Eichfelder, Gabriele; Gandibleux, Xavier; Geiger, Martin Josef; Jahn, Johannes;
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DOI: 10.22032/dbt.42198

URN: urn:nbn:de:gbv:ilm1-2020200317

Original published in:	Understanding Complexity in Multiobjective Optimization : report from Dagstuhl Seminar 15031 / Greco, Salvatore; Klamroth, Kathrin; Knowles, Joshua D.; Rudolph, Günter. - Wadern : Schloss Dagstuhl. - 5 (2015), 1, p. 121-129. (Dagstuhl Reports : Dokumentationen zu Dagstuhl-Seminaren und Dagstuhl-Perspektiven-Workshops / Schloss Dagstuhl, Leibniz-Zentrum für Informatik. - Wadern : Schloss Dagstuhl. - 5 (2015), 1, p. 96-163. - DOI: 10.4230/DagRep.5.1.96)
ISSN:	2192-5283
Original published:	2015-05-28
[Visited:	2020-03-12]

4.3 Heterogeneous Functions (WG3)

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4.3.1 Introduction

Observing the literature on real-world multiobjective optimization problems, one might notice that many practical applications exhibit considerable heterogeneity regarding the involved objective functions. This working group collected examples of such problems, characterized the kind of heterogeneity that may be found, and identified suitable benchmarks and potential challenges for respective optimization algorithms.

4.3.2 An example

Let $f^1, f^2 : \mathbb{R}^n \to \mathbb{R}$ be nonlinear (objective) functions and let $f^3 : \mathbb{R}^n \to \mathbb{R}$ be a linear objective function. Moreover, let $\Omega \subseteq \mathbb{R}^n$ be the constraint set. Based on these let us consider two multi-objective optimization problems:

$$\min\,(f^1(x), f^2(x)), \quad \text{s.t. } x \in \Omega \text{ and} \tag{P1}$$

$$\min\,(f^1(x), f^3(x)), \quad \text{s.t. } x \in \Omega. \tag{P2}$$

It is clear that both (P1) and (P2) are classified as nonlinear multi-objective optimization problems. If one applied a *weighted sum method* the scalarized single-objective function remains nonlinear. Therefore, there is no added difficulty (or simplicity) due to the heterogeneity of the objectives in (P2) compared with (P1). *Homotopy-based methods* [13], on the other hand, can use the linearity of objective f^3 in an efficient way, and therefore, (P2) can be solved using such methods in an easier way (compared to the nonlinear problem (P1)). (P2) can also be easier to solve using *population-based heuristics*. A well-known example is the benchmark problem ZDT1 (or ZDT4) where NSGA-II first finds the individual minima of the first, linear objective function, and then spreads along the efficient front.

4.3.3 Motivating Applications

Multiobjective capacitated arc routing problem. Lacomme et al. [19] and Mei et al. [23] consider the multiobjective version of capacitated arc routing problems (CARP). These find application in optimization of salting and removing the snow in the winter or in waste collection by a fleet of vehicles. They consider two objectives, namely the total cost (time) of the routes, which is related to minimization of the total operational costs, and the makespan, i. e., the length of the longest route, which is related to the satisfaction of the clients. Clearly the two objectives differ by mathematical form – sum or maximum of the routes' costs. This difference may also influence the landscapes of these objectives and thus influence their practical difficulty. Consider for example the typical insertion or swap moves for CARP. Such moves modify two routes at a given step. In order to improve the makespan objective the longest route has to be improved, so it has to be one of the modified routes. This means that there are in general less potential moves that could improve this objective and local search may stop at a local optimum very fast. For the total cost objective, on the other hand, many moves may result in an improvement. Please note that this situation is similar to the optimization of either linear (weighted sum) or Chebycheff scalarizing functions. The latter type of functions use a maximum operator. Jaszkiewicz [17] observed that linear functions are easier to optimize than Chebycheff ones in the framework of a multiple objective genetic local search algorithm.

