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Abstract

Demographic change and automation are two main structural trends shaping the macroeconomy in the next decades. We present a general equilibrium model with a tractable life-cycle structure that allows the investigation of the main transmission mechanisms by which demography and technology affect economic growth. Due to a trade-off between innovation and automation, lower fertility and population ageing lead to reductions in GDP per capita growth and the labour income share. During the demographic transition, the extent growth and factor shares are affected depends on alternative labour market configurations and scenarios for the integration of robots in economic activity.

Keywords: population ageing, automation, innovation.

JEL classification: O31, O40, J11.

Resumen

Los cambios demográficos y una nueva ola de innovación y de automatización son dos de las principales tendencias estructurales que configurarán el escenario macroeconómico en las próximas décadas. Utilizando un modelo de equilibrio general con una sencilla estructura demográfica, investigamos los principales mecanismos de transmisión por los cuales la demografía y la tecnología afectan al crecimiento económico. Debido a una disyuntiva entre innovación y automatización, una menor fertilidad y el envejecimiento poblacional generan una reducción en el crecimiento del PIB per cápita y en la participación de los salarios en el PIB. Durante la transición demográfica, la medida en la que el crecimiento y la participación de los factores se ven afectados depende de las diferentes configuraciones del mercado laboral y de escenarios para la integración de robots en la actividad económica.

Palabras clave: envejecimiento poblacional, automatización, innovación.

Códigos JEL: O31, O40, J11.

1 Introduction

Demographic – baby boomers reaching retirement age, a fall in fertility, and the continuous rise in longevity – and technological changes – the new wave of automation brought by developments in robotics and in artificial intelligence – are two structural trends that will frame the macroeconomic context in the next decades. The implications of these trends for economic growth are the subject of much debate. On the one hand, population ageing is found to be associated with lower interest rates, less innovation activity, and lower output growth (Aksoy et al. (2019), Gordon (2012) and Derrien et al. (2018)). On the other hand, Acemoğlu and Restrepo (2017b, 2018a) argue that it may give incentives to automation and, hence, to higher productivity and growth, although it may also decrease employment and the labour income share.[1]

We analyse the macroeconomic consequences of demographic and technological changes in a general equilibrium model in which both population dynamics and research and development (R&D) determine long-run growth. R&D comprises of two activities: innovation, which involves the creation of new products, and automation, which is the development of production processes that allow robots to replace labour. Although demographic changes, resulting from lower fertility and mortality, boost automation, we find that they eventually lead to lower growth of GDP per capita.

The primary source of growth in our framework is the origination of new goods that increase overall productivity (productivity effect). By creating new goods, innovation also creates new job opportunities (reinstatement effect). Eventually, as robots are more productive than labour, automation increases productivity but destroys jobs (displacement effect).

We identify three key channels through which demographic changes affect the economy. First, changes in labour supply affect factor prices (wages and the price of

[1]On the empirical literature on the employment and wage effects of automation see Graetz and Michaels (2018), Acemoğlu and Restrepo (2017a), Dauth et al. (2017) and Frey and Osborne (2017). On the relation between the labour income share and automation, see Martinez (2018) and Bergholt et al. (2019).

robots), altering the relative profitability of labour-intensive and automated sectors, and, hence, the incentives to innovate and automate. Second, demographic changes affect savings and the interest rate, altering the amount of resources available for investment in capital accumulation, innovation, and automation. Third, insofar as the efficiency of R&D may depend on the age structure of the population, the arrival rate of new goods is also affected.

Automation, by re-allocating production from a labour-intensive sector to an automated sector, can eventually generate an imbalance between the labour and robot income shares, preventing the economy from reaching a balanced growth path (BGP) — defined as a dynamic equilibrium with constant factor shares. We exclude divergent paths by imposing a restriction on the efficiency of robots production. Similarly to Aghion et al. (2017), we prevent a labour-free singularity by restricting the productivity gain of an essential input (robots), sustaining its relative price in terms of the final good. Ultimately, as the economy develops and gains in complexity, the robots that are capable of replacing labour in the production of an increasing variety of goods must also become harder to produce. Without this restriction, automation generates full robotisation of production with the price of robots converging to zero.

We show that if the economy is on a BGP, then a fall in fertility generates a new BGP with lower GDP per capita growth, a higher degree of automation, and a lower labour income share. Since on a BGP the pace of innovation and automation are matched, the creation of new goods, which initially must be produced employing labour, is the main source of technological change. A fall in labour supply growth leads to a decrease in the incentive to innovate, pushing output growth down. Furthermore, the incentive to automate increases as the labour supply growth falls, thus the robots income share increases and the labour income share falls. Even holding population constant, an increase in longevity entails lower interest rates, more automation, and a lower labour income share on the new BGP.

Embedding the demographic projections for the United States (US) and Europe for the next decades into our model allows us to quantify the contribution of demographic changes to medium-run economic trends in these regions. Lower fertility and higher longevity lead to higher automation both in the US and in Europe, with a stronger effect in Europe, which is consistent with the available data on robot density. Despite the positive effects of automation, as resources are diverted from innovation, population changes lead to lower output growth in the medium run, even without a fall in the efficiency of the R&D sector (as in Bloom et al. (2017)). Our results indicate that demographics reinforce three observed trends in the past decades: the fall in real interest rates (Aksoy et al. (2019) and Eggertsson et al. (2019)), the fall in labour income shares (Elsby et al. (2013), and Karabarbounis and Neiman (2014)) and the fall in the price of robots (Graetz and Michaels (2018)).

We extend the model to consider different labour market configurations and ways robots integrate in economic activity. We consider that (*i*) workers may move towards the R&D sector boosting labour supply in R&D during the demographic transition, (*ii*) the retirement age rises as longevity increases, (*iii*) automation also generates an increase in the relative productivity of robots; and (*iv*) robots also replace labour in R&D. Although in all cases the long-run effect on per-capita growth is the same as in the benchmark model, during the transition the relative fall in GDP per capita is mitigated, with the different labour market configurations having a more significant impact. When the retirement age increases to maintain the ratio of the duration of the working life and the retirement constant, demographic changes no longer generate an increase in automation, benefiting mostly older workers employed in production. In contrast, allowing young workers to migrate to the R&D sector leads to higher income and automation, benefiting mostly young workers employed in R&D.

We relax the restriction on the efficiency of the production of robots, which ensures that factor prices and ultimately factor income shares do not diverge, in the medium run only enforcing it in the long run. Thus, the efficiency of robots production initially increases as automation rises. Under this scenario, the price of robots falls more significantly, diverging from the path of wages. A reduction in robot prices further boosts automation. As a result, the share of output produced in the automated sector increases markedly while the labour income share falls. This happens at the cost of resources being diverted from innovation, which eventually leads to a sizeable fall in GDP per capita growth. Thus, a 'robocalypse scenario', resembling the immiseration equilibrium of Benzell et al. (2015), may arise.

A different outcome occurs when the individual effective labour supply is no longer constant as in the benchmark case but changes endogenously as a function of the degree of automation or the level of capital deepening (thus adding economy-wide capital-skill or automation-skill complementarities). The negative implications for growth of the demographic transition may be mitigated, both in the short and in the long run, although for that to happen labour skills must increase substantially to offset the fall in labour supply due to lower fertility. In such scenario automation no longer increases due to demographics.

The key mechanism driving the results in all specifications is the trade-off between innovation and automation. Automation crowds out innovation, and, as the former is a subsidiary activity of the latter, automation cannot progress indefinitely without innovation. In this regard, the assumption that newly created goods need to necessarily be produced employing labour implies that labour constrains the creation of new goods, ultimately controlling productivity gains and growth. Relaxing this assumption may generate demographic transitions in which the share of labour income falls and per-capita output growth increases, hence, exacerbating the inequality between the production factor remunerations.

In what follows, we describe the model (Section 2), discuss the characteristics of the *BGP* and present the comparative analysis results (Section 3). Section 4 focuses on transitional dynamics, looking at the medium-run effects of demographic changes in the US and Europe. Section 5 discusses a set of extensions of the baseline model. Section 6 concludes.

2 The Model

The model economy consists of three sectors, goods production, R&D, and robot production, and households. The goods production sector comprises of a final good producer, who aggregates a continuum of intermediate differentiated goods $i \in Z_t$, produced by combining inputs (final goods), capital, and either labour or robots.

R&D involves two activities: innovation and automation. Innovation creates new goods or varieties (Romer (1990) and Comin and Gertler (2006)) that are added to the set Z_t of intermediate goods, and which, initially, can only be produced by labour. Automation develops procedures such that existing intermediate good i could be produced by robots. The set of goods produced by robots is denoted $A_t \subset Z_t$. Robots are machines created in the robot production sector, that are then used in the production of intermediate goods.

As in Gertler (1999), households, who supply labour, accumulate assets and consume the final good, face two stages of life, mature (working) and old (retirement). Thus, fertility, longevity, and retirement drive population dynamics.

2.1 Households

There are N_t households, divided amongst two age groups: workers (w) and retirees (r). $\omega^y_{t,t+1} N^w_t$ new households are born every period as workers. Workers (N^w_t) retire with a probability $1 - \omega^w$, and retirees (N^r_t) die with a probability $1 - \omega^r_{t,t+1}$. Thus,

$$N^w_{t+1} = \omega^y_{t,t+1} N^w_t + \omega^w N^w_t, \text{ and } N^r_{t+1} = (1 - \omega^w) N^w_t + \omega^r_{t,t+1} N^r_t. \tag{1}$$

Households face two idiosyncratic risks: i) loss of wage income at retirement and ii) time of death. There is a perfect annuity market allowing retirees to insure against time of death by turning their wealth over to perfectly competitive financial intermediaries that invest the proceeds and pay back a return of $R_t/\omega^r_{t-1,t}$ for surviving retirees. Households are risk neutral, so that uncertainty about the time of retirement does not affect optimal choices. Nevertheless, there is consumption smoothing since preferences belong to the recursive utility family (Epstein and Zin (1989) and Farmer (1990)), such that risk neutrality coexists with a positive elasticity of intertemporal substitution.

For $z = \{w, r\}$, the household j selects consumption and asset holdings to maximise

$$V_t^{jz} = \left\{ (C^{jz})^{\upsilon} + \beta^z_{t,t+1} (E_t[V^j_{t+1} \mid z]^{\upsilon}) \right\}^{1/\upsilon} \tag{2}$$

$$\text{subject to } C_t^{jz} + FA_{t+1}^{jz} = R_t^z FA_t^{jz} + W_t^j I^z + d_t^z \tag{3}$$

where $\beta^z_{t,t+1}$ is the discount factor, which is equal to β for workers and $\beta\omega^r_{t,t+1}$ for retirees, R_t^z is the return on assets, which is equal to the real rate R_t for workers and $R_t/\omega^r_{t-1,t}$ for retirees, W_t^j is the real wage for worker j, and I^z is an indicator function that takes the value of one when $z = w$ and zero otherwise. Thus, we assume that retirees do not work and each worker's labour supply is fixed. FA_t^{jz} and d_t^z denote, respectively, the assets acquired and the dividends from the financial intermediary.

A fixed share Sw_{RD} of new workers $\omega^y_{t,t+1} N_t^w$ is employed in $R\&D$ and the remaining $(1 - Sw_{RD})$ supplies labour to intermediate firms. At every period a fraction $drop_{RD}$ of R&D workers, who do not retire, is no longer able to work in this sector, and, thus, start supplying labour to firms in the production sector.[2] Hence, employment in the R&D and labour-intensive sectors are, respectively:

$$N_{t+1}^{wRD} = \omega^y_{t,t+1} N_t^w Sw_{RD} + (1 - drop_{RD})\omega^w N_t^{wRD}, \text{ and} \tag{4}$$

$$N_{t+1}^{wL} = \omega^y_{t,t+1} N_t^w (1 - Sw_{RD}) + \omega^w N_t^{wL} + (drop_{RD})\omega^w N_t^{wRD}. \tag{5}$$

with W_t^{RD} and W_t denoting respectively wages in the R&D and in the production sectors.

The resulting consumption functions of workers and retirees are:[3]

$$C_{w,t} = \varsigma_t [R_t FA_{w,t} + H_{w,t} + D_{w,t}], \text{ and } C_{r,t} = \varepsilon_t \varsigma_t [R_t FA_{r,t} + D_{r,t}]. \tag{6}$$

[2] This is to reflect the fact that innovation productivity peaks during the first 10-15 years of a workers life (see Jones (2010)).

[3] All equilibrium conditions are described in the Appendix.

where, $H_{w,t}$ is the present value of human capital, $D_{z,t}$ is the present value of dividends for $z = \{w, r\}$. ς_t denotes the marginal propensity of consumption of workers and $\varepsilon_t \varsigma_t$ the one for retirees (where $\varepsilon_t > 1$). As marginal propensities to consume are different across ages, changes in the distribution of asset holdings as well as in the population age structure, affect aggregate demand. Moreover, the marginal propensities to consume are functions of fertility (ω^y), longevity (ω^r) and time of retirement (ω^w). Thus, through changes in savings, demographics affect the equilibrium interest rate.

Finally, labour supply is a function of fertility ($\omega^y_{t,t+1}$), the share of new workers entering the R&D sector (Sw_{RD}) and the retirement age (ω^w). In our benchmark specification changes in labour supply will solely be a function of fertility. In different extensions we analyse the impact of variations in labour supply due to changes in the share of workers entering the *R&D* sector and in the retirement age, and due to improvements in skill.

2.2 Production

A final producer combines intermediate goods (which are substitutes) according to

$$y_t = \left[\int_0^{Z_t} y_{i,t}^{\frac{\psi-1}{\psi}} di \right]^{\frac{\psi}{\psi-1}}, \text{where } \psi > 1. \tag{7}$$

Each firm $i \in [0, Z_t]$ produces a differentiated good that is sold to the final producer. A subset $i \in A_t$ can be produced using inputs ($\Upsilon_{i,t}$), rented capital ($K_{i,t}$) and robots ($M_{i,t}$) or labour ($L_{i,t}$). Robots are more productive than labour, and, thus, if a good can be produced by robots, the firm selects to do so. For the remaining goods, $i \in Z_t \setminus A_t$, production can only be done using inputs ($\Upsilon_{i,t}$), rented capital ($K_{i,t}$) and labour ($L_{i,t}$). Therefore,

$$\begin{cases} y_{i,t} = ((K_{i,t})^{\alpha}(\theta_t M_{i,t})^{1-\alpha})^{1-\gamma_I}\Upsilon_{i,t}^{\gamma_I} \text{ for } i \in A_t \text{ (the automated sector)} \\ y_{i,t} = ((K_{i,t})^{\alpha}(L_{i,t})^{1-\alpha})^{1-\gamma_I}\Upsilon_{i,t}^{\gamma_I} \text{ for } i \in Z_t \setminus A_t \text{ (the labour-intensive sector).} \end{cases} \tag{8}$$

θ_t denotes the relative productivity of robots versus labour, and $\alpha, \gamma_I \in (0,1)$ control the capital and input shares. The rental rate of capital, net of depreciation, is denoted $r_{k,t} - \delta$ and the relative price of robots, q_t. We initially set θ_t to be time invariant, $\theta_t = \bar{\theta}$.[4]

Since capital and labour are complements, capital biased technological progress may increase labour productivity and wages. On the contrary, robot biased technological progress (automation) substitutes workers, hence, it displaces labour and decreases wages. In our view, this is a crucial difference between automation and previous technological revolutions, which introduced new forms of capital that were complementary to (some) labour inputs instead of a new form of capital (robots) that replaces labour in the production process.

In this framework the ratio of profits ($\Pi_{i,t}$) in the automated and labour-intensive sector is given by

$$\frac{\Pi_{i\in A_t,t}}{\Pi_{i\in Z_t\setminus A_t,t}} = \left(\frac{W_t}{q_t/\theta_t}\right)^{(1-\alpha)(1-\gamma_I)(\psi-1)}. \tag{9}$$

The higher the real wage is relative to the price of robots, the larger is the profit differential in favour of the automated sector.

Under this production structure, economic growth is the result of i) the rise in the number of intermediate goods (Z_t grows), and ii) the introduction of robots that displace labour. These two forms of technological change come from R&D investments, described next.

2.3 Research and Development

R&D consists of the creation of goods (innovation), and the development of procedures that allow robots to be introduced in the production process (automation).

Let Z_t^p be the stock of goods for innovator p, who at each period spends S_t^p and employs labour ($L_{I,p,t}$) to invent $\varphi_t(S_{p,t})^{\kappa_{RD}}(L_{I,p,t})^{\kappa_L}$ new goods, where $\kappa_{RD}, \kappa_L \in$

[4] To ensure robots are more productive than labour we set $\bar{\theta}$ such that $W_t > q_t/\theta_t$ for all t. In an extension we allow productivity of robots relative to labour to increase as automation rises (θ_t is a function of A_t).

$[0,1]$ represents the relative weight of investment and labour for R&D. Thus, the stock of goods Z^p_{t+1} is

$$Z^p_{t+1} = \varphi_t (S_{p,t})^{\kappa_{RD}} (L_{I,p,t})^{\kappa_L} + \phi Z^p_t, \tag{10}$$

where ϕ is the intermediate good survival rate. Following Comin and Gertler (2006) and Aksoy et al. (2019) we set $\varphi_t \equiv \chi Z_t [\tilde{\Psi}^{\rho} (S_t)^{\kappa_{RD}-\rho} (N_t)^{\kappa_L}]^{-1}$. Since R&D productivity depends on the aggregate stock of goods (Z_t), there is a positive spillover as in Romer (1990). There is also a congestion externality via the factor $[\tilde{\Psi}^{\rho} (S_t)^{\kappa_{RD}-\rho} (N_t)^{\kappa_L}]$.[5] The R&D elasticity of new technology creation in equilibrium is ρ.

