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Constrained Dynamic Control of Traffic Junctions

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Abstract

Excessive traffic in our urban environments has detrimental effects on our health, economy and standard of living. To mitigate this problem, an adaptive traffic lights signalling scheme is developed and tested in this paper. This scheme is based on a state space representation of traffic dynamics, controlled via a dynamic programme. To minimise implementation costs, only one loop detector is assumed at each link. The comparative advantages of the proposed system over optimal fixed time control are highlighted through an example. Results will demonstrate the flexibility of the system when applied to different junctions. Monte Carlo runs of the developed scheme highlight the consistency and repeatability of these results.

Keywords: traffic junction model; state space modelling; adaptive traffic light control; dynamic programming.

1. Introduction

Modern urban areas are witnessing increasing traffic congestion due to high car ownership rates and a strong dependency on private cars. Junctions often pose the main bottleneck in urban traffic networks resulting in capacity flow on the network links and excessive delays. A continuous expansion of the infrastructure cannot remain the go-to solution to these problems due to land-use, environmental and financial limitations. An alternative is the use of dynamic traffic control strategies that are adaptable to prevailing traffic conditions. Such methods can provide a safe and feasible solution to traffic congestion problems through the efficient and intelligent use of the network.

Various types of adaptive control systems have already been implemented and have shown promising results. Two such widely used systems are SCOOT[1] and SCATS[2]. These systems use real time measurements to dynamically effect the split time, offsets and cycle time according to current traffic conditions. These commercially available systems are known to be very well optimised for under-saturated traffic conditions, however, their performance tends to deteriorate in heavy traffic[3]. Other widely used control systems including OPAC[4], PRODYN[5] and RHODES[6], implement model-based optimization and thus use current traffic measurements to identify in real-time an optimal control strategy.

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The last two decades also saw the introduction of the TUC system[7]. This system is based on the store-and-forward model as proposed by Gazis and Potts[8], where a multivariable linear regulator approach is used to optimize in real-time the network splits. The closed form solution obtained makes this system ideal for real time implementation, however such an approach cannot handle constraints and hence requires a final tuning procedure. Nevertheless, TUC has been successfully implemented in various places including Greece[9]. More recently, Tettamani *et al.*[10] proposed a traffic control system based on model predictive control. Other computationally expensive numerical solutions have also been proposed including genetically tuned controllers[11].

The work in this paper aims to provide a competing real-time adaptive control strategy which through the use of dynamic programming can natively handle the nonlinear constraints involved in the control of the traffic at a junction. This method is based on a novel model of traffic dynamics recently proposed by Pecherkova *et al.*[12]. Similarly to the store-and-forward models, this model can easily be extended to accommodate multiple junctions with any configuration. The implementation is kept flexible by allowing the use of different cost functions and most importantly, different detector configurations. It will be shown that through the use of Kalman filtering, the degradation in performance at the junction is only minimal if less sensors are used, thus allowing for a robust and cost-effective implementation.

This paper is divided into 4 Sections. Section 2 presents the model description of the traffic system used in this work and the development of the dynamic controller being proposed. In Section 3, an implementation of the dynamic controller at a junction is presented together with a discussion of the results obtained. Finally, Section 4 highlights the main results and draws some concluding remarks.

2. State Space Junction Modelling and Control

The state space model of a junction used in this work is based on the model proposed by Pecherkova *et al.*[12] which uses realistic non-linear dynamics to represent traffic flow. The traffic flow through a controlled intersection can be described by the following quantities with k being the cycle index:

- Queue length (ζ_k) is the number of cars queueing at a link to pass through the intersection at the start of each cycle (in unit vehicles [uv])
- Intensity (I_k) is the rate of incoming unit vehicles per cycle (in uv/period)
- Occupancy (O_k) is the portion of time during which the detector is occupied by a vehicle (in %)

Note that the cycle time can be set to any reasonable value by the user. Considering a single arm of the intersection, the queue length is described by the principle of conservation[13], where the queue during the next timing period ζ_{k+1} depends on the previous queue ζ_k, the incoming intensity I_k and the outgoing intensity I_k^π, through the relationship:

$$\zeta_{k+1} = \zeta_k + I_k - I_k^\pi(\zeta_k, I_k, z_k) + w_{1,k} \tag{1}$$

where the intensity of outgoing vehicles is given by the non-linear relationship:

$$I_k^\pi(\zeta_k, I_k, z_k) = S - S\, e^{-\frac{\zeta_k + I_k}{S z_k}} \tag{2}$$

with

- S being the saturated flow determined by the physical properties of the intersection,
- z_k being the ratio of green time per cycle and
- $w_{1,k}$ being a white, zero-mean, Gaussian noise process describing the random variations from this mean behaviour.

