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Fractional Order PID Control of a MIMO Distillation Column Process Using Improved Bat Algorithm

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Abstract In this paper, a new Bat Algorithm (BA) based on dynamic control parameters selection is presented. The Dynamic BA (DBA) uses a new mechanism to dynamically select the best performing combination of the pulse rate coefficient, the pulse frequency coefficient, and the population size. A fractional order PID (FOPID) controller based on the DBA is implemented to improve the performance of a distillation column process. The proposed FOPID controller is used to control the distillate and bottom mole fractions. The influence of the feed rate disturbance is considered for this model. The efficacy of the DBA-based FOPID is compared with the performance of the controllers based on the conventional BA, directional BA (dBA), enhanced BA (BA-IS), Genetic Algorithm (GA), and Particle Swarm Optimization (PSO) algorithm. The analyses and simulation results show the superiority of the proposed method.

Keywords Distillation column · Fractional order PID · Dynamic Bat algorithm

1 Introduction

Distillation columns are one of the most important separation process parts in chemical and petrochemical industries. In a typical process, a distillation column system is used for the separation and the purification of mixtures containing two or more components. The control of the distillation column process has some difficulties due to its highly nonlinear characteristics, its Multiple Inputs Multiple Outputs (MIMO) structure, and the presence of disturbances during operation (Bhattacharjee and Medhi 2012). The need for parameters estimation, accurate modeling, and control of the distillation column has led to several publications in the literature. Bhattacharjee and Medhi (2012) compare fuzzy logic and neuro-fuzzy controllers with the conventional PID and PI controllers for distillation column systems. A new method for the independent design of multi-loop PI and PID controllers is presented in Luan and Lee (2010). This paper uses an Internal Model Control (IMC)-based PID tuning approach to design a controller for Vinante and Luyben, Wood and Berry, and Ogunnaike and Ray column systems. Atashpaz-Gargari et al. (2008) apply Colonial Competitive Algorithm (CCA) to design a multivariable PID controller for a typical distillation column process. A real distillation column process is identified and modeled using artificial neural network by Sahraie et al. (2013). The Model Predictive Control (MPC) for controlling a distillation column is proposed by Manimaran et al. (2013). This paper shows that the MPC controller gives a very fast response and a quick setting time compared to the PID controller. In Rajabioun (2011) the Cuckoo Optimization Algorithm (COA) is used to tune the parameters of a multivariable PID controller for a distillation column process. This paper investigates the performance of the COA compared to Genetic Algorithm (GA) and Particle Swarm Optimization (PSO) algorithms. A decentralized PI control system based on the Nyquist stability analysis for Wood and Berry,

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Vinante and Luyben, and Alatiqi distillation column is presented by Chen and Seborg (2003).

The use of fractional calculus in the area of control theory combined with evolutionary and swarm intelligence algorithms further extends the system control performance of the conventional controllers. The tuning and auto-tuning of fractional order controllers for industry application is presented in Monje et al. (2008). Bettayeb and Mansouri (2014) propose a new fractional controller structure based on a fractional PID controller cascaded with a fractional order filter. This paper uses the IMC paradigm as design method. Moradi (2014) proposes a multivariable fractional order PID controller to control a multivariable process with time delay. A Genetic Algorithm (GA) is used to tune the gains of the fractional orders. A new method based on the Differential Evolution (DE) algorithm to tune the parameters of a fractional order $PI^{\lambda}D^{\mu}$ is proposed in Martín et al. (2015). The performance of the proposed method is considered based on a DC motor in real environment. Gao et al. (2014) discuss the robust stabilizing region with stability degrees of fractional order controllers for time delay fractional order systems. A set of tuning methods for optimal PID and fractional order PID controllers is considered in Padula and Visioli (2011). A detailed review of the fractional calculus concepts and its applications can be found in Podlubny (1999) and Monje et al. (2010).

The Bat Algorithm (BA) is a newly proposed stochastic global search algorithm, which has been used in different optimization fields and problems, such as optimization (Perez et al. 2017), earthquake prediction (Saba et al. 2016), travelling salesman problem (Osaba et al. 2016), brain tumor MR image classification (Kaur et al. 2017), photovoltaic system (Oshaba et al. 2017), visual tracking (Gao et al. 2016), and distributed generations (Yammani et al. 2016). A fuzzy PD-based speed controller for a brushless direct current motor is presented by Premkumar and Manikandan (2015). The controller gains are tuned using the bat algorithm. The performance of the BA-based controller is analyzed and compared with PSO and Cuckoo search algorithms. In Jaddi et al. (2015), a modified BA for both optimizing the weights and structure of an artificial neural network is proposed. A self adaptive BA-based intelligent strategy for multi-area load frequency control is presented in Khooban and Niknam (2015). Abd-Elazim and Ali (2016) propose a BA algorithm for the optimal tuning of a PI controller for a nonlinear interconnected power system. This paper shows the superiority of BA compared to the simulated annealing algorithm. The offline and online parameter estimation of a permanent magnet synchronous motor using self-adaptive learning BA is presented in Rahimi et al. (2016).

The BA shows a considrable success in solving several optimization problems, but the performance of the algorithm is highly dependent on the right combination of control parameters. In the present paper, a new dynamic parameters selection mechanism is proposed to improve the BA's convergence rate and minimization of cost function. The Dynamic BA (DBA) uses a dynamic mechanism to select the best performing combinations of the pulse rate coefficient, frequency coefficient, and population size. A fractional order PID (FOPID) controller based on the DBA algorithm is proposed to further enhance the performance of a distillation column process. The proposed controller is implemented to control the distillate and bottom mole fractions. The analyses and simulation results based on the Integral Squared Error (ISE), Integral Absolute Error (IAE), Integral Time Absolute Error (ITAE), and Integral Time Squared Error (ITSE) cost functions show the performance of the proposed controller with optimum gains.

This paper is organized as follows. In Section 2, the mathematical modeling and FOPID control scheme of the distillation column process is presented. A brief overview of the BA algorithm and the description of the proposed DBA are presented in Section 3. In Section 4, the simulation results and analyses of the designed controller for changes in the set point and the feed rate disturbance are provided. Finally, the paper is concluded in Section 5.

2 Modeling and Control Scheme

A distillation column is used to separate a mixture which contains two or more component. The distillation column is a nonlinear, non-stationary, and MIMO system with strong interactions between inputs and outputs. Some distillation system mathematical models from the literature are: Tyreus stabilizer (1979), Wood and Berry (1973), Vinante and Luyben (1972), Wardle and Wood (1969), Orgunnaike and Ray (1979), Tyreue (1982), Doukas and Luyben (1978), and Alatiqi (1985) (Luyben 1986). The model considered in this work is based on the Wood and Berry model (Wood and Berry 1973).

A very simple schematic diagram of a distillation column system is shown in Fig. 1. A distillation column process is mainly the combination of a vertical column where trays are installed, a reboiler to provide heat, a condenser to condense the overhead vapor (enriched vapor), and a reflux drum to hold the condensed vapor (Minh and Rani 2009). In Fig. 1, Feed is a liquid mixture of the two components to be separated. The simplified dynamic Wood and Berry's model is defined as:

$$\begin{bmatrix} X_D(s) \\ \\ X_B(s) \end{bmatrix} = \begin{bmatrix} G_{11} = \frac{12.8e^{-s}}{16.7s+1} & G_{12} = \frac{-18.9e^{-3s}}{21.0s+1} \\ \\ G_{21} = \frac{6.6e^{-7s}}{10.9s+1} & G_{22} = \frac{-19.4e^{-3s}}{14.4s+1} \end{bmatrix} \cdot \begin{bmatrix} R(s) \\ \\ S(s) \end{bmatrix} + \begin{bmatrix} G_{DF} = \frac{3.8e^{-8s}}{14.9s+1} \\ \\ G_{BF} = \frac{4.9e^{-3s}}{13.2s+1} \end{bmatrix} F(s), \quad (1)$$

where $X_D(s)$ (lb/min) and $X_B(s)$ (lb/min) are percentages of methanol in the distillate and bottom compositions, respectively. $R(s)$ and $S(s)$ are the reflux and reboiler vapor flow rates, respectively, and $F(s)$ (lb/min) is the feed flow rate disturbance. The closed-loop distillation column system with and without decoupling is represented in Fig. 2 and Fig. 3, respectively (Corriou 2004), where D_{21} and D_{12} are equal to

$$D_{12} = \frac{-G_{12}}{G_{11}}, \quad (2)$$

$$D_{21} = \frac{-G_{21}}{G_{22}}. \quad (3)$$

In these figures, X_{Dref} and X_{Bref} are the distillate and bottom product compositions references, respectively. As shown in Fig. 2 and Fig. 3, we are going to use a FOPID for tracking the control inputs X_{Dref} and X_{Bref} by the outputs $X_D(s)$ and $X_B(s)$, respectively.

The continuous transfer function of the FOPID or $PI^{\lambda}D^{\mu}$ controller is given as follows:

$$C(s)_i = K_{Pi} + K_{Ii}s^{-\lambda_i} + K_{Di}s^{\mu_i}, \qquad i = 1, 2 \quad (4)$$

where $\lambda_i, \mu_i > 0$ are the fractional orders of the integral and derivative actions, respectively. The $PI^{\lambda}D^{\mu}$ controller output $u(t)$ in time domain is:

$$u(t)_i = K_{Pi}e(t)_i + K_{Ii}D^{-\lambda_i}e(t)_i + K_{Di}D^{\mu_i}e(t)_i, \quad i = 1, 2 \quad (5)$$

where $e(t)_i$, K_{Pi}, K_{Ii} and K_{Di} are the error signal and the proportional, integral, and derivative gains, respectively. The controller gains $[K_{P1}\ \ K_{I1}\ \ K_{D1}\ \ \lambda_1\ \ \mu_1\ \ K_{P2}\ \ K_{I2}\ \ K_{D2}\ \ \lambda_2\ \ \mu_2]$ will be tuned using DBA, BA, PSO, and GA to minimize ISE, IAE, ITSE, and ITAE cost functions.

3 Bat Algorithm

The BA algorithm is a population-based meta-heuristic optimization method that was introduced by Yang (2010). This algorithm simulates the fascinating behavior of the bat to detect prey, avoid obstacles, and locate roosting crevices in the dark. A pseudo code of the BA is shown in Fig. 4. This algorithm is established upon three idealized rules: (1) Each bat uses echolocation characteristics to sense distance, and they know the difference between food (prey) and obstacle in some magical way; (2) Each bat flies randomly with a velocity v_i at position x_i with a fixed frequency f_{min}, varying wavelength λ and loudness A_0 to seek for prey. Every bat can automatically adjust the wavelength (or frequency) of the emitted pulses and adjust the rate of the pulse emission $r \in [0, 1]$ depending on the closeness of the targeted prey; (3) The loudness of each bat emission can vary in many ways; it is assumed that this loudness changes from a large (positive) A_0 to a minimum constant value A_{min}.

At each time step, the velocity v_i and position x_i of the ith bat in a D dimensional search space can be updated based on the following equations:

$$f_i = f_{min} + (f_{max} - f_{min})\beta, \quad (6)$$

$$v_i^t = v_i^{t-1} + (x_i^{t-1} - x_{gbest}^t)f_i, \quad (7)$$

$$x_i^t = x_i^{t-1} + v_i^t, \quad (8)$$

where $\beta \in [0, 1]$ is a uniform random number, f_i is the frequency of the ith bat that controls the range and speed of movement of the bats, and x_{gbest}^t is the current global best solution at time step t. The values f_{min} and f_{max} depend on the domain size of the problem of interest.