Innovators borrow S^p_t from the financial intermediary. Upon creation of a new good, they receive a fraction ϑ of the profits of the intermediate firm that produces it. Thus, the value of an invented good is $J_t = \vartheta \Pi_{i,t} + (R_{t+1})^{-1} \phi E_t J_{t+1}$, for $i \in Z_t \setminus A_t$, where $\Pi_{i,t}$ for $i \in Z_t \setminus A_t$ is the profit of the intermediate good firm. Innovator p will then invest $IS_{p,t} = (S_{p,t})^{\kappa_{RD}} (L_{I,p,t})^{\kappa_L}$ until the marginal cost equates the expected gain. Defining $\tau_{S,t}$ as the shadow price of $IS_{p,t}$, then $S_{p,t} = IS_{p,t} \tau_{S,t} \kappa_{RD}$, $L_{I,p,t} W_{RD,t} = IS_{p,t} \tau_{S,t} \kappa_L$ and $\phi E[J_{t+1}] = \frac{R_{t+1} \tau_{S,t}}{\varphi_t}$. Using (10),

$$S_t = \kappa_{RD} R^{-1}_{t+1} \phi E_t J_{t+1} (Z_{t+1} - \phi Z_t). \tag{11}$$

Automation investors (q) spend $\Xi_{q,t}$ and hire $L_{A,q,t}$ to transform a Z^q_t good into a A^q_t good, which then becomes part of the set of goods that can be produced by robots. This conversion process succeeds with probability $\lambda_t = \lambda \left(\frac{(Z^q_t - A^q_t)^{\kappa_{RD}+\kappa_L}}{\tilde{\Psi}^{\kappa_{RD}}_t N^{\kappa_L}_t} \Xi_{q,A,t} \right)$, with $\lambda'(\cdot) > 0$ and $\Xi_{q,A,t} = (\Xi_{q,t})^{\kappa_{RD}} (L_{A,q,t})^{\kappa_L}$. If unsuccessful, the good remains in the labour-intensive sector. Once automation is successful the investor earns a fraction ϑ of the profits of the robot-intensive intermediate producer. Thus, the

[5] N_t is included in the congestion factor since, as discussed in Jones (1995) and more recently Bloom et al. (2017), models of endogenous growth where growing employment in R&D (due to population growth) generates faster steady state output growth are inconsistent with the data.

value of an automated good is $V_t = \vartheta \Pi_{i,t} + (R_{t+1})^{-1} \phi E_t V_{t+1}$, for $i \in A_t$. Automation investors set investment and labour employed to maximise expected gains such that

$$\Xi_{q,t} = \epsilon_\lambda \lambda_t R_t^{-1} \phi E_t [V_{t+1} - J_{t+1}], \text{ and } L_{A,q,t} W_{RD,t} = \Xi_{q,t} \frac{\kappa_L}{\kappa_{RD}}. \tag{12}$$

where ϵ_λ is a function of the elasticity of λ_t to changes in its input.[6]

Since the stock of labour intensive goods at t for which automation is feasible is $(Z_t^q - A_t^q)$, the flow of the stock of automated goods is given by

$$A_{t+1}^q = \lambda_t \phi (Z_t^q - A_t^q) + \phi A_t^q. \tag{13}$$

From (11) and (12) the ratio of aggregate investment in innovation and automation is[7]

$$\frac{S_t}{\Xi_t} = \frac{E_t[\phi \kappa_{RD} (g_{Z,t+1} - \phi) J_{t+1}]}{E_t[\epsilon_\lambda (a_{z,t+1} g_{Z,t+1} - a_{z,t} \phi)(V_{t+1} - J_{t+1})]}, \text{ where, } g_{t+1}^Z = \frac{Z_{t+1}}{Z_t}, a_{z,t} = \frac{A_t}{Z_t} \tag{14}$$

This R&D set-up features trade-offs between automation and innovation. First, as the expected profits of firms in the automated sector increase relative to expected profits in the labour-intensive sector (thus, the ratio $E_t[J_{t+1}/(V_{t+1} - J_{t+1})]$ falls), automation investment increases relative to innovation investment. As the differential of profits is ultimately a function of factor prices, automation and innovation respond to changes in wages and the price of robots. Second, investment in automation is a negative function of $a_{z,t}$, the current level of the ratio of total number of automated goods to the total number of goods. As innovation decreases and this ratio increases, the pace of automation slows down. In this respect, automation is a subsidiary activity of innovation; without innovation, automation cannot progress indefinitely. Finally, innovators and automation investors compete for a limited supply of labour and loans to fund their activities.

[6] We assume that the elasticity of λ_t to changes in its input is constant and not greater than one, then we define $\epsilon_\lambda \equiv \frac{\lambda'}{\lambda_t} \frac{(Z_t^q - A_t^q)^{\kappa_{RD} + \kappa_L} \Xi_{q,A,t}}{\tilde{\Psi}_t^{\kappa_{RD}} N_t^{\kappa_L}} \frac{1}{\kappa_{RD}}$. See the Appendix for details.

[7] Aggregate investment in automation is $\Xi_{q,t}(Z_t - A_t)$ and since innovators are of measure 1, $S_{p,t} = S_t$.

2.4 Robots Production

Robot producers invest Ω_t final goods to produce $M_t = \varrho\Omega_t^\eta$ robots according to

$$\max_{\Omega,t} \Pi_{\Omega,t} = q_t M_t - \Omega_t \quad s.t. \quad M_t = \varrho\Omega_t^\eta. \tag{15}$$

where q_t is the relative price charged to intermediate good producers for each robot.

2.5 Financial Intermediary

A zero expected profit financial intermediary allocates assets among the households, and the production and R&D sectors and provides annuities to retirees. It sells assets to the households (FA_t^w, FA_t^r), owns capital (K_t) and rents it to firms and lends funds (B_{t+1}) to innovators and automation investors to finance their expenditures (S_t and Ξ_t, respectively). Finally, it owns R&D plants, robots and good producers and receives their dividends.

2.6 Market Clearing and Equilibrium

The market clearing conditions are: Final Good: $y_t = C_{w,t} + C_{r,t} + \int_0^{Z_t} \Upsilon_{i,t} di + \Omega_t + I_t$, Asset Flow Condition: $K_{t+1} = (1-\delta)K_t + I_t$, Credit Markets: $B_{t+1} = S_t + \Xi_t$, Capital Markets: $K_t = \int_0^{Z_t} K_{i,t} di$, Inputs: $\Upsilon_t = \int_0^{Z_t} \Upsilon_{i,t} di$, Robots Markets: $M_t = \int_0^{A_t} M_{i,t} di$, and Labour Markets: $N_t^{wR} = \int_q L_{A,q,t} di + \int_p L_{i,p,t} di$, and $N_t^{wL} = \int_0^{Z_t \setminus A_t} L_{i,t} di$.

The equilibrium consists of tuples of endogenous predetermined variables $\{FA_{t+1}^z$, K_{t+1}, A_{t+1}, Z_{t+1}, $B_{t+1}\}$ and of endogenous variables $\{C_t^z$, H_t^w, d_t^z, D_t^z, N_t^{wR}, N_t^{wL}, y_t, $y_{i,t}$, $y_{j,t}$, M_t, Ω_t, S_t, Ξ_t, $L_{A,t}$, S_t, $L_{I,t}$, V_t, J_t, λ_t, Π_t^i, Π_t^j, C_t, r_t^k, R_t, Π_t^{RD}, Π_t^A, W_t, $W_{RD,t}$, $P_{i,t}$, $P_{j,t}$, q_t, ε_t, $\varsigma_t\}$ for $z = \{w, r\}$, $i \in A_t$, $j \in Z_t \setminus A_t$ such that:

a. Workers and retirees maximise utility subject to their budget constraints; *b*. Intermediate and final firms maximise profits; *c*. Profits are also maximised in innovation, automation, and robot production ; *d*. The financial intermediary selects assets to maximise profits, and its profits are shared amongst retirees and workers according to their share of assets; and *e*. Consumption goods, capital, labour, robots, and asset markets clear.

3 Balanced Growth and Comparative Analysis

We define a BGP as an equilibrium in which factor shares, $(r_t^k + \delta)K_t/y_t$, W_tL_t/y_t and q_tM_t/y_t, and the interest rate, R_t, are constant, and consumption, C_t, and capital stock, K_t, grow at the same rate as output, Y_t.

First, to ensure that investment in innovation does not diverge, we use the current value of automated goods as a scaling factor, so that $\tilde{\Psi}_t \equiv V_tA_t$.[8] Comin and Gertler (2006), in a similar model in which the price of capital is determined at time t, assume that $\tilde{\Psi}_t$ equals the value of the stock of capital, and, hence, $\tilde{\Psi}_t$ fluctuates accordingly. Since in our model there is only one final good, the price of capital and the value of the capital stock are constant at t, which invalidates the choice of the value of the stock of capital as a scaling factor.

Second, in models with factor-augmenting technological progress, substitutability between factors of production may prevent the economy from reaching a BGP (see Acemoğlu (2003)). As such, automation may generate an imbalance between the labour and robot income shares. One option is to rely on wage adjustments to correct for such imbalances, while the effective labour supply is allowed to change to ensure that the capital output ratio is constant, as in Acemoğlu and Restrepo (2018b), where effective labour supply responds to wages and the return on capital ($L^s(W/RK)$). In such a framework, there is flexibility for wages to adjust to equilibrate the incentive to automate with the incentive to create new tasks. However, following this approach in our model would introduce a mechanism that directly offsets the demographic changes that alter labour supply, which is the main objective of our analysis. In our setting, each agent is endowed with a normalised one unit of effective labour supply (skill). In such an environment with a deterministic labour supply responding to demographic changes, wages must vary to ensure the capital output ratio is constant on a BGP. Thus, we need an alternative mechanism to ensure output shares of labour-intensive and automated sectors do not diverge. The

[8] We also verify the robustness of our results by setting $\tilde{\Psi}_t \equiv y_t$. Transition paths between BGPs are more persistent but the results are qualitatively similar.

introduction of a robot producing sector to determine the price of robots plays this role, as shown in Proposition 1.

Proposition 1. *Let* $g_{q,t} \equiv \frac{q_t}{q_{t-1}}$ *be the growth rate of the relative price of robots,* $g_{wg,t} \equiv \frac{W_t}{W_{t-1}}$ *be the growth rate of the real wage in production,* $g_t \equiv \frac{y_t}{y_{t-1}}$*, the growth rate of output,* $g_{n,t}$*, the population growth and* $g_t^Z \equiv \frac{Z_t}{Z_{t-1}}$*,* $g_t^A \equiv \frac{A_t}{A_{t-1}}$*, the growth rates of varieties* Z_t *and* A_t*, respectively. Then, with robots production given by* $M_t = \varrho \Omega_t^\eta$*, for*

$$\begin{cases} \eta < 1, & \text{there exists a } BGP \text{ where } (g_t)^{\eta-1} g_{q,t} = 1,\ g_{q,t} = \frac{g_t}{g_{n,t}} > 1 \\ & \text{and } g_t^Z = g_t^A = (g_t)^{(1-\eta)(\psi-1)(1-\alpha)(1-\gamma_I)} > 1 \\ \eta = 1, & \text{there exists a } BGP \text{ where } g_t = g_{n,t} \text{ and } g_t^Z = g_t^A = 1 \end{cases}$$

Proof

As the final good aggregates all varieties, with a constant elasticity of substitution (see equation (7)), the relative prices $(p_{i,t}/p_t)$, for automated goods $(i \in A_t)$, and $(p_{j,t}/p_t)$, for labour-intensive goods $(j \in Z_t \setminus A_t)$ must grow at the same rate in a *BGP* to ensure output shares in the automated sector $(\int_{i \in A_t} p_{i,t} y_{i,t} di / p_t y_t)$ and labour-intensive sector $(\int_{j \in Z_t \setminus A_t} p_{j,t} y_{j,t} dj / p_t y_t)$ do not diverge. Given the production functions for each sector (8), the relative prices are given by

$$\frac{p_{i,t}}{p_t} = \frac{\psi - 1}{\psi} \frac{(r_t^k + \delta)^{\alpha(1-\gamma_I)} q_t^{(1-\alpha)(1-\gamma_I)}}{(\alpha(1-\gamma_I))^{\alpha(1-\gamma_I)} \gamma_I^{\gamma_I} ((1-\alpha)(1-\gamma_I))^{(1-\alpha)(1-\gamma_I)}} \tag{16}$$

$$\frac{p_{j,t}}{p_t} = \frac{\psi}{\psi - 1} \frac{(r_t^k + \delta)^{\alpha(1-\gamma_I)} (W_t)^{(1-\alpha)(1-\gamma_I)}}{(\alpha(1-\gamma_I))^{\alpha(1-\gamma_I)} \gamma_I^{\gamma_I} ((1-\alpha)(1-\gamma_I))^{(1-\alpha)(1-\gamma_I)}} \tag{17}$$

Therefore, on a *BGP* with constant interest rates, the growth rate of the price of robots and of real wages must be equal $(g_{q,t} = g_{wg,t})$.

On a *BGP*, the labour income share $\left(ls_t = \frac{W_t L_t}{y_t}\right)$ is constant. We assume each agent is endowed with a (normalised) one unit of effective labour supply and thus $g_{L,t} = L_t / L_{t-1} = N_t^{wL} / N_{t-1}^{wL} = g_{n,t}$. Therefore, $g_{q,t} = g_{wg,t} = \left(\frac{g_t}{g_{n,t}}\right)$

Using the production function for robots and that inputs in robots production over output Ω_t / y_t and robots income share $\left(rs_t = \frac{q_t M_t}{y_t}\right)$ are constant on a *BGP*, the growth rate of the price of robots is such that $g_t^{\eta-1} g_{q,t} = 1$

Combining these two conditions gives: $g_t^\eta = g_{n,t}$.

Finally, from the final good production function we obtain that demand for automated goods is given by $y_{i,t} = \left(\frac{p_{i,t}}{p_t}\right)^{-\psi} y_t$. As all automated firms are identical, summing up across i, and as $y_{m,t} \equiv \int_{i \in A_t} p_{i,t} y_{i,t} di / p_t y_t$ and interest rate are constant, we obtain that on a BGP

$$\begin{aligned} g_t^A &= \left(\frac{y_{m,t}}{y_{m,t-1}}\right)^{(1+(1-\eta)(\psi-1)(1-\alpha)(1-\gamma_I))} \left(\frac{r_t^k - \delta}{r_{t-1}^k - \delta}\right)^{\alpha(1-\gamma_I)(\psi-1)} (g_t)^{(1-\eta)(\psi-1)(1-\alpha)(1-\gamma_I)} \\ &= (g_t)^{(1-\eta)(\psi-1)(1-\alpha)(1-\gamma_I)} \end{aligned}$$

If $\eta < 1$ then as $g_{n,t} > 1$, it follows that $g_{q,t} > 1$ and $g_t > g_{n,t}$. Hence, at the equilibrium with constant factor shares, output growth is greater than population growth. If $\eta = 1$ then $g_t^A = 1$ and $g_t = g_{n,t}$. The BGP in this case has no technological progress.

The intuition behind this proposition is as follows. When $\eta < 1$, increases in the stock of produced robots M_t require greater investment. Thus, as the economy grows and robots become more abundant, it becomes relatively harder to transform a complex final good into robots, so that the marginal profitability of robot production falls and the differential in output shares between the labour-intensive and the automated sectors does not increase. As a result, the labour and robot income shares do not diverge in the long run. Therefore, holding wages constant, for robots to become more abundant than labour their relative price with respect to the final good must decrease. Eventually, as robots are extensively used in production to the detriment of labour, such price decreases are no longer feasible, preventing the economy from reaching an equilibrium where only robots are used in production.[9]

Alternatively, were $\eta > 1$ an equilibrium with constant shares would necessarily imply $g_t^A < 1$, the number of goods with automated production would asymptotically approach zero, and $g_t < g_{n,t}$, with the innovation rate being smaller than the good's survival rate. When the economy grows faster than population, costs in

[9] We extend the model to relax the assumption that robots fully depreciate in one period. Transitions between BGPs are affected, but the restriction on η to ensure a BGP exists remains the same: BGP is still characterised by $g_t^{\eta} = g_{n,t}$. Details of this extension are presented in the Appendix.

the labour-intensive sector (relative to the automated sector) increase. With $\eta > 1$ a higher output in the automated sector requires a relatively smaller increase in investment (Ω_t), boosting the relative size of the robots-intensive sector. Hence, to ensure that in equilibrium $\frac{y_{m,t}}{y_{m,t-1}} = 1$, the number of automated goods must shrink, restricting the growth of robots to be consistent with constant factor shares. In this scenario, the price of robots (q_t) asymptotically converges to zero.

In sum, to avoid a labour-free singularity we restrict the efficiency gains in the transformation of final goods into robots. On a BGP the final good embeds an increasing set of intermediate goods, some produced employing the less efficient factor — labour. It is reasonable to assume that robots that are capable of replacing labour in production would also eventually increase in complexity to reflect the level of economic development. Robots production with $\eta < 1$ reproduces such an environment by requiring that as the number of varieties and output increase, the transformation of the final good into robots does not become more efficient. This is similar to Aghion et al. (2017), who introduce a 'bottleneck' to prevent a singularity by restricting the productivity gain of an essential input, sustaining its relative price in terms of the final good.[10]

We now explore the implications of Proposition 1 to comparative analysis across BGPs, restricting the focus to BGPs with technological progress where $\eta < 1$.

Corollary 1. Technological progress on the BGP is given by $g_t^Z = \chi \left(\frac{S_t}{\bar{\Psi}_t}\right)^{\rho} (L_{i,t}/N_t)^{\kappa_L} + \phi = (g_{n,t})^{(1-\eta)(\psi-1)(1-\alpha)(1-\gamma_I)(1/\eta)} = g_t^A$. As $g_{n,t}$ is exogenous, changes in the efficiency in innovation investment χ only affect economic growth in the short term.

As the efficiency of innovation investment (χ) increases, Z grows faster, and both the share of output produced with labour and wages increase. Higher wages lead to a fall in the relative profitability in the labour-intensive sector, increasing the incentives to automate. That diverts resources from innovation and as such, investment in new product creation (S_t) and labour employed in innovation ($L_{i,t}$)

[10] Aghion et al. (2017) discuss different alternatives to prevent singularities. In one of the cases, which they frame as a form of Baumol's cost disease, this is achieved by restricting the productivity gain of some tasks in a production framework where tasks are complementary. Thus, as income/output increases, the relative price of these tasks are sustained.

fall, reducing g_t^Z back to its initial position.[11] Thus, an increase in χ has only a temporary effect on growth. Automation, however, increases in the long run; with the resources diverted towards automation, the economy can sustain a higher degree of automation on a BGP. Therefore, the key driver of this result is the trade-off between innovation and automation. [12]

Corollary 2. Starting from an initial BGP equilibrium with technological progress, where $g_t^\eta = g_{n,t}$ and $\eta < 1$, if population growth falls, then the new BGP is characterised by lower per-capita output growth.

A lower growth of labour supply directly leads to lower output growth as it reduces one of the production's inputs. Moreover, it also reduces the incentives to innovate. This additional mechanism lowers output growth further such that output per capita growth on the new BGP falls. Since on a BGP the growth of factor prices equalise and robots and labour eventually grow at the same pace, the aggregate production function net of intermediate inputs (Υ_t) can be represented as a function of the level of capital/labour ratio or of the level of capital/output ratio as in Uzawa (1961) and Jones and Scrimgeour (2005). Therefore, balanced growth emerges in the presence of total factor productivity gains.[13] In contrast to the neoclassical growth model in Uzawa (1961), here technological growth is endogenous and is a function of the incentive to create new goods (Z). As automation and innovation are matched on a BGP and because of the assumption that newly created goods can only be produced with labour, technological gain is directly linked with the profitability in the labour-intensive sector, and ultimately, the labour market restrictions imposed by population growth end up determining the pace of technological growth and per capita growth. Confirming the insights from Grossman et al. (2017), who look at potential ways to offset the restrictions identified by Uzawa (1961) and Jones and Scrimgeour (2005) on balanced growth in neoclassical growth models, when

[11] The quantitative analysis of this transition, using the calibrated model, is shown in the appendix.

[12] In the model with demographic changes and innovation investment without automation developed in Aksoy et al. (2019) this trade-off is not present and an increase in χ leads to higher growth in the long run.

[13] On a BGP, the capital/robots ratio is a constant linear function of the capital/labour ratio. The assumption of Cobb-Douglas production functions in both labour and robots intensive sectors ensures the existence of a BGP in which total factor technical change can be represented as labour-augmenting.(see Jones and Scrimgeour (2005)).

we relax the assumption of constant labour skill and allow endogenous changes to skill/effective labour supply in section 5.3, output growth is no longer a function of population growth only.