The input intensity to the junction is modelled as a Markovian process with known mean and standard deviation given by:

$$I_{k+1} = I_k + w_{2,k} \tag{3}$$

The occupancy at any time period can also be described as a random process with mean given by a linear relationship dependent on the previous occupancy and the queue length with known standard deviation given by:

$$O_{k+1} = \kappa\zeta_k + \beta O_k + w_{3,k} \tag{4}$$

Given the junction dynamics of equations (1) to (4), the dynamic equation of a state space model describing the junction can be given by:

$$\begin{bmatrix} \zeta_{k+1} \\ I_{k+1} \\ O_{k+1} \end{bmatrix} = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 0 \\ \kappa & 0 & \beta \end{bmatrix} \begin{bmatrix} \zeta_k \\ I_k \\ O_k \end{bmatrix} + \begin{bmatrix} -I_k^\pi \\ 0 \\ 0 \end{bmatrix} z_k + \begin{bmatrix} w_{1,k} \\ w_{2,k} \\ w_{3,k} \end{bmatrix} \tag{5}$$

Such dynamics can be expanded based on the number of arms of the intersection, indexed by $i = 1, 2, \ldots, n$, to give:

$$\boldsymbol{x_{k+1}} = \begin{bmatrix} 1 & 0 & \cdots & 0 & 1 & 0 & \cdots & 0 & 0 & \cdots & \cdots & 0 \\ 0 & 1 & \cdots & 0 & 0 & 1 & \cdots & 0 & 0 & \cdots & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \ddots & \vdots & 0 & \cdots & \cdots & 0 \\ 0 & 0 & \cdots & 1 & 0 & 0 & \cdots & 1 & 0 & \cdots & \cdots & 0 \\ 0 & \cdots & \cdots & 0 & 1 & 0 & \cdots & 0 & 0 & \cdots & \cdots & 0 \\ 0 & \cdots & \cdots & 0 & 0 & 1 & \cdots & 0 & 0 & \cdots & \cdots & 0 \\ \vdots & \cdots & \cdots & \vdots & \vdots & \vdots & \ddots & \vdots & 0 & \cdots & \cdots & 0 \\ 0 & \cdots & \cdots & 0 & 0 & 0 & \cdots & 1 & 0 & \cdots & \cdots & 0 \\ \kappa_1 & 0 & \cdots & 0 & 0 & \cdots & \cdots & 0 & \beta_1 & 0 & \cdots & 0 \\ 0 & \kappa_2 & \cdots & 0 & 0 & \cdots & \cdots & 0 & 0 & \beta_2 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots & \cdots & \cdots & \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & \kappa_n & 0 & \cdots & \cdots & 0 & 0 & \cdots & \cdots & \beta_n \end{bmatrix} \boldsymbol{x_k} + \begin{bmatrix} -I_{k_1}^\pi & 0 & \cdots & 0 \\ 0 & -I_{k_2}^\pi & \cdots & 0 \\ \vdots & \vdots & \ddots & 0 \\ 0 & \cdots & 0 & -I_{k_n}^\pi \\ 0 & \cdots & \cdots & 0 \\ 0 & \cdots & \cdots & 0 \\ \vdots & \cdots & \cdots & \vdots \\ 0 & \cdots & \cdots & 0 \\ 0 & \cdots & \cdots & 0 \\ \vdots & \cdots & \cdots & \vdots \\ 0 & \cdots & \cdots & 0 \\ 0 & \cdots & \cdots & 0 \end{bmatrix} \boldsymbol{z_k} + \boldsymbol{w_k} \tag{6}$$

where $\boldsymbol{x_k} = \left[\zeta_{k_1} \ldots \zeta_{k_n}\ I_{k_1} \ldots I_{k_n}\ O_{k_1} \ldots O_{k_n}\right]^\top$ and $\boldsymbol{w_k} = \left[w_{1,k_1} \ldots w_{1,k_n}\ w_{2,k_1} \ldots w_{2,k_n}\ w_{3,k_1} \ldots w_{3,k_n}\right]^\top$.