Once a solution is selected among the current best solutions, a new solution for each bat is generated locally using random walk as follows:

$$x_{new} = x_{old} + \epsilon A^t, \quad (9)$$

where $\epsilon \in [-1, 1]$ is a random number, and A^t denotes the mean loudness of all bats so far. The loudness A_i and the rate r_i of pulse emission can be updated based on the following equations:

$$A_i^{t+1} = \alpha A_i^t, \quad (10)$$

$$r_i^{t+1} = r_i^0[1 - exp(-\gamma t)], \quad (11)$$

where r_i^0 is the initial pulse rate, α and γ are constants, and for any $0 < \alpha < 1$ and $\gamma > 0$, we have:

$$A_i^t \to 0,\ r_i^t \to r_i^0,\ as \quad t \to \infty. \quad (12)$$

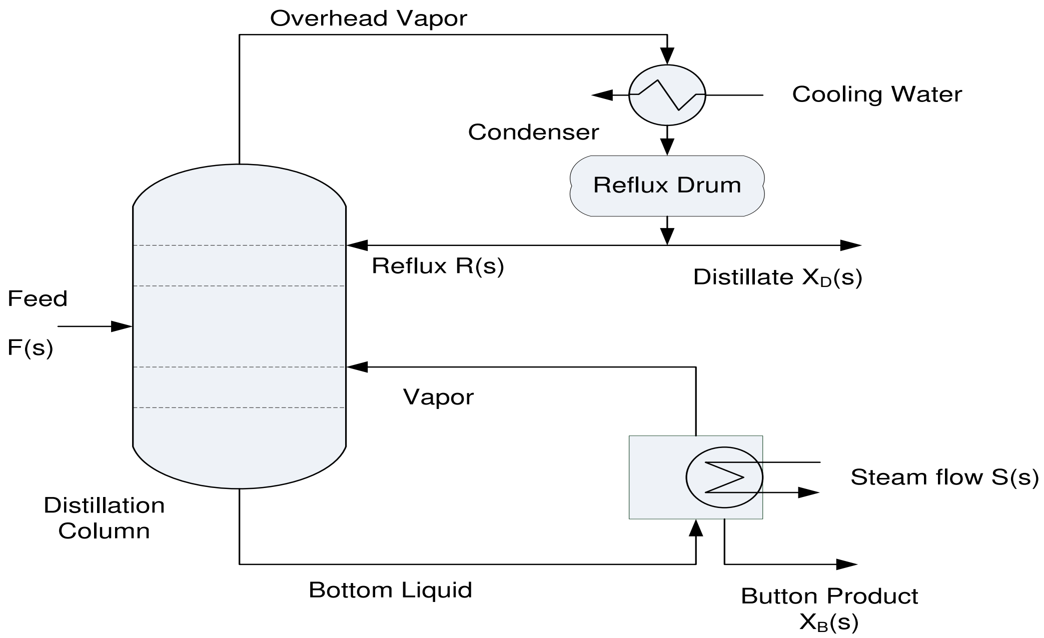

Fig. 1 Equivalent circuit diagram of a distillation column process

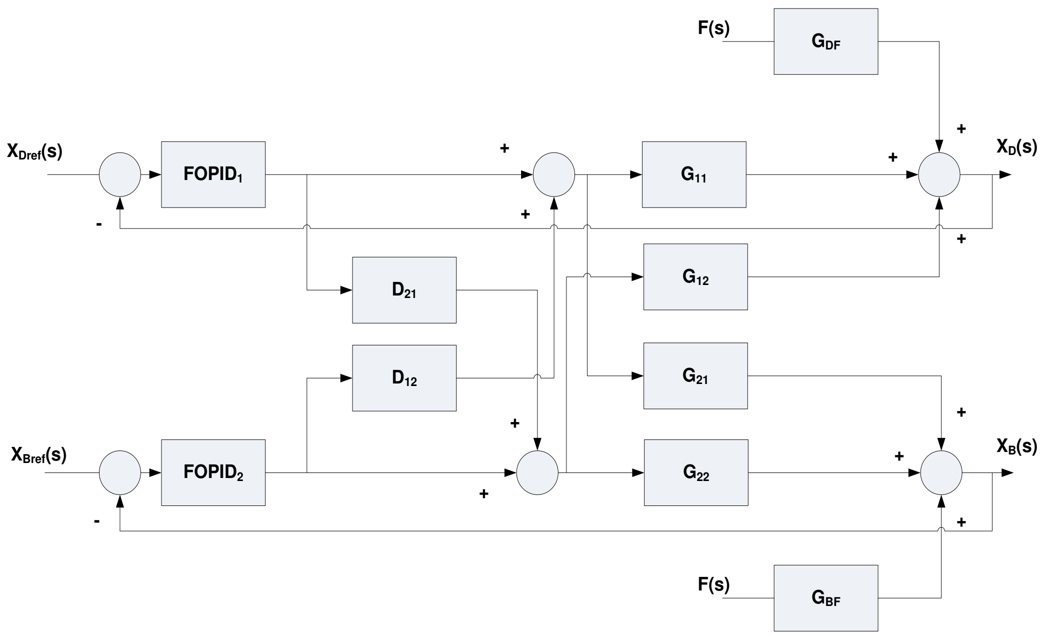

Fig. 2 Column control system representation with decoupling

3.1 Dynamic Bat Algorithm

In solving any optimization problem, the right choice of the control parameters plays an important role in the performance of the algorithm. For the case of the BA algorithm, a better combination of the frequency coefficient (f), the pulse rate coefficient (r), and the population size (PS) enhances the algorithm's flexibility and robustness. A dynamic parameters selection mechanism is implemented to dynamically select the best performing combination of the parameters (amplification factor, crossover rate, and the population size) for the DE algorithm by Sarker et al. (2014). This paper shows the superiority of the dynamic DE over other state-of-the-art algorithms. In previous works by the authors (see Haji Haji and Monje (2017a,b)), they propose a dynamic parameters selection mechanism to improve the

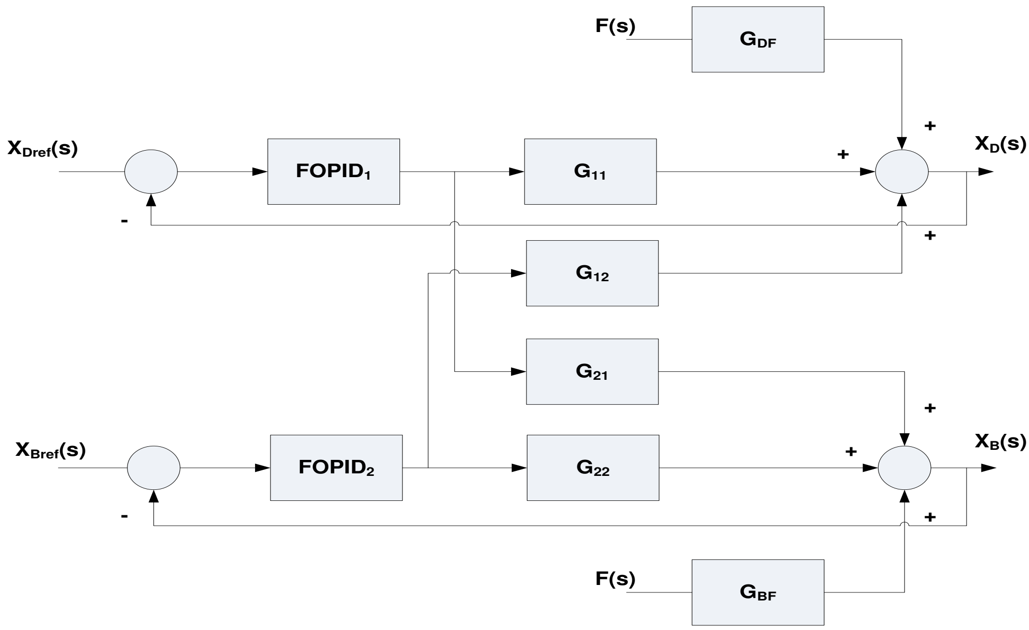

Fig. 3 Column control system representation without decoupling

Bat Algorithm

Objective function $f(x), x = (x_1, ..., x_d)^T$
Generate the bat population $x_i (i = 1, 2, .., n)$ and v_i
Define frequency f
Initialize r_i and A_i
while $(t < MaxGeneration)$
    Generate new solution by adjusting frequency and updating velocity and location through Eqns. (6) to (8)
    if $(rand > r_i)$
        Select a solution among the best solutions
        Generate a local solution around the best solution
    end if
    Generate a new solution by flying randomly
    if $(rand < A_i$ and $f(x_i) < f(x_{gbest}))$
        Accept the new solutions
        Increase r_i *and reduce* A_i *by Eqns. 10 and 11*
    end if
Rank the bats and find the current best x_{gbest}
end while

Fig. 4 Pseudo code of the Bat Algorithm.

performance of the PSO and FA algorithms, with very good results.

In this paper, a dynamic parameter selection mechanism is implemented to choose the best combination of the frequency coefficient, pulse rate coefficient, and the population size in a BA-based optimization problem. The pseudo code of the dynamic algorithm is shown in Fig. 5. The DBA starts with a random combination of f, r and PS for each bat in the population, where $f \in f_{set} = \{f_1, f_2, ..., f_{nf}\}$, $r \in r_{set} = \{r_1, r_2, ..., r_{nr}\}$, and $PS \in PS_{set} = \{PS_1, PS_2, ..., PS_{nps}\}$. Here, nf, nr, and nps refer to the cardinality of the frequency factor, the pulse rate factor, and the population size PS, respectively. The velocity and position of the ith bat in population can be updated by Eqn. 7 and Eqn. 8, where $f_i \in f_{set}$ and if the new position x_i is better than its previous position, the success rate SR of a combination y is increased by one, where y is a combination of the parameters, $y \in y_{set}$, and y_{set} is the combination of all f and r. The success rates of combinations are recorded for a certain number of generations CS. After CS generations, based on Eqn. (13)

the ranking of the combinations are calculated and the numbers of combinations are reduced to the half. Besides, the population size reduces to PS_{nps-1} and the remaining $PS_{nps} - PS_{nps-1}$ bats are archived (assuming $PS_{nps} > PS_{nps-1}$).

$$Rank_y = \frac{SR_y}{N_y}, \tag{13}$$

where N_y is the number of times a combination y is used, and the combinations with the highest $Rank_y$ are selected for the next generations. At the end of $CS \times nps$ generations, based on Eqn. (14) the best population size PS is chosen and used for the $(\eta - nps) \times CS$ next generations, where $\eta > nps$ and is calculated based on Eqn. (15).

$$Rank_{PS_i} = \frac{\sum_{1}^{CS} \sum_{k=1}^{TC} SR_y}{PS_k}, \tag{14}$$

$$\eta \approx \frac{log(TC)}{log(2)}, \tag{15}$$

where TC is the total combination of f and r. For the sake of illustration, if $TC = 32$, this means that the maximum possible value for η is 5 ($32 \to 16 \to 8 \to 4 \to 2 \to 1$) or TC can be divided by 2 for 5 times. Besides, in this paper, nps is 2; therefore $(\eta - nps)$ is a fixed coefficient that refers to remaining possible next generations and must be a positive number. Finally, after $\eta \times CS$ generations, the dynamic process restarts with all combinations of the pulse rate coefficient, the frequency coefficient, and the population size.

In the standard BA, bats can move toward a selected best solution using Eqn. 9. This structure can lead to a premature convergance issue (Yilmaz and Küçüksille (2014)). In order to remove this problem and improve the capability of the BA, inspired by the Differential Evolution (DE) algorithm (Sarker et al. (2014)), the cross over operator based on Eqn. 16 and Eqn. 17 is proposed.

$$z = x_{gbest}^t + \gamma * (x_1^t - x_2^t), \tag{16}$$

$$x_{new} = \begin{cases} z & \text{if } rand \leq CR \text{ or } j = j_{rand}, \\ x_i^t & \text{otherwise,} \end{cases} \tag{17}$$

where x_1, x_2 are two randomly selected bat in the population, $x_1 \neq x_2 \neq x_i$, $\gamma \in \{0.2, 0.8\}$, $rand \in [0, 1]$, j_{rand} is a randomly chosen number in the range $[1, D]$ (D refers to the problem dimension), and $CR = 0.3$.

In order to evaluate the performance of the proposed DBA compared to GA, PSO, BA, directional BA (dBA) (Chakri et al. (2017)) and enhanced BA (BA-IS) (Yilmaz and Küçüksille (2014)), the first 15 of the CEC 2005 benchmark functions (Lynn (2015)) are used. These functions are listed in Table 1, and the parameter settings are dimension (D) 10, function evaluations (FES) $100,000$, population size 20, and run time 20. As Table 2 shows, DBA provides the best results for 9 functions. In this table, BA and BA-IS cannot show any success in minimization of cost functions compared to the other algorithms.