Next we focus on the effects of demographic changes on automation ($a_{z,t} = A_t/Z_t$) and on the labour share (ls_t) on BGPs. Before stating the results, we make two assumptions.[14] First, ($A1$) we impose a upper bound on the level of automation on the BGP by setting $a_z \leqslant \frac{\rho\phi}{\epsilon_\lambda}$. The stock of goods for which automation is feasible is $Z_t - A_t$. As a_z increases, this stock falls. Given the survival rate of varieties ϕ, investment in automation and innovation may then be implausibly high such that a high degree of automation is maintained on a BGP. We therefore limit the initial level of automation to discard equilibria with such implausible levels of R&D investment. Second, ($A2$) we assume $\psi > 1 + \frac{1}{(1-\eta)(1-\alpha)(1-\gamma_I)}$ to restrict the firms' mark-ups such that an increase of one percent in the growth of varieties does not generate a more than 1% increase in output growth (recall that on the BGP, $g_t^Z = g_t^A = (g_t)^{(1-\eta)(\psi-1)(1-\alpha)(1-\gamma_I)}$). If ($A2$) does not hold, an equilibrium with a positive rate of output growth would also imply the number of varieties shrinks with time. Under our calibration ($A1$) restricts the degree of automation such that no more than 75% of the existing varieties are produced in the automated sector, and ($A2$) restricts the intermediate good firms' mark-up to be smaller than 23%.

Proposition 2. *Starting from a BGP, when population growth falls, the economy converges to a new BGP in which, when interest rates are lower, the degree of automation is higher and the labour income share is lower. The changes in the degree of automation and in the labour income share are respectively given by*

$$\frac{da_z}{a_z} = \frac{-\frac{1}{\eta}\left(d_1\frac{\kappa_{RD}}{\kappa_L}\Gamma_2 + \frac{\rho}{\kappa_L}\Gamma_1 d_2\right)\frac{dg_n}{g_n} - \left(d_1\frac{\kappa_{RD}}{\kappa_L} + \frac{\rho}{\kappa_L}d_2\right)\frac{dR}{R}}{c_2 d_1 - d_2 c_1} \tag{18}$$

$$\frac{dls}{ls} = (1-y_L)\frac{\frac{1}{\eta}\left(c_1\frac{\kappa_{RD}}{\kappa_L}\Gamma_2 + \frac{\rho}{\kappa_L}\Gamma_1 c_2\right)\frac{dg_n}{g_n} + \left(c_1\frac{\kappa_{RD}}{\kappa_L} + \frac{\rho}{\kappa_L}c_2\right)\frac{dR}{R}}{c_2 d_1 - d_2 c_1} \tag{19}$$

[14] Both assumptions are sufficient but not necessary for propositions 2 and 3 to hold. Proofs of both propositions are shown in the Appendix.

where $y_L \equiv \left(\int_{i\in Z\setminus A} p_i y_i di\right)/py$ *is the share of output produced using labour, and* c_1, c_2, d_1, $d_2 > 0$ *are functions of the initial* BGP. *If* $A1$ *holds then* $c_2d_1 - d_2c_1 > 0$ *and if* $A2$ *holds then parameters* Γ_1, $\Gamma_2 > 0$.

Proposition 3. *Starting from a* BGP, *the changes in the degree of automation and in the labour income share due to population ageing are respectively given by*

$$\frac{da_z}{a_z} = \frac{\frac{dRDpop}{RDpop}(d_1+d_2) - \left(d_1\frac{\kappa_{RD}}{\kappa_L} + \frac{\rho}{\kappa_L}d_2\right)\frac{dR}{R}}{c_2d_1 - d_2c_1} \tag{20}$$

$$\frac{dls}{ls} = (1-y_L)\frac{-\left(\frac{dRDpop}{RDpop}(c_1+c_2) - \left(c_1\frac{\kappa_{RD}}{\kappa_L} + \frac{\rho}{\kappa_L}c_2\right)\frac{dR}{R}\right)}{c_2d_1 - d_2c_1} \tag{21}$$

where $RDpop = \frac{N^{wRD}}{N}$ *is the share of R&D workers in the population and* c_1, c_2, d_1, $d_2 > 0$. *Thus, the lower the weight of labour on R&D activity* (κ_L) *is, the more likely it is that the BGP of a more aged economy, in which interest rates are smaller, is characterised by a higher degree of automation and a lower labour income share.*

In both cases, automation and the labour income share on the new $BGPs$ are directly affected, due to labour supply effects, and indirectly, due to changes in the equilibrium interest rate. A fall in the interest rate boosts greater investments in innovation and automation. However, as labour supply growth falls, relative costs in the labour-intensive sector increase, giving incentives for R&D investment to be diverted to automation, which leads to a decrease in the labour share. In the case of ageing, the direct effect of labour supply lead to a fall of both innovation and automation.

We find that the positive productivity effects of the increase in automation are not able to fully compensate for the effects of the fall in labour supply growth once the economy converges to the new BGP. Acemoğlu and Restrepo (2018b) highlight a long-run productivity effect occurring when the pace of automation increases. As the economy converges back to a new BGP, the return on capital (interest rate), which initially increased together with automation, must fall back. As a result, capital accumulates with the gains accruing to the relatively inelastic factor, namely, labour. In our model, capital is complementary to labour and robots. Thus, although capital accumulation also occurs after an ageing shock, the main driver is the saving effect due to the fall in marginal propensity to consume of households approaching retirement. In our setting, automation increases while interest rate falls and capital accumulation increases, in line with empirical evidence.[15]

4 Demographic Transition and Growth in Europe and in the US

In this section we focus on transition dynamics, analysing the consequences of demographic changes over the next decades predicted for the US and for Core Europe (defined as the aggregation of Germany, France, Italy and Spain). Before presenting the numerical results we briefly describe the choices of the parameter values.

4.1 Calibration

One period of the model corresponds to one year. Workers are defined as 20 to 65 years old individuals and retirees are defined as individuals above 65 years old. The parameters controlling the law of motion of population are calibrated to match the average share of workers and retirees in total population in 1993 in the US, and the number of working years the individual may live before retiring (45 years). This procedure delivers a birth rate of $\omega^y = 0.0265$, a probability of retirement of $1 - \omega^w = 0.022$, and a death probability of $1 - \omega^r = 0.07$. The same procedure is applied using data from Europe.

The share of workers in innovation (Sw_{RD}) is set to match the share of R&D workers in US population, and $drop_{RD}$ is set to make the average age of R&D workers to be 40 (slightly lower than the average age of employed scientists reported in the Survey of Doctorate Recipients (SDR) of the National Science Foundation - 2013).

For the R&D parameters, we closely follow Comin and Gertler (2006). Obsolescence (ϕ) and productivity in innovation (χ) are set so that growth of GDP per

[15] Adding an explicit capital deepening effect as a result of automation in the long run as in Acemoğlu and Restrepo (2018b) could further offset the negative effect of lower labour supply growth. Berg et al. (2018), employing a neoclassical growth model with robots, argue automation leads to higher growth and inequality. Technology in their model is exogenous thus the innovation and automation trade-off is not present.

working age person is 0.016 (as in the US from 1970 onwards) and the share of innovation expenditures in total GDP is 0.012. The mark-up for intermediate goods is 15%. The elasticity of intermediate goods with respect to $R\&D$ (ρ) is 0.9. The rate of automation is set to $\lambda = 0.1$. The elasticity of this rate to increasing intensity (ϵ_λ) is set to 1. Finally we set $\kappa_{RD} = 1$.

Regarding the link between demographics and innovation, which depends on the elasticity of invention to employed workers in $R\&D$ (κ_L), we follow Aksoy et al. (2019) who, reflecting the productivity of ideas of agents of different ages in Jones (2010), set $\kappa_L = 0.5$.

Finally we set the macro parameters in line with Comin and Gertler (2006). The discount factor $\beta = 0.96$; the capital share $\alpha = 0.33$; the yearly depreciation rate $\delta = 0.08$; and the share of intermediate goods $\gamma_I = 0.5$. Following Gertler (1999) we set the intertemporal elasticity of substitution $(1/(1-\upsilon)) = 0.25$. Given output and population growth and $(g_t)^{\eta-1}(g_t/g_{n,t}) = 1$, we obtain $\eta = 0.15$.[16]

4.2 Quantitative Results

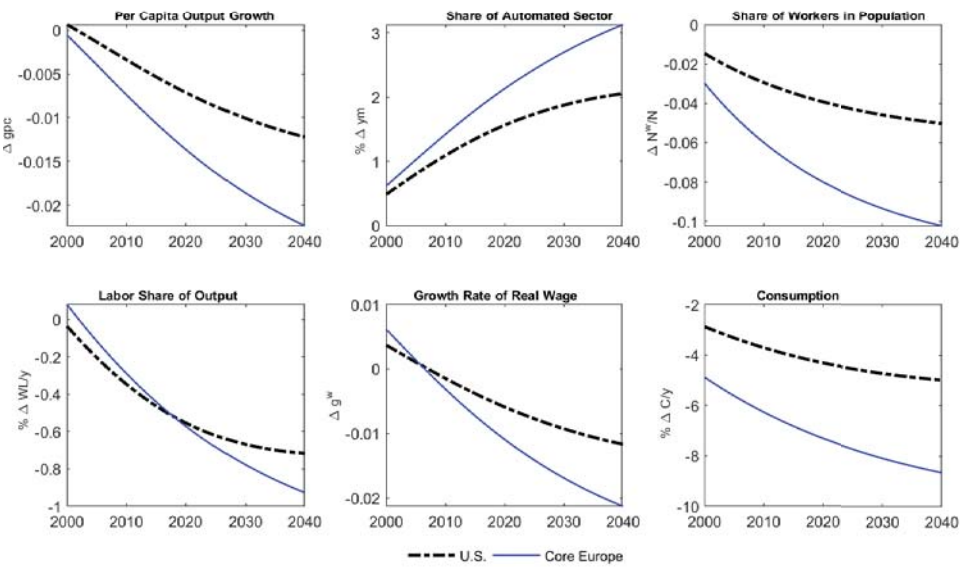
The initial steady state is set with a stationary population distribution, represented by fertility and survival probability rates, that match the population shares of workers (age 20-65) and retirees (age above 65) in 1993 for the US and Core Europe. We then use the same population shares from the United Nations (2016) projection data for 2055 for each region, to obtain the new fertility and survival probability rates that describe the final stationary population distribution (all demographic variables are shown in the Appendix). The demographic transition is then defined as the path of these two demographic parameters from 1993 until 2055. From 2055 onwards demographic variables no longer change and the economy slowly converges to a new BGP. As our interest is on the medium-run effects of this demographic transition we focus on the variation of economic variables produced by the demo-

[16] Tables listing all parameter values are in the Appendix.

graphic changes from 2000 up to 2040 (Figure 1), discarding the first seven years to decrease the dependence of the simulation to the initial steady state.[17]

When mortality decreases, savings increase and the interest rate falls, as in Aksoy et al. (2019) and Eggertsson et al. (2019). Hence, there are more resources for innovation, automation, and capital investments. As fertility decreases, the new cohort of workers entering in the labour market also decreases, pushing wages up. Higher wages depress the profitability of the labour-intensive sector, boosting automation. As robots are more productive than labour, productivity rises. Both of these mechanisms lead to an increase in output growth.[18] However, as the profits of the labour-intensive sector fall, the investment in innovation decreases. Moreover, a drop in fertility implies that the pool of workers available for innovation decreases.[19]

Figure 1: Demographic Transition: United States and Europe



Note: The figure plots the effects of the projected demographic changes in each region. For Per Capita Output growth = $(y_t/y_{t-1})(N_{t-1}/N_t)$, Share of Workers in Population - N_t^w/N_t and Growth Rate of real wage - W_t/W_{t-1} we show the Change relative to the initial BGP. For Share of Automated Sector - $y_{m,t}$, Labour Share of Output W_tL_t/y_t, R_t - Real Interest Rate and Consumption - C_t/y_t we show the percentage change relative to the initial BGP. For the price of robots we show the ratio of price of robots during the transition and on the BGP - q_t/q_{BGP}.

[17]Our results are not dependent on the years selected to calibrate the demographic variables. See the Appendix for alternative simulations.

[18]The change in the growth rate from 1993 till 2000 is positive, we show results from 2000 in figure 1.

[19]Using a cross-section data on patents and demographics, Acemoğlu and Restrepo (2018a) document an increase in the number of patents related to robots and a decrease in those related to computer, software, nanotechnology and pharmaceutics due to demographics, supporting the trade-off between innovation and automation present in our model.

As the growth of new goods Z_t decreases, overall growth is reduced, hampering the pace of automation in the future and, ultimately, delivering lower per capita growth. Thus, the initial effect of higher savings and lower interest rates wears off, and the reduction in invention of new differentiated goods outweighs the productivity gains from automation.[20]

The share of workers in Core Europe decreases faster (since fertility is considerably lower in Europe), boosting automation more than in the US. This result is consistent with the data. From 2000 to 2015, automation, measured as the stock of robots by thousand employees, increased from 1.55 to 2.7 in the four core European countries, with an increase from 2.28 to 4.24 in Germany, 0.79 to 1.6 in Spain, 0.81 to 1.17 in France, and from 1.7 to 2.5 in Italy, while in the US it increased from 0.64 to 1.55 (International Federation of Robotics (2017)).

Graetz and Michaels (2018) show that during1990-2005 the price of robots fell by roughly 20% on average across developed countries. In our set-up, on a BGP the price of robots q must be increasing, ensuring it does not diverge from the growing real wages. However, during the demographic transition, as the degree of automation increases, the growth of the price of robots falls. Figure 1 depicts the price of robots, q, relative to its BGP path. By 2030, q would be 10% lower due to demographic changes. A more substantial fall would require the framework to account also for technological progress that increases the efficiency in robot production (see section 5.2).

Despite the initial increase in wages, as the economy becomes more automated, eventually the labour share of income decreases by around 0.6 percent from 2000 to 2020, due to both a fall in wages and employment. Karabarbounis and Neiman (2014) show that from 2000 to 2015 the global labour income share fell around 4 percent (roughly from 0.615 to 0.59). The labour share has different drivers: price

[20]Gordon (2012) includes demographic changes as one of the "headwinds" preventing economic growth. Aksoy et al. (2019) estimate that from 2000 till 2015, population changes have lead to a reduction in output growth in the US of around 0.5 percentage points. The link between demographics and innovation is behind this negative effect. In a similar projection exercise, they report that demographic changes in the next decades lead to a more sizeable reduction in output than the results reported here. The key distinction of our analysis is the inclusion of automation, which has a positive offsetting effect on growth. Nonetheless, both studies give support to the "headwind" effect of demographics on growth.

of capital, changes in goods and labour market structures, and automation, which we show are influenced by demographic changes.[21]

5 Extensions

In an attempt to account for different ways in which labour markets and technological progress may evolve, we start this section by modifying labour markets and the integration of robots in economic activity. We then look at a case where the price of robots fall substantially during the transition and finally consider endogenous changes in labour skills.

5.1 Alternative labour market and robot configurations

We considering four extensions. The first two focus on labour market configurations. First, we allow new workers with idiosyncratic inherited talent to select to which sector they will supply labour (Labour Choice). Once this decision is taken, workers drop from the R&D sector and cannot re-join during their working lives as in the benchmark case. Thus, the share of new workers that join the R&D sector, Sw_{RD}, is a function of the wage differential between the R&D and the production sectors (W_t^{RD}/W_t). Second, we analyse the effects of delaying the retirement age (Late Retirement). We alter the retirement age to keep the ratio between the durations of working life and of retirement approximately constant.[22]

As for the role of robots in production and innovation, we first consider that robots could also be used to innovate and automate (Robots in R&D), so that as the economy becomes more automated ($a_{z,t}$ increases), robots replace a larger share of labour in R&D.[23] This specification resembles the artificial intelligence model of Aghion et al. (2017), but restricting efficiency gains in robots production to ensure the economy converges to a BGP. Second, we also consider the possibility (Robots

[21] Bergholt et al. (2019) also provide evidence that automation, or technological progress that leads to the substitution of labour by capital, may be behind the fall in labour income shares.

[22] Details on the extensions are in the Appendix. Under the UN projections, in the US life expectancy will increase by 12,7 years from 1993 to 2055. Thus, we simulate the effects of rising retirement age by 8 years, roughly two thirds of the increase in life expectancy.

[23] In this case, investment in innovation and automation are now, respectively, $IS_t = (S_{p,t})^{\kappa_{RD}}((1-a_{z,t})L_{I,t}^{\xi_{LM}} + a_{z,t}M_{I,t}^{\xi_{LM}})^{\kappa_L/\xi_{LM}}$ and $\Xi_{A,t} = (\Xi_{q,t})^{\kappa_{RD}}((1-a_{z,t})L_{A,t}^{\xi_{LM}} + a_{z,t}M_{A,t}^{\xi_{LM}})^{\kappa_L/\xi_{LM}}$, where $M_{I,t}$ and $M_{A,t}$ are robots used in $R\&D$, produced by a similar robot production sector as in the benchmark model, and ξ_{LM} is the elasticity of substitution of robots and labour.

Productivity) that the relative productivity of robots, instead of being constant, rises as the automated sector grows ($\theta_t = \bar{\theta} A_t^{\mu}$, $\mu = 0.05$).

We first focus on the long-run output growth comparison across extensions with the following proposition (the proof is shown in the Appendix).

Proposition 4. *After the demographic transition, the long-run effect on output growth is the same in all four extensions as in the benchmark model.*

In the three extensions, Labour Choice, Late Retirement and Robots in R&D the BGP, as in the benchmark case, is characterised by $g_t = g_{n,t}^{\frac{1}{\eta}}$*. The conditions that determine relative prices, labour supply growth and the price of robots in the robots production sector are unchanged. Across these cases,* η *is the same as in the benchmark model.*

In the case the relative productivity of robots rises as the automated sector grows (Robots Productivity) the BGP is characterised by $g_t = g_{n,t}^{\frac{1+\mu(\psi-1)(1-\alpha)(1-\gamma_I)}{\hat{\eta}+\mu(\psi-1)(1-\alpha)(1-\gamma_I)}}$*, given that the conditions that determine relative prices are altered. As the initial BGP in this extension has the same level of output and population growth as in the benchmark economy,* $\hat{\eta}$ *is set such that* $\eta_{Benchmark} = \frac{1+\mu(\psi-1)(1-\alpha)(1-\gamma_I)}{\hat{\eta}+\mu(\psi-1)(1-\alpha)(1-\gamma_I)}$*, thus the effect of changes in* $g_{n,t}$ *on* g_t *remains the same.*[24]

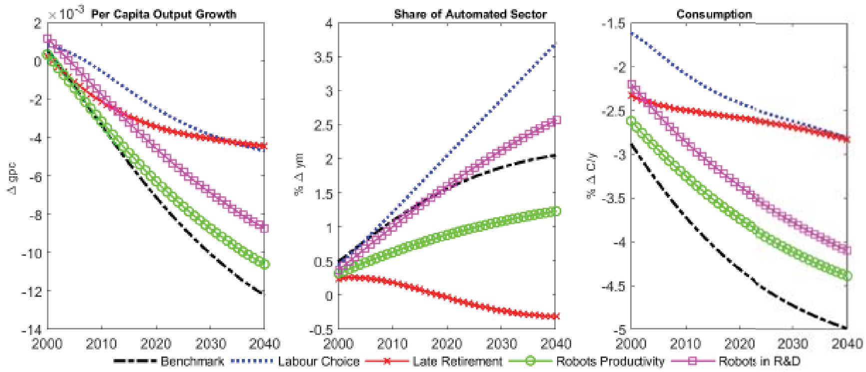
We now look at the effects of the demographic transition in the medium-run for the US under these alternative specifications. Results are shown in Figure 2. Allowing new workers to migrate to the R&D sector offsets the negative effects of the demographic transition on growth during the transition. As automation picks up, the wage in the production sector falls. Wages in the R&D sector, given the lack of substitutes, do not fall and thus Sw_{RD} increases. Labour employed in automation and innovation increase, with the former increasing more. The trade-off between innovation and automation is still present, but is less pronounced as the economy diverts its labour resources towards R&D.[25] Delaying the retirement age

[24]Note that, in the *Robots Productivity* case if $\hat{\eta} < 1$ then a fall in population growth continues to lead to a fall in output per capita for any value of $\mu > 0$ (thus, in general, the result in Corollary 2 remains valid for this extension as well).