The measurement equation of a state space model represents the type and number of detectors of the controlled intersection. In this work, only one strategic loop detector per link placed a few meters away from the stop line is considered thus minimising the cost of the proposed implementation. Hence the measurement equation is given by:

$$\boldsymbol{y_k} = [I_k\ O_k]^\top + \boldsymbol{v_k} \tag{7}$$

where $\boldsymbol{y_k}$ is the measured output of the system and $\boldsymbol{v_k} = \left[v_{k_1}\ v_{k_2} \ldots v_{k_n}\right]^\top$ represents zero mean, Gaussian measurement noise.

Given the state space model of equations (6) and (7), various well known control strategies can be implemented. In this work, dynamic programming[14] will be adopted due to its advantageous numerical performance and ability to identify strategies that minimise the system's states while adhering to known constrains. Such control strategies thus aim to minimise the cost function

$$J = \sum_{i=k+1}^{k+N} x_i^\top R x_i + \sum_{j=k}^{k+N-1} z_j^\top Q z_j \tag{8}$$

where N is the number of cycles the controller uses to reduce the states to the desired value and R and Q are the weighting matrices on the states and the inputs respectively. The dynamic program thus aims to minimise the cost function in (8) subject to the constraints:

$$\zeta_k, I_k > 0 \tag{9}$$

$$0 < O_k < 100 \tag{10}$$

$$z_{min} < z_k < z_{max} \tag{11}$$

$$\sum_{i=1}^{n} z_i = 1 \tag{12}$$

To minimise the computational burden and thus allow for a real-time implementation with minimum added hardware cost, a *constrained receding horizon procedure*[15] is used. Thus for the given dynamics the controller identifies the next control sequence such that $\zeta_{k+1} = a\zeta_k$ with $a < 1$.

I_k and O_k can be measured directly from a loop detector installed on the arms of the intersection. On the other hand, direct measurement of ζ_k cannot be obtained and is therefore estimated using a Kalman filter[16]. Note that, if the intersection is equipped with enough detectors such that full knowledge of the system states can be obtained, such a filter is not required.

3. Example

To demonstrate the performance of the dynamic controller, it was tested on a T-junction as depicted in Fig. 1. The junction is assumed to be equipped with one strategic detector on each link, placed a few meters away from the stop line. The parameters of the junction are specified in Table 1, where the saturation values are determined by the physical properties of the intersection while the parameters κ and β of each link can be obtained through linear regression based on measured values[12]. Test data was thus generated though the state space model given by (6) and (7).

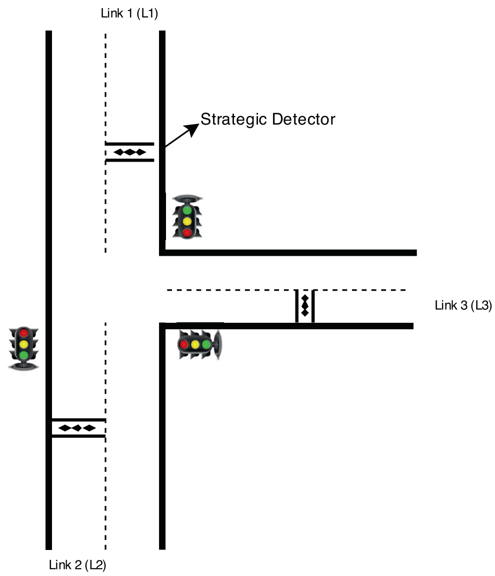
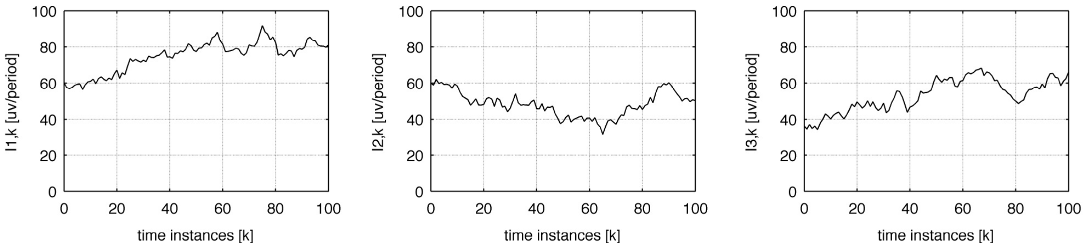