Multiobjective chemical formulation problem. Based on communications with Unilever plc., Allmendinger and Knowles motivated their recent work on heterogeneous evaluation times of objectives [1] using an example from a chemical formulation problem: "We wish to optimize the formulation of a washing powder, and our two objectives are washing excellence and cost. In this case, [...] assessing washing excellence may be a laborious process involving testing the powder, perhaps on different materials and at different temperatures. By contrast, the cost of the particular formulation can be computed very quickly by simply looking up the amounts and costs of constituent ingredients and performing the appropriate summation". Earlier work by the same authors [2] stated that heterogeneous evaluation times could be associated with other lengthy experimental processes such as fermentation, or might occur because of a need for subjective evaluations from experts. In both studies (ibid.), the authors consider a variety of algorithmic approaches to handling objectives with different "latency", including use of pseudofitness values, and techniques based on interleaving evaluations on the slower and the faster objective(s).

Multiobjective traveling salesman problem with profits. Jozefowiez et al. [18] consider the multiobjective traveling salesman problem with profits. The two objectives are minimization of the tour length and maximization of the collected profits. The tour does not have to include all nodes. TSP with profits is a well known combinatorial problem with multiple applications [10]. Although it is multiobjective by nature, it is usually solved by aggregation of the two objectives, which not only differ by mathematical form but also have different domains. The tour length depends on both the selected cities and the chosen tour, while the profit depends only on the selected cities. Furthermore, the two objectives correspond to two different classes of combinatorial problems. The authors used two sets of moves. The first set optimizes the tour while the second set modifies the set of visited nodes. An interesting observation is that the higher the number of selected nodes, the more difficult is the related TSP subproblem, i. e., optimization of the tour.

Multi-objective optimization in the Lorentz force velocimetry framework. Lorentz force velocimetry (LFV) is an electromagnetic non-contact flow measurement technique for electrically conducting fluids. It is especially suited for corrosive or extremely hot fluids (glass melts, acidic mixtures, etc) that can damage other measurement setups [30]. The magnetic flux density B is produced by permanent magnets and an electrically conducting (σ) fluid moves with a velocity v through a channel. The magnetic field interacts with the fluid and eddy currents develop. The resulting secondary magnetic field acts on the magnet system. The Lorentz force F_L breaks the fluid and an equal but opposite force deflects the magnet system, which can be measured. It holds that $F_L \sim \sigma \cdot \bar{v} \cdot \bar{B}^2$. Fluids with a small electrical conductivity produce only very small Lorentz forces. Thus, a sensitive balance system is necessary for measurement. This limits the weight of the magnet system (we use the magnetization M as surrogate) and causes external disturbances to have a high influence on the force signal. In order to increase the force to noise aspect ratio, the objective function has to take into account two conflicting goals: maximize the Lorentz force and minimize the magnetization.

$$\min \begin{pmatrix} f_1(x) \\ f_2(x) \end{pmatrix} = \begin{pmatrix} -F_L(\Phi, \Theta, M) \\ \sum_{k=1}^{8} M_k \end{pmatrix}$$
$$\text{such that}$$
$$\Phi_i \in [-\pi, \pi],\ i = 1, \dots, 8,$$
$$\Theta_j \in [0, \pi],\ j = 1, \dots, 8,$$
$$M_k \in [0, 10^6],\ k = 1, \dots, 8$$

The Lorentz force is thereby calculated by a time consuming (20–120 s) simulation run while the magnetization can be calculated analytically. In the above optimization problem $\Phi \in \mathbb{R}^8$ and $\Theta \in \mathbb{R}^8$ describe the direction of the magnetization vector. Both functions are assumed to be smooth. The derivatives of the second objective can be easily determined while already the first derivative of the first objective can only be approximated by numerical differentiation. As this requires in general many functional evaluations, it should be avoided. The second objective is even linear and also the feasible set is a linearly constrained set (there are only box constraints). The first objective is nonlinear and has locally optimal solutions which are not globally optimal.