[25]A caveat is in order: as Bloom et al. (2017) show, despite a sharp increase in labour employed in R&D, the production of new ideas is fairly constant; in their conclusion new ideas seem to be harder to find. If that is indeed the case migration of workers towards R&D might be less effective in dampening the effects of the demographic transition.

obviously delivers a lower fall in the share of workers to total population and, thus, the incentives to automate are lower. Although delaying retirement slows down the fall in working age population, it cannot avoid the negative impact of population ageing on innovation activity (less young workers involved in the creation of new goods depresses innovation). In this case, since automation eventually falls on the new BGP, wage growth is sustained and the labour share is higher.[26]

Figure 2: Demographic Transition: Alternative Scenarios



Note: The figure plots the effects of the projected demographic changes under different specifications. For Per Capita Output growth = $(y_t/y_{t-1})(N_{t-1}/N_t)$ we show the Change relative to the initial BGP. For Share of Automated Sector - $y_{m,t}$ and Consumption - C_t/y_t we show the percentage change relative to the initial BGP.

In both extensions that alter the way robots are integrated the negative effects of lower population growth on GDP per capita growth and on consumption during the transition are of smaller magnitude than in the benchmark model. As intermediate inputs are used in production, higher robot productivity increases total factor productivity (TFP), generating positive spillovers on the labour-intensive sector. That partially offsets the negative impact of lower labour supply, reducing the incentive to divert resources from innovation such that the share of the automated sector is not as large as in the benchmark case. When robots are used in R&D, the negative effects of resource reallocation on innovation are mitigated and thus automation increases more significantly.

[26]We do not explore whether the age structure within the working population has an effect on automation. If older workers are more/less replaceable relative to their younger counterparts our results may be altered. On this, see Acemoğlu and Restrepo (2018a).

Setting the income per capita equal across all scenarios in 2000, we find that the income per capita in 2040 would be around 15% lower in the benchmark model than in the two best case scenarios, when labour migrates towards the R&D sector (Labour Choice) and when the retirement age is delayed (Late Retirement). Thus, achieving a smoother transition towards the new BGP may generate significant gains. Finally, extending the retirement age decreases the pace of automation and increases the share of income of older workers, who are most likely employed in the production sector. With the transition of young workers towards R&D, older workers are more significantly negatively affected while most benefits accrue to R&D workers. In sum, although in the long run per capita output growth converges to the same level, these extensions mitigate the effect of demographics during the transition and impact automation and the shares of output assigned to labour and robots in the long run.

5.2 Divergence and Robocalypse

Conceivably, robots could be produced more efficiently as the economy becomes more automated. We incorporate this possibility by allowing TFP in the robots production sector to increase in the medium run.[27] Eventually (after more than 150 years in our simulation) TFP in robots production converges to a constant, so that in this scenario the efficiency restriction that ensures BGP convergence is in effect only in the long run.

Results are displayed in Figure 3. As demographics trigger automation, robots are produced more cheaply, further raising the incentives to automate (from 2000 until 2020 the price of robots, q, falls by 40%). As TFP increases together with the ratio $a_{z,t} = (A_t/Z_t)$, most of the output is produced by the automated sector. Eventually, as innovation investment is compromised, output growth is negatively affected despite the efficiency gains in the production of robots. Thus, if robots are produced more efficiently but cannot invent new varieties, then a demographic

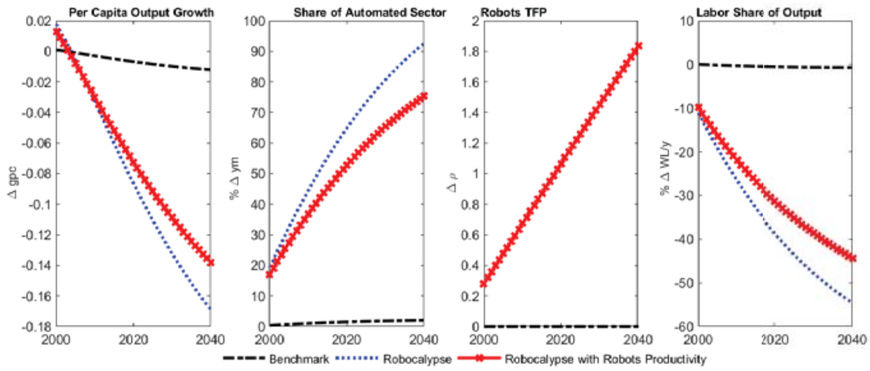
[27] The production of robots is given by $M_t = \varrho\Omega_t^\eta$. We allow ϱ to increase as automation increases. Eventually it converges to a constant. As output and investment Ω_t are growing this scenario is analogous to when $\eta > 1$.

transition that generates automation not only fails to increase output growth but a "robocalypse scenario", resembling the immiseration equilibrium of Benzell et al. (2015), may arise. This conclusion is also robust to the case when both TFP of robots production and productivity of robots in good's production (θ_t) increase during the transition (Robocalypse with Robots Productivity).

5.3 Labour Skill Improvements

One of the key features of the benchmark economy is that individuals are endowed with constant effective labour supply (skill). Even with skill upgrading, insofar as it

Figure 3: Demographics and Robocalypse



Note: The figure plots the effects of the projected demographic changes under different specifications. For Per Capita Output growth = $(y_t/y_{t-1})(N_{t-1}/N_t)$ and Robots TFP - ϱ_t/ϱ_{t-1} we show the Change relative to the initial BGP. For Share of Automated Sector - $y_{m,t}$ and Labour Share of Output $(W_tL_t)/y_t$ we show the percentage change relative to the initial BGP.

is exogenously given, results will not qualitatively differ. For instance, assume the effective labour supply is given by $H_tN_t^{wL} = \int_0^{Z_t \setminus A_t} L_{i,t}di$. Then $g_t^{\eta} = g_{L,t} = g_{n,t}g_{H,t}$, where $g_{H,t} \equiv \frac{H_t}{H_{t-1}}$. As long as $g_{H,t}$ is exogenous and $g_t^Z > 1$, and thus $\eta < 1$, the comparative analyses derived from the benchmark model are unchanged. [28]

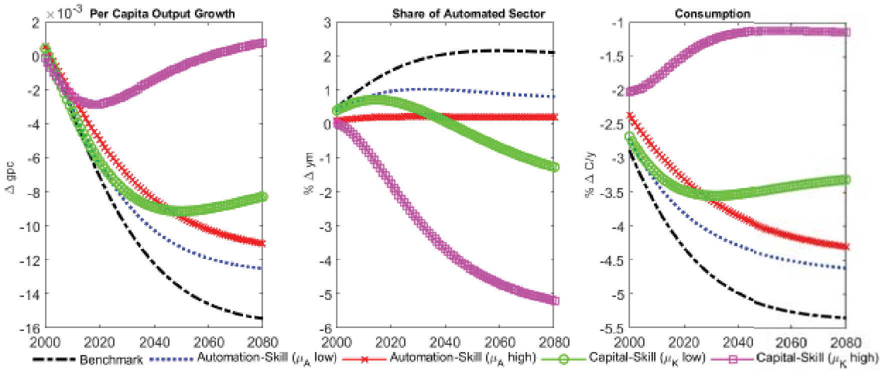
However, if the growth of effective labour depends on either automation or capital accumulation, then the effects of demographic changes may be very different. We consider two cases. In the first, Automation-Skill, we set $g_{H,t} \equiv (\overline{g_H})(a_{z,t})^{\mu_a}$, which implies that as the production in the labour-intensive sector becomes more specialised in a smaller subset of the current set of varieties, as a result of greater

[28] This is consistent with the findings of Grossman et al. (2017) on the conditions under which human capital accumulation alters the restrictions on technological progress under balanced growth.

automation, labour skills grow faster. In the second case, Capital-Skill, we assume skills grow faster as the capital/output ratio increases, reflecting some form of economy-wide capital-skill complementarity (see Grossman et al. (2017)). In this case we set $g_{H,t} \equiv (\overline{g_H}) \left(\frac{K_t}{y_t}\right)^{\mu_K}$. μ_a and μ_K control the intensity of the complementarity between labour skills and automation and capital deepening, respectively.

In figure 4, we show the implications of demographic changes when effective labour supply is endogenous, increasing with the degree of automation or capital deepening. We reset $\overline{g_H}$ such that in all specifications $g_{H,0} = 1$ and the initial BGP is the same as in the benchmark case.[29] Higher skill levels may offset the negative effect of lower fertility on total labour supply, and, ultimately, on wages. In the case of complementarity with automation, for high levels of μ_a, wages do not respond to the demographic change as much and the incentive to automate is significantly weakened. In the case of capital-skill complementarity, as ageing leads to a significant fall in interest rates, the capital stock increases substantially during the demographic transition. As a result, skill growth may be substantial, fully compensating for the fall in the number of workers such that automation ends up falling as a result of demographic changes. Only in this extreme scenario, growth per capita in the long run is not negatively affected.

Figure 4: Demographics and Effective Labour Supply



Note: The figure plots the effects of the projected demographic changes under different specifications. For Per Capita Output growth = $(y_t/y_{t-1})(N_{t-1}/N_t)$ we show the Change relative to the initial BGP. For Share of Automated Sector - $y_{m,t}$ and Consumption - C_t/y_t we show the percentage change relative to the initial BGP.

[29] To highlight that these extensions affect output growth also in the long run we show the transition for 80 years.

Finally, one of the features of the benchmark economy is that an increase in η, such that higher investment in robots production does not suffer from the same degree of scaling inefficiencies as before, leads to a fall in growth per capita. The main reason is that as η increases, despite the initial positive effect on growth, the price of robots relative to wages falls, promoting automation and decreasing the incentives to innovate. Since on a BGP automation and innovation gains are eventually matched, the creation of varieties through innovation is the main driver of growth. Consequently, a rise in η ultimately leads to lower growth in the long run. Allowing for higher automation or capital deepening to trigger higher levels of effective labour supply may offset this negative effect, mitigating the trade-off between innovation and automation and delivering higher per capita growth as η increases (in the appendix we show the effect of a shock to η in the benchmark model and in the model with skill complementarity).

6 Conclusion

Demographic changes are bound to shape the macroeconomic landscape of the next decades. Population ageing may affect the effectiveness of monetary and fiscal policies (Eggertsson et al. (2019) and Basso and Rachedi (2018)). In the medium run demographic changes may restrain economic growth (Aksoy et al. (2019)) and promote automation (Acemoğlu and Restrepo (2018a)).

We have analysed the main interactions between demographics and technology and their implications for economic growth. In our analysis we stress the importance of considering the trade-offs between the generation of new goods and the automation of the production of existing ones while studying the consequences of population ageing for R&D.

While keeping complementarities among inputs (intermediate goods, capital, and either labour and robots), we put at the front of our analysis the labour displacement effect of automation, and leave the creation of new job opportunities (the reinstatement effect of technological changes) only to innovation. This may be an extreme case but still a good starting point for the analysis of the consequences of demographic and technological changes.

Admittedly, it may be too early to conjecture how the new developments from robotics and artificial intelligence will change the production of goods and R&D. Thus, we consider several alternative specifications of how innovation and automation come about. The main conclusion is that, even though lower population growth and population ageing increase automation and, initially, raise productivity growth, in the medium run they are detrimental to economic growth. When using population forecasts for US and Europe, the model predicts a fall in output per capita growth, an increase in automation, and a fall in the labour income share and in interest rates, reinforcing the economic trends already observed in the last decades. Finally, our model indicates that instead of relying on machines, increases in effective labour supply may be the force that offsets the impact of ageing and lower fertility.

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Appendix - "From Secular Stagnation to Robocalypse? Implications of Demographic and Technological Changes"

Appendix A. Equilibrium Conditions

We start by looking at the final and intermediate producers.

Goods Production Sector

Intermediate good firms $j \in A_t$ select capital, robots and inputs to minimise total costs, $TC = p_t q_t M_t^j + (r_t^k + \delta)K_t^j + P_t \Upsilon_t^j$ given a level of production $Y_t^j = \left[(K_t^j)^\alpha(\theta_t M_t^j)^{(1-\alpha)}\right]^{(1-\gamma_I)}\left[\Upsilon_t^j\right]^{\gamma_I}$. Let ν_t^j be the real marginal cost for firm j. Then

$$\nu_t^j = \frac{(r_t^k+\delta)^{\alpha(1-\gamma_I)} q_t^{(1-\alpha)(1-\gamma_I)}}{(\alpha(1-\gamma_I))^{\alpha(1-\gamma_I)}\gamma_I^{\gamma_I}((1-\alpha)(1-\gamma_I))^{(1-\alpha)(1-\gamma_I)}} \tag{A.1}$$

$$K_t^j = \nu_t^j \frac{\alpha(1-\gamma_I)}{(r_t^k+\delta)} y_{j,t} \tag{A.2}$$

$$\Upsilon_t^j = \nu_t^j \gamma_I y_{j,t} \tag{A.3}$$

$$M_t^j = \nu_t^j \frac{(1-\alpha)(1-\gamma_I)}{q_t} y_{j,t}\theta_t \tag{A.4}$$

And given the final good production function,

$$\frac{p_{j,t}}{p_t} = \frac{\psi-1}{\psi}\nu_t^j \tag{A.5}$$

$$y_{j,t} = \left(\frac{p_{j,t}}{p_t}\right)^{\psi} y_t \tag{A.6}$$

$$\Pi_t^j = \left[\frac{p_{j,t}}{p_t} - \nu_t^j\right] y_{j,t} = \frac{1}{\psi-1}\nu_t^j y_{j,t} \tag{A.7}$$

Intermediate good firms $i \in Z_t \setminus A_t$ select capital, labour and inputs to minimise total costs, $TC = W_t L_t^i + (r_t^k + \delta)K_t^i + P_t \Upsilon_t^i$ given a level of production $Y_t^i = \left[(K_t^i)^\alpha(L_t^i)^{(1-\alpha)}\right]^{(1-\gamma_I)}\left[\Upsilon_t^i\right]^{\gamma_I}$. Let ν_t^i be the real marginal cost for firm i. Then

$$\nu_t^i = \frac{(r_t^k+\delta)^{\alpha(1-\gamma_I)} (W_t)^{(1-\alpha)(1-\gamma_I)}}{(\alpha(1-\gamma_I))^{\alpha(1-\gamma_I)}\gamma_I^{\gamma_I}((1-\alpha)(1-\gamma_I))^{(1-\alpha)(1-\gamma_I)}} \tag{A.8}$$

$$K_t^i = \nu_t^i \frac{\alpha(1-\gamma_I)}{(r_t^k+\delta)} y_{i,t} \tag{A.9}$$

$$\Upsilon_t^i = \nu_t^i \gamma_I y_{i,t} \tag{A.10}$$

$$L_t^i = \nu_t^i \frac{(1-\alpha)(1-\gamma_I)}{(W_t)} y_{i,t} \tag{A.11}$$

And given the final good production function,

$$\frac{p_{i,t}}{p_t} = \frac{\psi}{\psi-1}\nu_t^i \tag{A.12}$$

$$y_{i,t} = \left(\frac{p_{i,t}}{p_t}\right)^{\psi} y_t \tag{A.13}$$

$$\Pi_t^i = \left[\frac{p_{i,t}}{p_t} - \nu_t^i\right] y_{i,t} = \frac{1}{\psi-1}\nu_t^i y_{i,t} \tag{A.14}$$

Robots Production Sector

Optimisation of robots producers imply

$$\Pi_{\Omega,t} = q_t p_t M_t - p_t \Omega_t \quad (A.15)$$
$$M_t = \varrho \Omega_t^{\eta} \quad (A.16)$$
$$q_t = \frac{\Omega_t}{M_t \eta} \quad (A.17)$$

Innovation Process

One can easily determine the flow of the stock of goods (Z_t) and goods for which robots can be employed in the production process (A_t), which are given by

$$\frac{Z_{t+1}}{Z_t} = \chi \left(\frac{S_t}{\tilde{\Psi}_t} \right)^{\rho} (L_{I,t}/N_t)^{\kappa_L} + \phi, \text{ and} \quad (A.18)$$
$$\frac{A_{t+1}}{A_t} = \lambda \left(\frac{(Z_t - A_t)^{\kappa_{RD}+\kappa_L} (\Xi_t)^{\kappa_{RD}} (L_{A,t})^{\kappa_L}}{\tilde{\Psi}_t^{\kappa_{RD}} N_t^{\kappa_L}} \right) \phi[Z_t/A_t - 1] + \phi \quad (A.19)$$

Investment in R&D (S_t) and labour demand in product creation is determined by (11) which using (10) becomes

$$S_t = \kappa_{RD} R_{t+1}^{-1} \phi E_t J_{t+1} (Z_{t+1} - \phi Z_t). \quad (A.20)$$
$$L_{I,t} W_{RD,t} = \frac{S_t \kappa_L}{\kappa_{RD}} \quad (A.21)$$

Profits are given by the total gain in seeling the right to goods invented as a result of the previous period investment S_{t-1} to adopters minus the cost of borrowing for that investment. Thus,

$$\Pi_{RD,t} = \vartheta \int_{i \in Z_t \setminus A_t} \Pi_t^i di - S_{t-1} R_t - L_{I,t} W_{RD,t}$$

Let $\tau_{A,t}$ be the shadow price of $\Xi_{q,A,t}$, then automation investors solve

$$\max_{\Xi_{q,A,t}, \Xi_{q,t}, L_{A,q,t}} -\tau_{A,t} \Xi_{q,A,t} + (R_{t+1})^{-1} \phi E_t [\lambda_t V_{t+1} + (1 - \lambda_t) J_{t+1}]. \quad (A.22)$$

Thus,

$$\frac{(Z_t^q - A_t^q)^{\kappa_{RD}+\kappa_L}}{\tilde{\Psi}_t^{\kappa_{RD}} N_t^{\kappa_L}} \lambda' R_t^{-1} \phi [V_{t+1} - J_{t+1}] = 1 \quad (A.23)$$

Assuming the elasticity of λ_t to changes in its input (denoted $\tilde{\epsilon}_\lambda$) is constant and smaller than one, we define $\epsilon_\lambda = \frac{\lambda'}{\lambda_t} \frac{(Z_t^q - A_t^q)^{\kappa_{RD}+\kappa_L} \Xi_{q,A,t}}{\tilde{\Psi}_t^{\kappa_{RD}} N_t^{\kappa_L}} \frac{1}{\kappa_{RD}} = \frac{\tilde{\epsilon}_\lambda}{\kappa_{RD}}$, then we obtain[30]

$$\Xi_t = \epsilon_\lambda \lambda_t R_t^{-1} \phi [V_{t+1} - J_{t+1}] (Z_t - A_t) \quad (A.24)$$
$$L_{A,t} W_{RD,t} = \Xi_t \frac{\kappa_L}{\kappa_{RD}} \quad (A.25)$$

[30] We aggregate across automation investors to obtain $\Xi_t = X_{q,t}(Z_t - A_t)$ and $Ł_{A,t} = L_{A,q,t}(Z_t - A_t)$. Also note that $\tilde{\epsilon}_\lambda = \epsilon_\lambda \kappa_{RD} \leqslant 1$. We use this result in the proof of proposition 3 and 4.