Fig. 1: T-junction

Table 1: T-junction Parameters

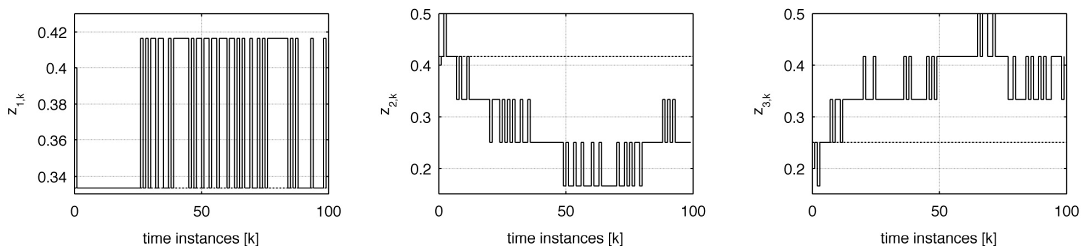
Parameter	Value
Cycle time	180 seconds
Min green time	15 seconds
Max green time	150 seconds
Mean intensity on L1	60 uv/c
Mean intensity on L2	60 uv/c
Mean intensity on L3	36 uv/c
Saturation on L1 (S_1)	300 uv/c
Saturation on L2 (S_2)	250 uv/c
Saturation on L3 (S_3)	180 uv/c
κ_1, κ_2, κ_3	0.1
β_1, β_2, β_3	0.9
R_1,R_2, R_3,Q_1,Q_2,Q_3	1
w_1,w_2,w_3,w_7,w_8,w_9	$N(0, 0.32)$
w_4,w_5,w_6	$N(0, 2.24)$
v_1,v_2,v_3,v_4,v_5,v_6	$N(0, 0.32)$
a	0.95

To quantify the effectiveness of the dynamic controller, comparisons with an optimised fixed time signalling strategy applied to the same intersection are drawn. This fixed time strategy was chosen to minimise the same cost function (8), with steady state queue lengths replacing ζ_{k+1_1}, ζ_{k+1_2} and ζ_{k+1_3}, for all possible signalling ratios. Given the typical input intensities shown in Fig. 2, the split timings given by the dynamic controller and the optimal fixed strategy are shown in Fig. 3. These traffic light timings result in a build up of queues at each link, shown in Fig. 4 for the optimal fixed timing scheme and shown in Fig. 5 for the adaptive scheme being proposed.



(a) Input Intensity on Link 1 (b) Input Intensity on Link 2 (c) Input Intensity on Link 3

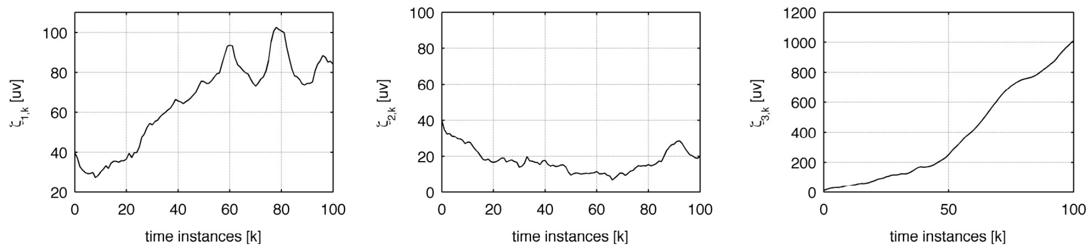
Fig. 2: Input intensities



(a) Green Time on Link 1 (b) Green Time on Link 2 (c) Green Time on Link 3

Fig. 3: Ratio of green times per cycle allocated by the dynamic controller (solid) and by the fixed time plan (dotted)

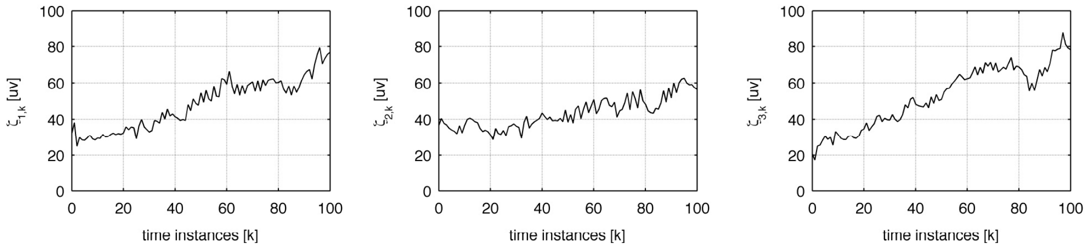
Note that, the different input intensities and saturation values of each link result in the fixed timing inputs shown in Fig. 3. Since an identical cost function was chosen for the adaptive scheme, its timing patterns oscillate around the same value with adjustments reflecting the queue build-up due to the stochastic variations in the input intensity.