4 Simulation Results

In this section, the considered FOPID is designed and implemented in control loops of the coupling and decoupling distillation column models. The gains of the controller are tuned using DBA, BA, dBA, BA-IS, GA, and PSO algorithms. In order to assess the performance of each algorithm, all the algorithms have been run five times, and the best, worst, and average results have been discussed in detail. The control values for the DBA-based optimization are: maximum number of iterations = 100, $CS = 50$, $\eta = 3$, $A = 0.9$, $PS \in \{4, 6\}$ $r \in \{0.1, 0.2, 0.3, 0.5\}$, and $f \in \{1.5, 1.6, 1.7, 1.8, 1.9, 2\}$. The parameters settings of the BA, dBA, BA-IS, GA, and PSO algorithms are given in Table 3. The fitness functions ITSE, ITAE, ISE, and IAE are used to tune the gains of the controllers using GA, PSO, BA, dBA, BA-IS, and DBA algorithms.

$$J_1 = ITSE = \int_0^\infty t(e_1^2(t) + e_2^2(t))dt, \tag{18}$$

$$J_2 = ITAE = \int_0^\infty t(|e_1(t)| + |e_2(t)|)dt, \tag{19}$$

$$J_3 = ISE = \int_0^\infty (e_1^2(t) + e_2^2(t))dt, \tag{20}$$

$$J_4 = IAE = \int_0^\infty (|e_1(t)| + |e_2(t)|)dt, \tag{21}$$

where $e_1(t)$ and $e_2(t)$ are the error signals for outputs X_D and X_B, respectively.

4.1 Distillation Column with Decoupling

The tuned controllers gains and the corresponding fitness function parameters of the proposed FOPID controllers obtained using different evolutionary and swarm intelligence algorithms for the distillation column with decoupling are presented in Tables 4-7, where M_p, T_r, T_s, Of, and TE refer to the maximum peak, rise time (minute), settling time (minute), objective function, and total error ($e_1(t) + e_2(t)$), respectively. Figure 6 shows the convergence rate characteristics of the DBA, BA, dBA, BA-IS, PSO, and GA algorithms for the cost

```
Dynamic Bat Algorithm
Define parameters, i_pop = nps, f_set, r_set, and PS_set
Objective function f(x), x = (x_1, ..., x_d)^T
Generate initial random population of bats x_i (i = 1, 2, .., n) and v_i
while (t < MaxGeneration)
Assign a random combination (y) of parameters
  for i = 1 : n all n bats
    Generate new solution through Eqns. (7) and (8)
    if (rand > r_set), then Generate a solution using Eqns. 16 and 17; end if
    if new vector is better than its previous vector, then SR_y ← SR_y + 1; end if
    if (rand < A and f(x^i_new) < f(x^i_old))
      Accept the new solution
    end if
  end for i
Rank the bats and find the current best x_gbest
period ← period + 1
PS_prd ← PS_prd + 1
if mod(period, CS) == 0 and period < (η ∗ CS)
  Select the best half combination based on the rankings using Eqn. 13 and update y_set
else if mod(period, η ∗ CS) == 0
  Set SR_y ← 0 and period ← 0
end if
if mod(PS_prd, CS) == 0 and i_pop > 0
  Calculate Rank_{PS_{i_pop}} using (14)
  set i_pop ← i_pop − 1
    if i_pop ∼= 0
      Archive the worst (PS_{i_pop+1} − PS_{i_pop}) individuals
      Set PS ← PS_{i_pop}
    end if
end if
if i_pop == 0 and PS_prd == nps ∗ CS
  Set PS to the population size with the best ranking
  Use required individuals from the archive
end if
if PS_prd == η ∗ CS
  Set PS_prd ← 0, i_pop ← nps, and PS ← PS_{i_pop}
  Use required individuals from the archive
  Clear the archive
end if
end while
```

Fig. 5 Pseudo code of the Dynamic Bat Algorithm.

Table 1 CEC 2005 benchmark functions.

Functions	Range	Optimum
F01 Shifted sphere function	$[-100, 100]^D$	-450
F02 Shifted Schwefel's problem 1.2	$[-100, 100]^D$	-450
F03 Shifted rotated high conditioned elliptic function	$[-100, 100]^D$	-450
F04 Shifted Schwefel's problem 1.2 with noise in fitness	$[-100, 100]^D$	-450
F05 Schwefel's problem 2.6 with global optimum on bounds	$[-100, 100]^D$	-310
F06 Shifted Rosenbrock's function	$[-100, 100]^D$	390
F07 Shifted rotated Griewank's function without bounds	$[-600, 600]^D$	-180
F08 Shifted rotated Ackley's function with global optimum on bounds	$[-32, 32]^D$	-140
F09 Shifted Rastrigin's function	$[-5, 5]^D$	-330
F10 Shifted rotated Rastrigin's function	$[-5, 5]^D$	-330
F11 Shifted rotated Weierstrass function	$[-0.5, 0.5]^D$	90
F12 Schwefel's problem 2.13	$[-100, 100]^D$	-460
F13 Expanded extended Griewank's plus Rosenbrock's function(F8F2)	$[-3, 1]^D$	-130
F14 Shifted rotated expanded Scaffer's F6	$[-100, 100]^D$	-300
F15 Hybrid composition function	$[-5, 5]^D$	120

Table 2 Comparison of the avarage results for 10 dimensional CEC 2005 benchmark functions.

Functions	DBA	*dBA*	*BA–IS*	BA	GA	PSO
F01	**0.000E+00**	0.015E+00	0.163E+00	0.072E+00	0.015E+00	**0.000E+00**
F02	**0.000E+00**	0.039E+00	0.099E+00	0.025E+00	3.154E+02	**0.000E+00**
F03	1.989E+05	**1.496E+04**	1.837E+05	5.199E+04	2.809E+06	5.359E+04
F04	**0.000E+00**	0.054E+00	5.779E+02	0.128E+00	3.934E+03	**0.000E+00**
F05	**0.000E+00**	1.397E+02	3.755E+02	2.361E+02	9.354E+03	**0.000E+00**
F06	**1.835E+00**	3.905E+01	2.619E+03	1.976E+01	7.149E+02	8.702E+00
F07	1.483E+00	0.975E+00	3.335E+01	6.911E+01	2.370E+00	**0.280E+00**
F08	2.041E+01	**2.023E+01**	2.034E+01	2.036E+01	2.037E+01	2.028E+01
F09	5.621E+00	5.971E+01	3.693E+01	3.202E+01	**0.003E+00**	2.487E+01
F10	**9.927E+00**	1.154E+02	4.049E+01	3.339E+01	3.111E+01	2.915E+01
F11	**5.503E+00**	1.159E+01	9.043E+00	8.871E+00	8.436E+00	6.146E+00
F12	9.707E+03	**5.528E+02**	1.039E+04	6.521E+03	3.353E+03	3.705E+03
F13	**0.456E+00**	2.009E+01	6.334E+00	5.576E+00	0.477E+00	0.981E+00
F14	**2.957E+00**	4.589E+00	3.874E+00	3.890E+00	3.747E+00	3.571E+00
F15	2.898E+02	8.659E+02	4.626E+02	4.942E+02	**1.196E+02**	3.796E+02
Best	**9**	**3**	**0**	**0**	**2**	**5**
Rank	**1**	**3**	**5**	**5**	**4**	**2**

functions ITSE, ITAE, ISE, and IAE. The simulation results achieved by the different algorithms in five runs are presented in Table 8 and Table 9. As tables show, DBA offers, in general, a better average for all objective functions. In the case of the ITSE function, from Fig. 6 and Table 6, it is clear that the GA algorithm provides a very fast convergence rate, but it fails in the minimization of the objective function compared to the DBA. As for the ITAE function, as shown in Fig. 6 and Table 6, the PSO algorithm presents a very low objective function, but not lower than DBA. In the case of the IAE and ISE functions, from Fig. 6 and Table 7, the DBA provides a very fast convergence rate and a very low final error.

The dynamic responses for X_D and X_B outputs with the optimally tuned FOPID controllers based on these algorithms are shown in Fig. 7 and Fig. 8, respectively. In the case of the ITSE function and X_D output, from Fig. 7 and Table 6, it is evident that the DBA provides a very fast rise time and a very low cost function, while the overshoot and the settling time are bigger than those based on the BA and BA-IS algorithms. As for the X_B output, from Fig. 8 and Table 6, it is clear that the PSO- and BA-IS-based FOPID provides the best and the worst settling time, respectively. In the case of the ITAE function and X_D output, from Fig. 7 and Table 6, the GA algorithm shows a very low overshoot and the best settling time. As for the X_B output, From Fig. 8 and Table 6, the BA provides the worst objective function.

From Fig. 7 and Table 7, regarding the ISE function and X_D output, although the GA algorithm provides the best overshoot, it fails in the minimization of the objective function compared to the PSO, BA-IS, dBA, and DBA algorithms. As for the X_B output, based on Fig. 8 and Table 7, the BA-IS algorithm represents the best rise time. In the case of the IAE function and X_D output, from Fig. 7 and Table 7, the BA presents the worst maximum overshoot. As for the X_B output, from Fig. 8 and Table 7, the DBA-based controller shows the best cost function, but it fails in minimization of the rise time and settling time aspects.

The dynamic responses of the X_D and X_B outputs for an instantaneous 0.2 input feed rate variation are depicted in Fig. 9 and Fig. 10, respectively. These figures show that all FOPID controllers have the ability to bring back the X_D and X_B outputs in a desired interval within a minimum time. The variation of best control parameters f and r during the entire evolution process are depicted in Fig. 11. The figures show that there is not a fixed best combination for f and r control parameters during the whole evolution process.

According to the simulation results shown and discussed in this section, it is clear that the dynamic mechanism proposed here improves the BA's minimization of the cost function, specially for the ITSE, ISE, and IAE cases. It is also noticeable that other algorithms like GA, PSO, BA-IS and BA are not that effective in this minimization.

4.2 Distillation Column without Decoupling

The distillation column transfer function is known with uncertainty and time delays. Therefore, in practice, coupling effects could be observed. The tuned controller gains and the corresponding fitness function parameters of the proposed FOPID controller obtained using

Table 3 Parameters settings in simulation

Algorithm	Parameters value
GA	Population size = 6, Mutation probability = 0.05, Crossover probability = 0.95
PSO	Population size = 6, Velocity coefficients = 1.4962, Inertia weight = 0.7298
BA	Population size = 6, A = 0.9, r = 0.1, Frequency = 0-2
dBA	Population size = 6, A = 0.9, r = 0.1, Frequency = 0-2
BA-IS	Population size = 6, A = 0.95, r = 0.85, Frequency = 0-1

Table 4 Optimum values of FOPID gains obtained via minimizing J_1 and J_2 for distillation column with decoupling.

Algorithms	K_{P1}	K_{I1}	K_{D1}	λ_1	μ_1	K_{P2}	K_{I2}	K_{D2}	λ_2	μ_2
$J_1 = ITSE$										
GA	0.1858	0.0591	-0.0189	0.8926	0.5115	-0.0993	-0.0262	0.0079	0.8791	0.2498
PSO	0.2772	0.0799	0.0075	0.9264	0.2981	-0.062	-0.0367	-0.0761	0.8032	0.4507
BA	0.2997	0.0799	0.0797	0.8517	0.998	-0.0948	-0.0306	-0.0799	0.998	0.5092
BA-IS	0.2996	0.0797	-0.0294	0.7782	0.4556	-0.0873	-0.0230	-0.0799	0.998	0.4461
dBA	0.2995	0.07964	0.0056	0.9229	0.1297	-0.104	-0.0307	-0.0799	0.8898	0.7644
DBA	0.2973	0.0795	0.0735	0.8941	0.6728	-0.1033	-0.0315	-0.0799	0.8827	0.998
$J_2 = ITAE$										
GA	0.2396	0.0513	-0.0220	0.8966	0.2225	-0.1027	-0.0286	0.0433	0.8333	0.0284
PSO	0.2761	0.0679	-0.0230	0.8805	0.0422	-0.0777	-0.0252	-0.0326	0.8947	0.6361
BA	0.2995	0.0388	-0.0799	0.995	0.3673	-0.1458	-0.0174	0.0799	0.996	0.1519
BA-IS	0.2571	0.0799	0.0798	0.998	0.6681	-0.1222	-0.0361	-0.0799	0.997	0.998
dBA	0.1259	0.0747	0.0344	0.8487	0.2795	-0.0887	-0.0207	0.0187	0.9202	0.5890
DBA	0.2997	0.0689	0.0377	0.9026	0.998	-0.0714	-0.0296	-0.0740	0.8763	0.6567

Table 5 Optimum values of FOPID gains obtained via minimizing J_3 and J_4 for distillation column with decoupling.