Finally, the value of labour intensive goods and automated goods are given by

$$J_t = \vartheta \Pi_t^j + (R_{t+1})^{-1} \phi E_t[J_{t+1}], \text{ and} \tag{A.26}$$
$$V_t = \vartheta \Pi_t^i + (R_{t+1})^{-1} \phi E_t V_{t+1} \tag{A.27}$$

Profits for adopters are given by the gain from marketing specialised intermediated goods net the amount paid to inventors to gain access to new goods and the expenditures on loans to pay for adoption intensity.

$$\Pi_{A,t} = \vartheta \int_{j \in A_t} \Pi_t^j dj - \Xi_{t-1} R_t - L_{A,t} W_{RD,t}$$

Household Sector

Retiree j decision problem is

$$\max V_t^{jr} = \left\{ (C_t^{jr})^{\upsilon} + \beta \omega_{t,t+1}^r ([V_{t+1}^{jr}]^{\upsilon}) \right\}^{1/\upsilon} \tag{A.28}$$

subject to

$$C_t^{jr} + FA_{t+1}^{jr} = \frac{R_t}{\omega_{t-1,t}^r} FA_t^{jr} + d_t^{jr}. \tag{A.29}$$

The first order condition and envelop theorem are

$$(C_t^{jr})^{\upsilon-1} = \beta \omega_{t,t+1}^r \frac{\partial V_{t+1}^{jr}}{\partial FA_{t+1}^{jr}} (V_{t+1}^{jr})^{\upsilon-1}, \tag{A.30}$$
$$\frac{\partial V_t^{jr}}{\partial FA_t^{jr}} = (V_t^{jr})^{1-\upsilon} (C_t^{jr})^{\upsilon-1} \frac{R_t}{\omega_{t-1,t}^r}. \tag{A.31}$$

Combining these conditions above gives the Euler equation

$$C_{t+1}^{jr} = (\beta R_{t+1})^{1/(1-\upsilon)} C_t^{jr} \tag{A.32}$$

Conjecture that retirees consume a fraction of all assets (including financial assets, profits from financial intermediaries), such that

$$C_t^{jr} = \varepsilon_t \varsigma_t \left[\frac{R_t}{\omega_{t-1,t}^r} FA_t^{rj} + D_t^{rj} \right]. \tag{A.33}$$

Combining these and the budget constraint gives

$$FA_{t+1}^{jr} = \frac{R_t}{\omega_{t-1,t}^r} FA_t^{jr} (1 - \varepsilon_t \varsigma_t) + d_t^{jr} - \varepsilon_t \varsigma_t (D_t^{rj}).$$

Using the condition above the Euler equation and the solution for consumption gives

$$(\beta R_{t+1})^{1/(1-\upsilon)} \varepsilon_t \varsigma_t [\frac{R_t}{\omega_{t-1,t}^r} FA_t^{rj} + D_t^{rj}] = \tag{A.34}$$
$$\varepsilon_{t+1} \varsigma_{t+1} \left[\frac{R_{t+1}}{\omega_{t,t+1}^r} \left(\frac{R_t}{\omega_{t-1,t}^r} FA_t^{jr} (1 - \varepsilon_t \varsigma_t) + d_t^{jr} - \varepsilon_t \varsigma_t D_t^{rj} \right) + D_{t+1}^{jr} \right].$$

Collecting terms we have that

$$
\begin{aligned}
1-\varepsilon_t\varsigma_t &= \frac{(\beta R_{t+1})^{1/(1-\upsilon)}\omega^r_{t,t+1}}{R_{t+1}}\frac{\varepsilon_t\varsigma_t}{\varepsilon_{t+1}\varsigma_{t+1}}, && \text{(A.35)}\\
D_t^{jr} &= d_t^{jr}+\frac{\omega^r_{t,t+1}}{R_{t+1}}D^{jr}_{t+1}. && \text{(A.36)}
\end{aligned}
$$

One can also show that $V_t^{jr}=(\varepsilon_t\varsigma_t)^{-1/\upsilon}C_t^{jr}$. Worker j decision problem is

$$
\max V_t^{jw}=\left\{(C_t^{jw})^{\upsilon}+\beta[\omega^w V^{jw}_{t+1}+(1-\omega^w)V^{jr}_{t+1}]^{\upsilon}\right\}^{1/\upsilon} \tag{A.37}
$$

subject to

$$
C_t^{jw}+FA^{jw}_{t+1}=R_tFA_t^{jw}+W_t\xi_t+d_t^{jw}-\tau_t^{jw}. \tag{A.38}
$$

First order conditions and envelop theorem yield

$$
\begin{aligned}
(C_t^{jw})^{\upsilon-1} &= \beta[\omega^w V^{jw}_{t+1}+(1-\omega^w)V^{jr}_{t+1}]^{\upsilon-1}\left[\omega^w\frac{\partial V^{jw}_{t+1}}{\partial FA^{jw}_{t+1}}+(1-\omega^w)\frac{\partial V^{jr}_{t+1}}{\partial FA^{jw}_{t+1}}\right],\\
\frac{\partial V_t^{jw}}{\partial FA_t^{jw}} &= (V^{jw}_{t+1})^{1-\upsilon}(C_t^{jw})^{\upsilon-1}R_t, \text{ and} && \text{(A.39)}
\end{aligned}
$$

$$
\frac{\partial V_t^{jr}}{\partial FA_t^{jw}} = \frac{\partial V_t^{jr}}{\partial FA_t^{jr}}\frac{\partial FA_t^{jr}}{\partial FA_t^{jw}}=\frac{\partial V_t^{jr}}{\partial FA_t^{jr}}\frac{1}{\omega^r_{t-1,t}}=(V_t^{jr})^{1-\upsilon}(C_t^{jr})^{\upsilon-1}R_t. \tag{A.40}
$$

$\frac{\partial FA_t^{jr}}{\partial FA_t^{jw}}=\frac{1}{\omega^r_{t-1,t}}$ since as households are risk neutral with respect to labour income they select the same asset profile independent of their worker/retiree status, adjusting only for expected return due to probability of death. Combining these conditions above, and using the conjecture that $V_t^{jw}=(\varsigma_t)^{-1/\upsilon}C_t^{jw}$, gives the Euler equation

$$
\begin{aligned}
C_t^{jw} &= \left((\beta R_{t+1}\mathfrak{z}_{t+1})^{1/(1-\upsilon)}\right)^{-1}[\omega^w C^{jw}_{t+1}+(1-\omega^w)\varepsilon_{t+1}^{\frac{-1}{\upsilon}}C^{jr}_{t+1}] && \text{(A.41)}\\
&\text{where } \mathfrak{z}_{t+1}=(\omega^w+(1-\omega^w)\varepsilon_{t+1}^{(\upsilon-1)/\upsilon}).
\end{aligned}
$$

Conjecture that retirees consume a fraction of all assets (including financial assets, human capital and profits from financial intermediaries), such that

$$
C_t^{jw}=\varsigma_t[R_tFA_t^{jw}+H_t^{jw}+D_t^{jw}]. \tag{A.42}
$$

Following the same procedure as before we have that

$$
\begin{aligned}
&\varsigma_t[R_tFA_t^{jw}+H_t^{jw}+D_t^{jw}](\beta R_{t+1}\mathfrak{z}_{t+1})^{1/(1-\upsilon)}= && \text{(A.43)}\\
&\quad\omega^w\varsigma_{t+1}\Big[R_{t+1}\Big(R_tFA_t^{jw}(1-\varsigma_t)+W_t\xi_t+d_t^{jw}-\varsigma_t(H_t^{jw}+D_t^{jw})\Big)+H^{jw}_{t+1}+D^{jw}_{t+1}\Big]+\\
&\quad\varepsilon_{t+1}^{\frac{-1}{\upsilon}}(1-\omega^w)\varepsilon_{t+1}\varsigma_{t+1}\Big[R_{t+1}\Big(R_tFA_t^{jw}(1-\varsigma_t)+W_t\xi_t+d_t^{jw}-\varsigma_t(H_t^{jw}+D_t^{jw})\Big)+D^{jr}_{t+1}\Big].
\end{aligned}
$$

Collecting terms and simplifying we have that

$$\begin{aligned} \varsigma_t &= 1 - \frac{\varsigma_t}{\varsigma_{t+1}} \frac{(\beta R_{t+1} \mathfrak{Z}_{t+1})^{1/(1-\upsilon)}}{R_{t+1} \mathfrak{Z}_{t,t+1}} && (A.44) \\ H_t^{jw} &= (W_t^j) + \frac{\omega^w}{R_{t+1}\mathfrak{Z}_{t,t+1}} H_{t+1}^{jw} \text{ and} && (A.45) \\ D_t^{jw} &= d_t^{jw} + \frac{\omega^w}{R_{t+1}\mathfrak{Z}_{t,t+1}} D_{t+1}^{jw} + \frac{(1-\omega^w)\varepsilon_{t+1}^{(\upsilon-1)/\upsilon}}{R_{t+1}\mathfrak{Z}_{t,t+1}} D_{t+1}^{jr}. && (A.46) \end{aligned}$$

Aggregation across households

Assume that for any variable X_t^{jz} we have that $X_t^z = \int_0^{N_t^z} X_t^{jz}$ for $z = \{w, r\}$, then

$$\begin{aligned} L_t &= N_t^{wL}, && (A.47) \\ L_{I,t} + L_{A,t} &= N_t^{wRD}, && (A.48) \\ H_t^w &= (W_t)N_t^{wL} + (W_t^{RD})N_t^{wRD} + \frac{\omega^w}{R_{t+1}\mathfrak{Z}_{t,t+1}} H_{t+1}^w \frac{N_t^w}{N_{t+1}^w}, && (A.49) \\ D_t^w &= d_t^w + \frac{\omega^w}{R_{t+1}\mathfrak{Z}_{t,t+1}} \frac{D_{t+1}^w N_t^w}{N_{t+1}^w} + \frac{(1-\omega^w)\varepsilon_{t+1}^{(\upsilon-1)/\upsilon}}{R_{t+1}\mathfrak{Z}_{t,t+1}} \frac{D_{t+1}^r N_t^w}{N_{t+1}^r}, && (A.50) \\ C_t^w &= \varsigma_t[R_t FA_t^w + H_t^w + D_t^w - T_t^w], && (A.51) \\ D_t^r &= d_t^r + \frac{\omega_{t,t+1}^r}{R_{t+1}} D_{t+1}^r \frac{N_t^r}{N_{t+1}^r}, && (A.52) \\ C_t^r &= \varepsilon_t \varsigma_t [R_t FA_t^r + D_t^r]. && (A.53) \end{aligned}$$

Note that $\omega_{t,t+1}^r$ is not shown in the last equation due to the perfect annuity market for retirees, allowing for the redistribution of assets of retirees who died at the end of the period.

Financial Intermediary

The profits of the financial intermediary are

$$\begin{aligned} \Pi_t^F &= [r_t^k + 1]K_t + R_t B_t - R_t(FA_t^w + FA_t^r) - K_{t+1} - B_{t+1} + FA_{t+1}^w + FA_{t+1}^r + \\ &\quad + (\Pi_{A,t} + \Pi_{RD,t} + (1-\vartheta)\left(\int_{j\in A_t} \Pi_t^j dj + \int_{i \in Z_t \setminus A_t} \Pi_t^i di\right) + \Pi_{\Omega,t}), && (A.54) \end{aligned}$$

where $B_{t+1} = S_t + \Xi_t$ and $FA_t = FA_t^w + FA_t^r$.

The financial intermediaries selects capital and bonds such that it maximise profits and thus we obtain the standard arbitrage conditions whereby all assets must pay the same expected return, thus

$$E_t\left[r_{t+1}^k + 1\right] = R_t. \quad (A.55)$$

Also note that under a perfect foresight solution, by ensuring the financial intermediary behaves under perfect competition, this equality holds without expectations, $\Pi_t^F = 0$ and thus $d_t^r = d_t^w = 0$. If $\Pi_t^F \neq 0$, then we assume profits are divided based on the ratio of assets. As such, $d_t^r = \Pi_t^F \frac{FA_t^r}{FA_t^r+FA_t^w}$ and $d_t^w = \Pi_t^F \frac{FA_t^w}{FA_t^r+FA_t^w}$. The flow of capital is then given by

$$K_{t+1} = K_t(1-\delta) + I_t. \tag{A.56}$$

Where I_t is the investment in capital made by the financial intermediary.

Asset Markets

Asset Market clearing implies

$$FA_{t+1} = FA_{t+1}^w + FA_{t+1}^r = K_{t+1} + B_{t+1} \tag{A.57}$$

Finally, the flow of assets are given by

$$FA_{t+1}^r = R_t FA_t^r + d_t^r - C_t^r + (1-\omega^w)(R_t FA_t^w + W_t \xi_t L_t + d_t^w - C_t^w - \tau_t) \tag{A.58}$$
$$FA_{t+1}^w = \omega^w (R_t FA_t^w + W_t \xi_t L_t + d_t^w - C_t^w - \tau_t) \tag{A.59}$$

Clearing conditions

$$y_t = C_{w,t} + C_{r,t} + \Upsilon_t + \Omega_t + I_t \tag{A.60}$$
$$K_{t+1} = (1-\delta)K_t + I_t \tag{A.61}$$
$$K_t = \int_{j\in A_t} k_t^j dj + \int_{i\in Z_t\setminus A_t} k_t^i di \tag{A.62}$$
$$\Upsilon_t = \int_{j\in A_t} \Upsilon_t^j dj + \int_{i\in Z_t\setminus A_t} \Upsilon_t^i di \tag{A.63}$$
$$M_t = \int_{j\in A_t} M_t^j dj \tag{A.64}$$
$$N_t^{wR} = \int_q L_{A,q,t} di + \int_p L_{i,q,t} di N_t^{wL} = \int_{i\in Z_t\setminus A_t} L_t^i di \tag{A.65}$$
$$\tag{A.66}$$

Appendix B. Detrended equilibrium conditions

This section shows the detrended equilibrium conditions. Note that $\bar{x}$ denotes the steady state of variable x_t.

$$w_t = ls_t + li_t + la_t \tag{A.67a}$$
$$h_t^w = w_t + \frac{\omega^w}{R_{t+1}\mathfrak{Z}_{t,t+1}} \frac{g_{t+1} h_{t+1}^w}{g_{t+1}^w} \text{ where } h_t^w = \frac{H_t^w}{Y_t}, g_{t+1} = \frac{Y_{t+1}}{Y_t}, g_{t+1}^w = \frac{N_{t+1}^w}{N_t^w} \tag{A.67b}$$
$$\tilde{D}_t^r = \tilde{d}_t^r + \frac{\omega_{t,t+1}^r}{R_{t+1}} g_{t+1} \frac{\tilde{D}_{t+1}^r \zeta_t^r}{\zeta_{t+1}^r g_{t+1}^w} \text{ where } \tilde{D}_t^r = \frac{D_t^r}{Y_t}, \tilde{d}_t^r = \frac{d_t^r}{Y_t} \tag{A.67c}$$
$$\tilde{D}_t^w = \tilde{d}_t^w + \frac{\omega^w}{R_{t+1}\mathfrak{Z}_{t,t+1}} \frac{g_{t+1}\tilde{D}_{t+1}^w}{g_{t+1}^w} + \frac{(1-\omega^w)\varepsilon_{t+1}^{(\upsilon-1)/\upsilon}}{R_{t+1}\mathfrak{Z}_{t,t+1}} \frac{g_{t+1}\tilde{D}_{t+1}^r}{\zeta_{t+1}^r g_{t+1}^w} \text{ where } \tilde{D}_t^w = \frac{D_t^w}{Y_t}, \tilde{d}_t^w = \frac{d_t^w}{Y_t} \tag{A.67d}$$

$$c_t^w = \varsigma_t[R_t \frac{fa_t^w}{g_t} + h_t^w + \tilde{D}_t^w] \text{ where } fa_t^w = \frac{FA_t^w}{Y_{c,t-1}}, c_t^w = \frac{C_t^w}{Y_t} \tag{A.67e}$$

$$c_t^r = \varepsilon_t \varsigma_t[R_t \frac{fa_t^r}{g_t} + \tilde{D}_t^r] \text{ where } fa_t^r = \frac{FA_t^r}{Y_{c,t-1}}, c_t^r = \frac{C_t^w}{Y_t} \tag{A.67f}$$

$$1 - \varepsilon_t \varsigma_t = \frac{(\beta R_{t+1})^{1/(1-\upsilon)} \omega_{t,t+1}^r}{R_{t+1}} \frac{\varepsilon_t \varsigma_t}{\varepsilon_{t+1} \varsigma_{t+1}} \tag{A.67g}$$

$$\varsigma_t = 1 - \frac{\varsigma_t}{\varsigma_{t+1}} \frac{(\beta R_{t+1} \mathfrak{Z}_{t+1})^{\frac{1}{(1-\upsilon)}}}{R_{t+1} \mathfrak{Z}_{t,t+1}} \tag{A.67h}$$

$$\mathfrak{Z}_{t+1} = (\omega^w + (1-\omega^w) \varepsilon_{t+1}^{(\upsilon-1)/\upsilon}) \tag{A.67i}$$

$$\zeta_{t+1}^{wRD} g_{t+1}^w = \omega_{t,t+1}^y Sw_{RD} + (1 - drop_{RD}) \omega^w \zeta_t^{wRD} \text{ where } \zeta_t^{wRD} = \frac{N_t^{wRD}}{N_t^w} \tag{A.68a}$$

$$\zeta_{t+1}^{wL} g_{t+1}^w = \omega_{t,t+1}^y (1 - Sw_{RD}) + \omega^w \zeta_t^{wL} + (drop_{RD}) \omega^w \zeta_t^{wRD} \text{ where } \zeta_t^{wL} = \frac{N_t^{wL}}{N_t^w} \tag{A.68b}$$

$$g_{t+1}^w = \omega^w + (1 - \omega^y) \tag{A.68c}$$

$$\zeta_{t+1}^r = \left((1-\omega^w) + \omega_{t,t+1}^r \zeta_t^r\right) (\omega^w + (1-\omega^y))^{-1} \text{ where } \zeta_t^r = \frac{N_t^r}{N_t^w} \tag{A.68d}$$

$$g_{t+1}^n = \frac{(1+\zeta_{t+1}^r)}{(1+\zeta_t^r)} g_{t+1}^w \text{ where } g_{t+1}^n = \frac{N_{t+1}}{N_t} \tag{A.68e}$$