(a) Queue Length on Link 1 (b) Queue Length on Link 2 (c) Queue Length on Link 3

Fig. 4: Queue build-up on all links using the fixed time plan

Comparing the queue lengths of Fig. 4 and Fig. 5, significant improvements can be noted for the junction equipped with the dynamic controller. With increasing input intensities, as shown in Fig. 2a and Fig. 2c, the fixed time plan

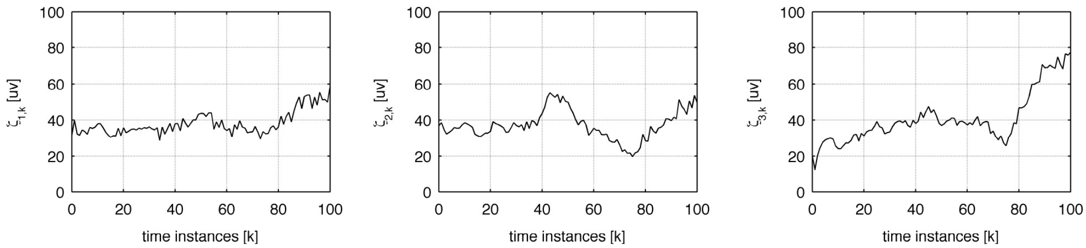


(a) Queue Length on Link 1 (b) Queue Length on Link 2 (c) Queue Length on Link 3

Fig. 5: Queue build-up on all links using the dynamic controller

does not alter its timings and hence the queue lengths increase. This is especially significant for link 3, due to its generally low vehicle intensity and its short green time. On the other hand, when comparing with the results in Fig. 5, it can be noted that with increasing input intensities, the controller adapted its timing to minimize the queues on each link.

Due to its adaptive nature, the proposed dynamic controller is expected to further outperform any fixed timing scheme as the deviation from the expected input intensity increases. This assumption is tested by increasing the stochastisity on the input intensities and thus Fig. 6 shows the resulting queue lengths with w_4, w_5 and $w_6 \sim N(0, 2.65)$. From the results in Fig. 6, it is clear that the controller dealt well with the higher input variability and still managed to alleviate any momentarily build up of queues.

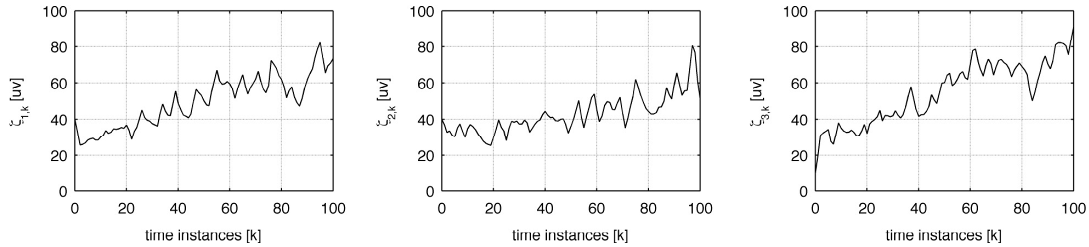


(a) Queue Length on Link 1 (b) Queue Length on Link 2 (c) Queue Length on Link 3

Fig. 6: Queue build up on all links with increased stochasticity on input intensities using the dynamic controller

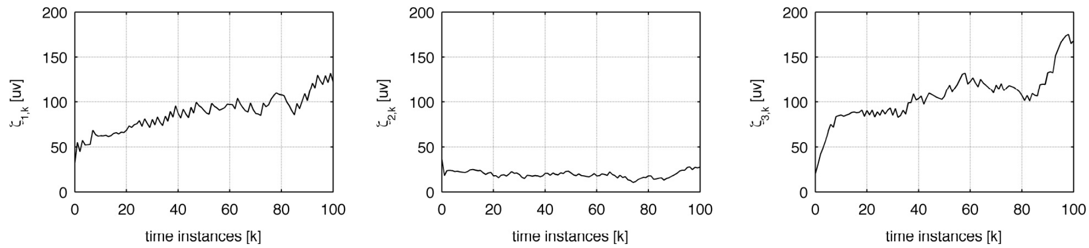
Using only one detector per link results in a significant reduction in implementation costs. Nevertheless, this cost saving comes at the expense of limited information about the current state of the junction. In particular, one loop sensor at each link gives no direct readout of the current queue length. To overcome this problem, the use of a Kalman filter was proposed in Section 2, nevertheless, any inaccuracies in the estimates of the Kalman filter may have severe effects on the efficiency of the adaptive controller. To test if the deterioration in performance when using a Kalman filter is substantial, the results of the latter were compared with the results of a system where all states (including the queue lengths) were assumed to be perfectly known. The queue lengths obtained in each case are depicted in Fig. 7 and clearly indicate when compared to Fig. 5, that the deterioration in performance is minimal.