Portfolio optimization. The portfolio optimization problem is formulated as a bi-criterion optimization problem, where the reward (mean of return) of a portfolio is maximized, while the risk (variance of return) is to be minimized. Practical portfolio optimization problems use extensions to the Markowitz model, and these often use several risk measures, e. g., quantile-based risk measures [3]. These measures replace variance in the standard mean-variance model, thus leading to an entire family of mean-risk portfolio selection models. This makes the problem heterogeneous as the first objective is linear and the second objective has stochastic terms. Many other practical portfolio optimization formulations even use a tri-objective problem so as to find trade-offs between risk, return, and the number of securities in the portfolio [4], which is even more heterogeneous (continuous, stochastic, and integer-valued functions are involved). An overview on extended Markowitz models for further reading can be found in [29]. Conditional values at risk and satisficing constraints can also be incorporated.

Multi-objective inventory routing. The *inventory routing problem* (IRP) describes a generalization of the classical vehicle routing problem (VRP), in such that delivery volumes, i. e., the quantities of the products delivered to customers in a logistics network, are considered to be additional variables. While early research on this problem can be traced back to the 1980s [9], it has only recently been investigated in its true formulation as a multi-objective problem [12]. The bi-objective formulation of [12] introduces two objectives: the inventory levels held by the customers in the network are to be minimized (a typical consideration in just-in-time logistics), and the costs for transporting the goods to the customers are minimized. Obviously, the two criteria are in conflict with each other. A decision support system for this biobjective IRP is visualized in [16]. There, it could be observed that the minimization of the inventory levels is of lower practical difficulty than the minimization of the routing costs. The reasoning behind this is based on the observation that delivery volumes simply are the setting of a single variable value for each customer, and the subsequently held inventories are directly affected by the amount of delivered products. However, the solution of the resulting VRP is difficult even for small data-sets, and in practical cases with reasonable running time restrictions, only (meta-)heuristics appear to be promising solution approaches [15].

Interventional radiology in medical engineering. An essential component of interventional radiology is the procedure of minimally invasive therapeutic interventions, for example in the vasculature. Since the line of sight is interrupted, the interventional material used in these procedures, e. g., catheters, guide wires, stents, and coils, are tracked by imaging techniques. In this application we consider the deformable 3D-2D registration for CT. With the considered method the patient motion during the intervention can be corrected. Only such a procedure can reconstruct artifact-free volumes showing the true position of the interventional material. A bicriterial approach is taken in [11], which is based on raw data and adapts the position of the prior volume immediately to the position included in the raw data without a reconstruction. One objective is the sum of squared differences in raw data domain and the other is a regularization term which originates from physical models for fluids and diffusion processes. An application of a gradient method to this bicriterial problem would require the solution of an implicit differential equation for the computation of a gradient direction. In order to reduce the inhomogeneity of the objectives the bicriteria optimization is done in an alternating manner. The raw data fidelity is minimized by a conjugate gradient descent and the resulting vector fields are then convolved with Gaussian kernels to realize regularization. This alternation between the two objectives is only possible using a special linking term combining both objectives. With this technique one gets the required images with high quality in a faster way.

4.3.4 Aspects of Heterogeneity

Functions of multi-objective problems may differ in several, usually interconnected aspects, of which the following could be identified:

Scaling. An objective function's range of values may be quite different from the range for other objective functions of the problem.

Landscape. Objective functions may differ quite considerably in basic features, as their degree of multi-modality, presence of plateaus, separability, or smoothness. An even richer description can be achieved by calculating empirical summary characteristics such as fitness-distance correlation, auto-correlation, or the numerous features developed under the term exploratory landscape analysis (ELA) [24]. These require evaluating a space-filling sample drawn from the domain of the multi-objective problem. Such features may be less intuitive than theoretical properties, but nonetheless they are designed to correspond to the practical performance of heuristic optimization methods, and thus provide valuable information about the function. However, current ELA features are designed for individual objectives and the design of specific features capturing the multiobjective problem characteristics, like e.g. front shape, local fronts etc., is still an open research topic. The relationship between the individual ELA features and multiobjective problem characteristics would be very helpful in assessing the influence of objective heterogeneity.