Note that all firms $j \in A_t$ take the same decisions, then $\int_{j \in A_t} k_t^j dj = A_t k_t^j$. A similar argument holds for firms $i \in Z_t \setminus A_t$.

$$k_{m,t} = \frac{\alpha(1-\gamma_I)}{(r_t^k + \delta)} \frac{\psi - 1}{\psi} y_{m,t} g_t \text{ where } k_{m,t} = \frac{A_t k_t^j}{Y_{t-1}}, y_{m,t} = \frac{(p_t^j/p_t) y_t^j A_t}{Y_t} \tag{A.69a}$$

$$\Upsilon_{m,t} = \gamma_I \frac{\psi - 1}{\psi} y_{m,t} \text{ where } \Upsilon_{m,t} = \frac{A_t \Upsilon_t^j}{Y_t} \tag{A.69b}$$

$$m_t = (1-\alpha)(1-\gamma_I) \frac{\psi - 1}{\psi} y_{m,t} \text{ where } m_t = \frac{A_t m_t^j q_t}{Y_t} = \frac{q_t M_t}{Y_t} \tag{A.69c}$$

$$g_{pm,t} = \left(\frac{(r_t^k + \delta)}{(r_{t-1}^k + \delta)}\right)^{\alpha(1-\gamma_I)} \left(\frac{\theta_{t-1}}{\theta_t}\right)^{(1-\alpha)(1-\gamma_I)} g_{q,t}^{(1-\alpha)(1-\gamma_I)} \text{ where } g_{pm,t} = \frac{(p_{j,t}/p_t)}{(p_{j,t-1}/p_{t-1})}, g_{q,t} = \frac{q_t}{q_{t-1}} \tag{A.69d}$$

$$\frac{y_{m,t}}{y_{m,t-1}} = g_t^A g_{pm,t}^{1-\psi}, \text{ where } g_t^A = \frac{A_t}{A_{t-1}} \tag{A.69e}$$

$$\pi_{m,t} = \frac{1}{\psi} y_{m,t} \text{ where } \pi_{m,t} = \frac{A_t \Pi_t^j}{Y_t} \tag{A.69f}$$

$$k_{L,t} = \frac{\alpha(1-\gamma_I)}{(r_t^k + \delta)} \frac{\psi - 1}{\psi} y_{L,t} g_t \text{ where } k_{L,t} = \frac{(Z_t - A_t) k_t^i}{Y_{t-1}}, y_{L,t} = \frac{(p_{i,t}/p_t) y_t^i (Z_t - A_t)}{Y_t} \tag{A.69g}$$

$$\Upsilon_{L,t} = \gamma_I \frac{\psi - 1}{\psi} y_{L,t} \text{ where } \Upsilon_{L,t} = \frac{(Z_t - A_t) \Upsilon_t^i}{Y_t} \tag{A.69h}$$

$$ls_t = (1-\alpha)(1-\gamma_I)\frac{\psi - 1}{\psi}y_{L,t} \text{ where } ls_t = \frac{(W_t)N_t^{wL}}{Y_t} \tag{A.69i}$$

$$ls_t/ls_{t-1} = lspop_t/lspop_{t-1}(g_t^{wg}g_{t-1}^n)/g_t \text{ where } g_t^{wg} = \frac{W_t}{W_{t-1}} \tag{A.69j}$$

$$\frac{y_{L,t}}{y_{L,t-1}} = g_t^{ZA}g_{pL,t}^{1-\psi}, \text{ where } g_t^{ZA} = \frac{(Z_t - A_t)}{(Z_{t-1} - A_{t-1})} \text{ where } g_{pL,t} = \frac{(p_{i,t}/p_t)}{(p_{i,t-1}/p_{t-1})} \tag{A.69k}$$

$$g_{pL,t} = \left(\frac{(r_t^k + \delta)}{(r_{t-1}^k + \delta)}\right)^{\alpha(1-\gamma_I)}\left(\frac{ls_t}{ls_{t-1}}\right)^{(1-\alpha)(1-\gamma_I)}\left(\frac{g_t}{g_{L,t}}\right)^{(1-\alpha)(1-\gamma_I)} \text{ where } g_{L,t} = \frac{L_t}{L_{t-1}} = g_{w,t} \tag{A.69l}$$

$$\pi_{L,t} = \frac{1}{\psi}y_{L,t} \text{ where } \pi_{L,t} = \frac{(Z_t - A_t)\Pi_t^i}{Y_t} \tag{A.69m}$$

$$m_t = \frac{\tilde{\Omega}_t}{\eta} \text{ where } \tilde{\Omega}_t = \frac{\Omega_t}{Y_t} \tag{A.69n}$$

$$\pi_{\Omega,t} = m_t - \tilde{\Omega}_t \text{ where } \pi_{\Omega,t} = \frac{\Pi_{\Omega,t}}{Y_t} \tag{A.69o}$$

$$\frac{m_t}{m_{t-1}} = \left(\frac{\tilde{\Omega}_t}{\tilde{\Omega}_{t-1}}\right)^{\eta}(g_t)^{\eta-1}g_{q,t} \tag{A.69p}$$

$$g_{t+1}^Z = \chi\left(\frac{s_t}{\Psi_t}\right)^{\rho}(lipop_t)^{\kappa_L} + \phi \text{ where } g_t^Z = \frac{Z_t}{Z_{t-1}}, s_t = \frac{S_t}{Y_t}, \Psi_t = \frac{\tilde{\Psi}_t}{Y_t}, lipop_t = \frac{L_{I,t}}{N_t} \tag{A.70a}$$

$$g_{t+1}^A = \lambda_t\phi[1/a_{z,t} - 1] + \phi \text{ where } a_{z,t} = \frac{A_t}{Z_t} \tag{A.70b}$$

$$g_t^{ZA} = g_t^Z\frac{1 - a_{z,t}}{1 - a_{z,t-1}} \tag{A.70c}$$

$$az_t = az_{t-1}\frac{g_t^A}{g_t^Z} \tag{A.70d}$$

$$s_t = \kappa_{RD}g_{t+1}R_{t+1}^{-1}\phi j_{t+1}\left(\frac{g_{t+1}^Z - \phi}{g_{t+1}^Z(1 - a_{z,t+1})}\right) \text{ where } j_t = \frac{J_t(Z_t - A_t)}{Y_t} \tag{A.70e}$$

$$li_t = s_t\frac{\kappa_L}{\kappa_{RD}} \text{ where } li_t = \frac{L_{I,t}W_{RD,t}}{Y_t} \tag{A.70f}$$

$$li_t/li_{t-1} = lipop_t/lipop_{t-1}(g_t^{wrd}g_{t-1}^n)/g_t \text{ where } g_t^{wrd} = \frac{W_{RD,t}}{W_{RD,t-1}} \tag{A.70g}$$

$$v_t = \vartheta\pi_{m,t} + (R_{t+1})^{-1}\phi\frac{g_{t+1}}{g_{t+1}^A}v_{t+1} \text{ where } v_t = \frac{V_tA_t}{Y_t} \tag{A.70h}$$

$$j_t = \vartheta\pi_{L,t} + (R_{t+1})^{-1}\phi\frac{g_{t+1}}{g_{t+1}^{ZA}}j_{t+1} \tag{A.70i}$$

$$\varpi_t = \epsilon_\lambda\lambda_tR_{t+1}^{-1}\phi g_{t+1}\left[\frac{v_{t+1}}{g_{t+1}^A}[1/a_{z,t} - 1] - \frac{j_{t+1}}{g_{t+1}^{ZA}}\right] \text{ where } \varpi_t = \frac{\Xi_t}{Y_t} \tag{A.70j}$$

$$la_t = \varpi_t\frac{\kappa_L}{\kappa_{RD}} \text{ where } la_t = \frac{L_{A,t}W_{RD,t}}{Y_t} \tag{A.70k}$$

$$la_t/la_{t-1} = lapop_t/lapop_{t-1}(g_t^{wrd}g_{t-1}^n)/g_t \tag{A.70l}$$

$$\lambda_t = \lambda\left(\left(\frac{\varpi_t}{\Psi_t}\right)^{\kappa_{RD}} lapop_t^{\kappa_L}\right) \approx \bar{\lambda}\left(1+\epsilon_\lambda\left(\kappa_{RD}\frac{\varpi_t-\bar{\varpi}}{\bar{\varpi}}-\kappa_{RD}\frac{\Psi_t-\bar{\Psi}}{\bar{\Psi}}+\kappa_L\frac{lapop_t-\bar{lapop}}{\bar{lapop}}\right)\right) \tag{A.70m}$$

$$\pi_t^A = \vartheta\pi_{m,t} - R_t\varpi_{t-1}/g_t - li_t \tag{A.70n}$$

$$\pi_t^{RD} = \vartheta\pi_{L,t} - R_t s_{t-1}/g_t - la_t \tag{A.70o}$$

where ϵ_λ is the elasticity of $\lambda(\cdot)$

$$r_{t+1}^k + 1 = R_{t+1} \tag{A.71a}$$

$$\tilde{d}_t^r = \pi_t^F\frac{fa_t^r}{fa_t} \text{ where } \pi_t^F = \frac{\Pi_t^F}{Y_t} \tag{A.71b}$$

$$\tilde{d}_t^w = \pi_t^F\frac{fa_t^w}{fa_t} \tag{A.71c}$$

$$b_{t+1} = s_t + \varpi_t \text{ where } b_{t+1} = \frac{B_{t+1}}{Y_t} \tag{A.71d}$$

$$\pi_t^F = (r_t^k+1)\frac{k_t}{g_t} + \frac{R_t}{g_t}b_t - \frac{R_t}{g_t}(fa_t) - k_{t+1} - b_{t+1} + (fa_{t+1}) + \pi_t^A + \pi_t^{RD} + (1-\vartheta)(\pi_{m,t}+\pi_{L,t}) \tag{A.71e}$$

$$lspop = \frac{\zeta_t^{wL}}{1+\zeta_t^r} \tag{A.72a}$$

$$lipop_t + lapop_t = \frac{\zeta_t^{wRD}}{1+\zeta_t^y+\zeta_t^r} \text{ where } \zeta_t^{wRD} = \frac{N_t^{wRD}}{N_t^w} \tag{A.72b}$$

$$k_{t+1} = (1-\delta)\frac{k_t}{g_t} + i_t \text{ where } i_t = \frac{I_t}{Y_t} \tag{A.72c}$$

$$k_t = k_{m,t} + k_{L,t} \tag{A.72d}$$

$$\tilde{\Upsilon}_t = \Upsilon_{m,t} + \Upsilon_{L,t} \tag{A.72e}$$

$$1 = y_{m,t} + y_{L,t} \tag{A.72f}$$

$$1 = c_t + i_t + s_t + \varpi_t + \tilde{\Omega}_t + \tilde{\Upsilon}_t \text{ where } c_t = \frac{C_t}{Y_t} \tag{A.72g}$$

$$c_t = c_t^w + c_t^r \tag{A.72h}$$

$$fa_{t+1}^w + fa_{t+1}^r = k_{t+1} + b_{t+1} \tag{A.72i}$$

$$fa_{t+1}^r = \frac{R_t}{g_t}fa_t^r + \tilde{d}_t^r - c_t^r + (1-\omega^w)\left(\frac{R_t}{g_t}fa_t^w + w_t + \tilde{d}_t^w - c_t^w\right) \tag{A.72j}$$

$$fa_{t+1} = fa_{t+1}^w + fa_{t+1}^r \tag{A.72k}$$

$$\Psi_t = v_t \tag{A.72l}$$

$$fa_{t+1}^w = \omega^w\left(\frac{R_t}{g_t}fa_t^w + w_t + \tilde{d}_t^w - c_t^w\right)$$

Appendix C. Comparative Analysis

In this section of the appendix we present the proofs of Proposition 2 and 3. For both propositions we use the main detrended equilibrium conditions from firms, innovators and automation investors optimisation problems depicted above.

Total differentiation around the BGP equilibrium of the core equilibrium conditions for R&D, (A.70), using $\Psi_t = v_t$ and (A.69a) to replace for π_L and π_m we obtain

$$\frac{dg^Z}{g^Z+\phi} = \rho\frac{ds}{s} - \rho\frac{dv}{v} + \kappa_L\frac{dlipop}{lipop} \tag{A.73a}$$

$$\frac{ds}{s} = -\frac{dR}{R} + \frac{dj}{j} + \frac{a_z}{(1-a_z)}\frac{da_z}{a_z} + \frac{dg}{g} + \frac{\phi}{g^Z}\frac{dg^Z}{g^Z+\phi} \tag{A.73b}$$

$$\frac{dg^Z}{g^Z+\phi} = \frac{d\lambda}{\lambda} - \frac{1}{(1-a_z)}\frac{da_z}{a_z} \tag{A.73c}$$

$$\frac{d\lambda}{\lambda} = \epsilon_\lambda\kappa_{RD}\left(\frac{d\varpi}{\varpi} - \frac{dv}{v}\right) + \epsilon_\lambda\kappa_L\frac{dlapop}{lapop} \tag{A.73d}$$

$$\frac{d\varpi}{\varpi} = \frac{d\lambda}{\lambda} - \frac{dR}{R} + \frac{dg}{g} + \frac{dg^Z}{g^Z} + \frac{v[1/a_z-1]}{v[1/a_z-1]-j}\frac{dv}{v} - \frac{j}{v[1/a_z-1]-j}\frac{dj}{j} - \frac{v[1/a_z-1]}{v[1/a_z-1]-j}\frac{1}{1-a_z}\frac{da_z}{a_z} \tag{A.73e}$$

$$\frac{dv}{v} = -\frac{dy_L}{(1-y_L)} - \Gamma\frac{dR}{R} - \Gamma\frac{dg^Z}{g^Z} + \Gamma\frac{dg}{g},\ \text{where } \Gamma = \frac{\frac{\phi g}{g^Z R}}{\left(1-\frac{\phi g}{g^Z R}\right)^2} \tag{A.73f}$$

$$\frac{dj}{j} = \frac{dy_L}{y_L} - \Gamma\frac{dR}{R} - \Gamma\frac{dg^Z}{g^Z} + \Gamma\frac{dg}{g} \tag{A.73g}$$

$$\frac{dlapop}{lapop} - \frac{dlipop}{lipop} = \frac{d\varpi}{\varpi} - \frac{ds}{s} \tag{A.73h}$$

$$\frac{dg}{g} = \frac{1}{\eta}\frac{dg_n}{g_n} \tag{A.73i}$$

$$\frac{dg^z}{g^z} = \frac{(1-\eta)(\psi-1)(1-\alpha)(1-\gamma_I)}{\eta}\frac{dg_n}{g_n} \tag{A.73j}$$

$$dlapop + dlipop = dRDpop \tag{A.73k}$$

where $RDpop \equiv \frac{N_t^{wRD}}{N_t}$

Proof of Proposition 2:

Proposition 2 focuses on changes on population growth (dg_n), maintaining demographic structure (age shares) constant and thus $dRDpop = 0$. Combining (A.73) we obtain two conditions linking labour output share and the degree of automation with changes in population growth and changes in interest rates

$$-\frac{1}{\eta}\frac{\rho}{\kappa_L}\Gamma_1\frac{dg_n}{g_n} - \frac{\rho}{\kappa_L}\frac{dR}{R} = -c_1\frac{da_z}{a_z} - d_1\frac{dy_L}{y_L(1-y_L)}$$

$$-\frac{1}{\eta}\frac{\kappa_{RD}}{\kappa_L}\Gamma_2\frac{dg_n}{g_n} - \frac{\kappa_{RD}}{\kappa_L}\frac{dR}{R} = c_2\frac{da_z}{a_z} + d_2\frac{dy_L}{y_L(1-y_L)}$$

where

$$
\begin{aligned}
c_1 &\equiv \left(\frac{lapop}{RDpop}\frac{(v+j)}{v[1/a_z-1]-j}+\frac{\rho}{\kappa_L}\frac{a_z}{(1-a_z)}\right)>0\\
c_2 &\equiv \left(\frac{(RDpop-lapop)}{RDpop}\frac{(v+j)}{v[1/a_z-1]-j}+\frac{1}{\epsilon_\lambda\kappa_L}\left(1+\epsilon_\lambda\kappa_{RD}\frac{j}{v[1/a_z-1]-j}\right)\frac{1}{(1-a_z)}\right)>0\\
d_1 &\equiv \left(\frac{lapop}{RDpop}\frac{v[1/a_z-1]}{v[1/a_z-1]-j}+\frac{\rho}{\kappa_L}\right)>0\\
d_2 &\equiv \left(\frac{\epsilon_\lambda\kappa_{RD}}{\epsilon_\lambda\kappa_L}\frac{j}{v[1/a_z-1]-j}+\frac{(RDpop-lapop)}{RDpop}\frac{v[1/a_z-1]}{v[1/a_z-1]-j}\right)>0\\
\Gamma_1 &\equiv \left(\frac{\frac{g^Z}{\phi}-\rho}{(g^Z+\phi)}(1-\eta)(\psi-1)(1-\alpha)(1-\gamma_I)-1\right)>0\\
\Gamma_2 &\equiv \left(\frac{\frac{g^Z}{\epsilon_\lambda\kappa_{RD}}-\phi}{(g^Z+\phi)}(1-\eta)(\psi-1)(1-\alpha)(1-\gamma_I)-1\right)>0
\end{aligned}
$$

The first four inequalities follow from the fact that on any BGP, $\varpi>0\Rightarrow[v[1/a_z-1]-j]>0$, and $a_z<1$ and the last two since $\phi,\kappa_{RD}\leqslant 1$ and from assumption 2 ($A2$).