Different cost functions can also be utilised to deal with different local conditions such as limited buffer lengths upstream of the traffic light junction. For example, to minimise the queue length on link 2, and thus minimise the blockage upstream of this link, the cost function parameters can be modified with $R_2 = 20$. Such a choice has the effect of prioritising small queue lengths on link 2 as opposed to the other two links (which are assumed to have larger buffer lengths). Such results as obtained by this cost function are shown in Fig. 8.



(a) Queue Length on Link 1 (b) Queue Length on Link 2 (c) Queue Length on Link 3

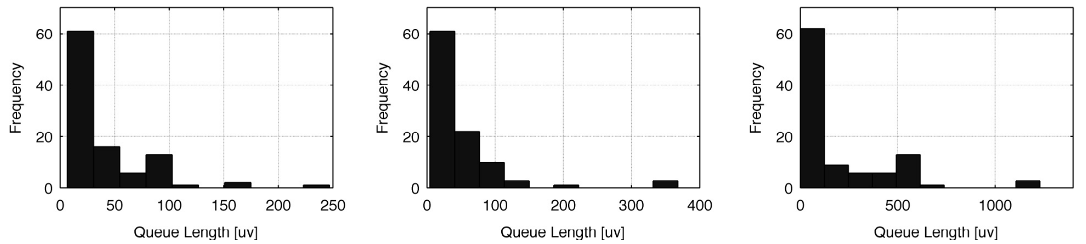
Fig. 7: Queue build-up on all links using the dynamic controller with full knowledge of the states



(a) Queue Length on Link 1 (b) Queue Length on Link 2 (c) Queue Length on Link 3

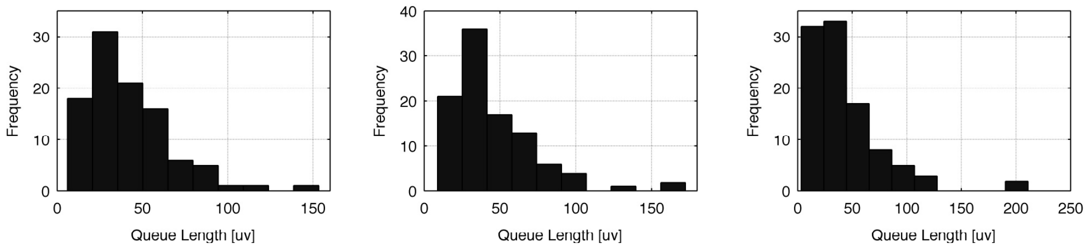
Fig. 8: Queue build-up using the dynamic controller with added weighting on link 2 in the cost function

Finally, to test the consistency of this improved performance over the optimal fixed timing scheme, a Monte Carlo run of 100 iterations was performed for each implementation. The resulting queue lengths on each link are summarised by the histograms of Fig. 9 and Fig. 10. (Note the different scales for each plot.).Clearly from these plots the dynamic controller developed consistently outperformed the fixed time scheme with significantly lower means and less frequent and smaller outliers.



(a) Histogram of queues on Link 1 (b) Histogram of queues on Link 2 (c) Histogram of queues on Link 3

Fig. 9: Histograms representing the results for 100 realisations using a optimal fixed time plan



(a) Histogram of queues on Link 1 (b) Histogram of queues on Link 2 (c) Histogram of queues on Link 3

Fig. 10: Histograms representing the results for 100 realisations using the dynamic controller

4. Conclusions

In view of the increasing traffic in our urban roads, improved control of the flow through busy junctions is indispensable. The results in this paper demonstrate that the proposed dynamic controller can in real time adapt to changing traffic conditions, thus minimising the traffic at any junction. This is obtained through minimal infrastructural investment, since only one loop detector is required at each controlled link. Moreover, the results show the ability of the proposed scheme to adapt to highly changing stochastic variations in the incoming vehicle intensity and an adaptability to different geographical constraints such as limited buffer zones upstream of the junction. Future work will focus on the analysis of the performance of this scheme on larger traffic networks, the adoption of different control strategies and the joint estimation in real-time of the control action and model parameters.

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