Algorithms	K_{P1}	K_{I1}	K_{D1}	λ_1	μ_1	K_{P2}	K_{I2}	K_{D2}	λ_2	μ_2
$J_3 = ISE$										
GA	0.2804	0.0559	0.0111	0.8296	0.2911	-0.0483	-0.0557	-0.0628	0.6729	0.6423
PSO	0.2679	0.0667	0.0275	0.9465	0.5072	-0.0775	-0.0346	-0.0717	0.8648	0.5949
BA	0.2995	0.0796	-0.0799	0.997	0.5664	-0.1345	-0.0318	-0.0799	0.998	0.997
BA-IS	0.2997	0.0798	0.0799	0.998	0.8460	-0.1221	-0.0241	-0.07998	0.998	0.3694
dBA	0.2998	0.0781	-0.0405	0.9020	0.6774	-0.1457	-0.0238	-0.0798	0.9971	0.9564
DBA	0.2995	0.0633	0.0796	0.998	0.1421	-0.1438	-0.0255	-0.0799	0.998	0.997
$J_4 = IAE$										
GA	0.2159	0.0576	0.0090	0.8939	0.0902	-0.0948	-0.0313	-0.0055	0.8202	0.1951
PSO	0.2341	0.0797	0.0442	0.8644	0.4074	-0.0239	-0.0568	-0.0799	0.6736	0.5122
BA	0.2996	0.0795	0.0797	0.998	0.4651	-0.0751	-0.0403	-0.0798	0.997	0.6963
BA-IS	0.2996	0.0604	-0.0380	0.999	0.998	-0.1084	-0.0336	-0.0799	0.998	0.996
dBA	0.2612	0.0797	-0.0061	0.8757	0.2281	-0.0864	-0.0295	-0.0749	0.8819	0.8734
DBA	0.2995	0.07998	0.0798	0.8744	0.4203	-0.0894	-0.0308	-0.0798	0.8732	0.997

Table 6 Optimum time indices parameters and objective function for J_1 and J_2 for distillation column with decoupling.

Algorithms	M_{p1}(%)	T_{s1}	T_{r1}	O_{f1}	M_{p2}(%)	T_{s2}	T_{r2}	O_{f2}	TE
$J_1 = ITSE$									
GA	2.9317	29.1219	4.7950	**5.8645**	12.8827	29.3280	4.4548	**15.3450**	**21.2095**
PSO	15.6823	28.3406	3.4834	**3.9806**	11.7207	25.7932	4.1919	**12.7048**	**16.6855**
BA	9.0865	16.0461	3.8498	**3.4980**	23.8717	36.9234	3.7040	**13.2013**	**16.6993**
BA-IS	10.4990	22.6891	3.4882	**5.3396**	8.4383	39.1734	4.2591	**12.8901**	**18.2297**
dBA	16.0519	27.9340	3.3511	**3.8937**	14.9776	25.8994	4.0608	**11.1359**	**15.0297**
DBA	11.1792	25.9410	3.5952	**3.3543**	12.0370	26.9116	4.4667	**11.0188**	**14.3731***
$J_2 = ITAE$									
GA	1.3659	15.8689	4.4804	**22.3518**	10.1774	17.3451	5.1532	**49.4813**	**71.8331**
PSO	6.4702	18.5156	4.0162	**18.2747**	3.5639	30.2647	5.0425	**33.4526**	**51.7274**
BA	2.1883	29.7596	4.6767	**34.8514**	5.8565	36.4753	5.2525	**52.0839**	**86.9354**
BA-IS	16.0014	29.4743	3.7136	**25.1853**	33.8622	29.2929	3.7240	**45.3965**	**70.5818**
dBA	3.2842	25.4469	4.7511	**19.6439**	4.0528	30.6888	5.0578	**37.7328**	**57.3767**
DBA	8.2921	17.5298	3.8473	**17.6097**	4.1105	30.0357	4.8530	**31.3768**	**48.9865***
$*$ = Best result									

Table 7 Optimum time indices parameters and objective function for J_3 and J_4 for distillation column with decoupling.

Algorithms	$M_{p1}(\%)$	T_{s1}	T_{r1}	O_{f1}	$M_{p2}(\%)$	T_{s2}	T_{r2}	O_{f2}	TE
$J_3 = ISE$									
GA	0.5863	22.8862	4.2707	**2.6624**	21.6898	22.5972	4.1054	**4.9081**	**7.5706**
PSO	8.6791	29.5263	3.9025	**2.4443**	13.4493	27.6699	4.2055	**4.6103**	**7.0546**
BA	26.8039	30.3503	3.1336	**2.7406**	30.2288	27.9002	3.6706	**4.4509**	**7.1916**
BA-IS	16.7998	29.1684	3.5683	**2.2081**	26.9275	47.5924	3.2042	**4.6585**	**6.8666**
dBA	16.7517	28.0952	3.3132	**2.5151**	20.1420	36.5980	3.7410	**4.3140**	**6.8292**
DBA	11.2215	32.9463	3.3552	**2.2267**	22.0456	36.8700	3.7622	**4.3123**	**6.5391***
$J_4 = IAE$									
GA	1.7814	15.056	4.6381	**4.5056**	17.8278	33.2409	4.1064	**7.8543**	**12.3599**
PSO	9.3277	18.8350	3.8681	**3.9421**	18.4811	23.2475	4.3089	**7.9220**	**11.8642**
BA	17.2361	28.9912	3.3072	**4.0493**	40.8556	26.0068	3.9500	**8.5420**	**12.5913**
BA-IS	13.1591	30.8970	3.5637	**4.5676**	27.0569	30.3074	4.1097	**7.3450**	**11.9127**
dBA	12.3460	19.6152	3.6787	**4.0746**	4.2421	28.5379	4.8379	**6.0963**	**10.1710**
DBA	10.6416	16.7730	3.4027	**3.7766**	5.7780	27.5183	4.9175	**6.0514**	**9.8281***

$*$ = Best result

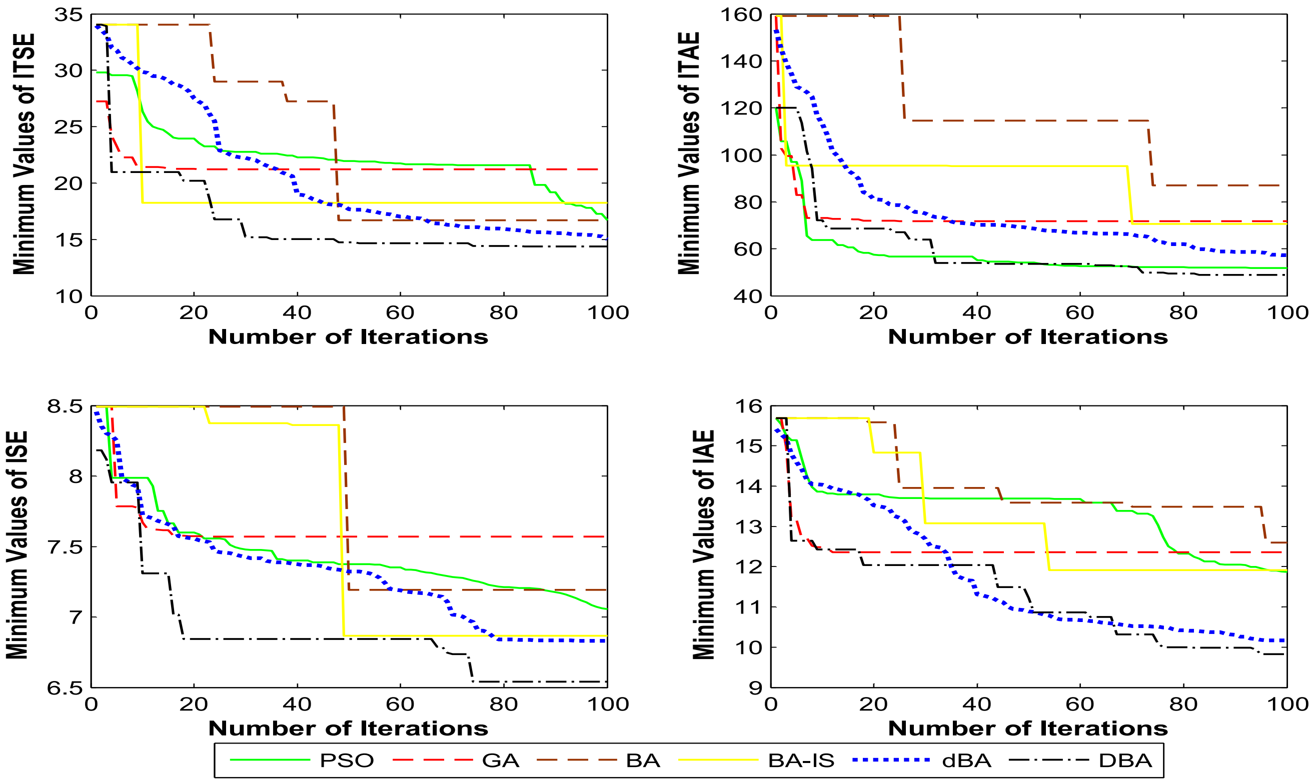

Fig. 6 Convergence characteristics of the DBA, BA, dBA, BA-IS, GA, and PSO algorithms for distillation column with decoupling.

Table 8 Optimum objective function for J_1 and J_2 using different evolutionary algorithms.

	Objective Function(J_1)			Objective Function(J_2)		
Algorithms	Best	Average	Worst	Best	Average	Worst
GA	**21.2095**	25.2414	28.6057	**71.8331**	96.7770	152.6311
PSO	**16.6855**	21.6112	23.7978	**51.7274**	54.1042	55.4096
BA	**16.6993**	21.0960	28.9771	**86.9354**	121.5415	152.5123
BA-IS	**18.2297**	26.6526	34.0246	**70.5818**	95.5040	141.6357
dBA	**15.0297**	15.4826	16.2511	**57.3767**	59.4419	62.5856
DBA	**14.3731***	15.0784	15.9156	**48.9865***	52.4100	55.2621

$*$ = Best result

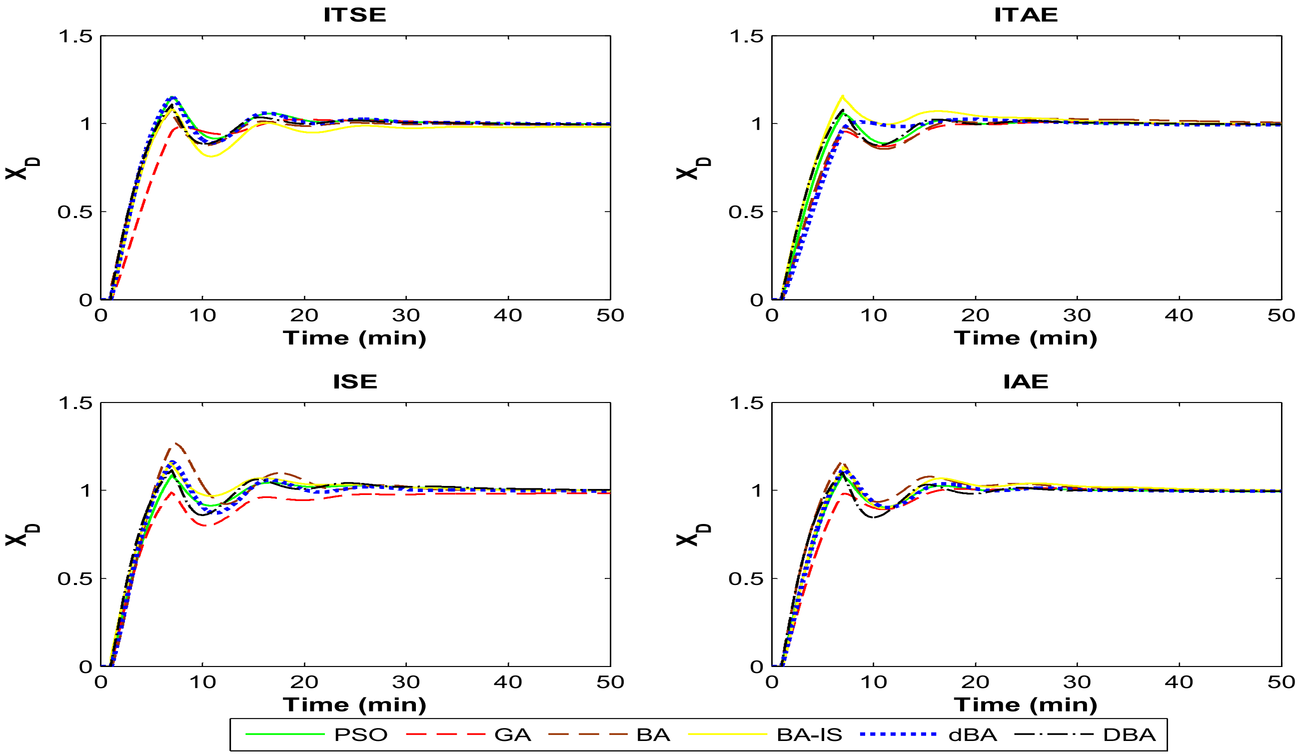

Fig. 7 X_D step responses using DBA, BA, dBA, BA-IS, GA, and PSO algorithms for distillation column with decoupling.