Then, as the labour income share is given by $ls_t=(1-\alpha)(1-\gamma_I)\frac{\psi-1}{\psi}y_{L,t}$ we have that

$$
\frac{da_z}{a_z} = \frac{-\frac{1}{\eta}\left(d_1\frac{\kappa_{RD}}{\kappa_L}\Gamma_2+\frac{\rho}{\kappa_L}\Gamma_1 d_2\right)\frac{dg_n}{g_n}-\left(d_1\frac{\kappa_{RD}}{\kappa_L}+\frac{\rho}{\kappa_L}d_2\right)\frac{dR}{R}}{c_2d_1-d_2c_1} \tag{A.74}
$$

$$
\frac{dls}{ls} = (1-y_L)\frac{\frac{1}{\eta}\left(c_1\frac{\kappa_{RD}}{\kappa_L}\Gamma_2+\frac{\rho}{\kappa_L}\Gamma_1 c_2\right)\frac{dg_n}{g_n}+\left(c_1\frac{\kappa_{RD}}{\kappa_L}+\frac{\rho}{\kappa_L}c_2\right)\frac{dR}{R}}{c_2d_1-d_2c_1} \tag{A.75}
$$

To conclude the proof of Proposition 2 we need to ensure the denominator is positive. From the definitions of c_1, c_2, d_1, and d_2 and as $\epsilon_\lambda\kappa_{RD}\leqslant 1$ we have that

$$
\begin{aligned}
c_2d_1 &> \tilde{c}_2d_1=\frac{1}{\epsilon_\lambda\kappa_L(1-a_z)}\left(\frac{v[1/a_z-1]}{v[1/a_z-1]-j}\right)\frac{\rho}{\kappa_L}+\frac{(RDpop-lapop)}{RDpop}\frac{(v+j)}{v[1/a_z-1]-j}\frac{\rho}{\kappa_L}\\
&+\frac{1}{\epsilon_\lambda\kappa_L(1-a_z)}\left(\frac{v[1/a_z-1]}{v[1/a_z-1]-j}\right)\frac{lapop}{RDpop}\frac{v[1/a_z-1]}{v[1/a_z-1]-j}\\
&+\frac{(RDpop-lapop)}{RDpop}\frac{(v+j)}{v[1/a_z-1]-j}\frac{lapop}{RDpop}\frac{v[1/a_z-1]}{v[1/a_z-1]-j}\\
c_1d_2 &= \frac{\rho}{\kappa_L}\frac{a_z}{(1-a_z)}\frac{\epsilon_\lambda\kappa_{RD}}{\epsilon_\lambda\kappa_L}\frac{j}{v[1/a_z-1]-j}+\frac{\rho}{\kappa_L}\frac{a_z}{(1-a_z)}\frac{(RDpop-lapop)}{RDpop}\frac{v[1/a_z-1]}{v[1/a_z-1]-j}\\
&+\frac{lapop}{RDpop}\frac{(v+j)}{v[1/a_z-1]-j}\frac{(RDpop-lapop)}{RDpop}\frac{v[1/a_z-1]}{v[1/a_z-1]-j}\\
&+\frac{lapop}{RDpop}\frac{(v+j)}{v[1/a_z-1]-j}\frac{\epsilon_\lambda\kappa_{RD}}{\epsilon_\lambda\kappa_L}\frac{j}{v[1/a_z-1]-j}
\end{aligned}
$$

Note that $\frac{(RDpop-lapop)}{RDpop} = \frac{lipop}{RDpop} = \frac{lapop}{RDpop}\frac{lipop}{lapop} = \frac{lapop}{RDpop}\frac{s}{\varpi}$.

As $\frac{s}{\varpi} = \frac{\kappa_{RD} g R^{-1}\phi j\left(\frac{g^Z-\phi}{g^Z(1-a_z)}\right)}{\epsilon_\lambda \frac{g^Z-\phi}{\phi[1/a_z-1]} R^{-1}\phi\frac{g}{g^Z}[v[1/a_z-1]-j]} = \frac{\kappa_{RD}\phi j}{\epsilon_\lambda a_z[v[1/a_z-1]-j]}$ then $\frac{(RDpop-lapop)}{RDpop} = \frac{lapop}{RDpop}\frac{\phi j}{\epsilon_\lambda a_z[v[1/a_z-1]-j]}$, and thus

$$\begin{aligned}
\tilde{c}_2 d_1 - c_1 d_2 &= \frac{\rho}{\epsilon_\lambda \kappa_L^2(1-a_z)}\left(\frac{v[1/a_z-1]-a_z j}{v[1/a_z-1]-j}\right) \\
&+\frac{1}{a_z\epsilon_\lambda\kappa_L}\frac{lapop}{RDpop}\left(\frac{v(v[1/a_z-1])-\epsilon_\lambda\kappa_{RD}a_z j)}{(v[1/a_z-1]-j)^2}\right) \\
&+\frac{\kappa_{RD}}{\epsilon_\lambda\kappa_L}\frac{lapop}{RDpop}\left(\frac{\rho\phi}{a_z}-\epsilon_\lambda\right)\frac{j^2}{(v[1/a_z-1]-j)^2}
\end{aligned}$$

As $v[1/a_z-1]-j > v[1/a_z-1]-a_z j > 0$ it is sufficient that $a_z \leqslant \frac{\rho\phi}{\epsilon_\lambda}$ to ensure $c_2 d_1 - c_1 d_2 > \tilde{c}_2 d_1 - c_1 d_2 > 0$. Note that given that the first two terms are positive, and the first increases as a_z increases, even when $A1$ does not hold, and the third term is negative the denominator may still be positive.

Proof of Proposition 3:

Proposition 3 assumes population growth is keep constant, $dg_n = 0$ (which implies $dg = dg^Z = 0$), and focuses on changes in the demographic structure particularly considering an increase the share of retirees (ageing) and thus $dRDpop < 0$.

Combining (A.73) we obtain two conditions linking labour output share and the degree of automation with changes in demographic structure and changes in interest rates. The system of equation, using the definitions of c_1, c_2, d_1 and d_2, becomes

$$\frac{dRDpop}{RDpop} - \frac{\rho}{\kappa_L}\frac{dR}{R} = -c_1\frac{da_z}{a_z} - d_1\frac{dy_L}{y_L(1-y_L)}$$

$$\frac{dRDpop}{RDpop} - \frac{\kappa_{RD}}{\kappa_L}\frac{dR}{R} = c_2\frac{da_z}{a_z} + d_2\frac{dy_L}{y_L(1-y_L)}$$

As the labour income share is given by $ls_t = (1-\alpha)(1-\gamma_I)\frac{\psi-1}{\psi}y_{L,t}$ we have that

$$\frac{da_z}{a_z} = \frac{\frac{dRDpop}{RDpop}(d_1+d_2) - \left(d_1\frac{\kappa_{RD}}{\kappa_L}+\frac{\rho}{\kappa_L}d_2\right)\frac{dR}{R}}{c_2 d_1 - d_2 c_1} \tag{A.76}$$

$$\frac{dls}{ls} = (1-y_L)\frac{-\left(\frac{dRDpop}{RDpop}(c_1+c_2) - \left(c_1\frac{\kappa_{RD}}{\kappa_L}+\frac{\rho}{\kappa_L}c_2\right)\frac{dR}{R}\right)}{c_2 d_1 - d_2 c_1} \tag{A.77}$$

As the denominator is positive that concludes the proof of proposition 3.

Appendix D. More on Calibration

This Section reports the values of the set of parameters of the model.

Table A.1: Calibration

Parameter	Value	Target/Source
Time Discount Factor	$\beta = 0.96$	Standard Value
Elasticity Intertemporal Substitution	$\upsilon = -3$	EIS = 0.25 (Gertler(1999))
Capital Depreciation Rate	$\delta = 0.08$	Standard Value
Capital Share in Production	$\alpha = 0.33$	Standard Value
Intermediate Share in Production	$\gamma_I = 0.5$	Comin and Gertler(2006)
Elasticity Substitution of Varieties	$\psi = 8$	Standard Value
Obsolescence	$\phi = 0.85$	Growth per Working age person
Productivity Innovation	$\chi = 5.67$	Share of innovation expenditure in GDP
Elasticity of Investment to Innovation	$\rho = 0.9$	Comin and Gertler (2006)
Elasticity of Final Goods to R&D Investment	$\kappa_{RD} = 1$	Comin and Gertler (2006)
Elasticity of Labour to R&D Investment	$\kappa_L = 0.5$	Aksoy et al. (2018)
Rate of Automation	$\lambda = 0.1$	Share of Automated Varieties
Robots Production Function	$\eta = 0.15$	Balanced Growth
Probability Transition from Mature to Old	$1 - \omega_w = 0.022$	Avg. Number of Years as Worker: 45y
Share of Workers in R&D	$Sw_{RD} = 0.07$	Share of R&D workers in Population
Probability Workers leaves R&D	$drop_{RD} = 0.07$	Average age of R&D workers

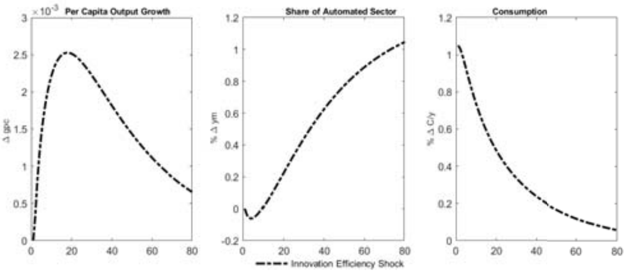
Table A.2: Calibration - Demographic Transition

Country	US	Year	1993	
Parameter		Value	Target/Source	
Birth Rate		$\omega_n = 0.0265$	Share of Workers in Population	
Death Probability of Old Agents		$1 - \omega_o = 0.07$	Share of Old in Population	Country
Country	US	Year	2055	
Parameter		Value	Target/Source	
Birth Rate		$\omega_n = 0.0236$	Share of Workers in Population	
Death Probability of Old Agents		$1 - \omega_o = 0.037$	Share of Old in Population	Country
Country	Europe	Year	1993	
Parameter		Value	Target/Source	
Birth Rate		$\omega_n = 0.0253$	Share of Workers in Population	
Death Probability of Old Agents		$1 - \omega_o = 0.06$	Share of Old in Population	Country
Country	Europe	Year	2055	
Parameter		Value	Target/Source	
Birth Rate		$\omega_n = 0.0206$	Share of Workers in Population	
Death Probability of Old Agents		$1 - \omega_o = 0.024$	Share of Old in Population	

Appendix E. Additional Simulation Results

First we show the impulse response to a shock to χ, the efficiency of innovation investment, discussed in *Corollary 1*.

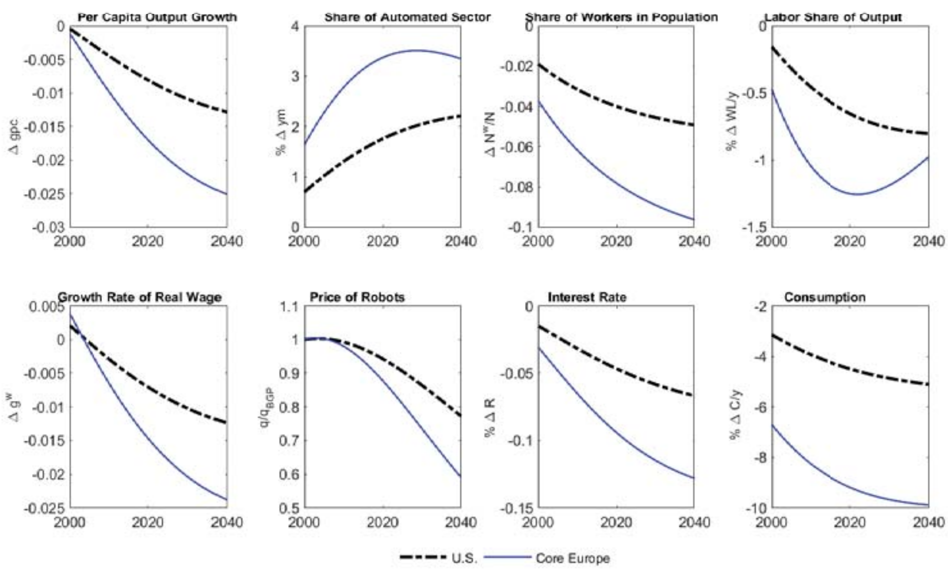
Figure A.1: Response to Permanent Increase in Investment Innovation Efficiency (χ)



Note: The figure plots the effects of the projected demographic changes in each region. For Per Capita Output growth = $(y_t/y_{t-1})(N_{t-1}/N_t)$ we show the Change relative to the initial BGP, for the Share of Automated Sector - $y_{m,t}$ and Consumption - C_t/y_t we show the percentage change relative to the initial BGP.

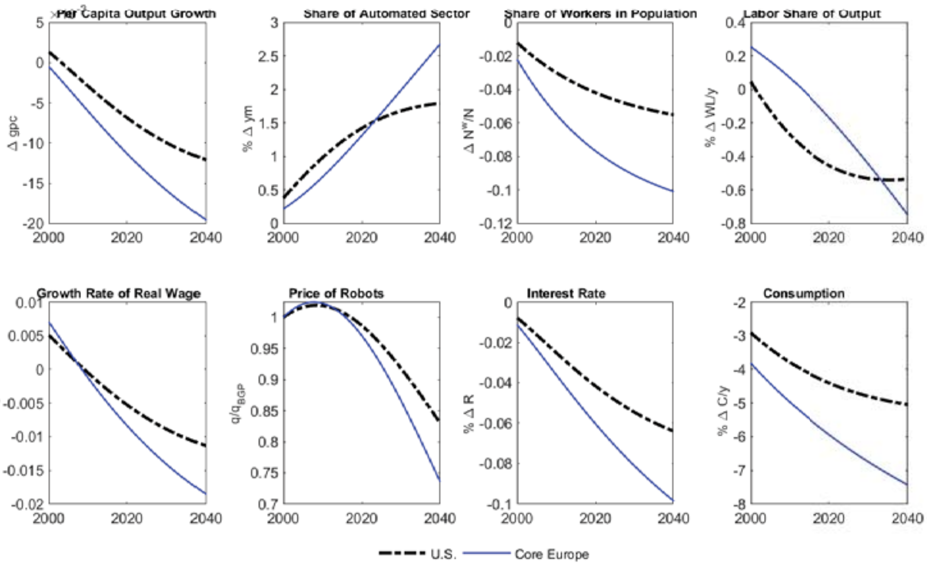
Then, we present additional simulation considering one case where the start year is 1990 instead of 1993, thus the demographic transition occurs from 1990 till 2055 and one case the start year is 1995 and the end year is 2060 instead of 2055. In both cases we depict the simulated variables from 2000 till 2040. In the first case the first 10 years of simulated data are discarded and in the second the first 5 years are discarded.

Figure A.2: Demographic Transition: United States and Europe



Note: Demographic Calibration: initial point 1990 - End point of 2055
The figure plots the effects of the projected demographic changes in each region. For Per Capita Output growth = $(y_t/y_{t-1})(N_{t-1}/N_t)$, Share of Workers in Population - N_t^w/N_t and Growth Rate of real wage - W_t/W_{t-1} we show the Change relative to the initial BGP. For Share of Automated Sector - $y_{m,t}$, Labour Share of Output W_tL_t/y_t, R_t - Real Interest Rate and Consumption - C_t/y_t we show the percentage change relative to the initial BGP. For the price of robots we show the ratio of price of robots during the transition and on the BGP - q_t/q_{BGP}.

Figure A.3: Demographic Transition: United States and Europe



Note: Demographic Calibration: initial point 1995 - End point of 2060
The figure plots the effects of the projected demographic changes in each region. For Per Capita Output growth = $(y_t/y_{t-1})(N_{t-1}/N_t)$, Share of Workers in Population - N_t^w/N_t and Growth Rate of real wage - W_t/W_{t-1} we show the Change relative to the initial BGP. For Share of Automated Sector - $y_{m,t}$, Labour Share of Output W_tL_t/y_t, R_t - Real Interest Rate and Consumption - C_t/y_t we show the percentage change relative to the initial BGP. For the price of robots we show the ratio of price of robots during the transition and on the BGP - q_t/q_{BGP}.

Appendix F. Model Extensions

We present the three more elaborate extensions of the model and then present the proof of proposition **??**.

F.1 Extension - Labour Choice Model

Under this extension, $Sw_{RD,t}$, the share of new workers that enter the economy and work in the R&D sector, is endogenous. In order to obtain that we assume a household, when entering her working life selects in which labour market (R&D or intermediate good production) to participate. At entry she is randomly assigned an efficiency level in R&D activity, denoted $\xi\tilde{\nu}_t^i$, where $\tilde{\nu}_t^i$ is drawn from a Pareto distribution with shape parameter $\epsilon > 1$ and support $[1, \infty)$. We denote the cumulative distribution by $F(\nu)$. The household then compares the human capital gain under the $R\&D$ sector (H_t^{RD}) which is a function of the wage W^{RD} and the average efficiency of workers in the sector, denoted $\nu_{m,t}$, and the human capital gain in the production sector (H_t, which is a function of the wage W) and selects in which labour market to be active in.

There exists a cut-off point ν_t^* such that given H_t^{RD} and H_t the household is indifferent between choosing each sector. Then, the share of households in $R\&D$ is given by

$$Sw_{RD,t} = \int_{\nu_t^*}^{\infty} dF(\nu) = \int_{\nu_t^*}^{\infty} \frac{\epsilon 1^{\epsilon}}{\nu^{\epsilon+1}} d\nu = \int_{\nu_t^*}^{\infty} \epsilon \nu^{-(\epsilon+1)} = (\nu_t^*)^{\epsilon}$$

The average efficiency of entrants in the R&D labour market is

$$\nu_{E,t} = \frac{\int_{\nu_t^*}^{\infty} \xi \nu dF(\nu)}{1 - F(\nu_t^*)} = \frac{\int_{\nu_t^*}^{\infty} \xi \epsilon \nu^{-(\epsilon)} d\nu}{1 - F(\nu_t^*)} = \xi \frac{\epsilon}{\epsilon - 1} \nu_t^*$$

The average efficiency of all workers in the R&D sector is then given by

$$\nu_{m,t} = \frac{Sw_{RD,t} \omega_{t,t+1}^{y} N_t^w}{N_{t+1}^{wRD}} \nu_{E,t} + (1 - drop_{RD}) \omega^w N_t^{wRD} N_{t+1}^{wRD} \nu_{m,t-1}$$

Defining

$$\begin{aligned} H_t^{jw} &= (W_t) + \frac{\omega^w}{R_{t+1} \mathfrak{z}_{t,t+1}} H_{t+1}^{jw}, \text{ where } j \text{ works in production} \\ H_t^{iw_{RD}} &= (\nu_{m,t} W_t^{RD}) + \frac{\omega^w}{R_{t+1} \mathfrak{z}_{t,t+1}} H_{t+1}^{iw_{RD}}, \text{ where } i \text{ works in R\&D} \end{aligned}$$

And since $\nu_{m,t}$ is a function of ν_t^*, ν_t^* is such that $H_t^{jw} = H_t^{iw_{RD}}$. Finally, we calibrate ϵ and ξ to obtain the same effective wage in R&D and Sw_{RD} at steady state as in the benchmark model.[31]

F.2 Labour Supply - Intensive Margin

In this extension we assume that all households also decide how much labour to supply (we allow retirees to also supply labour, although

Retiree j decision problem is

$$\max V_t^{jr} = \left\{ (C_t^{jr})^{\upsilon \mu_L} (\chi_r - l_t^{jr})^{\upsilon(1-\mu_L)} + \beta \omega_{t,t+1}^r ([V_{t+1}^{jr}]^{\upsilon}) \right\}^{1/\upsilon}$$

subject to

$$C_t^{jr} + FA_{t+1}^{jr} = \frac{R_t}{\omega_{t-1,t}^r} FA_t^{jr} + \xi W_t l_t^{jr} + d_t^{jr}.$$

Following similar steps as in the benchmark model we get

$$\begin{aligned} C_t^{jr} &= \varepsilon_t \varsigma_t \left[\frac{R_t}{\omega_{t-1,t}^r} FA_t^{rj} + H_t^{jr} + D_t^{rj} \right] \\ 1 - \varepsilon_t \varsigma_t &= \frac{\left(\beta R_{t+1} \left(\frac{W_t}{W_{t+1}} \right)^{(1-\mu_L)\upsilon} \right)^{1/(1-\upsilon)} \omega_{t,t+1}^r}{R_{t+1}} \frac{\varepsilon_t \varsigma_t}{\varepsilon_{t+1} \varsigma_{t+1}} \\ D_t^{jr} &= d_t^{jr} + \frac{\omega_{t,t+1}^r}{R_{t+1}} D_{t+1}^{jr}. \\ H_t^{jr} &= \xi W_t l_t^{jr} + \frac{\omega_{t,t+1}^r}{R_{t+1}} H_{t+1}^{jr}. \end{aligned}$$

[31] We also considered a case where we set $\kappa_L = 0$, so that the R&D sector does not need labour to innovate or automate. Results are similar to the *Labour Choice* case indicating that by allowing workers to migrate to research while young the labour supply constraint on research activity is almost fully mitigated.

$$
\begin{aligned}
(\chi_r - l_t^{jr}) &= \frac{\mu_L C_t^{jr}}{\xi W_t (1-\mu_L)} \\
V_t^{jr} &= (\varepsilon_t \varsigma_t)^{-1/\upsilon} C_t^{jr} (\chi_r - l_t^{jr})
\end{aligned}
$$

With endogenous labour supply wages affect the marginal propensity to consume. As a result we can no longer solve a single problem for all workers.