Evaluation time. Each objective or constraint function of a multi-objective problem may take a different amount of time to evaluate. These differences may result from different theoretical complexity of the functions, different size of the domain of the functions (see Domains below), or other differences. In practical problems, the heterogeneity of evaluation times could be large, for example if one objective function was a simple sum while the other one was evaluated by conducting a physical experiment [1, 2]. A further point related to evaluation time is that some functions may be computed more quickly if another solution, whose function value is known and differing in the values of a small number of decision variables, is available. In some cases the ability to evaluate efficiently the objective functions by computing the difference (or delta) from an existing solution is very important (e. g. in symmetric TSP) for local search methods.

Theoretical and practical difficulty. Some functions may be more or less difficult to optimize in terms of the number of solutions that must be explored in order to find an optimum (e. g., using a local search or other iterative search method). Differences in practical difficulty between the objectives could be a result of different theoretical complexity of the functions, or different domain sizes, or different properties of the fitness landscape.

Domains. Let us consider the binary relation "intersects with" between all pairs of domains of the objective functions and constraints as a graph. This graph may have only one connected component, or there would be no conflict between some of the functions. However, the domains do not necessarily have to be completely identical, either. This holds especially for constraints, which usually concern only a subset of the variables. Consequently, not all functions have to be defined on variables of the same data type.

Parallelization. Each objective function could have different restrictions regarding the amount of parallelization. E. g., some objective functions might require physical equipment or software licenses, which restrict the number of function evaluations that can be executed in parallel.

Problem class. It may be known that some objective belongs to a different problem class than another. Examples are the aforementioned TSP and shortest path.

Analytic form vs. black box. Some objective function may be available in analytic form, while another may be available only as a black box. This usually implies that the evaluation time differs considerably between the objective functions (see above). Moreover, while for the analytic functions the derivatives can be calculated, they can only be approximated for black-box functions using numerical differentiation.

Determinism. Some objective functions of a problem may be stochastic, while others might be deterministic.

4.3.5 Benchmarks

For investigating this topic in controlled experiments, "artificial" benchmark problems are a useful tool. Here we argue which existing benchmarks exhibit heterogeneity and how even more heterogeneous ones could be constructed.

Continuous benchmarks. In the area of evolutionary multi-objective optimization a large number of continuous test instances are collected in [14]. These have different landscapes as for instance one objective is linear and the second one is highly nonlinear. This is used to create convex, non-convex, mixed convex-concave, and multi-modal problems. The objectives in ZDT, SZDT, RZDT, and WFG test problem instances are heterogeneous. One of the test functions is linear (or piecewise linear) while the other objective(s) are highly nonlinear and multi-modal. DTLZ test problem instances, on the other hand, use similar objective functions (using sine and cosine terms) and hence are not heterogeneous at first sight. They might differ in terms of ELA features, however. Simple benchmark functions like e.g. the Schaffer or Binh problems are homogeneous, though. Instances with differing evaluation times can be easily constructed by inserting a time delay in the respective functions. Moreover, noise can be added to a subset of the objectives in order to address heterogeneity in terms of determinism as discussed above.

