LIFTED CODES AND THE MULTILEVEL CONSTRUCTION FOR CONSTANT DIMENSION CODES

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Abstract: Constant dimension codes are e.g. used for error correction and detection in random linear network coding, so that constructions for these codes have achieved wide attention. Here, we improve over 150 lower bounds by describing better constructions for subspace distance 4.

Keywords: constant dimension codes, multilevel construction, Echelon–Ferrers construction, linkage, network coding

1. Introduction

Let q be a prime power and $\mathbb{F}_q$ be the finite field with q elements. For two integers $0 \le k \le n$ we denote by $\mathcal{G}_q(n,k)$ the set of all k-dimensional subspaces in $\mathbb{F}_q^n$. The so-called subspace distance $d_S(U,W) := \dim(U) + \dim(W) - 2\dim(U \cap W) = 2k - 2\dim(U \cap W)$ defines a metric on $\mathcal{G}_q(n,k)$. A subset $\mathcal{C} \subseteq \mathcal{G}_q(n,k)$ is called a *constant dimension code* (CDC), its elements are also called codewords, and $d_S(\mathcal{C}) = \min\{d_S(U,W) : U,W \in \mathcal{C}, U \neq W\}$ is the corresponding *minimum (subspace) distance*. We also call $\mathcal{C}$ an $(n,M,d,k)_q$ CDC if $\mathcal{C}$ has cardinality M and $d_S(\mathcal{C}) \ge d$. The maximum possible cardinality of an $(n,M,d,k)_q$ CDC is denoted by $A_q(n,d;k)$. Constant dimensions codes are e.g. applied in random linear network coding, see e.g. [14], and the determination of bounds for $A_q(n,d;k)$ is one of the main problems. Here we improve more than 150 of the previously best known constructions for CDCs. An online table for bounds for $A_q(n,d;k)$ can be found at `subspacecodes.uni-bayreuth.de`, see the corresponding technical manual [11].

The remaining part of this paper is structured as follows. In Section 2 we introduce the necessary preliminaries and describe constructions for CDCs from the literature. Our theoretical and algorithmical results are the topic of Section 3. The resulting numerical improvements for lower bounds for $A_q(n,d;k)$ are listed in Appendix A. Extensive computational data about the details of the underlying constructions are given in Appendix B.

2. Preliminaries

Given a CDC $\mathcal{C}$ we first consider the question how to represent its codewords, i.e., k-dimensional subspaces $U \in \mathcal{G}_q(n,k)$. Starting from a generator matrix whose k rows form a basis of U the application of the Gaussian elimination algorithm gives a unique

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generator matrix in *reduced row echelon form* denoted by $E(U)$. In the other direction we write $U = \langle E(U) \rangle$. By $v(U) \in \mathbb{F}_2^n$ we denote the characteristic vector of the pivot columns in $E(U)$, which is also called *identifying vector*. The *Ferrers tableaux* $T(U)$ of U arises from $E(U)$ by removing the zeroes from each row of $E(U)$ left to the pivots and afterwards removing all pivot columns. If we then replace all remaining entries by dots we obtain the *Ferrers diagram* $\mathcal{F}(U)$ of U which only depends on the identifying vector $v(U)$. As an example we consider

$$U = \left\langle \begin{pmatrix} 1 & 0 & 1 & 1 & 0 & 1 & 0 & 1 & 0 & 1 \\ 1 & 0 & 0 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 0 & 0 & 0 & 1 & 1 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 0 & 1 \end{pmatrix} \right\rangle \in \mathcal{G}_2(10, 4),$$

where we have

$$E(U) = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 1 & 0 & 0 & 1 & 1 & 1 \\ 0 & 0 & 0 & 1 & 1 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 0 & 1 \end{pmatrix},$$

$v(U) = 1011001000 \in \mathbb{F}_2^{10}$, and

$$\mathcal{F}(U) = \begin{array}{cccccc} \bullet & \bullet & \bullet & \bullet & \bullet & \bullet \\ & \bullet & \bullet & \bullet & \bullet & \bullet \\ & \bullet & \bullet & \bullet & \bullet & \bullet \\ & & & \bullet & \bullet & \bullet \end{array}.$$

The *Hamming distance* $d_H(u, w) = \#\{1 \le i \le n \,:\, u_i \ne w_i\}$, for $u, w \in \mathbb{F}_q^n$, can be used to lower bound the subspace distance between two codewords $U, W \in \mathcal{G}_q(n, k)$:

Lemma 1. *(*[5, Lemma 2]*)*
For $U, W \in \mathcal{G}_q(n, k)$ we have $d_S(U, W) \ge d_H(v(U), v(W))$.

If the identifying vectors of two codewords coincide, then we can utilize the *rank distance* $d_R(A, B) := \operatorname{rank}(A - B)$ for matrices $A, B \in \mathbb{F}_q^{m \times l}$:

Lemma 2. *(*[20, Corollary 3]*)*
For $U, W \in \mathcal{G}_q(n, k)$ with $v(U) = v(W)$ we have $d_S(U, W) = 2 d_R(E(U), E(W))$.

Since d_R is a metric, we call a subset $C \subseteq \mathbb{F}_q^{m \times l}$ of matrices a *rank-metric code*. If C is a linear subspace of $\mathbb{F}_q^{m \times l}$ we call the code *linear*. Given a Ferrers diagram $\mathcal{F}$ with m dots in the rightmost column and l dots in the top row, we call a rank-metric code $C_{\mathcal{F}}$ a *Ferrers diagram rank-metric* (`FDRM`) code if for any codeword $M \in \mathbb{F}_q^{m \times l}$ of $C_{\mathcal{F}}$ all entries not in $\mathcal{F}$ are zero. By $d_R(C_{\mathcal{F}})$ we denote the minimum rank distance, i.e., the minimum of the rank distance between pairs of different codewords.

Definition 1. ([21])
Let $\mathcal{F}$ be a Ferrers diagram and $C_{\mathcal{F}} \subseteq \mathbb{F}_q^{k \times (n-k)}$ be an `FDRM` code. The corresponding

lifted FDRM *code* $\mathcal{C}_{\mathcal{F}}$ is given by

$$\mathcal{C}_{\mathcal{F}} = \{U \in \mathcal{G}_q(n,k) \,:\, \mathcal{F}(U) = \mathcal{F}, T(U) \in C_{\mathcal{F}}\}\,.$$

We remark that the bijection between the codewords of the CDC $\mathcal{C}_{\mathcal{F}}$ and the FDRM code $C_{\mathcal{F}}$ generalizes the construction of lifted *maximum rank distance* (MRD) codes. An MRD code corresponds to the case of a Ferrers diagram $\mathcal{F}$ with k dots in each column and $n-k$ dots in each row (more details below).

Directly from Lemma 2 and Definition 1 we can conclude:

Lemma 3. *(*[5, Lemma 4]*)*
Let $C_{\mathcal{F}} \subseteq \mathbb{F}_q^{k\times(n-k}$ be an FDRM *code with minimum rank distance δ, then the lifted* FDRM *code $\mathcal{C}_{\mathcal{F}} \subseteq \mathcal{G}_q(n,k)$ is an $(n, \#C_{\mathcal{F}}, 2\delta, k)_q$* CDC*.*

Let $v(\mathcal{F})$ be the identifying vector of a given Ferrers diagram $\mathcal{F}$. In general, we denote by $A_q(n,d;k;v)$ the maximum cardinality M of an $(n,M,d,k)_q$ CDC where all codewords have $v \in \mathbb{F}_2^n$ as identifying vector. We also speak of an $(n,M,d,k,v)_q$ CDC. With this (and Lemma 3) the upper bound for the cardinality of $C_{\mathcal{F}}$ from [5, Theorem 1][1] can be rewritten to:

Theorem 1.

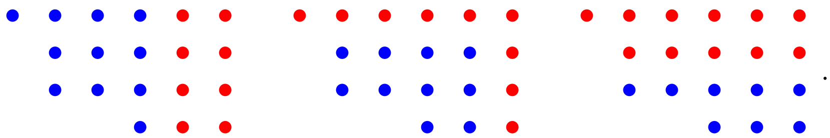
$$A_q(n,d;k;v(\mathcal{F})) \le q^{\min\{\nu_i \,:\, 0 \le i \le d/2-1\}},$$

where ν_i is the number of dots in $\mathcal{F}$, which are neither contained in the first i rows nor contained in the last $\frac{d}{2}-1-i$ columns.

If we choose a minimum subspace distance of $d=6$, then we obtain

$$A_2(10,6;4;1011001000) \le 2^8$$

due to



The Hamming weight of a vector $v \in \mathbb{F}_2^n$ is the Hamming distance $d_H(v,\mathbf{0})$ of v to the zero vector.

Theorem 2. *(*[5, Theorem 3]*)*
If $\mathcal{S} \subseteq \mathbb{F}_2^n$ is a set of binary vectors with Hamming weight k that has minimum Hamming distance d and $\mathcal{C}_v \subseteq \mathcal{G}_q(n,k)$ is an $(n,\star,d,k,v)_q$ CDC *for each $v \in \mathcal{S}$, then $\mathcal{C} = \cup_{v\in\mathcal{S}}\mathcal{C}_v$ is an $(n,\star,d,k)_q$* CDC *with cardinality $\sum_{v\in\mathcal{S}} \#\mathcal{C}_v$.*

[1]In the cited paper the upper bound was stated for linear FDRM codes only. However, the statement is also true without this assumption, as observed by e.g. the same authors later.

Choosing the $\mathcal{C}_v$ as lifted FDRM codes, the underlying construction is called *multilevel construction* in [5] and *Echelon-Ferrers construction* in some other papers. The binary code $\mathcal{S} \subseteq \mathbb{F}_2^n$ is called *skeleton code*. Using our notation, a given skeleton code $\mathcal{S}$ with minimum Hamming distance $\frac{d}{2}$ gives

$$A_q(n,d;k) \ge \sum_{v \in \mathcal{S}} A_q(n,d;k;v).$$

The upper bound of Theorem 1 is attained in many cases including $d \le 4$ and rectangular Ferrers diagrams. For other cases we refer e.g.. to [1, 18] and the references mentioned therein. Indeed, for finite fields no strict improvement of Theorem 1 is known so that it is conjectured that the upper bound can always be attained. If $2 \le 2k \le n$ and $\mathcal{F}$ is the rectangular Ferrers diagrams with k dots in each column and $n-k$ dots in each row, then a rank-metric code $C_{\mathcal{F}} \subseteq \mathbb{F}_q^{k\times(n-k)}$ attaining the maximum possible cardinality $q^{(n-k)(k-d/2+1)}$ for a given minimum subspace distance $d \le 2k$ is called *maximum rank distance* (MRD) code. Even linear MRD codes exist for all parameters, so that lifting gives the well-known lower bound

$$A_q(n,d;k) \ge q^{(n-k)(k-d/2+1)}$$

(assuming $2k \le n$), which is at least half the optimal value for $d \ge 4$, see e.g. [12, Proposition 8].