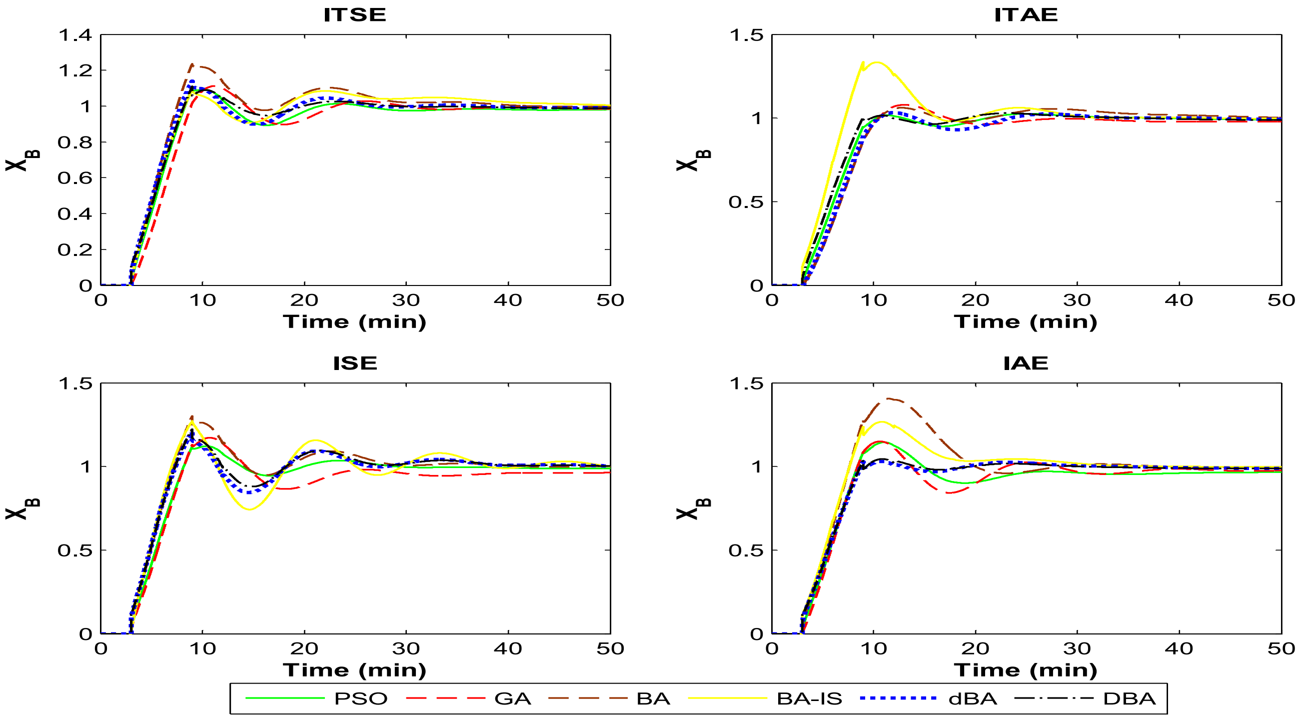

Fig. 8 X_B step responses using DBA, BA, dBA, BA-IS, GA, and PSO algorithms for distillation column with decoupling.

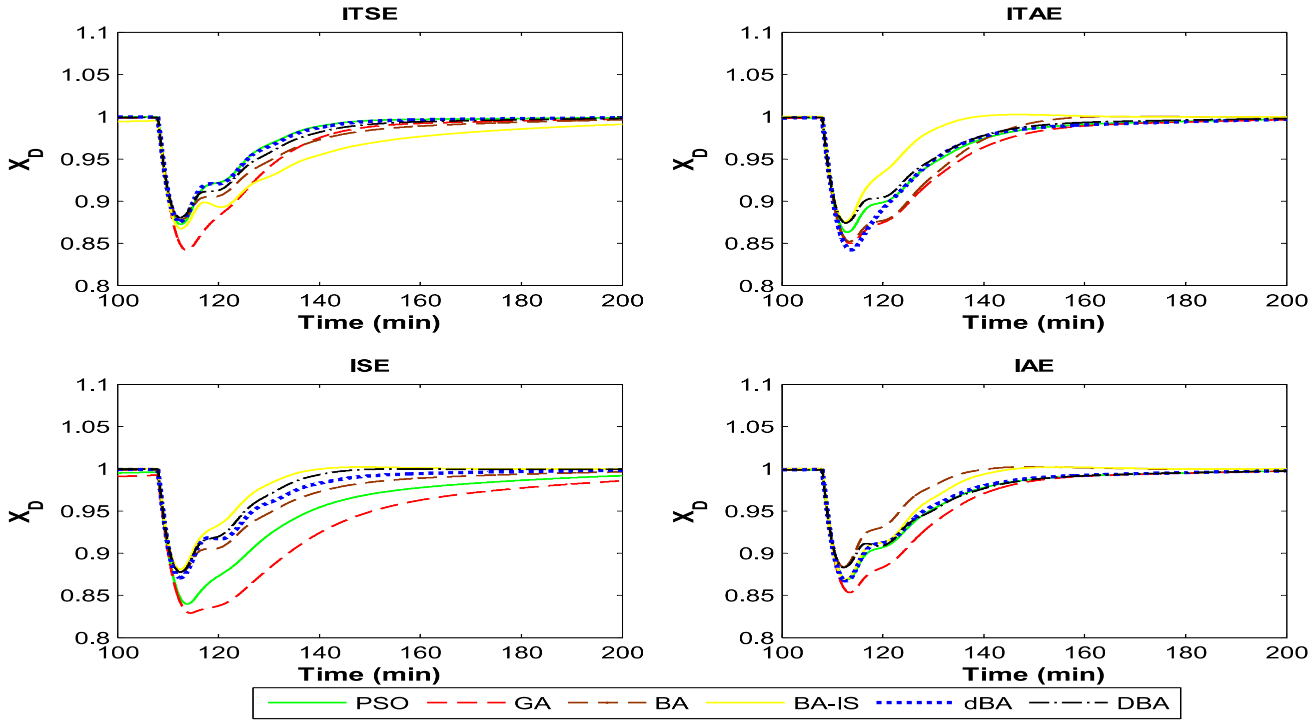

Fig. 9 X_D dynamic responses for 0.2 input feed rate disturbance using DBA, BA, dBA, BA-IS, GA and PSO algorithms for the distillation column with decoupling.

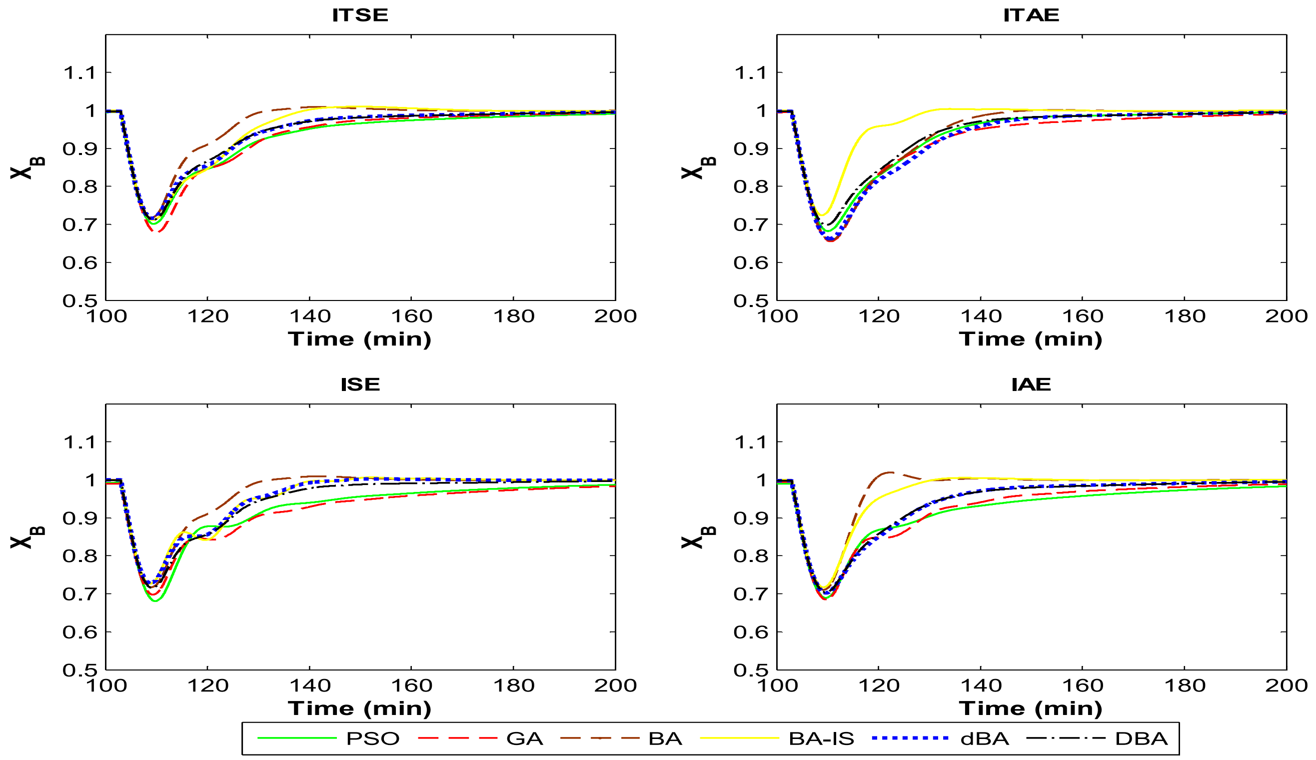

Fig. 10 X_B dynamic responses for 0.2 input feed rate disturbance using DBA, BA, dBA, BA-IS, GA and PSO algorithms for the distillation column with decoupling.

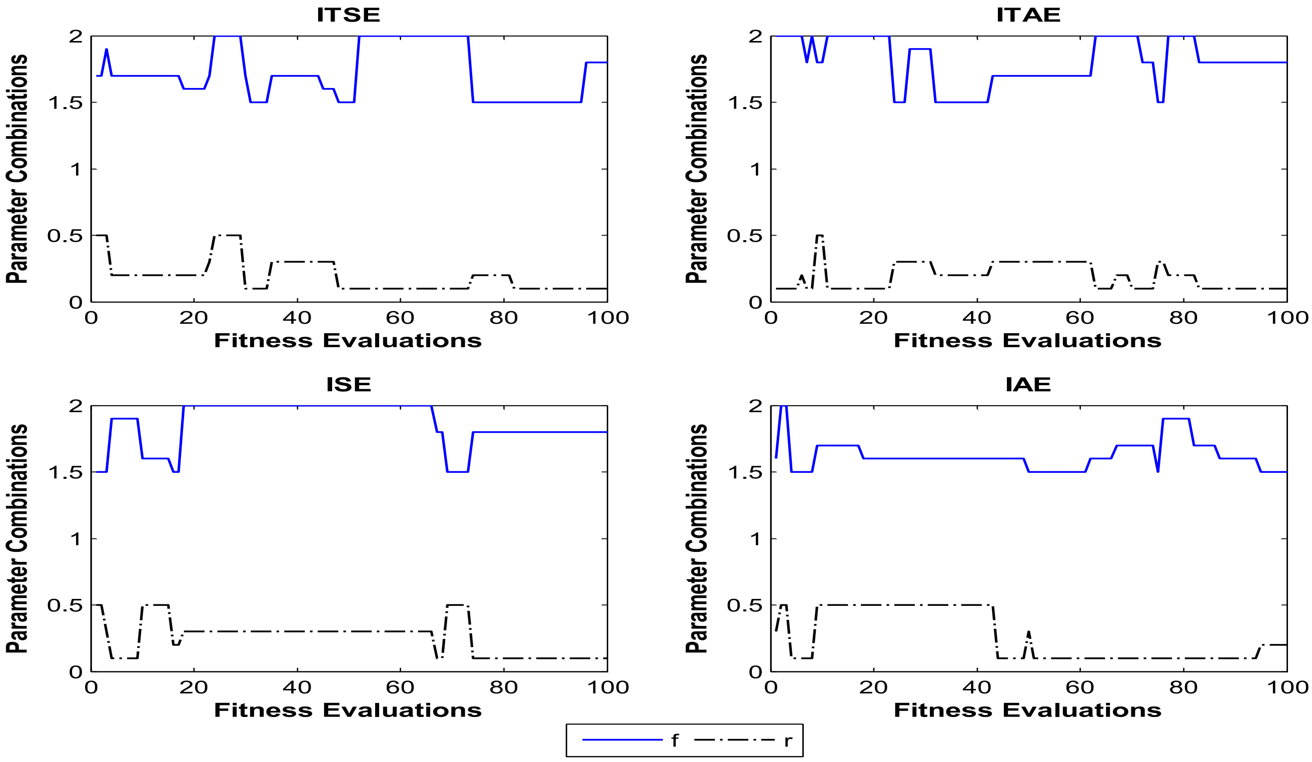

Fig. 11 Best parameters values during the evolution process for distillation column with decoupling.

