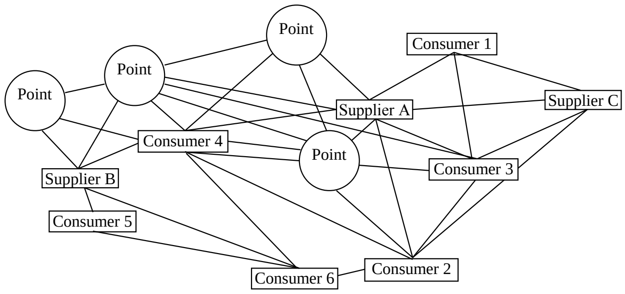

Figure 2.Graphic location scheme elements of logistics network (example)

Table 1

Initial data for modeling the supply process (example)

Parameters	Supplier A	Supplier B	Supplier C	Consumer 1	Consumer 2	Consumer 3	Consumer 4	Consumer 5	Consumer 6
W, t	2500	4000	1600	-	-	-	-	-	-
μ	0,5	0,3	0,2	-	-	-	-	-	-
V, t	-	-	-	800	1200	1500	700	1400	2300
β	-	-	-	0,14	0,18	0,1	0,3	0,08	0,2
q, t	20	20	20	10	10	10	10	10	10
γ	0,87	0,87	0,87	0,9	0,9	0,9	0,9	0,9	0,9
T_0, CMU/t·km	7	7	7	10	10	10	10	10	10

The scope of total stock reserve of goods, which should always be maintained in warehouses of the logistics network, is equal to 60 tons.

Storage costsconsist ofcosts relatedthe Exploitationthe warehouse,the cost of maintainingstocksand administrativecosts.Suppose that the functionstorage costsisdiscrete. Change instorage costs, depending on the capacity of warehouse for four possibleoptions shown in Table 2.

Table2

Change instorage costs, depending on the capacity of warehouse, CMU (example)

Capacity of warehouse, t	15	20	30	60
Costs relatedthe Exploitationthe warehouse, CMU	52000	57000	67000	146000
Costs of maintainingstocks, CMU	20000	24000	34000	68000
Administrativecosts, CMU	24000	27000	36000	64000
Storage costs, CMU	96000	108000	1370000	278000

The matrix of distances between the points of logistics network $D = \|d_{ij}\|_{13,13}$ is shownin Table 3.

The matrix of shortest distances between all points of the logistics network $D^* = \|d^*_{ij}\|_{13,13}$ is shownin Table 4.

Table 3
The distances between the points of logistics network, km

The participants logistics network	Supplier A	Supplier B	Supplier C	Point I	Point II	Point III	Point IV	Consumer 1	Consumer 2	Consumer 3	Consumer 4	Consumer 5	Consumer 6
Supplier A	0	∞	76	111	40	41	∞	42	121	51	106	∞	∞
Supplier B	∞	0	∞	70	∞	∞	68	∞	∞	∞	45	29	72
Supplier C	76	∞	0	∞	∞	∞	∞	68	∞	39	∞	∞	∞
Point I	111	70	∞	0	95	108	22	∞	∞	160	50	∞	∞
Point II	40	∞	∞	95	0	51	∞	∞	∞	∞	84	∞	∞
Point III	41	∞	∞	108	51	0	∞	∞	97	∞	97	∞	∞
Point IV	∞	68	∞	22	∞	∞	0	∞	∞	∞	69	∞	∞
Consumer 1	42	∞	68	∞	∞	∞	∞	0	∞	63	∞	∞	∞
Consumer 2	121	∞	∞	∞	∞	97	∞	∞	0	151	90	∞	31
Consumer 3	51	∞	39	160	∞	∞	∞	63	151	0	156	∞	∞
Consumer 4	106	45	∞	50	84	97	69	∞	90	156	0	∞	73
Consumer 5	∞	29	∞	∞	∞	∞	∞	∞	∞	∞	∞	0	81
Consumer 6	∞	72	∞	∞	∞	∞	∞	∞	31	∞	73	81	0