Instead of starting with an FDRM code $C_{\mathcal{F}}$ and lift it to a CDC $\mathcal{C}_{\mathcal{F}}$ one can also start from an $(m,N,d,k)_q$ CDC $\mathcal{C}$ and an MRD code $\mathcal{M} \subseteq \mathbb{F}_q^{k\times(n-m)}$ with minimum rank distance $d/2$. With this we can construct a CDC

$$\mathcal{C}' = \{\langle E(U)|M\rangle \,:\, U \in \mathcal{C}, M \in \mathcal{M}\} \subseteq \mathcal{G}_q(n,k)$$

with $d_S(\mathcal{C}') = d$ and $\#\mathcal{C}' = \#\mathcal{C} \cdot \#\mathcal{M}$, where $A|B$ denotes the concatenation of two matrices A and B with the same number of rows. This lifting variant was called *Construction D* in [21, Theorem 37], cf. [6, Theorem 5.1]. By construction, the identifying vectors of the codewords of $\mathcal{C}'$ contain their k ones in the first m positions. More generally, we denote by

$$\binom{n_1}{k_1}\dots\binom{n_l}{k_l}$$

the set of binary vectors which contain exactly k_i ones in positions $1+\sum_{j=1}^{i-1} n_j$ to $\sum_{j=1}^{i} n_j$ for all $1 \le i \le l$. With this, we write $A_q\Big(n,d;k;\binom{n_1}{k_1}\dots\binom{n_l}{k_l}\Big)$ for the maximum cardinality of an $(n,\star,d,k)_q$ CDC whose codewords all have identifying vectors in this set and state:

Theorem 3. *For each* $0 \le \Delta < n$ *we have*

$$A_q(n,d;k) \ge A_q\left(n,d;k;\binom{n-\Delta}{k},\binom{\Delta}{0}\right) \ge q^{\Delta(k-d/2+1)} A_q(n-\Delta,d;k).$$

The special structure of the identifying vectors can be used to add further codewords. In our notation the *linkage construction* from [7, Theorem 2.3], [21, Corollary 39] can be written as

$$A_q(n,d;k) \ge A_q\left(n,d;k;\binom{n-\Delta}{k},\binom{\Delta}{0}\right) + A_q\left(n,d;k;\binom{n-\Delta}{0},\binom{\Delta}{k}\right),$$

which was improved to

$$\begin{aligned} A_q(n,d;k) \;\ge\; & A_q\left(n,d;k;\binom{n-\Delta}{k},\binom{\Delta}{0}\right) \\ & + A_q\left(n,d;k;\binom{n-\Delta-k+d/2}{0},\binom{\Delta+k+d/2}{k}\right) \end{aligned}$$

in [12, Theorem 18, Corollary 4], taking Theorem 3 and $A_q(n,d;k;\binom{n-m}{0},\binom{m}{k}) = A_q(m,d;k)$ into account. By using the notation $\binom{n-n'}{\le k-k'},\binom{n'}{\ge k'}$ for the set of vectors in $\mathbb{F}_2^n$ with at most $k-k'$ ones in the first $n-n'$ positions and at least k' ones in the last n' positions we can denote by $A_q(n,d;k;\binom{n-n'}{\le k-k'},\binom{n'}{\ge k'})$ the maximum cardinality of an $(n,\star,d,k)_q$ `CDC` whose codewords have identifying vectors in this set, so that Lemma 1 gives:

Lemma 4. *For each* $0 \le \Delta < n$ *we have*

$$\begin{aligned} A_q(n,d;k) \;\ge\; & A_q\left(n,d;k;\binom{n-\Delta}{k},\binom{\Delta}{0}\right) \\ & + A_q\left(n,d;k;\binom{n-\Delta}{\le k-d/2},\binom{\Delta}{\ge d/2}\right). \end{aligned}$$

We remark that Lemma 4 was implicitly contained in the proofs of many papers improving lower bounds for $A_q(n,d;k)$, see e.g. [3, 22] and the references cited therein. In [16] the quantity $A_q\left(n,d;k;\binom{n-\Delta}{\le k-d/2},\binom{\Delta}{\ge d/2}\right)$ was introduced as $B_q(n,\Delta,d;k)$. For the special case $\Delta = k$ a lower bound for $A_q\left(n,d;k;\binom{n-\Delta}{\le k-d/2},\binom{\Delta}{\ge d/2}\right)$ was constructed in [22] via

$$\{\langle M|I_k\rangle \,:\, M \in \mathcal{M}, \operatorname{rank}(M) \le k-d/2\},$$

where I_k denotes the $k \times k$ unit matrix and $\mathcal{M} \subseteq \mathbb{F}_q^{k\times(n-k)}$ is a rank metric code with $d_R(\mathcal{M}) \ge d/2$. By replacing I_k by $E(U)$ for all codewords of a $(\Delta,\star,d,k)_q$ `CDC` we obtain yet another variant of the lifting idea. One of the most general versions can be found in [3, Lemma 4.1].

Of course we can also utilize the multilevel/Echelon-Ferrers construction from Theorem 2 to obtain lower bounds for $A_q\left(n,d;k;\binom{n-\Delta}{\le k-d/2},\binom{\Delta}{\ge d/2}\right)$ by restricting the skeleton code $\mathcal{S}$ to subsets of the set $\binom{n-\Delta}{\le k-d/2},\binom{\Delta}{\ge d/2}$ of identifying vectors. More generally, we can define the Hamming distance $d_H(\mathcal{S},\mathcal{S}')$ between two sets $\mathcal{S},\mathcal{S}' \subseteq \mathbb{F}_2^n$ as

$$d_H(\mathcal{S},\mathcal{S}') = \min\{d_H(v,v') \,:\, v \in \mathcal{S}, v' \in \mathcal{S}'\}$$

and slightly generalize Theorem 2 to:

Theorem 4. *Let $\mathcal{S}_i \subseteq \mathbb{F}_2^n$ and $\mathcal{C}_i$ be $(n,\star,d,k,\mathcal{S}_i)_q$ CDCs for all $1 \le i \le n$ such that $d_H(\mathcal{S}_i,\mathcal{S}_j) \ge d$ for all $1 \le i < j \le l$. Then $\mathcal{C} = \cup_{1\le i\le l}\mathcal{C}_i$ is an $(n,\star,d,k)_q$ CDC with cardinality $\#\mathcal{C} = \sum_{i=1}^{l} \#\mathcal{C}_i$.*

If we choose $n = n_1 + n_2$, $\mathcal{S}_1 = \left(\binom{n_1}{k},\binom{n_2}{0}\right) \subseteq \mathbb{F}_2^n$, $\mathcal{S}_2 = \left(\binom{n_1}{0},\binom{n_2}{k}\right) \subseteq \mathbb{F}_2^n$, and $\mathcal{S}_3,\dots,\mathcal{S}_l \subseteq \binom{n}{k} \cap \left(\binom{n_1}{\ge d/2},\binom{n_2}{\ge d/2}\right) \subseteq \mathbb{F}_2^n$ of cardinality $\#\mathcal{S}_j = 1$ (for $3 \le j \le l$) with $d_H(\mathcal{S}_3,\dots,\mathcal{S}_l) \ge d$, then the conditions of Theorem 4 are satisfied. This is [17, Theorem 3.1].[2] More examples where constant dimension codes with different sets of identifying vectors are combined can be found in [13].

3. Results

We want to apply Theorem 4 in order to obtain improved lower bounds for $A_q(n,d;k)$. In order to avoid to explicitly deal with the existence and construction of FDRM codes we restrict ourselves to subspace distance $d = 4$, where the upper bound of Theorem 1 can be always attained. As an introductory example we state:

Proposition 1. *We have $A_2(13,4;5) \ge 4\,796\,825\,069$ and $A_q(13,4;5) \ge q^{32} + q^{28} + q^{26} + 8q^{24} + q^{23} + 3q^{22} + q^{21} + 4q^{20} + 4q^{19} + 5q^{18} + q^{17} + 9q^{16} + 8q^{15} + 9q^{14} + 6q^{13} + 7q^{12} + 5q^{11} + q^{10} + 5q^9 + 3q^8 + q^7 + 3q^6 + 4q^5 + 3q^4 + q^3 + 3q^2$.*

Proof. We apply Theorem 2 with skeleton codes $\mathcal{S}^1_{13,4,5}$ and $\mathcal{S}^2_{13,4,5}$, see Appendix B. □

For $q \in \{2,3\}$ the previously best lower bounds were $A_2(13,4;5) \ge 4\,796\,417\,559$ and $A_3(13,4;5) \ge 1\,880\,918\,023\,783\,990$ [8], while our parametric lower bound yields $A_3(13,4;5) \ge 1\,880\,918\,252\,176\,932$. The only algorithmical challenge is to find good skeleton codes. Note that $\mathcal{S}^1_{13,4,5}$ gives $A_q(13,4;5) \ge q^{32} + q^{28} + q^{26} + 8q^{24} + q^{23} + 2q^{22} + 3q^{21} + 5q^{20} + 3q^{19} + 3q^{18} + 3q^{17} + 8q^{16} + 8q^{15} + 9q^{14} + 5q^{13} + 8q^{12} + 9q^{11} + q^{10} + 7q^9 + 2q^8 + 2q^7 + 2q^6 + q^5 + 3q^4 + 2q^3 + 3q^2 + q^0$, i.e., the coefficient for q^{22} is only 2 instead of 3, which pays off for $q = 2$ since the coefficient for q^{21} is 3 instead of 1 and the coefficient for q^{20} is 5 instead of 4.