KP benchmarks. We carried out some preliminary experiments to construct heterogeneous discrete problems. The bi-objective unidimensional 01 knapsack problem (KP) was used as a basis for these investigations. Its objective is to optimize $\vec{f} = (\max \sum_{j=1}^{n} c_j^1 x_j, \max \sum_{j=1}^{n} c_j^2 x_j)^T$ under the side constraints $\sum_{i=1}^{n} w_j x_j \leq \omega$ and $x_j \in \{0,1\}$. Four families (A/B/C/D) of instances are already provided by the MOCOlib [25]. Among them are family A, where c_j^1, c_j^2 are randomly generated for $i = 1, \ldots, n$ ($1 \leq c_j^1, c_j^2 \leq 100$), and family C, which contains patterns (plateaus where l_i is the length and v_i is the value) created by choosing v_i randomly in $\{1, \ldots, 100\}$, $c_1^1 = c_2^1 = \ldots = c_{l_1}^1 = v_1$, and $c_{l_1+1}^1 = c_{l_1+2}^1 = \ldots = c_{l_1+l_2}^1 = v_2$. In [8] it was observed that the patterns tend to make the MOCO problem harder to be solved. So, our preliminary impression is that the patterns provide a way to introduce a form of heterogeneity in functions.

We also constructed some new families by combining different existing ones, e. g., by taking objective 1 and resource constraint from family A and objective 2 from family C. This way, we obtained five new families, called AC, AL, AZ1, AZ12, and AZ3. In preliminary experiments with a solver taken from [5, 6], the comparison of results obtained on A, AZ12, and AZ3 indicated that the presence of "null" plateaus seems to affect the performance of the solver negatively. More research on this topic shall follow.

Constraint satisfaction benchmarks. Max-SAT-ONE [28, 22] is an example of a bi-criterion constraint satisfaction problem with objectives heterogeneous in their (assumed) computational complexity class. The first objective is NP-hard, while the other objective is a simple sum over variables and is hence linear.

Max-SAT-ONE is a relative of the logical Satisfiability (SAT) problem, an archetypal decision problem with a central role in theoretical computer science as the first to be proved NP-Complete [7]. In an instance of the SAT problem a number c of logical clauses involving a number n of Boolean variables are presented. The problem is to determine whether there is an assignment to the variables that satisfies all the clauses. The optimization form of the problem, known as MAX-SAT, is also well-known. The problem, the subject of intensive research for a number of years, follows the same form as SAT but for the objective, which is now to maximise the number of satisfied clauses. The problem is NP-hard, and examples of techniques developed for the problem can be found in [20, 27].

Max-SAT-ONE has been studied in the context of constraint programming [22] and decomposition methods in multiobjective optimization [28]. The first objective is that of MAX-SAT, while the second one is to maximize the number of variables with an assignment of TRUE. This leads to a discrete Pareto front with at most n distinct Pareto optimal points.

TSP benchmarks. One of the possibilities is to use a MOCO problem with objectives defined mathematically in the same way, but with different distribution of parameters. Paquete [26] and Lust and Teghem [21] proposed a set of travelling saleperson (TSP) instances with various classes of objective functions:

- Euclidean instances: the costs between the edges correspond to the Euclidean distance between two points in a plane, randomly located from a uniform distribution.
- Random instances: the costs between the edges are randomly generated from a uniform distribution.
- Clustered instances: the points are randomly clustered in a plane, and the costs between the edges correspond to the Euclidean distance.

They also proposed mixed instances: the first cost comes from the Euclidean instance while the second cost comes from the random instance. They observed some differences in behavior of the multiobjective algorithms for these instances. The Lin-Kerninghan heuristic used in the first phase required significantly more time for random than for Euclidean instances. The Pareto local search used in the second phase was on the other hand faster on Euclidean instances due to much lower number of efficient solutions. The time performance of mixed instances was in between in both cases.

The above mentioned multiobjective traveling salesman problem with profits [18] is an interesting candidate for discrete benchmark problem with heterogeneous objectives. It is relatively simple in definition, based on well studied TSP problem, and contains several aspects of heterogeneity – different mathematical definitions, different difficulty, different domains.

4.3.6 Conclusions and Outlook

Our study suggests that heterogeneity between the objectives of a multiobjective optimization problem is both common and yet little understood (or even considered) in the literature. We have made a modest start on providing motivating examples and beginning a characterization of this complex feature. There seems to be a rich vein to investigate further, and much work to do in proposing and testing suitable methods.

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