Table 9 Optimum objective function for J_3 and J_4 using different evolutionary algorithms.

	Objective Function (J_3)			Objective Function (J_4)		
Algorithms	Best	Average	Worst	Best	Average	Worst
GA	**7.5706**	7.9123	8.0759	**12.3599**	13.1729	13.9721
PSO	**7.0546**	7.4005	7.8784	**11.8642**	13.4288	14.6854
BA	**7.1916**	7.6716	8.2626	**12.5913**	13.9981	14.9946
BA-IS	**6.8666**	7.5406	8.4934	**11.9127**	13.2952	15.0834
dBA	**6.8292**	6.9349	7.1037	**10.1710**	10.3875	10.5307
DBA	**6.5391***	6.6767	6.8276	**9.8281***	10.2939	10.4912

$*$ = Best result

the previously cited algorithms for the distillation column without decoupling are presented in Tables 10-13. The convergence rate characteristics of the DBA, BA, dBA, BA-IS, PSO, and GA algorithms for the fitness functions ITSE, ITAE, ISE, and IAE are depicted in Fig. 12. Tables 14-15 show the simulation results obtained using different algorithms in five runs. In the case of the ITSE function, from Fig. 12 and Table 12, the GA algorithm fails in the minimization of the total cost function compared to the other algorithms. As for the ITAE function, the PSO algorithm provides the best average for objective function compared to the other algorithms. In the case of the ISE, from Fig. 12 and Table 13, the DBA-based FOPID controller, in terms of the objective function minimization, outperforms the GA, BA, dBA and PSO algorithms. As for IAE function, from Table 13, PSO can offer the best average for objective function.

The dynamic responses of the X_D and X_B outputs for DBA, BA, dBA, BA-IS, GA, and PSO-based FOPID controllers are presented in Fig. 13 and Fig. 14, respectively. From Fig. 13 and Table 12, according to the ITSE function and X_D output, the BA algorithm provides the best rise time but fails in the minimization of the cost function and maximum overshoot. As for the X_B output, from Fig. 14 and Table 12, the DBA and GA algorithms provide the worst overshoot and rise time, respectively. In the case of the ITAE function and for the X_D output, from Fig. 13 and Table 12, the dynamic PSO presents a very low rise time and overshoot compared to those from the BA, BA-IS, dBA, and DBA algorithms. For the X_B output, from Fig. 14 and Table 12, the GA provides the worst rise time and min-

Table 10 Optimum values of FOPID gains obtained via minimizing J_1 and J_2 for distillation column without decoupling.

Algorithms	K_{P1}	K_{I1}	K_{D1}	λ_1	μ_1	K_{P2}	K_{I2}	K_{D2}	λ_2	μ_2
J_1 = ITSE										
GA	0.2818	0.0794	0.0379	0.7607	0.3027	-0.1042	-0.0138	-0.0175	0.9749	0.8124
PSO	0.2902	0.0687	0.0799	0.7692	0.2739	-0.1170	-0.0175	-0.0798	0.9233	0.8812
BA	0.2997	0.0585	-0.0799	0.9990	0.9990	-0.1350	-0.0129	-0.0799	0.9990	0.9990
BA-IS	0.2997	0.0799	-0.0150	0.9216	0.1829	-0.0740	-0.0157	-0.0797	0.9990	0.3956
dBA	0.2995	0.0693	0.0524	0.8953	0.1752	-0.1110	-0.0170	-0.0786	0.9919	0.9248
DBA	0.2996	0.0781	0.0791	0.8387	0.4856	-0.1113	-0.0178	-0.0799	0.9990	0.9205
J_2 = ITAE										
GA	0.2124	0.0744	0.0564	0.7504	0.3389	-0.0619	-0.0133	0.0024	0.9777	0.8883
PSO	0.2970	0.0666	0.0799	0.9767	0.0309	-0.0779	-0.0191	-0.0799	0.9373	0.8616
BA	0.2787	0.0400	0.0797	0.9990	0.7108	-0.0676	-0.0170	-0.0799	0.9980	0.3693
BA-IS	0.2997	0.0505	0.0799	0.9980	0.7882	-0.1119	-0.0095	-0.0799	0.9990	0.7881
dBA	0.2813	0.0440	0.0667	0.9970	0.0845	-0.0789	-0.0165	-0.0322	0.9533	0.8538
DBA	0.2942	0.0782	0.0799	0.9300	0.2610	-0.0729	-0.0186	-0.0799	0.9704	0.7399

Table 11 Optimum values of FOPID gains obtained via minimizing J_3 and J_4 for distillation column without decoupling.

Algorithms	K_{P1}	K_{I1}	K_{D1}	λ_1	μ_1	K_{P2}	K_{I2}	K_{D2}	λ_2	μ_2
J_1 = ISE										
GA	0.2470	0.0354	0.0339	0.7908	0.2222	-0.1342	-0.0173	0.0022	0.8472	0.2413
PSO	0.2995	0.0398	0.0528	0.8972	0.2997	-0.1426	-0.0146	-0.0112	0.9935	0.3695
BA	0.2913	0.0799	0.0799	0.5441	0.9990	-0.0524	-0.0735	-0.0799	0.3488	0.9990
BA-IS	0.2996	0.0596	0.0799	0.8439	0.9413	-0.1081	-0.0191	-0.0740	0.9990	0.9990
dBA	0.2997	0.0373	0.0681	0.9990	0.0659	-0.1146	-0.0153	-0.0360	0.9901	0.4265
DBA	0.2603	0.0400	0.0679	0.9561	0.4571	-0.1365	-0.0130	-0.0799	0.9990	0.8574
J_2 = IAE										
GA	0.2080	0.0301	0.0316	0.8367	0.0812	-0.1518	-0.0219	-0.0082	0.7509	0.2037
PSO	0.2940	0.0312	0.0699	0.9988	0.2006	-0.1303	-0.0165	-0.0796	0.9935	0.9054
BA	0.2996	0.0649	0.0240	0.9980	0.9990	-0.0965	-0.0105	-0.0799	0.9980	0.2409
BA-IS	0.2997	0.0425	0.0799	0.9970	0.9990	-0.0939	-0.0247	-0.0498	0.9990	0.4767
dBA	0.2995	0.0426	0.0402	0.9960	0.1736	-0.0826	-0.0151	-0.0280	0.9631	0.4346
DBA	0.2996	0.0467	0.0799	0.9560	0.3414	-0.1098	-0.0125	-0.0206	0.9970	0.9990

Table 12 Optimum time indices parameters and objective function for J_1 and J_2 for distillation column without decoupling.

Algorithms	M_{p1}(%)	T_{s1}	T_{r1}	O_{f1}	M_{p2}(%)	T_{s2}	T_{r2}	O_{f2}	TE
J_1 = ITSE									
GA	28.3264	126.9956	3.8637	**69.8317**	5.5439	120.6849	81.6832	**480.1475**	**549.9793**
PSO	29.4332	124.0461	4.0207	**72.7996**	6.6614	118.0742	80.6678	**414.9279**	**487.7275**
BA	37.7322	122.5546	3.4012	**77.5303**	4.1083	125.0055	80.3727	**450.5866**	**528.1168**
BA-IS	32.9469	123.0852	3.4429	**66.5760**	8.1066	130.2979	81.2362	**461.1417**	**527.7177**
dBA	31.1523	109.8925	3.5075	**59.8147**	7.9618	126.1112	80.9647	**418.5985**	**478.4131**
DBA	30.4028	110.4377	3.6563	**61.2598**	9.9876	126.4710	80.8890	**411.8380**	**473.0978***
J_2 = ITAE									
GA	24.6788	121.8154	4.6912	**659.3943**	3.5990	121.8452	84.0529	**927.5416**	**1586.9359**
PSO	25.8651	109.0704	3.1779	**274.7633**	3.1744	112.9469	82.0944	**757.0654**	**1031.8287***
BA	34.4942	112.7389	5.1487	**411.1283**	7.4182	129.0136	81.3691	**788.4316**	**1199.5599**
BA-IS	30.7610	109.3070	4.4467	**308.6163**	0.0000	129.4594	83.0736	**1105.9475**	**1414.5639**
dBA	28.8026	113.8352	4.1132	**365.9694**	5.5065	108.7228	82.4422	**807.8939**	**1173.8633**
DBA	26.1022	109.6826	3.2796	**286.2907**	5.0620	116.9933	82.0729	**788.0578**	**1074.3486**

$*$ = Best result

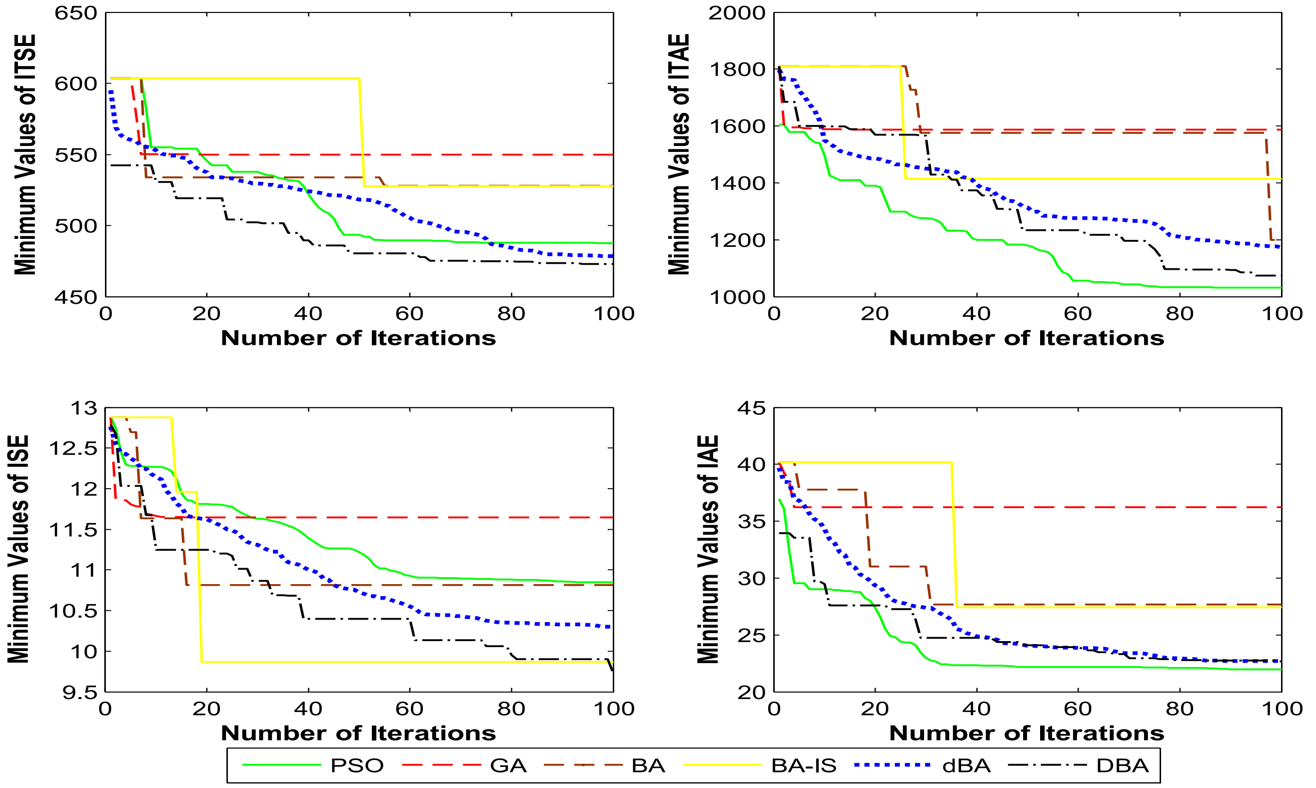

Fig. 12 Convergence characteristics of the DBA, BA, dBA, BA-IS, GA, and PSO algorithms for distillation column without decoupling.