If the maximum sizes of the FDRM codes are known, as it is the case for subspace distance $d = 4$, for given parameters n, d, k, and q the problem of determining the best lower bound for $A_q(n,d;k)$ based on Theorem 2 can be easily formulated as maximum weighted clique problem. To this end we denote by $\mathcal{G}_{n,d,k,q} = (\mathcal{V}_{n,d,k,q},\mathcal{E}_{n,d,k,q})$ the graph consisting of vertices corresponding to the binary vectors in $\mathbb{F}_2^n$ with Hamming weight k. W.l.o.g. we label the elements of $\mathcal{V}_{n,d,k,q}$ from 1 to $\binom{n}{k}$ such that $w(1) \ge w(2) \ge \dots$, where $w(v)$ denotes the weight of v that is given by the maximum possible cardinality of the corresponding FDRM code. For two different vertices $v, v' \in \mathcal{V}_{n,d,k,q}$ the edge $\{v,v'\}$ is in $\mathcal{E}_{n,d,k,q}$ iff the Hamming distance between v and v' is at least d. With

[2] In [9] it was claimed that [17, Theorem 3.1] is incorrect. However, the stated "counterexample" is flawed since the example for C_2 is not of the form specified in [17, Theorem 3.1] since there are e.g. non-zero entries in the first four columns of a_1.

this, the feasible skeleton codes are in bijection to the cliques of $\mathcal{G}_{n,d,k,q}$ and we are searching for the maximum weight cliques. Looping over all cliques or all inclusion maximal cliques of $\mathcal{G}_{n,d,k,q}$ becomes computationally intractable even for moderate sized parameters due to the quickly increasing number of vertices. Given an upper bound ub for the maximum clique size in $\mathcal{G}_{n,d,k,q}$, see e.g. [2] and `https://www.win.tue.nl/~aeb/codes/Andw.html`, we can compute an upper bound on the weight $w(\mathcal{S}') := \sum_{v\in\mathcal{S}'} w(v)$ of every clique that contains a subclique:

Lemma 5. *Let $\mathcal{S}, \mathcal{S}'$ be cliques in $\mathcal{G}_{n,d,k,q}$ with $\mathcal{S} \subseteq \mathcal{S}'$ and $\max\{w(v) : v \in \mathcal{S}\} < \min\{w(v) : w \in \mathcal{S}'\backslash\mathcal{S}\}$, where $w(i) \ge w(j) \ge 0$ for all $i \le j$. Then, we have $w(\mathcal{S}') \le \Omega$, where Ω is the value computed by Algorithm 1 applied to $\mathcal{G}_{n,d,k,q}$, w, and $\mathcal{S}$.*

Proof. By $cand$ we denote the set of vertices $m+1 \le v \le \#\mathcal{V}$ with $\Big\{ \{x,v\} : x \in \mathcal{S}\Big\} \subseteq \mathcal{E}$, where m, $\mathcal{V}$, and $\mathcal{E}$ are as in Algorithm 1. Since $\mathcal{S} \subseteq \mathcal{S}'$ and $\mathcal{S}'$ is a clique, we have $\Big\{ \{x,v\} : x \in \mathcal{S}\Big\} \subseteq \mathcal{E}$ for all vertices $v \in \mathcal{S}'\backslash\mathcal{S}$. Due to our assumption $\max\{w(v) : v \in \mathcal{S}\} < \min\{w(v) : w \in \mathcal{S}'\backslash\mathcal{S}\}$ we also have $m+1 \le v \le \#\mathcal{V}$ for all $v \in \mathcal{S}'\backslash\mathcal{S}$. Thus, we have $\mathcal{S}' \subseteq \mathcal{S} \cup cand$. If $\#\mathcal{S} + \#cand \le ub$, then $\hat{\mathcal{S}} = \mathcal{S} \cup cand$ and $w(\mathcal{S}') \le w(\mathcal{S} \cup cand) = w(\hat{\mathcal{S}}) = \Omega$. Otherwise we have $\#\mathcal{S}' \le ub = \#\hat{\mathcal{S}}$, set $s = \#\left(\mathcal{S}'\backslash\mathcal{S}\right)$, , $\hat{s} = \#\left(\hat{\mathcal{S}}\backslash\mathcal{S}\right)$, and write $\mathcal{S}' = \mathcal{S} \cup \{a_1, \dots, a_s\}$, $\hat{\mathcal{S}} = \mathcal{S} \cup \{b_1, \dots, b_{\hat{s}}\}$, where we assume that the sequences a_i and b_i are increasing. Due to our assumption on the weight function w and the choice of $\hat{\mathcal{S}}\backslash\mathcal{S}$ as those vertices with the smallest elements in $cand$ we have $w(a_i) \le w(b_i)$ for all $1 \le i \le s$, so that

$$w(\mathcal{S}') = w(\mathcal{S}) + \sum_{i=1}^{s} w(a_i) \le w(\mathcal{S}) + \sum_{i=1}^{s} w(b_i) \le w(\mathcal{S}) + \sum_{i=1}^{\hat{s}} w(b_i) = w(\hat{\mathcal{S}}) = \Omega.$$

□

We can use Algorithm 1 to determine a maximum weight clique of $\mathcal{G}_{n,d,k,q}$ without explicitly traversing all cliques:

Proposition 2. *Algorithm 2 determines a maximum weight clique $\mathcal{U}$ in $\mathcal{G}_{n,d,k,q}$.*

Proof. Let $\mathcal{U}'$ be a maximum weight clique in $\mathcal{G}_{n,d,k,q}$ and $\mathcal{U}_i'$ be the subset of the smallest i elements of $\mathcal{U}$ for $1 \le i \le \#\mathcal{U}'$. Now let m be the largest index such that `Dive` is called with $\mathcal{S} = \mathcal{U}_m'$. Since `Dive` is initially called with $\mathcal{S} = \emptyset = \mathcal{U}_0'$, $0 \le m \le \#\mathcal{U}'$ is well defined. If $m = \#\mathcal{U}'$ then either $\mathcal{U}$ is set to $\mathcal{U}'$ or we already have $w(\mathcal{U}) \ge w(\mathcal{U}')$. Note that every replacement of $\mathcal{U}$ strictly increases the value of $w(\mathcal{U})$. In the remaining cases we assume $m < \#\mathcal{U}$ and set $v' = \mathcal{U}_{m+1}'\backslash\mathcal{U}_m'$. By construction we have $l \le v'$, so that the for loop of Algorithm 3 attains $v = v'$. Since $\mathcal{U}_{m+1}'$ is a clique we have $\Big\{ \{x,v\} : x \in \mathcal{S}\Big\} \subseteq \mathcal{E}$ and since `Dive` is not called with $\mathcal{U}_{m+1}' = \mathcal{S} \cup \{v\}$ we have $\texttt{UB}(\mathcal{G}, \mathcal{S} \cup \{v\}, ub) \le w(\mathcal{U})$, so that Lemma 5 gives $w(\mathcal{U}) \ge w(\mathcal{U}')$. Since

Input: graph $\mathcal{G} = (\mathcal{V}, \mathcal{E})$ with weight function $w\colon \mathcal{V} \to \mathbb{N}$ such that $\mathcal{V} \subseteq \mathbb{N}$ and $w(i) \ge w(j)$ if $i \le j$, a clique $\mathcal{S}$ in $\mathcal{G}$, and an upper bound ub on the maximum clique size in $\mathcal{G}$

Output: An upper bound Ω for the weight of every clique extension of $\mathcal{S}$

$\Omega \longleftarrow w(\mathcal{S})$;
$\hat{\mathcal{S}} \longleftarrow \mathcal{S}$;
$m \longleftarrow 0$;
if $\mathcal{S} \neq \emptyset$ **then**
  $m \longleftarrow \max\{v : v \in \mathcal{S}\}$;
for v *from* $m+1$ *to* $\#\mathcal{V}$ **do**
  if $\Big\{ \{x, v\} : x \in \mathcal{S} \Big\} \subseteq \mathcal{E}$ ***and*** $\#\hat{\mathcal{S}} < ub$ **then**
    $\Omega \longleftarrow \Omega + w(v)$;
    $\hat{\mathcal{S}} \longleftarrow \hat{\mathcal{S}} \cup \{v\}$;
return Ω;

Algorithm 1: `UB`: upper bound for the weight of an extended clique

every replacement of $\mathcal{U}$ strictly increases the value of $w(\mathcal{U})$ the proposed statement follows. □

Input: graph $\mathcal{G} = (\mathcal{V}, \mathcal{E})$ with weight function $w\colon \mathcal{V} \to \mathbb{N}$ such that $\mathcal{V} \subseteq \mathbb{N}$ and $w(i) \ge w(j)$ if $i \le j$, a clique $\mathcal{S}$ in $\mathcal{G}$, and an upper bound ub on the maximum clique size in $\mathcal{G}$

Output: a weight maximum clique $\mathcal{U}$ with respect to w

`// global data structures:`
$\mathcal{U} \longleftarrow \emptyset$;
`// local data structures:`
$\mathcal{S} \longleftarrow \emptyset$;
`Dive` $(\mathcal{G}, w, \mathcal{S}, ub)$;
return $\mathcal{U}$;

Algorithm 2: Framework for the maximum weight clique algorithm

As an application of Proposition 2 we remark that for $(n, d, k) = (13, 4, 5)$ and $q \in \{2, \dots, 9\}$ the lower bound stated in Proposition 1 is indeed the optimal multilevel/Echelon-Ferrers construction. So, based on Lemma 5, an exhaustive search is indeed possible, while in [8] only a heuristic was used. However, we also have to use heuristics for larger parameters and replace Algorithm 2 by Algorithm 4. (The enlargement of $\mathcal{L}'$ has to be implemented in a modified version of `Dive` to be technically correct.)

