Polyhedral study of the 2-dominating set polytope of cycles and cactus graphs*

G. Argiroffo[1], M. Escalante[1,2] and M. E. Ugarte[1]
garua,mariana,ugarte@fceia.unr.edu.ar

[1] FCEIA Universidad Nacional de Rosario
[2] CONICET

Domination and its variations arise in many applications, in particular in those involving strategic placement of items at vertices of a network. For general graphs these problems are NP-hard, however, domination in graphs has been shown to be polynomially solvable in several graph classes. In this work we consider a generalization of this problem called *k-domination* in graphs.

Given a graph $G = (V, E)$, we denote by $N[i]$ the *closed neighborhood* of i, i.e., the set of vertices of V adjacent to vertex i and i itself. The minimum cardinality of all neighbourhoods of vertices of G is $\delta(G)$.

If an integer number k is such that $k \leq \delta(G) + 1$, a *k-dominating set* in G is a set $S \subset V$ such that $|S \cap N[i]| \geq k$ for every vertex $i \in V$. In the applications we are usually interested in the k-dominating sets of minimum cardinality. In this work we focus on some polyhedral aspects of this problem. For this purpose we consider $N[G]$ the $\{0,1\}$-matrix whose rows are the incidence vectors of $N[i]$ for every $i \in V$. Then, the *k-dominating set polytope* is defined as:

$$Q_k^*(G) = \text{conv}\left(\{x \in \{0,1\}^V : \ N[G]x \geq k\mathbf{1}\}\right),$$

where $\mathbf{1}$ is the vector of all ones (of appropriate dimension) and $\text{conv}(T)$ stands for the convex hull of the elements of T.

A compact description of the k-dominating set polytope of a graph is not known for general graphs. The family of cycles is the simplest family of graphs to begin our polyhedral study of k-domination when k is not equal to one. In this work we present two important results. The first one is a complete description of the 2-dominating set polytope of cycles. Since in this description there is an exponential size family of facets, we analyse the separation problem of this family. This allows us to show that the minimum weight 2-domination problem is solvable in polynomial time for cycles. The second important result is on the 2-dominating set polytope of a more general family of graphs called cacti, which can be obtained as 1-sum of cycles and edges. In this case, we present some valid inequalities for its 2-dominating set polytope that can be obtained using the description of the 2-dominating set polytope of cycles.

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