Table 4
The shortest distances between the points of logistics network, km

The participants logistics network	Supplier A	Supplier B	Supplier C	Point I	Point II	Point III	Point IV	Consumer 1	Consumer 2	Consumer 3	Consumer 4	Consumer 5	Consumer 6
Supplier A	0	151	76	111	40	41	133	42	121	51	106	180	152
Supplier B	151	0	227	70	129	142	68	193	103	201	45	29	72
Supplier C	76	227	0	187	116	117	209	68	190	39	182	256	221
Point I	111	70	187	0	95	108	22	153	140	160	50	99	123
Point II	40	129	116	95	0	51	117	82	148	91	84	158	157
Point III	41	142	117	108	51	0	130	83	97	92	97	171	128
Point IV	133	68	209	22	117	130	0	175	159	182	69	97	140
Consumer 1	42	193	68	153	82	83	175	0	163	63	148	222	194
Consumer 2	121	103	190	140	148	97	159	163	0	151	90	112	31
Consumer 3	51	201	39	160	91	92	182	63	151	0	156	230	182
Consumer 4	106	45	182	50	84	97	69	148	90	156	0	74	73
Consumer 5	180	29	256	99	158	171	97	222	112	230	74	0	81
Consumer 6	152	72	221	123	157	128	140	194	31	182	73	81	0

The model was implemented in Solver Add-In for MS Excel 2010.

As a result of the modeling, the optimum variant of a storage facilities organization for logistics network was determined. It is the variant with two warehouses located at the points III and II.

The values of the traffic volume indicators between the logistics network elements and storage capacity for three variants considered within the research are shown in the Table 5.

Table 5
Indicatorstraffic volumes andcapacityof warehouses, tons

The elements ofthe logistics network	Onewarehouse (Point III)	Two warehouses		Three warehouses		
		(Point III)	(Point II)	(Point III)	(Point II)	(Point I)
Supplier A	400	400	-	390	10	-
Supplier B	600	-	600	-	-	600
Supplier C	260	260	-	-	260	-
Consumer 1	80	-	80	-	80	-
Consumer 2	250	250	-	250	-	-
Consumer 3	170	-	170	-	170	-
Consumer 4	150	-	150	-	-	150
Consumer 5	50	-	50	-	-	50
Consumer 6	500	380	120	120	-	380
Capacity of warehouse	1260	660	600	390	270	600

The dynamics of change of the transportation work values for the considered variants of the logistics network construction are shown in Fig. 3.

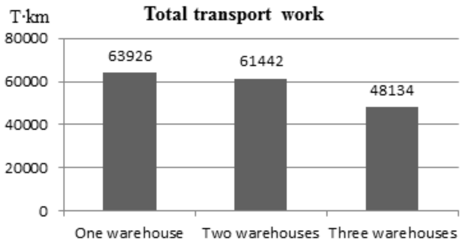

Figure 3.Dynamics ofchanges in the valuesof the transportwork

The dynamics of change of the logistics costs values for the considered variants of the logistics network construction are shown in Fig. 4.

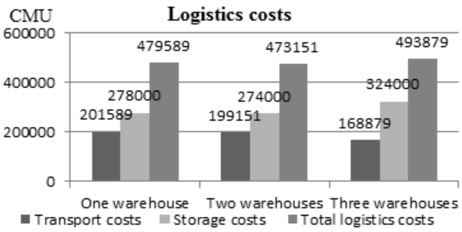

Figure 4.Dynamics ofchanges in the valuesof logistics costs

The best routes of cargo delivery from suppliers to customers through warehouses for the selected option of constructing a logistics network shown in Fig. 5.

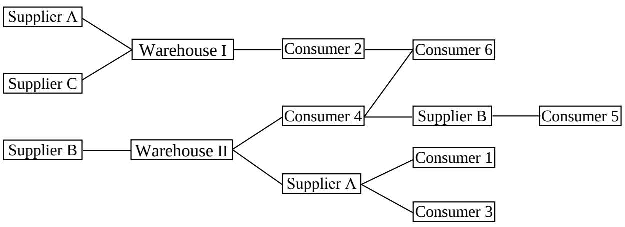

Figure 5.The best routesof cargo delivery tologistics network

CONCLUSIONS

The best logistics network (serving a larger territory with a larger number of spatially distributed links – providers, warehouses/distribution centers, consumers) is a network ensuring a high level of customer service at minimum total logistics costs.

The mathematical model and method suggested in the paper allow the definition of:

- ➢ the optimal number of warehouses for the logistics network under review, with set locations for suppliers and consumers;
- ➢ the optimal location of warehouses in points of logistics network, based on actual transportation services and a set level of total reserve stock at network warehouses;
- ➢ the optimal number of warehouses and their locations regarding the other influencing factors (reflected by weight factors of suppliers and consumers);
- ➢ total logistics costs, total transportation work and capacity of network warehouses;
- ➢ the shortest through routes for delivery of goods (in the logistics network).

This method can serve as a basis for development of methods defining the optimal number of warehouses and their locations in multi-format logistics networks (with developed transportation services), with additional restrictions on location of warehouses and regarding the multi-nomenclature character of goods, diverse nature of vehicles, and other factors influencing the optimal arrangement of a warehouse network.

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