We can iteratively apply Algorithm 4 in order to heuristically find a clique of large weight in $\mathcal{G}_{n,d,k,q}$. Starting from $\mathcal{L} = \emptyset$ we can use the determined list $\mathcal{L}'$ in the next

Input: clique $\mathcal{S} \subseteq \mathcal{V}$ and the input data from Algorithm 2
Output: -
if $w(\mathcal{S}) > w(\mathcal{U})$ **then**
  $\mathcal{U} \longleftarrow \mathcal{S}$;
Let $1 \le l \le \#V$ be the smallest index such that the elements in $\mathcal{S}$ have strictly smaller indices; return if no such index exists;
for v *from* l *to* $\#\mathcal{V}$ **do**
  if $\Big\{ \{x, v\} \,:\, x \in \mathcal{S} \Big\} \subseteq \mathcal{E}$ **then**
    if `UB` *(*$\mathcal{G}$*,* w*,* $\mathcal{S} \cup \{v\}$*,* ub*)*$> w(\mathcal{U})$ **then**
      `Dive` ($\mathcal{G}$, w, $\mathcal{S} \cup \{v\}$, ub);
return;

Algorithm 3: Subroutine `Dive`

Input: graph $\mathcal{G} = (\mathcal{V}, \mathcal{E})$ with weight function $w\colon \mathcal{V} \to \mathbb{N}$ such that $\mathcal{V} \subseteq \mathbb{N}$ and $w(i) \ge w(j)$ if $i \le j$, a clique $\mathcal{S}$ in $\mathcal{G}$, a list $\mathcal{L}$ of cliques in $\mathcal{G}$, and parameters Δ_1, Δ_2, ub
Output: a weight maximum clique $\mathcal{U}$ with respect to w and a list $\mathcal{L}'$ of cliques
`// global data structures:`
$\mathcal{U} \longleftarrow \emptyset$;
$\mathcal{L}' \longleftarrow \emptyset$;
for *each* $\mathcal{S} \in \mathcal{L}$ **do**
  `Dive` ($\mathcal{G}$, w, $\mathcal{S}$, ub);
  if `Dive` is called with $\#\mathcal{S} \ge ub - \Delta_2$ then add the smallest $ub - \Delta_2 - \Delta_1$ elements of $\mathcal{S}$ as a clique to $\mathcal{L}'$;
return $\mathcal{U}$;

Algorithm 4: Framework a heuristic for the maximum weight clique algorithm

round increasing ub incrementally, say by 10 in each iteration, until no further improvement is found. The advantage of that approach is that using the parameters Δ_1 and Δ_2 we can control the extend to which we want to perform an exhaustive search. The iterative process also partially prevents from stalling in local neighborhoods of partial cliques that are very diverse to a global optimum. As an example for a result obtained by the application of Algorithm 4 we state:

Proposition 3. $A_q(17,4;6) \ge q^{55} + q^{51} + q^{49} + 8q^{47} + 3q^{45} + 3q^{44} + 5q^{43} + q^{42} + 5q^{41} + 9q^{40} + 22q^{39} + 7q^{38} + 11q^{37} + 13q^{36} + 19q^{35} + 3q^{34} + 17q^{33} + 15q^{32} + 69q^{31} + 20q^{30} + 49q^{29} + 22q^{28} + 33q^{27} + 15q^{26} + 23q^{25} + 20q^{24} + 38q^{23} + 17q^{22} + 29q^{21} + 24q^{20} + 40q^{19} + 19q^{18} + 20q^{17} + 15q^{16} + 28q^{15} + 15q^{14} + 13q^{13} + 8q^{12} + 7q^{11} + 5q^{10} + 3q^{9} + 10q^{8} + q^{7} + 2q^{6} + q^{5} + 2q^{4} + q^{3} + q^{2} + q^{1} + q^{0}$.

Proof. We apply Theorem 2 with skeleton codes $\mathcal{S}_{17,4,6}$, see Appendix B. □

In Table 1 we list the cases where Algorithm 2 or Algorithm 4 yields a skeleton code such that Theorem 2 gives a strictly better constructive lower bound for $A_q(n,4;k)$. In all cases we obtain improvements for all $q \in \{2,\ldots,9\}$, see Table 3 in Appendix A. The utilized skeleton codes are listed in Appendix B. Some of them consist of over 1000 identifying vectors. Whenever we state two skeleton codes, for given parameters n and k, the first code gives the lower bound for $A_2(n,4;k)$ and the second code gives the lower bound for general field sizes q, which is larger for $q \geq 3$, see Proposition 1 for an example. Whenever we only list one skeleton code, it yields the utilized lower bound for all field sizes $q \geq 2$, see Proposition 3 for an example. We remark that for $d = 4$ all explicitly stated numerical results of [8, 19] have been strictly improved. (The other results of [8] depend on the unproven assumption, nevertheless the author states otherwise, that the upper bound of Theorem 1 is attained for $d = 6$.)

k	n
5	13, 14
6	14, 15, 16, 17
7	16, 17, 18, 19
8	18, 19
9	19

TABLE 1. Parameters where improved codes for $A_q(n,4;k)$ have been found using Algorithm 2 or Algorithm 4.

Another approach, to obtain improved lower bounds, is based on Theorem 3 and Lemma 4:

Proposition 4. $A_q(11,4;3) \geq q^8 \cdot A_q(7,4;3) + q^4 + q^3 + 2q^2 + q + q^0$.

Proof. We apply Theorem 3 with $\Delta = 4$ to deduce $A_q\left(11,4;3;\binom{7}{3},\binom{4}{0}\right) \geq q^8 \cdot A_q(7,4;3)$ and Theorem 2 with the skeleton code $\mathcal{S}_{11,7,4,3}$, see Appendix B, to deduce $A_q\left(11,4;3;\binom{7}{\leq 1},\binom{4}{\geq 2}\right) \geq q^4 + q^3 + 2q^2 + q + q^0$. With this, the stated lower bound follows from Lemma 4. □

For $q \geq 3$ Proposition 4 yields a strictly larger lower bound than the previous record from [10]. Using $\Delta = 3$ and skeleton code

$$\mathcal{S}_{10,7,4,3} = \{0000100110, 0000010101, 0000001011\} \mathrel{\hat{=}} \{38, 21, 11\},$$

where the integers give the binary identifying vectors reading them in base 2 representation, gives $A_q(10,4;3) \geq q^6 \cdot A_q(7,4;3) + q^2 + q + 1$, which equals the lower bound from [10]. For the naming of the skeleton codes $\mathcal{S}_{n,m,d,k}$ for $A_q(n,d;k)$ we use $m = n - \Delta$ as an abbreviation.

k	n
3	11(7), 12(7), 13(7), 14(11,7), 15(11,7), 16(13,7), 17(13,7), 18(13,7), 19(13,7)
4	13(8), 14(8), 15(8), 17(12), 18(12), 19(12)

TABLE 2. Parameters where improved codes for $A_q(n,4;k)$ have been found using Theorem 3 and Lemma 4.

In Table 2 we list the parameters n and k where the approach based on Theorem 3 and Lemma 4 yields strict improvements. The corresponding value of m is stated in brackets, where the last is for general field sizes q and the last but one for $q = 2$. In some cases, when n is rather small, better lower bounds for $A_2(n,4;k)$ have been found using integer linear programming and prescribed automorphisms, see e.g. [15] for an introductory paper and [11] for the precise reference per specific instance. The numerical improvements are listed in Table 6 in Appendix A and the corresponding skeleton codes in Appendix B. We remark that for $k = 4$ and $n \in \{12, 16\}$ the obtained codes are inferior compared to those from [3]. We remark that for $d = 4$ all numerical results of [17] have been strictly improved.

In order to find the skeleton codes Algorithm 2 and Algorithm 4 are applied by modifying the input graphs $\mathcal{G}_{n,m,d,k,q}$, where $m = n - \Delta$, accordingly. Since Lemma 5 and Algorithm 1 significantly speed up the solution process by cutting parts of the search tree as early as possible, we also want to use "good" upper bounds on the maximum clique size in $\mathcal{G}_{n,m,d,k,q}$. While of cause the maximum clique size in $\mathcal{G}_{n,d,k,q}$ is an upper bound this can usually be improved.

Proposition 5. *Let $1 \le k \le m \le n$ and $2 \le d/2 \le k$ be integers. Then, the maximum cardinality of a set $\mathcal{S} \subseteq \mathbb{F}_2^n$ of binary vectors with Hamming weight k that have at most $k - d/2$ ones in their first m coordinates with minimum Hamming distance d is upper bounded by $\sum_{j\in J} c_j$, where $J = \{j \in \mathbb{N} : j \ge k - n + m, j \le k - d/2\}$ and the c_j are integers satisfying the constraints*

$$\sum_{j\in J} \binom{j}{i} \cdot \binom{k-j}{k-d/2+1-i} c_j \le \binom{m}{i}\binom{n-m}{k-d/2+1-i}$$

for all $0 \le i \le k - d/2 + 1$. Moreover, we have $c_0 \le A_1(n - m, d; k)$, which denotes the maximum cardinality of a subset of binary vectors in $\mathbb{F}_2^{n-m}$ with Hamming weight k and minimum Hamming distance d.

Proof. Given $\mathcal{S}$ we let c_j be the number of elements in $\mathcal{S}$ that have exactly j ones in their first m coordinates. It can be easily checked that $j \in J$ implies $0 \le j \le m$, $0 \le k - j \le n - m$, and $j \le k - d/2$, i.e., the counts c_j are well defined. Since $\mathcal{S}$ has minimum Hamming distance d every subset $I \subseteq \mathbb{F}_2^n$ of cardinality $k - d/2 + 1$ is contained in the support of at most one element in $\mathcal{S}$. For a given integer $0 \le i \le k - d/2 + 1$ we consider all those sets I containing exactly i elements in $\{1, \ldots, m\}$.

There are exactly $\binom{m}{i}\binom{n-m}{k-d/2+1-i}$ of those sets and every element of $\mathcal{S}$ with j ones in the first m coordinates has a support that contains exactly $\binom{j}{i} \cdot \binom{k-j}{k-d/2+1-i}$ of those sets. Thus, the stated inequalities are valid. The additional upper bound for c_0 follows directly from our choice of the count c_0. □

Given fixed parameters the integer linear program of Proposition 5 can be solved easily. As an example we will show that also parametric upper bounds can be concluded. To this end, let $k = 4$, $d = 4$, and $n - m \geq 4$, so that $J = \{0, 1, 2\}$. For $i \in \{0, 1, 2\}$ the constraints of Proposition 5 read

$$\begin{aligned} 4c_0 + c_1 &\leq \binom{m}{0}\binom{n-m}{3} = \binom{n-m}{3}, \\ 3c_1 + 2c_2 &\leq \binom{m}{1}\binom{n-m}{2} = \frac{m(n-m)(n-m-1)}{2}, \\ 2c_2 &\leq \binom{m}{2}\binom{n-m}{1} = \frac{m(m-1)(n-m)}{2}, \end{aligned}$$

noting that that the constraint for $i = 3 = k - d/2 + 1$ is trivially satisfied. Since c_1 and c_2 are non-negative integers, dividing the second constraint by two gives

$$c_1 + c_2 \leq \left\lfloor \frac{m(n-m)(n-m-1)}{4} \right\rfloor,$$

so that $c_0 \leq A_1(n-m, 4; 4)$ gives

$$c_0 + c_1 + c_2 \leq \left\lfloor \frac{m(n-m)(n-m-1)}{4} \right\rfloor + A_1(n-m, 4; 4).$$

For $(n, m) = (13, 8)$ we obtain an upper bound of $40 + 1 = 41$, which indeed is attained by $\mathcal{S}_{13,8,4,4}$, see Appendix B, while $A_1(13, 4; 4) = 65$. Similarly, for $(n, m) = (14, 8)$ we obtain an upper bound of $60 + 3 = 63$, which indeed is attained by $\mathcal{S}_{14,8,4,4}$, while $A_1(14, 4; 4) = 91$ is much larger.

