On the Complexity of $\{k\}$ -domination for Chordal Graphs

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Due to its large range of applications, many variations and extensions of the classical domination problem in graphs have been defined and studied during the past fourty years. Given a graph G = (V, E), $A \subseteq \mathbb{R}$ and $B = \{b_1, \ldots, b_{|V|}\}$, an A, B-dominating function of G is a function $f : V \mapsto A$ such that $f(N[v_i]) \geq b_i$ for all $v \in V$, where $f(U) = \sum_{u \in U} f(u)$, for $U \subseteq V$ and N[v] is the closed neighborhood of v. The weigth of f is given by w(f) = f(V). This work is focused in two variations of the problem. Let $k \in \mathbb{Z}_+$ and $b_i = k$ for all $i \in \{1, \ldots, |V|\}$. When $A = \{0, 1\}$, f is a k-tuple dominating function and $\gamma_{\times k}(G)$ is the k-tuple domination number of G [3]. When $A = \{0, 1, \ldots, k\}$, f is a $\{k\}$ -dominating function and $\gamma_{\{k\}}(G)$ is the $\{k\}$ -domination number of G [1]. As usual, these definitions induce the study of the following decision problems, for fixed $k \in \mathbb{Z}_+$:

k-TUPLE DOMINATING FUNCTION (k-DOM)

Instance: $G = (V, E), j \in \mathbb{N}$

Question: Does G have a k-tuple dominating function of weight at most j?

 $\{k\}$ -DOMINATING FUNCTION ($\{k\}$ -DOM)

Instance: $G = (V, E), j \in \mathbb{N}$

Question: Does G have a $\{k\}$ -dominating function of weight at most j?

In this work we obtain a new graph class where $\{k\}$ -DOM is NP-complete: the class of chordal graphs. We also identify some maximal subclasses for which it is polynomial time solvable. By relating this problem with k-DOM, we prove that $\{k\}$ -DOM is polynomial time solvable for strongly chordal graphs. Besides, by expressing the property involved in k-DOM in Counting Monadic Secondorder Logic, we obtain that both problems are linear time solvable for bounded tree-width graphs. In this way we enlarge the family of graphs for which k-DOM is polynomial time solvable (see [2]).

References

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