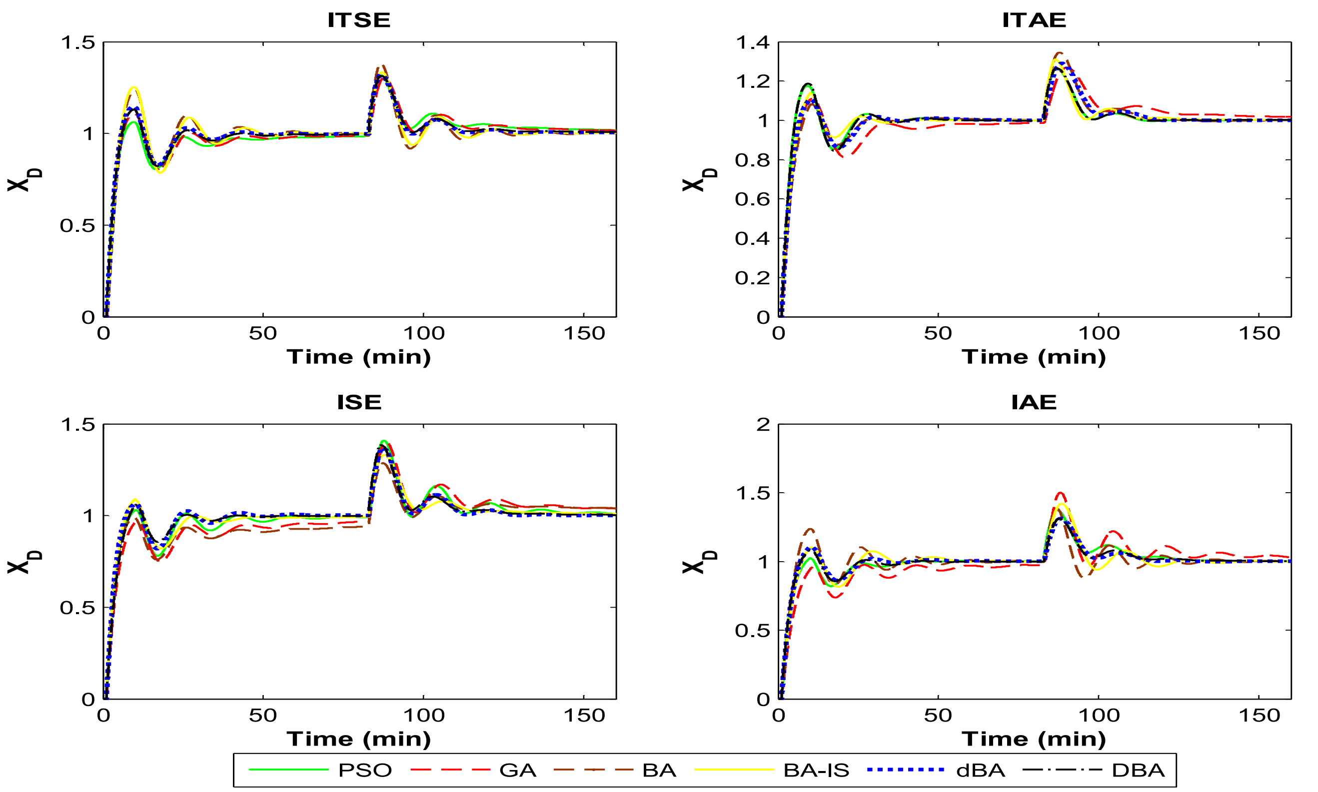

Fig. 13 X_D step responses using DBA, BA, dBA, BA-IS, GA, and PSO algorithms for distillation column without decoupling.

Table 13 Optimum time indices parameters and objective function for J_3 and J_4 for distillation column without decoupling.

Algorithms	$M_{p1}(\%)$	T_{s1}	T_{r1}	O_{f1}	$M_{p2}(\%)$	T_{s2}	T_{r2}	O_{f2}	TE
J_3 = ISE									
GA	35.1307	128.4233	7.3655	**4.6917**	18.9546	137.9572	80.0809	**6.9557**	**11.6474**
PSO	39.5278	138.4507	4.9483	**3.7742**	19.0263	133.5458	80.0182	**7.0714**	**10.8456**
BA	23.5895	123.1134	5.8119	**3.6584**	20.0721	137.6834	79.8843	**7.1553**	**10.8137**
BA-IS	31.7900	114.0321	4.6429	**3.2944**	14.4329	115.1481	81.0011	**6.5758**	**9.8702**
dBA	36.7459	123.7199	4.2172	**3.3074**	12.3598	124.7498	80.5469	**6.9910**	**10.2984**
DBA	37.9496	122.4806	5.2603	**3.5102**	4.2603	119.2213	80.3180	**6.2142**	**9.7244***
J_4 = IAE									
GA	45.7899	141.1647	7.8932	**17.2727**	30.1656	150.1783	79.0210	**18.9397**	**36.2123**
PSO	37.1174	123.4478	4.9784	**10.1506**	10.3820	117.1212	80.2513	**11.8052**	**21.9558***
BA	38.7608	123.1583	3.7407	**10.6486**	2.5235	133.8273	80.4055	**17.0444**	**27.6930**
BA-IS	41.9148	122.0103	4.9587	**10.6663**	33.9796	134.1972	80.3204	**16.7774**	**27.4436**
dBA	30.9574	113.3922	4.2736	**8.5850**	4.5419	108.0169	82.1012	**14.1204**	**22.7054**
DBA	30.6258	113.0370	4.2981	**8.6174**	2.8910	120.6250	81.8410	**14.0294**	**22.6467**

$*$ = Best result

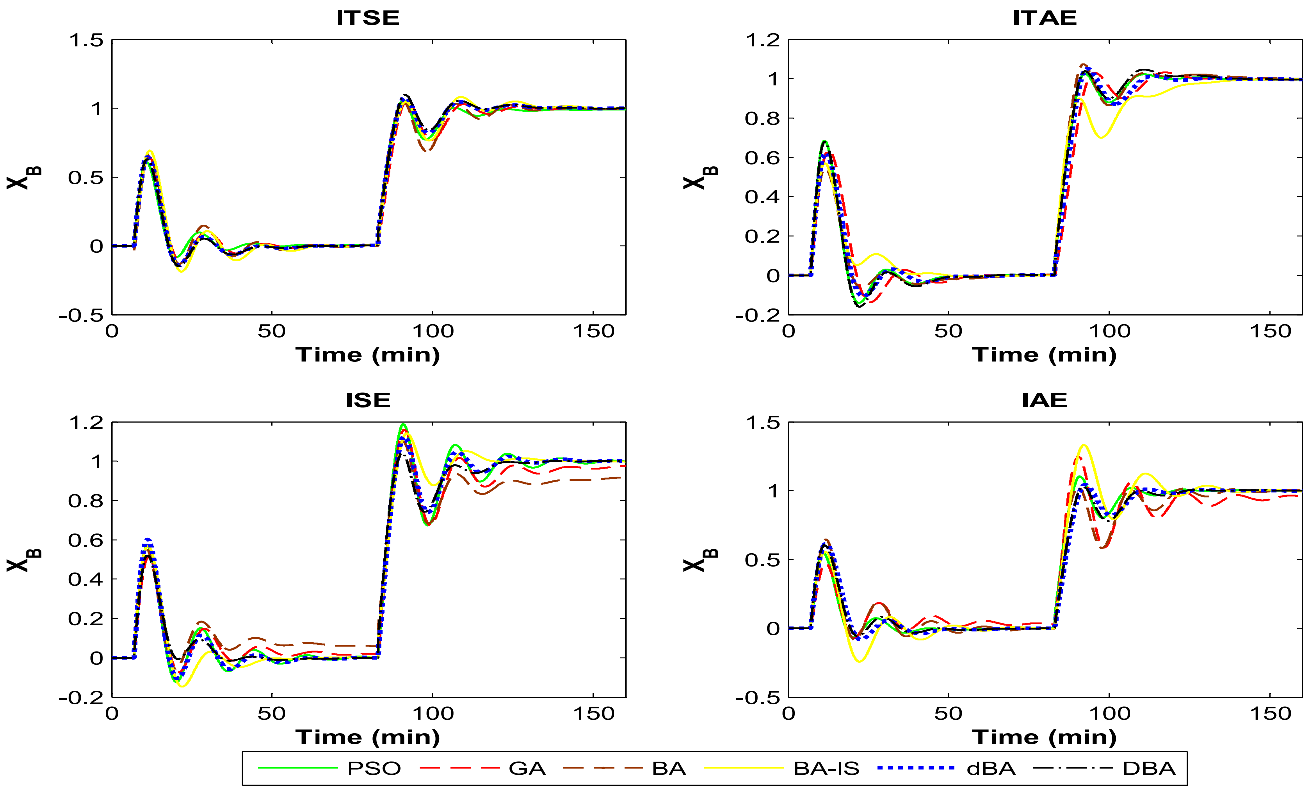

Fig. 14 X_B step responses using DBA, BA, dBA, BA-IS, GA, and PSO algorithms for distillation column without decoupling.

imization of objective function compared to the other algortihms.

In the case of the ISE function and for the X_D output, according to Fig. 13 and Table 13, although the BA algorithm provides the best overshoot, it fails in the minimization of the objective function and rise time. As for the X_B output, from Fig. 14 and Table 13, the DBA algorithm presents the best overshoot and minimization of the cost function. For IAE function and the X_D output, from Fig. 13 and Table 13, the BA-based FOPID controller provides a very fast rise time. As for the X_B output, from Fig. 14 and Table 13, the BA-IS and BA provide the worst and the best overshoot, respectively.

The dynamic resposnses of the X_D and X_B outputs for an instantaneous 0.2 input feed rate variation are presented in Fig. 15 and Fig. 16, respectively. From these figures, it is evident that all FOPID controllers are able to track the reference signal within a very short time. The variation of best control parameters f and r during the entire evolution process are depicted in Fig. 17. It is clear that best combination for f and r control parameters changes during the whole evolution process.

According to the simulation results shown and discussed in this section, it is clear that the dynamic mechanism proposed here effectively improves the BA's convergence rate and the minimization of the cost function, specially for the ITSE and IAE cases. Interestingly, in the case of the ITAE cost function, the PSO-based FOPID outperforms those controllers based on the DBA, BA, BA-IS, dBA and GA algorithms.

Even if the PSO-based controller outperforms the controller tuned with our DBA algorithm, the differences are really slight, and our proposal keeps its competitiveness in comparison with the rest of performances.

In any case, the optimization problem is very dependent on the particular fitness or objective function to choose and the selection of the control parameters. The fact that an optimization algorithm provides good results for a specific objective function does not guarantee the same success when applied to a different objective function. In our case, we can conclude that: 1) even if for some specific objective functions the performance of the DBA approach is better than for others, the differences are very slight; and 2) for every particular objective function considered, the DBA algorithm outperforms in general terms the rest of algorithms used for comparison, both for the coupled and decoupled cases.

5 Conclusions

This paper proposes a new Bat Algortihm based on a dynamic parameters selection mechanism. This DBA dynamically selects the best performing combinations of the frequency coefficient, the pulse rate coefficient, and the population size. The analyses carried out based on the ITSE, ITAE, ISE, and IAE fitness functions show that the proposed dynamic mechanism improves the convergence rate of the algorithm and the minimization of the cost function compared to the conventional BA, dBA, BA-IS, GA, and PSO algorithms.

A fractional order PID controller has been used to control the distillate and bottom mole fractions of a distillation column system. The gains of this controller are calculated successfully using DBA, BA, dBA, BA-IS, GA, and PSO algorithms. The simulation results clearly show that the proposed FOPID controller based on the DBA algorithm improves the performance of the distillation column system during a change in the set point or any feed rate disturbance.