We remark that Proposition 5 mimics [16, Lemma 4.1], which proves an upper bound for $A_q\left(n, d; k; \binom{n-\Delta}{\leq k-d/2}, \binom{\Delta}{\geq d/2}\right)$. Possibly the quantity $A_1(n, m, d; k)$ upper bounded in Proposition 5, generalizing $A_1(n, d; k)$, is interesting on its own and similar techniques as for the (partial) determination of $A_1(n, d; k)$ can be applied. To this end, we remark that the upper bound of Proposition 5 can surely be improved in most cases but was sufficiently good for our purpose.

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APPENDIX A. IMPROVED LOWER BOUNDS

In Table 3 we compare the numerical improvements for $A_q(n,4;k)$ based on Theorem 2. The previously best known lower bounds are mostly from [8]. However, the lower bounds for $A_q(19,4;7)$ from [8] are flawed, i.e., some magnitudes too small.

$A_q(n,4;k)$	New	Old
$A_2(13,4;5)$	4796825069	4796417559 [8]
$A_3(13,4;5)$	1880918252176932	1880918023783990 [8]
$A_4(13,4;5)$	18525690519076963184	18525690479132333173 [8]
$A_5(13,4;5)$	23322304251254415373950	23322304248923865096456 [8]
$A_7(13,4;5)$	1104898620940893642387898640	1104898620939789578683671514 [8]
$A_8(13,4;5)$	79247846163928378274466378432	79247846163915655208442806985 [8]
$A_9(13,4;5)$	3434214279120463268840947142394	3434214279120353599762054717228 [8]
$A_2(14,4;5)$	76749681496	76745404672 [8]
$A_3(14,4;5)$	152354381482802889	152354354408240436 [8]
$A_4(14,4;5)$	4742576773448941977744	4742576757171205745408 [8]
$A_5(14,4;5)$	14576440157067948253027775	14576440154794852120820500 [8]
$A_7(14,4;5)$	2652861588879102858398037208053	2652861588875282767080909163052 [8]
$A_8(14,4;5)$	324599177887450844354761706030144	324599177887378338095324360943616 [8]
$A_9(14,4;5)$	22531879885309361371329789150871659	22531879885308389971852673089028988 [8]
$A_2(13,4;6)$	38327432465	38325131657 [8]
$A_3(13,4;6)$	50782269101589019	50782269101569336 [8]
$A_4(13,4;6)$	1185639430145591548865	1185639430145591024577 [8]
$A_5(13,4;6)$	2915286427121720403833501	2915286427121720397974126 [8]
$A_7(13,4;6)$	378980216844611802379905892367	378980216844611802379704124332 [8]
$A_8(13,4;6)$	40574896910456482568428114359809	40574896910456482568427309053441 [8]
$A_9(13,4;6)$	2503542202545027727509322118254705	2503542202545027727509319406311282 [8]
$A_2(14,4;6)$	1227267234053	1227203232293 [8]
$A_3(14,4;6)$	12340234566815274820	12340234566810426241 [8]
$A_4(14,4;6)$	1214095649227435851637265	1214095649227435312865809 [8]
$A_5(14,4;6)$	9110270832108553596766282526	9110270832108553578429954401 [8]
$A_7(14,4;6)$	6369520523839151727825310046433656	6369520523839151727821917674342793 [8]
$A_8(14,4;6)$	1329558223045414729174436187566776385	1329558223045414729174409793499570241 [8]
$A_9(14,4;6)$	147831663555770209444761899682581639650	147831663555770209444761739522908440329 [8]
$A_2(15,4;6)$	39273527139056	39267675031563 [8]
$A_3(15,4;6)$	2998677038028861083854	2998676636295383433055 [8]
$A_4(15,4;6)$	1243233944887303700963929029	1243233943362040432057180581 [8]
$A_5(15,4;6)$	28469596350369158409165204552256	28469596349440811610995019309681 [8]
$A_7(15,4;6)$	107052531444164864809019626364096185842	107052531444149851093995761837649623043 [8]
$A_8(15,4;6)$	43566963852752158504511827652740340200969	43566963852751451455533741586485379191945 [8]
$A_9(15,4;6)$	87293119013046753015079265888857783791854 44	87293119013046541123889721019479429050233 81 [8]
$A_2(16,4;6)$	1256765678235469	1256703351587805 [8]
$A_3(16,4;6)$	728678523485248028210242	728678523483522880513165 [8]
$A_4(16,4;6)$	1273071559584675915665964748625	1273071559584674249524907514705 [8]
$A_5(16,4;6)$	88967488594921579438014390756229526	88967488594921579094529973785226401 [8]
$A_7(16,4;6)$	17992318959820794052190487867406638758853 38	17992318959820794052179837126816751848892 05 [8]
$A_8(16,4;6)$	14276022715269827610082997266445121659360 67137	14276022715269827610082737902847157431165 87585 [8]

TABLE 3. Improvements based on Theorem 2.

$A_q(n,4;k)$	New	Old
$A_9(16,4;6)$	5154571384601397730287595638954067515856079 78830	5154571384601397730287591300333304652935777 20745 [8]
$A_2(17,4;6)$	40214593296350543	40210734642430233 [8]
$A_3(17,4;6)$	17706888124530311690233954 6	177068857538981556600415147 [8]
$A_4(17,4;6)$	130362527701515426804753058800597 3	13036252754160145629780423288891 21 [8]
$A_5(17,4;6)$	27802340185913059485814361134634777015 6	27802340185006505130000103396342638005 1 [8]
$A_7(17,4;6)$	3023969047577080860475188125680128048826831 1358	3023969047576656762486652218945332498636702 4243 [8]
$A_8(17,4;6)$	4677967123339617111605053771369641637526181 3580361	4677967123339541192988109893334610569638870 8610177 [8]
$A_9(17,4;6)$	3043722856893279345773594379914249147185590 9627410848	3043722856893271957593877642162439701004004 2569507531 [8]
$A_2(15,4;7)$	313939996903443	313923840120169 [8]
$A_3(15,4;7)$	809623907356805726683 48	80962387333738514962426 [8]
$A_4(15,4;7)$	795668637761460890928730590 65	79566863724904828874349525569 [8]
$A_5(15,4;7)$	35586990308382671094689082311485 86	355869903075037543196636766848887 6 [8]
$A_7(15,4;7)$	3671901811167589276646725300523023447280 0	3671901811166948536632605172623651127013 4 [8]
$A_8(15,4;7)$	2230628546549055386800205460016664942615931 3	2230628546549001382437710508355500600532224 1 [8]
$A_9(15,4;7)$	6363668373716727919196827279641011245211439 310	6363668373716701033966177001673180121197368 344 [8]
$A_2(16,4;7)$	20093092605969267	20090530823175168 [11]
$A_3(16,4;7)$	5902159990181063084238456 4	590215919070980962386487 17 [8]
$A_4(16,4;7)$	325905875863710195123556939265157	325905875503966895183219736444928 [8]
$A_5(16,4;7)$	55604672371466910516074031340231456506	55604672369921077974140644073486328125 [8]
$A_7(16,4;7)$	4319955761831483147392111643370405301140304 392	4319955761830991620286839274911971343679804 101 [8]
$A_8(16,4;7)$	5847458897067883289789161262085069896599613 506569	5847458897067807309822598445696990677571597 238272 [8]
$A_9(16,4;7)$	3381914284196654468636744719790551399185806 891914222	3381914284196647958706309890495614396898957 540722053 [8]
$A_2(17,4;7)$	1285973764408635208	1285780755925958656 [8]
$A_3(17,4;7)$	43026746493711590523056074275	43026740586030433477849264332 [8]
$A_4(17,4;7)$	133491046755498571404771556159619436 8	133491046607515354591777884239770419 2 [8]
$A_5(17,4;7)$	8688230058043081366965061690776076764763 75	8688230057801169780083000659942626953125 00 [8]
$A_7(17,4;7)$	5082384754237122694106062733081158653196752 71592683	5082384754236544286588065373763554392106735 71016516 [8]
$A_8(17,4;7)$	1532876265112963221100230608239000778944194 589626442240	1532876265112943302025972486322657421692282 703427141632 [8]
$A_9(17,4;7)$	1797287909107754250279842359793367953350214 940243153381903	1797287909107750790631435583972090558986192 090028734124332 [8]
$A_2(18,4;7)$	82302571633443282819	82291970549255555624 [11]
$A_3(18,4;7)$	31366498204356725852831211333673	31366481390109307710095048570592 [8]
$A_4(18,4;7)$	5467793275108157387543615361694729718789	5467793218060404819993105682513015078912 [8]
$A_5(18,4;7)$	1357535946569236568489855246745354062759843 0631	1357535945899618013799772597849369049072265 6250 [8]
$A_7(18,4;7)$	5979374839512432491438423629603821088645709 0991018946109	5979374839483504026532674648217830203768951 6829976915564 [8]
$A_8(18,4;7)$	4018343156417726306779719721598697301069358 44085497257345033	4018343156414647599769044241394647941280954 21048885275000832 [8]
$A_9(18,4;7)$	9551524837041340265310654836054497707429084 2137526889056874 9019	9551524837039904386035921158457210995678555 4364524375603467 1222 [8]

TABLE 4. Improvements based on Theorem 2 cont.

$A_q(n,4;k)$	New	Old
$A_2(19,4;7)$	5267367924445148864092	5058097205000347197549 [21]
$A_3(19,4;7)$	2286617719062189275767922318915925	22813524065165704375162588681706875 [21]
$A_4(19,4;7)$	22396081254840251014738769405257047429902630	22388542052518188728938813857333763516548177 [21]
$A_5(19,4;7)$	212114991651442904801790962946768806466026993016307	212098136800904512722363506327420535880394984750151 [21]
$A_7(19,4;7)$	7034674704937981691536939532038341471264547397843085217471817	7034607564860042802726763060199636693617578982540335678343419 [21]
$A_8(19,4;7)$	105338454839596844486744190127909498074561619935428942997712801930	105338015236067410645641833582747817345721344118067187280665018945 [21]
$A_9(19,4;7)$	507607191092208691189655287695910007652613420811027477177039803795175	507606165043191151744342958417924108474489256852919544839333945104727 [21]
$A_2(18,4;8)$	1316667538397101428149	1264601087568682942805 [21]
$A_3(18,4;8)$	2540681546251523549771895302399518	2534836537121138399731153735006570 [21]
$A_4(18,4;8)$	1399755003650511313993312179188667483412753	1399283883495040631375076946731142842749201 [21]
$A_5(18,4;8)$	8484599639617517977059919631386329281416928756276	8483925473426183151796067249961197376251221110026 [21]
$A_7(18,4;8)$	143564789890828673950879999130406072192091727223436678152394	143563419691232957462860102908032188890771110979148690800890 [21]
$A_8(18,4;8)$	1645913356856762202620793478078317759855408541359722889311735873	1645906488063927527123120350869468572360596441738049974310277185 [21]
$A_9(18,4;8)$	6266755445572949210771538046911060250703242571284112409913460035888	6266742778311275807883809715767905942364252612630126994566337005626 [21]
$A_2(19,4;8)$	168534060204346081643054	161868939208791416732918[21]
$A_3(19,4;8)$	5556470543990117093920796733024932097	5543687506683929680212033218489009029[21]
$A_4(19,4;8)$	22933585979825739912419408751656551857477750740	22925867147182745704449260695243044338837225537[21]
$A_5(19,4;8)$	662859346845121791249456607590251642324227507808667905	662806677611420558734067753903218545019626771492187626[21]
$A_7(19,4;8)$	118231777761062718962220892216025575894811105121482204632859666129	118230649342777063487836197729189552935672313049101078765671318222[21]
$A_8(19,4;8)$	3451730488158872567254861778301003471443124750791181363571603492943432	3451716083256241741361306090066607755463169548983834979971097956450817[21]
$A_9(19,4;8)$	29973697026756603314019362623281023278094694159378234061636310178009770385	29973636439636704539558217472416705317244006954378905881075605724274378842[21]
$A_2(19,4;9)$	1348002146261417447406857	1289520797394170812563456 [4]
$A_3(19,4;9)$	150024595081884393007012600338236544880	149656439781495352647490043484854812672 [4]
$A_4(19,4;9)$	1467749485360498225634923971135876693953433631137	1467233016300822815028029538272531565587680198656 [4]
$A_5(19,4;9)$	82857418316609426935359109104035114463034035070083878426	82850622250904212594169271527415901448337330052257546240 [4]
$A_7(19,4;9)$	40553499771898134480982178003920834382400027150097709434199076454452	40553105686976525598537903185027802877404377818341526587313754210304 [4]
$A_8(19,4;9)$	1767286009936419984391665170809196907855921985408931899768042741180322945	1767278529263067694187078101991773742855576219388246358086384262766919680 [4]
$A_9(19,4;9)$	21850825132503488679494454968297144272741734751304289430201060161991357689262	21850780456810680205404210364952225732272903279105184451471542312700846538752 [4]

TABLE 5. Improvements based on Theorem 2 cont.

In Table 3 we compare the numerical improvements for $A_q(n,4;k)$ based on Theorem 2, Theorem 3, and Lemma 4. We also list a few cases which result in the same lower bound that was previously known. For consistency reasons we also list the obtained lower bounds for $A_q(12,4;4)$ and $A_q(16,4;4)$ which are inferior to the lower bounds obtained in [3].

$A_q(n,4;k)$	New	Old
$A_3(10,4;3)$	5086975	5086975 [10]
$A_4(10,4;3)$	273727509	273727509 [10]
$A_5(10,4;3)$	6162421906	6162421906 [10]
$A_7(10,4;3)$	680487816156	680487816156 [10]
$A_8(10,4;3)$	4407724867657	4407724867657 [10]
$A_9(10,4;3)$	22911698562814	22911698562814 [10]
$A_3(11,4;3)$	45782788	45782686 [10]
$A_4(11,4;3)$	4379640165	4379639873 [10]
$A_5(11,4;3)$	154060547681	154060547001 [10]
$A_7(11,4;3)$	33343902991701	33343902989195 [10]
$A_8(11,4;3)$	282094391530121	282094391525889 [10]
$A_9(11,4;3)$	1855847583588025	1855847583581293 [10]
$A_3(12,4;3)$	412045132	412044676 [10]
$A_4(12,4;3)$	70074242725	70074241065 [10]
$A_5(12,4;3)$	3851513692181	3851513687561 [10]
$A_7(12,4;3)$	1633851246593749	1633851246571461 [10]
$A_8(12,4;3)$	18054041057928329	18054041057886353 [10]
$A_9(12,4;3)$	150323654270630845	150323654270557225 [10]
$A_3(13,4;3)$	3708406309	3708405100 [10]
$A_4(13,4;3)$	1121187883941	1121187877725 [10]
$A_5(13,4;3)$	96287842305306	96287842283141 [10]
$A_7(13,4;3)$	80058711083096502	80058711082943685 [10]
$A_8(13,4;3)$	1155458627707417737	1155458627707087193 [10]
$A_9(13,4;3)$	12176215995921105826	12176215995920451445 [10]
$A_2(14,4;3)$	6241671	6241671 [10]
$A_3(14,4;3)$	33375657145	33375648396 [10]
$A_4(14,4;3)$	17939006144421	17939006056956 [10]
$A_5(14,4;3)$	2407196057636556	2407196057148120 [10]
$A_7(14,4;3)$	3922876843071748206	3922876843065022206 [10]
$A_8(14,4;3)$	73949352173274772617	73949352173255598072 [10]
$A_9(14,4;3)$	986273495669609638336	986273495669561209956 [10]
$A_3(15,4;3)$	300380915398	300380802505 [10]
$A_4(15,4;3)$	287024098316197	287024096598921 [10]
$A_5(15,4;3)$	60179901440933431	60179901426410736 [10]
$A_7(15,4;3)$	192220965310515799351	192220965310140305694 [10]
$A_8(15,4;3)$	4732758539089585747081	4732758539088207997713 [10]
$A_9(15,4;3)$	79888153149238381303087	79888153149234028941436 [10]
$A_2(16,4;3)$	102223687	102223687 [10]
$A_3(16,4;3)$	2703428238582	2703427322125 [10]
$A_4(16,4;3)$	4592385573059152	4592385547188501 [10]
$A_5(16,4;3)$	1504497536023335775	1504497535674194406 [10]
$A_7(16,4;3)$	9418827300215274168199	9418827300197242691478 [10]
$A_8(16,4;3)$	302896546501733487813184	302896546501646667812937 [10]
$A_9(16,4;3)$	6470940405088308885550047	6470940405087960642352846 [10]

TABLE 6. Improvements based on Theorem 2, Theorem 3, and Lemma 4.

$A_q(n,4;k)$	New	Old
$A_2(17,4;3)$	408894755	408894729 [10]
$A_3(17,4;3)$	24330854147239	24330847680853 [10]
$A_4(17,4;3)$	73478169168946433	73478168809292217 [10]
$A_5(17,4;3)$	37612438400583394376	37612438392939375961 [10]
$A_7(17,4;3)$	461522537710548434241752	461522537709756797516943 [10]
$A_8(17,4;3)$	193853789761109432200437777	193853789761059155009916977 [10]
$A_9(17,4;3)$	52414617281215301972955380 8	524146172812127278980234877 [10]
$A_2(18,4;3)$	1635579035	1635578957 [10]
$A_3(18,4;3)$	218977687325155	218977629126520 [10]
$A_4(18,4;3)$	1175650706703142933	1175650700948669781 [10]
$A_5(18,4;3)$	940310960014584859406	940310959823484378906 [10]
$A_7(18,4;3)$	2261460434781687327784585 6	226146043477780830781906 14 [10]
$A_8(18,4;3)$	12406642544711003660828017 37	12406642544707785920631650 01 [10]
$A_9(18,4;3)$	424558399977843945980938584 58	424558399977823095973984207 06 [10]
$A_2(19,4;3)$	6542316171	6542316059 [10]
$A_3(19,4;3)$	1970799185926408	1970798662145206 [10]
$A_4(19,4;3)$	18810411307250286949	18810411215178781533 [10]
$A_5(19,4;3)$	23507774000364621485181	23507773995587109861266 [10]
$A_7(19,4;3)$	1108115613043026790614447001	1108115613041126070837091728 [10]
$A_8(19,4;3)$	7940251228615042342929931124 1	7940251228612982989205931042 5 [10]
$A_9(19,4;3)$	34389230398205359624456025351 89	34389230398203670773893150739 58 [10]
$A_2(12,4;4)$	19674269	19676797 [3]
$A_3(12,4;4)$	288648673507	288648887023 [3]
$A_4(12,4;4)$	283104148286289	283104153226065 [3]
$A_5(12,4;4)$	59732550564570151	59732550620930151 [3]
$A_7(12,4;4)$	191677878196845899475	191677878199060649103 [3]
$A_8(12,4;4)$	47237229505049081247 37	47237229505144234440 33 [3]
$A_9(12,4;4)$	797804410207202373083 59	797804410207546805638 15 [3]
$A_2(13,4;4)$	157396313	157332190 [3]
$A_3(13,4;4)$	7793514240823	7793495430036 [3]
$A_4(13,4;4)$	18118665490931521	18118664249474716 [3]
$A_5(13,4;4)$	7466568820575245751	7466568787180077320 [3]
$A_7(13,4;4)$	657455122215182132089 51	657455122165552896141 88 [3]
$A_8(13,4;4)$	24185461506585131790955 53	24185461506221269214774 96 [3]
$A_9(13,4;4)$	58159941504105053602711 351	58159941503893673245551 936 [3]
$A_2(14,4;4)$	1259181253	1258757174 [3]
$A_3(14,4;4)$	210424885316173	210424421624298 [3]
$A_4(14,4;4)$	1159594591440766481	1159594516050838620 [3]
$A_5(14,4;4)$	933321102572187566901	933321098538702991570 [3]
$A_7(14,4;4)$	2255071069198076197705411 7	2255071069030902876467149 8 [3]
$A_8(14,4;4)$	1238295629137158820145963073	1238295629118788686643907448 [3]
$A_9(14,4;4)$	423985973564925843706982381 41	423985973563402044449578485 30 [3]
$A_2(15,4;4)$	10073483841	10071464646 [3]
$A_3(15,4;4)$	5681471907358915	5681463153275925 [3]
$A_4(15,4;4)$	742140538523340567 93	742140501691015483 68 [3]
$A_5(15,4;4)$	11666513782152541784766 1	11666513741527966102765 0 [3]
$A_7(15,4;4)$	77348937673494014924850755 43	77348937668570152587692895 66 [3]
$A_8(15,4;4)$	6340073621182253166433198782 25	6340073621099867758588340106 88 [3]
$A_9(15,4;4)$	309085774728830940094938701255 53	309085774727842869893999409571 38 [3]
$A_2(16,4;4)$	80596312221	80596320222 [3]
$A_3(16,4;4)$	153399853228893616	153399853246113244 [3]
$A_4(16,4;4)$	47496995291759148675 37	47496995291771780985 13 [3]
$A_5(16,4;4)$	1458314224143819491027327 6	1458314224143823012288327 6 [3]
$A_7(16,4;4)$	2653068562231495127604331943392	2653068562231495132921600978204 [3]
$A_8(16,4;4)$	32461176940518520138695497645728 1	32461176940518520142592843508588 9 [3]
$A_9(16,4;4)$	22532352977741502934357344934823692	22532352977741502934583323007263948 [3]