6 Compliance with Ethical Standards

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Conflict of Interest: The authors declare that they have no conflict of interest.
Ethical approval: This article does not contain any studies with human participants or animals performed by any of the authors.

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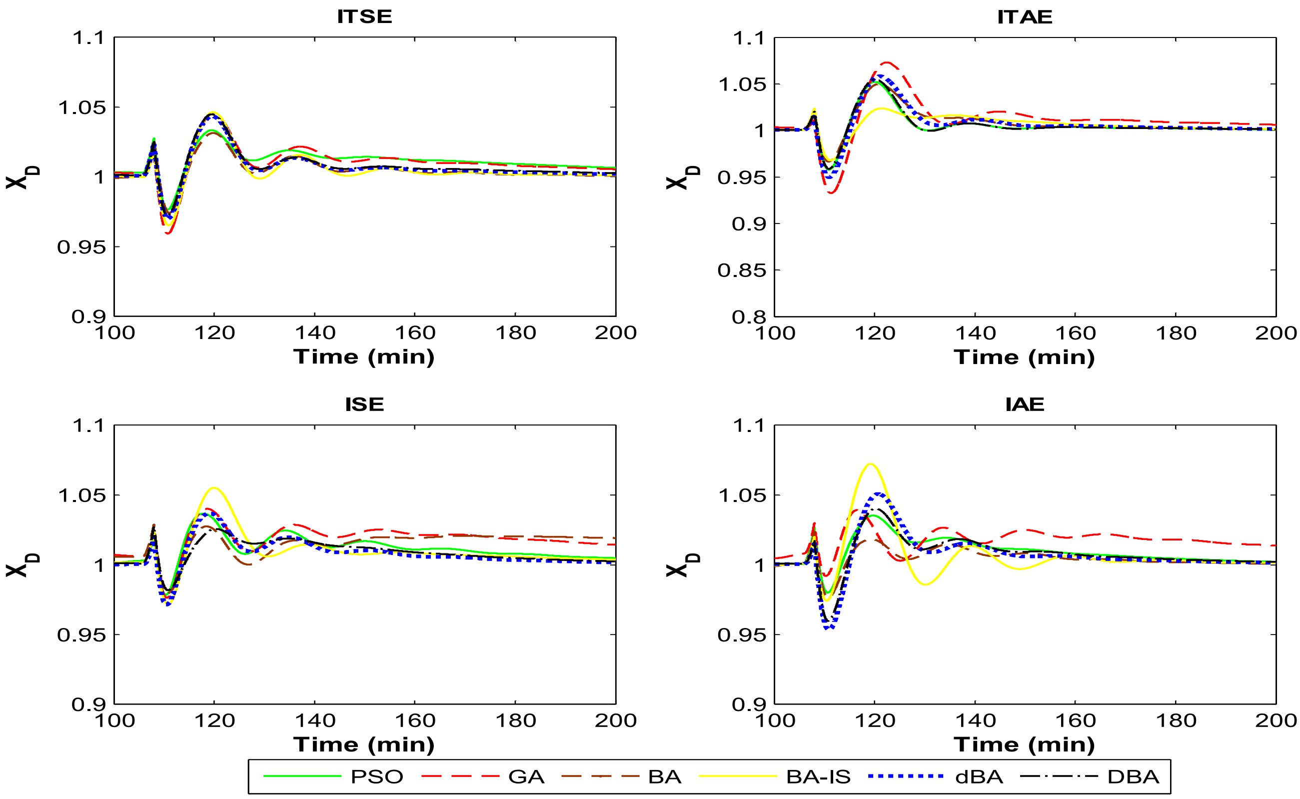

Fig. 15 X_D dynamic responses for 0.2 input feed rate disturbance using DBA, BA, dBA, BA-IS, GA and PSO algorithms for the distillation column without decoupling.

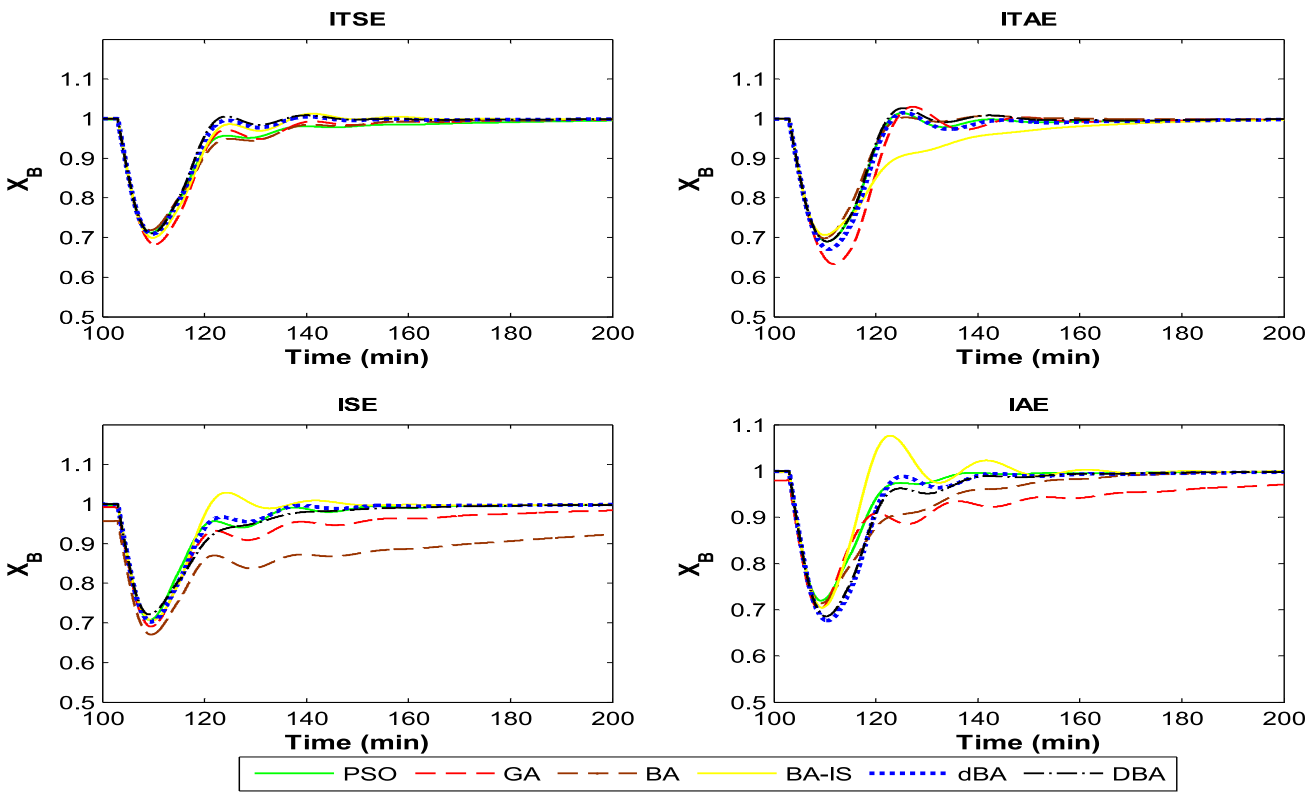

Fig. 16 X_B dynamic responses for 0.2 input feed rate disturbance using DBA, BA, dBA, BA-IS, GA and PSO algorithms for the distillation column without decoupling.

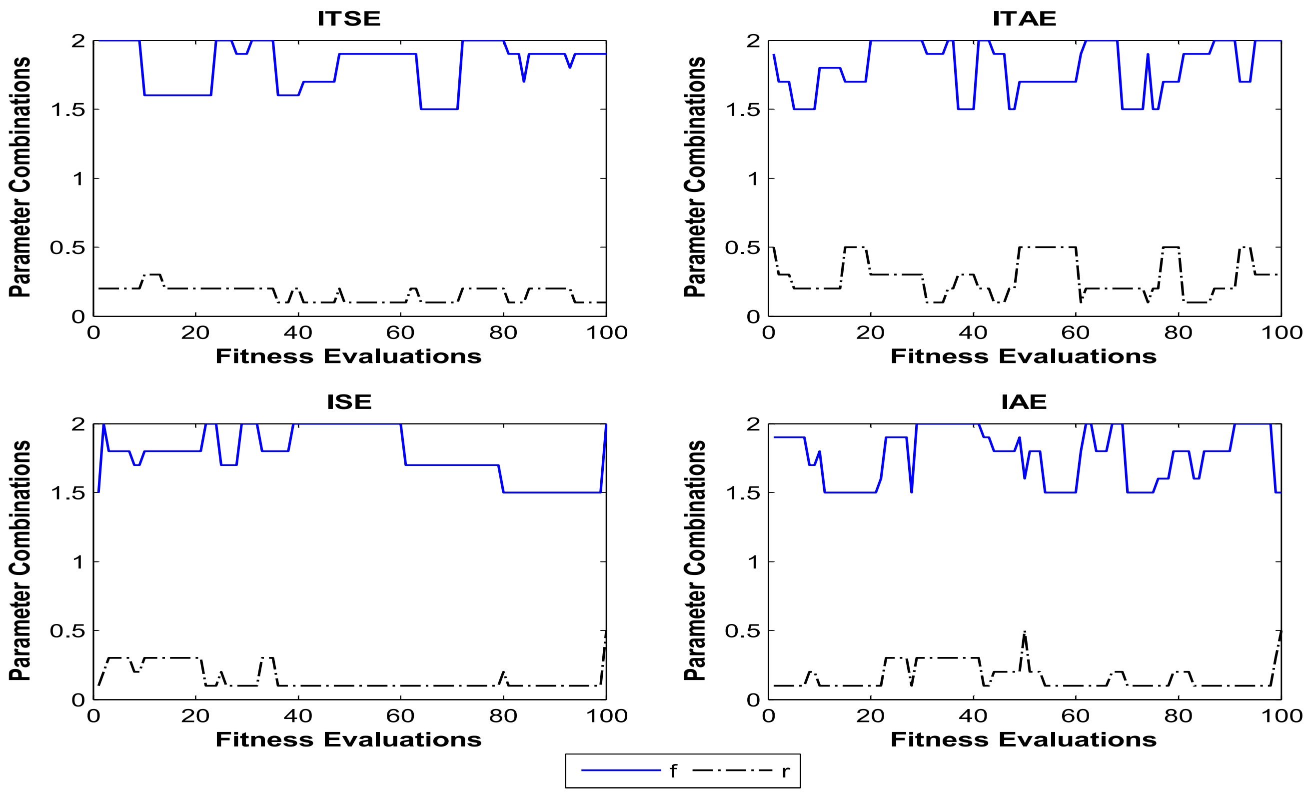

Fig. 17 Best parameters values during the evolution process for distillation column without decoupling.

Table 14 Optimum objective function for J_1 and J_2 without decoupling and using different evolutionary.

	Objective Function(J_1)			Objective Function(J_2)		
Algorithms	Best	Average	Worst	Best	Average	Worst
GA	**549.9793**	557.1102	570.5866	**1586.9359**	1686.7534	1809.6992
PSO	**487.7275**	519.6377	559.7173	**1031.8287***	1043.5210	1051.2656
BA	**528.1168**	564.9880	603.6296	**1199.5599**	1687.6713	1809.6992
BA-IS	**527.7177**	568.9104	603.6296	**1414.5639**	1730.6721	1809.6992
dBA	**478.4131**	481.6674	486.1626	**1173.8633**	1221.1082	1268.0359
DBA	**473.0978***	483.8802	499.6774	**1074.3486**	1212.8895	1274.1767

$*$ = Best result

Table 15 Optimum objective function for J_3 and J_4 without decoupling and using different evolutionary.

	Objective Function(J_3)			Objective Function(J_4)		
Algorithms	Best	Average	Worst	Best	Average	Worst
GA	**11.6474**	12.2694	12.8762	**36.2123**	37.5629	38.2257
PSO	**10.8456**	11.8101	12.4939	**21.9558***	25.5113	29.4325
BA	**10.8137**	11.5112	12.8762	**27.6930**	30.8951	34.6447
BA-IS	**9.8702**	11.8932	12.8762	**27.4436**	31.3027	35.3713
dBA	**10.2984**	10.4664	10.7586	**22.7054**	22.9177	23.1012
DBA	**9.7244***	10.2053	10.8530	**22.6467**	23.0968	23.5495

$*$ = Best result

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