TABLE 7. Improvements based on Theorem 2, Theorem 3, and Lemma 4 cont.

$A_q(n,4;k)$	New	Old
$A_2(17,4;4)$	644770526929	644769492958 [3]
$A_3(17,4;4)$	4141796037183639766	4141796035658667783 [3]
$A_4(17,4;4)$	3039807698672586700433933	3039807698669408018756122 [3]
$A_5(17,4;4)$	18228927801797743656863443376	18228927801797534926757804445 [3]
$A_7(17,4;4)$	910002516845402828768417638272482	910002516845402816852352967620731 [3]
$A_8(17,4;4)$	16620122593545482311012166568798208 1	166201225935454822961083064550157688 [3]
$A_9(17,4;4)$	1642608532077355563914650767366956510 8	1642608532077355563775963885865459153 7 [3]
$A_2(18,4;4)$	5158164354661	5158157544758 [3]
$A_3(18,4;4)$	111828493003995506044	111828492966430770027 [3]
$A_4(18,4;4)$	194547692715045570757309611	194547692714852569599519641 [3]
$A_5(18,4;4)$	2278615975224717957648777512 76	2278615975224692748305658821 95 [3]
$A_7(18,4;4)$	3121308632779731702675744107262053 00	3121308632779731662537422715466101 47 [3]
$A_8(18,4;4)$	8509502767895286943238234329290978924 9	8509502767895286935713827198363990719 2 [3]
$A_9(18,4;4)$	1197461619884392206093780437802048887443 6	1197461619884392205993803966201279942005 9 [3]
$A_2(19,4;4)$	41265315376833	41265282958278 [3]
$A_3(19,4;4)$	3019369311108187930600	3019369310399000457648 [3]
$A_4(19,4;4)$	12451052333762916848349771113	12451052333753487629738955504 [3]
$A_5(19,4;4)$	2848269969030897447184201620703 6	2848269969030872056759489735577 5 [3]
$A_7(19,4;4)$	1070608861043447974017783454548525973 60	1070608861043447962195587933023967117 73 [3]
$A_8(19,4;4)$	4356865417162386914937976275020128770926 5	4356865417162386911563469768601956669497 6 [3]
$A_9(19,4;4)$	8729495208957219182423659412931786239553508	8729495208957219181775386539037823737728819 [3]

TABLE 8. Improvements based on Theorem 2, Theorem 3, and Lemma 4 cont.

APPENDIX B. SKELETON CODES FOR THE MULTILEVEL CONSTRUCTION

In this appendix we list the used skeleton codes from the results of Section 3. For the ease of a more compact representation we replace each vector $v \in \mathbb{F}_2^n$ by the integer $\sum_{i=1}^{n} v_i \cdot 2^{n-i}$. As an example, the integer 6168 corresponds to the vector $1100000011000 \in \mathbb{F}_2^{13}$. Starting from an integer, the value of n needs to be clear from the context.

$\mathcal{S}^1_{13,4,5}$ = {7936, 7360, 6816, 1984, 3488, 3680, 5776, 6496, 6544, 6736, 7216, 3720, 5456, 5704, 3400, 5000, 5508, 2948, 4912, 5416, 5668, 7180, 2856, 3604, 4932, 1816, 2882, 6666, 1826, 3346, 4802, 2753, 4336, 5218, 5281, 6406, 6409, 6661, 7171, 1256, 1380, 2706, 2833, 3217, 4514, 4545, 5258, 740, 929, 1618, 2264, 2388, 2636, 3206, 3333, 4705, 696, 1236, 2274, 2442, 3114, 1417, 1669, 2228, 4300, 4426, 4636, 4867, 5189, 376, 466, 841, 1202, 1329, 1577, 2598, 5142, 5145, 428, 618, 790, 1347, 1550, 2161, 2217, 4389, 1084, 2339, 405, 4154, 597, 4243, 651, 563, 2078, 2123, 118, 199, 109, 283, 1063, 4111}

$\mathcal{S}^2_{13,4,5}$ = {7936, 7360, 6816, 3008, 3488, 3680, 5776, 6496, 6544, 6736, 7216,3720, 5456, 5512, 5704, 3400, 1840, 1924, 5668, 7180, 3604, 4904, 4932, 4994, 1858, 2840, 2852, 5410, 6666, 3346, 1264, 1698, 4516, 4801, 5281, 6406, 6409, 6661, 7171, 1380, 1473, 2706, 3217, 4328, 4706, 4748, 4881, 740, 929, 1801, 2264, 2388, 2444, 2636, 3333, 5254, 696, 1617, 2274, 3114, 3142, 4308, 1228, 2228, 2609, 2819, 4426, 5146, 5189, 376, 466, 714, 1308, 4274, 4630, 426, 1550, 2217, 2245, 4209, 409, 617, 4156, 661, 355, 1078, 1081, 309, 333, 2131, 4235, 391, 1099,2078, 583, 110, 539, 2087}

$\mathcal{S}^1_{14,4,5}$ = {15872, 14720, 13632, 3968, 7456, 11072, 11456, 12992, 13088, 13472, 14432, 11536, 6816, 7312, 5904, 6920, 10896, 3680, 6736, 7240, 9992, 14360, 5768, 9808, 11304, 9860, 13332, 5508, 5700, 6468, 10788, 992, 2864, 3608, 9602, 12812, 12818, 13322, 14342, 1488, 1712, 2760, 3394, 4994, 5232, 5666, 6338, 9508, 10530, 3236, 4528, 4552, 6418, 8872, 9089, 9368, 9761, 10508, 10762, 12561, 1448, 1730, 1857, 4712, 5313, 6284, 7173, 8560, 8644, 8980, 10324, 10401, 920, 2288, 2408, 3590, 4756, 6196, 9314, 11267, 12425, 2452, 2497, 3210, 3337, 6659, 9292, 3122, 3153, 4324, 4450, 5164, 6186, 10313, 12357, 844, 850, 2821, 5379, 4657, 8372, 8402, 8522, 8774, 12323, 628, 721, 810, 1308, 1329, 2601, 5145, 4393, 8729, 678, 1557, 4188, 4250, 421, 1114, 1129, 1174, 2150, 2201, 4366, 234, 345, 1187, 1547, 8250, 8334, 403, 611, 653, 310, 4179, 8455, 542, 2183, 8237, 1095}

$\mathcal{S}^2_{14,4,5} = \{$15872, 14720, 13632, 6016, 6976, 7360, 11552, 12992, 13088, 13472, 14432, 7440, 10912, 11024, 11408, 6800, 3680, 3848, 11336, 14360, 7208, 9808, 9864, 9988, 3716, 5680, 5704, 10820, 13332, 6692, 992, 3396, 9032, 9602, 10562, 12812, 12818, 13322, 14342, 2760, 2946, 3457, 5412, 6434, 8656, 9412, 9496, 9762, 1480, 1858, 3602, 4528, 4776, 4888, 5272, 6412, 9089, 10433, 10762, 12561, 1392, 3234, 4548, 6228, 6305, 6665, 8616, 10380, 11269, 1729, 2288, 2408, 2456, 2468, 2849, 5218, 5638, 6282, 7171, 8852, 10292, 10505, 12425, 1428, 1809, 2616, 5385, 8548, 9313, 852, 908, 4705, 5201, 9260, 12357, 2641, 3100, 3121, 4434, 4869, 5253, 8418, 1234, 8312, 8498, 8753, 12323, 690, 1322, 1577, 4739, 710, 806, 4204, 8771, 10259, 618, 2326, 4262, 8346, 451, 665, 677, 2138, 2150, 8462, 220, 233, 316, 345, 1166, 2197, 4154, 8278, 779, 1078, 2567, 589, 1287, 4149, 542, 2093, 4171, 8327, 115, 8221, 1051$\}$

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$\mathcal{S}^2_{13,4,6} = \{$8064, 7776, 7504, 6624, 6864, 6960, 6984, 7344, 7368, 7464, 7704, 3908, 6824, 2016, 5828, 5924, 5954, 3748, 3778, 3874, 6552, 7686, 5794, 3521, 3857, 5524, 7429, 1944, 2964, 3468, 3474, 5057, 5537, 5777, 5897, 2977, 3284, 3380, 3721, 4948, 5004, 5010, 5452, 5514, 6264, 6534, 6789, 6915, 7299, 2898, 2954, 3402, 5330, 5426, 2764, 2860, 4788, 5236, 5292, 1656, 1925, 2676, 2738, 3180, 3186, 3242, 4810, 4906, 4716, 4722, 5226, 6246, 2666, 1478, 1814, 2396, 2417, 3161, 6229, 504, 1265, 1638, 4316, 4412, 4442, 4457, 4697, 5177, 934, 1381, 1621, 1678, 2236, 2266, 2281, 2362, 2617, 6174, 6189, 6195, 6219, 867, 4282, 4325, 1363, 1581, 1587, 1611, 3143, 723, 821, 845, 1206, 1229, 1326, 4679, 5159, 469, 1309, 2599, 435, 459, 683, 795, 374, 429, 669, 1118, 1179, 2327, 414, 4247, 4367, 238, 574, 2191, 123$\}$

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$\mathcal{S}^1_{15,4,6} = \{$32256, 31104, 30016, 26496, 27456, 27840, 27936, 29376, 29472, 29856, 30816, 8064, 15632, 23200, 23312, 23696, 23816, 27280, 15112, 15496, 26208, 30744, 11936, 13968, 14928, 7776, 14084, 14980, 15428, 22096, 22152, 23112, 27176, 29716, 13896, 20228, 22274, 23588, 23618, 12034, 14512, 15394, 19792, 20016, 21808, 21892, 22852, 25056, 26136, 27660, 27666, 29196, 29202, 29706, 30726, 11592, 11652, 13608, 13698, 14628, 14658, 19336, 20098, 26788, 26900, 6624, 7704, 10064, 11056, 12752, 13424, 14881, 22818, 28945, 3792, 10952, 11796, 11841, 12968, 13185, 14529, 18248, 19624, 19841, 20936, 21608, 22049, 26890, 3880, 7489, 18864, 19140, 21313, 21697, 22668, 24984, 25250, 26118, 27141, 28809, 2016, 5828, 6850, 9648, 10664, 12900, 17360, 20720, 21140, 25172, 25745, 25865, 26705, 5040, 6552, 7329, 7686, 9922, 11320, 18084, 18792, 18882, 19233, 19977, 21048, 21090, 23043, 28741, 5794, 9156, 10017, 10850, 12684, 13827, 19553, 20882, 20897, 22577, 24908, 24914, 25162, 26755, 2980, 3428, 3490, 3724, 3857, 4968, 5464, 6708, 6801, 8944, 9448, 9665, 10642, 12514, 12641, 13445, 18648, 19026, 24696, 24966, 25137, 25347, 28707, 1944, 2904, 3009, 5474, 5897, 6356, 6917, 12594, 17128, 17252, 20780, 25637, 25667, 3658, 4824, 4948, 5304, 5521, 5676, 6264, 6534, 6698, 9428, 9778, 9865, 13337, 14357, 17634, 17802, 18972, 2744, 3252, 5002, 5330, 10356, 10778, 17612, 17962, 18982, 20650, 20678, 1844, 3356, 4833, 6441, 9004, 9542, 10396, 10438, 10565, 12380, 12442, 14347, 17202, 17524, 17989, 19477, 21517, 21523, 24678, 1656, 1925, 2508, 2886, 3186, 3273, 3621, 9033, 9498, 10521, 17702, 20570, 1713, 2668, 3226, 4902, 5398, 5413, 5653, 6246, 8650, 9382, 10345, 10531, 18582, 1452, 1859, 2673, 6419, 8982, 9493, 11271, 17050, 17177, 18490, 18701, 20534, 20747, 1393, 4332, 4549, 5262, 8869, 12334, 12371, 12551, 17091, 17166, 17497, 18510, 24606, 504, 937, 1638, 1686, 2282, 2723, 3339, 6221, 8846, 2453, 3118, 4686, 4749, 6174, 8851, 9293, 16572, 1253, 2358, 5195, 8409, 16725, 16739, 17543, 4659, 9259, 16941, 486, 725, 2259, 4277, 8733, 8775, 1563, 2221, 4381, 16563, 1181, 2583, 619, 1118, 365, 411, 574, 207, 1079}

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$\mathcal{S}_{10,7,4,3} = \{38, 21, 11\}$

$\mathcal{S}_{11,7,4,3} = \{44, 74, 25, 134, 69, 35\}$

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$\mathcal{S}_{16,13,4,3}$ = {38, 21, 11}

$\mathcal{S}_{16,7,4,3}$ = {896, 1344, 2240, 2336, 1184, 4368, 608, 8336, 8456, 4232, 16464, 16644, 32816, 32840, 32900, 33026, 4164, 16424, 16514, 536, 8228, 8258, 1044, 4130, 2060, 2066, 1034, 518}

$\mathcal{S}_{17,13,4,3}$ = {44, 74, 25, 134, 69, 35}

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$\mathcal{S}_{18,13,4,3}$ = {56, 84, 140, 146, 74, 273, 38, 521, 1029, 2051}

$\mathcal{S}_{18,7,4,3}$ = {3584, 5376, 8960, 9344, 4736, 17472, 2432, 33344, 33824, 16928, 65856, 66576, 131264, 131360, 131600, 132104, 16656, 65696, 66056, 2144, 32912, 33032, 4176, 16520, 8240, 8264, 4136, 2072, 38, 21, 11}

$\mathcal{S}_{19,13,4,3}$ = {112, 168, 280, 292, 148, 546, 76, 1042, 1057, 529, 2058, 4102, 4105, 2053, 67}

$\mathcal{S}_{19,7,4,3}$ = {7168, 10752, 17920, 18688, 9472, 34944, 4864, 66688, 67648, 33856, 131712, 133152, 262528, 262720, 263200, 264208, 33312, 131392, 132112, 4288, 65824, 66064, 8352, 33040, 16480, 16528, 8272, 4144, 44, 74, 25, 134, 69, 35}

$\mathcal{S}_{12,8,4,4}$ = {3084, 780, 1546, 2314, 2566, 2569, 204, 1286, 1289, 1541, 3075, 2309, 170, 771, 105, 60, 90, 102, 150, 153, 165, 195, 85, 51, 15}

$\mathcal{S}_{13,8,4,4}$ = {6168, 1560, 3092, 4628, 5132, 5138, 408, 2572, 2578, 3082, 6150, 4618, 212, 785, 1542, 120, 308, 332, 338, 1169, 1289, 172, 178, 202, 390, 649, 2129, 2309, 298, 4145, 4169, 4229, 4355, 581, 2089, 2179, 102, 1061, 1091, 547, 15}

$\mathcal{S}_{14,8,4,4}$ = {12336, 3120, 6184, 9256, 10264, 10276, 816, 5144, 5156, 6164, 12300, 9236, 240, 424, 1570, 3084, 616, 664, 676, 2338, 2578, 2593, 344, 356, 404, 780, 1298, 1313, 1553, 4258, 4618, 12291, 596, 2321, 8290, 8338, 8353, 8458, 8710, 8713, 204, 1162, 3075, 4178, 4193, 4241, 4358, 4361, 4613, 2122, 2182, 2185, 8273, 8453, 771, 1094, 1097, 1157, 2117, 60, 195, 51, 15}

$\mathcal{S}_{15,8,4,4}$ = {24672, 6240, 12368, 18512, 20528, 20552, 1632, 10288, 10312, 12328, 24600, 18472, 480, 848, 3140, 6168, 1232, 1328, 1352, 4676, 5156, 5186, 688, 712, 808, 1560, 2596, 2626, 3106, 8516, 9236, 9281, 24582, 1192, 4642, 16580, 16676, 16706, 16916, 16961, 17420, 17426, 17441, 408, 2324, 2369, 3089, 6150, 8356, 8386, 8482, 8716, 8722, 8737, 9226, 12293, 4244, 4289, 4364, 4370, 4385, 4625, 5129, 16546, 16906, 18437, 20483, 1542, 2188, 2194, 2209, 2314, 2569, 8465, 10243, 120, 4234, 16529, 16649, 390, 773, 8329, 1157, 1283, 643, 101, 86, 51, 46, 75, 29}

$\mathcal{S}_{16,12,4,4}$ = {49164, 12300, 24586, 36874, 40966, 40969, 3084, 20486, 20489, 24581, 49155, 36869, 780, 1546, 12291, 2314, 2566, 2569, 204, 1286, 1289, 1541, 3075, 2309, 170, 771, 105, 60, 90, 102, 150, 153, 165, 195, 85, 51, 15}

$\mathcal{S}_{17,12,4,4}$ = {98328, 24600, 49172, 73748, 81932, 81938, 6168, 40972, 40978, 49162, 98310, 73738, 1560, 3092, 12305, 24582, 4628, 5132, 5138, 18449, 20489, 408, 2572, 2578, 3082, 6150, 10249, 33809, 36869, 4618, 66065, 66569, 67589, 69635, 212, 1542, 9221, 33289, 34819, 120, 308, 332, 338, 16901, 17411, 172, 178, 202, 390, 8707, 298, 101, 15}

$\mathcal{S}_{18,12,4,4}$ = {196656, 49200, 98344, 147496, 163864, 163876, 12336, 81944, 81956, 98324, 196620, 147476, 3120, 6184, 24610, 49164, 9256, 10264, 10276, 36898, 40978, 40993, 816, 5144, 5156, 6164, 12300, 20498, 20513, 24593, 67618, 73738, 196611, 9236, 36881, 132130, 133138, 133153, 135178, 139270, 139273, 240, 424, 3084, 18442, 49155, 66578, 66593, 67601, 69638, 69641, 73733, 616, 664, 676, 33802, 34822, 34825, 132113, 135173, 344, 356, 404, 780, 12291, 17414, 17417, 18437, 596, 33797, 204, 3075, 771, 60, 195, 51, 15}

$\mathcal{S}_{19,12,4,4}$ = {393312, 98400, 196688, 294992, 327728, 327752, 24672, 163888, 163912, 196648, 393240, 294952, 6240, 12368, 49220, 98328, 18512, 20528, 20552, 73796, 81956, 81986, 1632, 10288, 10312, 12328, 24600, 40996, 41026, 49186, 135236, 147476, 147521, 393222, 18472, 73762, 264260, 266276, 266306, 270356, 270401, 278540, 278546, 278561, 480, 848, 6168, 36884, 36929, 49169, 98310, 133156, 133186, 135202, 139276, 139282, 139297, 147466, 196613, 1232, 1328, 1352, 67604, 67649, 69644, 69650, 69665, 73745, 81929, 264226, 270346, 294917, 327683, 688, 712, 808, 1560, 24582, 34828, 34834, 34849, 36874, 40969, 135185, 163843, 1192, 67594, 264209, 266249, 408, 6150, 12293, 133129, 18437, 20483, 1542, 10243, 120, 390, 773, 1157, 1283, 643, 101, 86, 51, 46, 75, 29}