Production workers j decision problem is

$$
\max V_t^{jw} = \left\{ (C_t^{jw})^{\mu_L \upsilon} (\chi_w - l_t^{jw})^{\upsilon(1-\mu_L)} + \beta [\omega^w V_{t+1}^{jw} + (1-\omega^w) V_{t+1}^{jr}]^{\upsilon} \right\}^{1/\upsilon}
$$

subject to

$$
C_t^{jw} + FA_{t+1}^{jw} = R_t FA_t^{jw} + W_t l_t^{jw} + d_t^{jw}
$$

Following the same procedure as before we have that

$$
\begin{aligned}
C_t^{jw} &= \varsigma_t [R_t FA_t^{jw} + H_t^{jw} + D_t^{jw}] \\
\varsigma_t &= 1 - \frac{\varsigma_t}{\varsigma_{t+1}} \frac{\left(\beta R_{t+1} \mathfrak{Z}_{t+1} \left(\frac{W_t}{W_{t+1}} \right)^{(1-\mu_L)\upsilon} \right)^{1/(1-\upsilon)}}{R_{t+1} \mathfrak{Z}_{t,t+1}} \\
H_t^{jw} &= (W_t l_t^{jw}) + \frac{\omega^w}{R_{t+1} \mathfrak{Z}_{t,t+1}} H_{t+1}^{jw} + \frac{(1-\omega^w)(1/\xi)^{1-\mu_L} \varepsilon_{t+1}^{(\upsilon-1)/\upsilon}}{R_{t+1} \mathfrak{Z}_{t,t+1}} H_{t+1}^{jw} \\
D_t^{jw} &= d_t^{jw} + \frac{\omega^w}{R_{t+1} \mathfrak{Z}_{t,t+1}} D_{t+1}^{jw} + \frac{(1-\omega^w)(1/\xi)^{1-\mu_L} \varepsilon_{t+1}^{(\upsilon-1)/\upsilon}}{R_{t+1} \mathfrak{Z}_{t,t+1}} D_{t+1}^{jr} \\
(\chi_w - l_t^{jw}) &= \frac{\mu_L C_t^{jw}}{W_t (1-\mu_L)} \\
V_t^{jw} &= (\varepsilon_t \varsigma_t)^{-1/\upsilon} C_t^{jw} (\chi_w - l_t^{jw}) \\
\mathfrak{Z}_{t+1} &= (\omega^w + (1-\omega^w) \varepsilon_{t+1}^{(\upsilon-1)/\upsilon}).
\end{aligned}
$$

R&D workers j decision problem is

$$
\begin{aligned}
\max V_t^{jwR} &= \Big\{ (C_t^{jwR})^{\mu_L \upsilon} (\chi_{wR} - l_t^{jwR})^{\upsilon(1-\mu_L)} + \beta [\omega^w (1 - drop_{RD}) V_{t+1}^{jwR} \\
&\quad + \omega^w (drop_{RD}) V_{t+1}^{jw} + (1-\omega^w) V_{t+1}^{jr}]^{\upsilon} \Big\}^{1/\upsilon}
\end{aligned}
$$

subject to

$$
C_t^{jwR} + FA_{t+1}^{jwR} = R_t FA_t^{jwR} + W_t^{RD} l_t^{jwR} + d_t^{jwR}
$$

Following the same procedure as before we have that

$$
\begin{aligned}
C_t^{jwR} &= \varsigma_t o_t [R_t FA_t^{jwR} + H_t^{jwR} + D_t^{jwR}] \\
\frac{1 - o_t \varsigma_t}{o_t} &= 1 - \frac{\varsigma_t}{\varsigma_{t+1}} \frac{\left(\beta R_{t+1} \mathfrak{Z}^{\mathfrak{RD}}{}_{t+1} \left(\frac{W_t^{RD}}{W_{t+1}} \right)^{(1-\mu_L)\upsilon} \right)^{1/(1-\upsilon)}}{R_{t+1} \mathfrak{Z}^{\mathfrak{RD}}{}_{t,t+1}} \\
H_t^{jwR} &= (W_t^{RD} l_t^{jwRD}) + \frac{\omega^w}{R_{t+1} \mathfrak{Z}^{\mathfrak{RD}}{}_{t,t+1}} H_{t+1}^{jw} + \frac{(1-\omega^w)(1/\xi)^{1-\mu_L} \varepsilon_{t+1}^{(\upsilon-1)/\upsilon}}{R_{t+1} \mathfrak{Z}^{\mathfrak{RD}}{}_{t,t+1}} H_{t+1}^{jw}
\end{aligned}
$$

$$+\frac{(1-\omega^w)drop_{RD}\left(\frac{W_{t+1}}{W^{RD}_{t+1}}\right)^{(1-\mu_L)\upsilon} o^{(\upsilon-1)/\upsilon}_{t+1}}{R_{t+1}\mathfrak{Z}^{\mathfrak{RD}}{}_{t,t+1}} H^{jwRD}_{t+1}$$

$$D^{jwR}_t = (W^{RD}_t l^{jwRD}_t) + \frac{\omega^w}{R_{t+1}\mathfrak{Z}^{\mathfrak{RD}}{}_{t,t+1}} D^{jw}_{t+1} + \frac{(1-\omega^w)(1/\xi)^{1-\mu_L}\varepsilon^{(\upsilon-1)/\upsilon}_{t+1}}{R_{t+1}\mathfrak{Z}^{\mathfrak{RD}}{}_{t,t+1}} D^{jw}_{t+1}$$

$$+\frac{(1-\omega^w)drop_{RD}\left(\frac{W_{t+1}}{W^{RD}_{t+1}}\right)^{(1-\mu_L)\upsilon} o^{(\upsilon-1)/\upsilon}_{t+1}}{R_{t+1}\mathfrak{Z}^{\mathfrak{RD}}{}_{t,t+1}} D^{jwRD}_{t+1}$$

$$(\chi_{wR} - l^{wR}_t) = \frac{\mu_L C^{jwR}_t}{W_t(1-\mu_L)}$$

$$V^{jwR}_t = (\varepsilon_t\varsigma_t)^{-1/\upsilon} C^{jwR}_t (\chi_{wR} - l^{jwR}_t)$$

$$\mathfrak{Z}^{\mathfrak{RD}}{}_{t+1} = ((1-\omega^w)drop_{RD} o^{(\upsilon-1)/\upsilon}_{t+1}\left(\frac{W_{t+1}}{W^{RD}_{t+1}}\right)^{1-\mu_L}\omega^w drop_{RD} + (1-\omega^w)\varepsilon^{(\upsilon-1)/\upsilon}_{t+1}).$$

Finally, in order to ensure unique transition path we assume innovators and automation investors pay a cot to adjust labour demand given by $\frac{\epsilon}{2}(L_{X,t} - g_n L_{X,t-1})^2$, for $X = I, A$.

Allowing for adjustment of labour supply in the intensive margin affects the impact of the demographic transition in the medium run. However, as the economy converges to a new steady state, labour supply of each workers converges to a constant and thus the growth rate of labour employed in production becomes equal to the population growth $g_{L,t} = g_{n,t}$. Thus, the individual effective labour supply remains constant on a BGP and the conditions driving the result in proposition 1 remain constant.

F.3 Depreciation of Robots

We assume at every period robots producers start with $(1-\delta_R)M_t$ amount of robots and invest Ω_t and get $I^R_t = \varrho(\Omega_t)^\eta$. Robots are rented to firms at a price q_t. Problem of robots producers is

$$\max_{\Omega,t}\sum_{t=0}^{\infty}\beta^t\Pi_{\Omega,t} = q_t M_t - \Omega_t \quad s.t. \quad M_t = \varrho\Omega^\eta_t + (1-\delta_R)M_{t-1}. \tag{A.78}$$

Maximisation conditions are

$$\frac{\eta\varrho q_t}{\Omega^{1-\eta}_t} = 1-(1-\delta_R)\frac{\Omega^{1-\eta}_{t+1}}{\Omega^{1-\eta}_{t+1}} \tag{A.79}$$

$$M_t = \varrho\Omega^\eta_t + (1-\delta_R)M_{t-1} \tag{A.80}$$

If (A.79) holds then on a BGP, $(g_t)^{\eta-1}g_{q,t} = 1$ and $\frac{\Omega_t}{y_t}$ is constant. Thus, (**??**) in proposition 1 continues to hold and thus restriction on η to ensure BGP exists is unchanged in this extension.

F.4 Long-run Effect Under Alternative Scenarios

Proof of Proposition ??: Using the optimisations conditions of firms and robot's producers we set the system of equations that determine output growth. As we have seen from proposition one, this set of equations are made of the two equations that determine the growth of relative prices in each sector ($g_{pm,t}$, $g_{pL,t}$), the two demand equation for output in each sector coming from the final good optimisation conditions which determine the change in the relative output share in each sector ($y_{m,t}$, $y_{L,t}$) and the production function in the robots production sector that determines the change in robots-output ratio (m_t). Formally,

$$g_{pm,t} = \left(\frac{(r_t^k + \delta)}{(r_{t-1}^k + \delta)}\right)^{\alpha(1-\gamma_I)} \left(\frac{\theta_{t-1}}{\theta_t}\right)^{(1-\alpha)(1-\gamma_I)} g_{q,t}^{(1-\alpha)(1-\gamma_I)} \tag{A.81a}$$

$$\frac{y_{m,t}}{y_{m,t-1}} = g_t^A g_{pm,t}^{1-\psi} \tag{A.81b}$$

$$g_{pL,t} = \left(\frac{(r_t^k + \delta)}{(r_{t-1}^k + \delta)}\right)^{\alpha(1-\gamma_I)} \left(\frac{ls_t}{ls_{t-1}}\right)^{(1-\alpha)(1-\gamma_I)} \left(\frac{g_t}{g_{L,t}}\right)^{(1-\alpha)(1-\gamma_I)} \tag{A.81c}$$

$$\frac{y_{L,t}}{y_{L,t-1}} = g_t^{ZA} g_{pL,t}^{1-\psi} \tag{A.81d}$$

$$\frac{m_t}{m_{t-1}} = \left(\frac{\tilde{\Omega}_t}{\tilde{\Omega}_{t-1}}\right)^{\eta} (g_t)^{\eta-1} g_{q,t}. \tag{A.81e}$$

First consider the two extensions that alters the function of labour markets. The first extension, *Labour Choice*, allow young workers who enter the labour market to select which sector to supply labour. This decision is function of the ratio of wage in R&D, W_{RD}, and the wage in the labour-intensive sector W. During the demographic transition as the ratio of wages changes, Sw_{RD}, the share of workers in R&D, increases, labour supply in production falls and thus $g_{L,t} = \frac{L_t}{L_{t-1}}$ falls relative to the benchmark model. However, as the economy approaches the new BGP with a stationary demographic composition, the growth rate of W_{RD} and W are matched and thus Sw_{RD} is constant on a BGP. To see this note that from (A.69f) and (A.69k) $li_t + la_t = (L_{I,t} + L_{A,t})W_{RD}/Y_t = (N_t^{wRD})W_{RD}/Y_t$ is constant on a BGP and from (A.68i), $ls_t = N_t^{wL} W_t/Y_t$ is also constant. As $N_t^{wRD} + N_t^{wL} = N_t^w$, a BGP with constant output shares implies $g_t^w = \frac{N_t^w}{N_{t-1}^w} = g_t^{wRD} = \frac{N_t^{wRD}}{N_{t-1}^{wRD}} = g_t^{wL} = \frac{N_t^{wL}}{N_{t-1}^{wL}}$. As such the growth rate of labour supply in this extension is the same as in the benchmark case and is given by $g_{L,t} = \frac{L_t}{L_{t-1}} = g_{w,t} = \frac{N_t^w}{N_{t-1}^w} = g_{n,t}$. Thus, the set of conditions above deliver the same result as in the benchmark case: $g_t^\eta = g_{L,t} = g_{n,t}$.

A similar argument is developed for the second extension, *Late Retirement*. In this extension, we assume that while longevity is increasing (ω^r), we assume ω^w, also increases, such that the portion of life in retirement does not increase relative to life as a workers. Once the economy approaches to a stationary demographic composition, the share of workers and retirees remains constant and their growth is matched and is a function of fertility and longevity, which are the same in all extensions. Thus, once again, $g_{L,t} = g_{n,t}$, and $g_t^\eta = g_{n,t}$.

The extension that allows robots to be used in R&D, *Robots in R&D*, alters the creation of goods and automation activity such that it does not rely on labour but on labour and robots. The main effect of such change is to curtail the increase in W_{RD} that hampers innovation and automation during the demographic transition as the share of young entering the economy and supply labour in the $R\&D$ sector decreases. Nonetheless, as discuss in *Corollary 1*, on the BGP, g_Z is ultimately a function of population growth. Also note that robots and labour growth at the same rate on a BGP. Thus, on the BGP the set of conditions A.81 continue to hold and $g_t^{\eta} = g_{n,t}$.

Finally, when the productivity of robots changes, *Robots Productivity*, $\theta_t = \bar{\theta} A_t^{\mu}$ is no longer constant and as such using the system of equation A.80 we have that

$$g_{pM,t} = \left(g_{q,t} \frac{\bar{\theta} A_{t-1}^{\mu}}{\bar{\theta} A_t^{\mu}} \right)^{(1-\alpha)(1-\gamma_I)} = \left(\frac{g_t}{g_{n,t}} \right)^{(1-\alpha)(1-\gamma_I)} = g_{pL,t}, \tag{A.82}$$

$$(g_t)^{\eta-1} g_{q,t} = 1, \tag{A.83}$$

$$g_t^A = (g_t/g_{n,t})^{(\psi-1)(1-\alpha)(1-\gamma_I)} \tag{A.84}$$

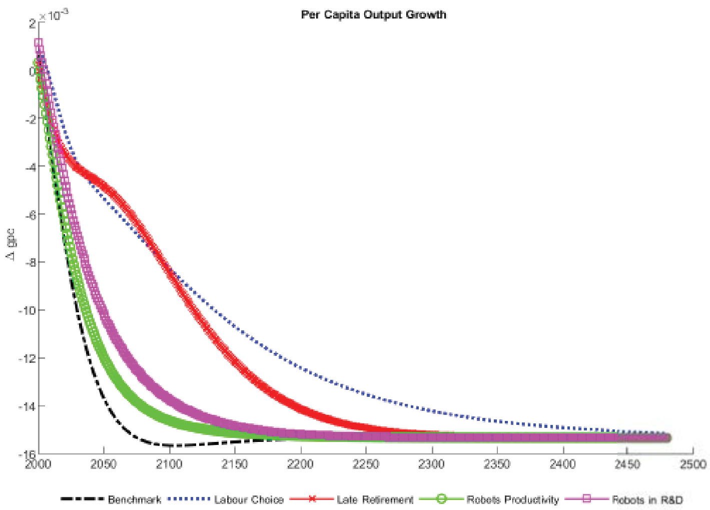
Combining (A.82), (A.83) and (A.84)

$$g_t^{\frac{\hat{\eta}+\mu(\psi-1)(1-\alpha)(1-\gamma_I)}{1+\mu(\psi-1)(1-\alpha)(1-\gamma_I)}} = g_{n,t} \tag{A.85}$$

Since the benchmark in all extensions have the same initial level of output growth and population growth, the calibration procedure implies $\frac{\hat{\eta}+\mu(\psi-1)(1-\alpha)(1-\gamma_I)}{1+\mu(\psi-1)(1-\alpha)(1-\gamma_I)} = \eta_{Benchmark}$. As a result, due to the calibration, $g_t^{\eta} = g_t^{\frac{\hat{\eta}+\mu(\psi-1)(1-\alpha)(1-\gamma_I)}{1+\mu(\psi-1)(1-\alpha)(1-\gamma_I)}} = g_{n,t}$, and as $g_{n,t}$ changes the effect on g_t is the same in all extensions.

In order to illustrate this result we show the long-run effect on per capita growth of the demographic changes projected for the US using the calibrated versions of the benchmark and four extensions in Figure A.4. Note that in this long-run simulation, the demographic transition continues to occur from 1993 until 2055. Thus from 2055 fertility and longevity no longer change. However, the economy has takes considerably longer to reach the new BGP.

Figure A.4: Demographic Transition: Alternative Scenarios - Long Run Effect

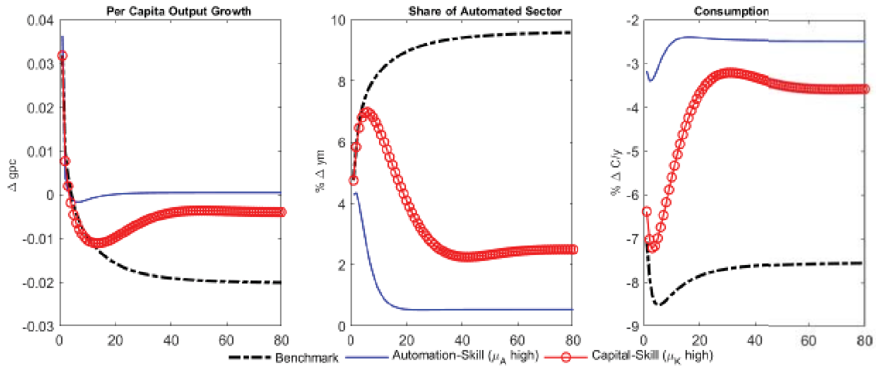


Note: The figure plots the effects of the projected demographic changes under different specifications. For Per Capita Output growth = $(y_t/y_{t-1})(N_{t-1}/N_t)$ we show the Change relative to the initial BGP.

F.5 Shock to η - Robots Production

In this section we show the effect of a shock to η in the benchmark model and in the model with automation and capital skill complementarities.

Figure A.5: Impulse response to η Shock



Note: The figure plots the effects of the projected demographic changes under different specifications. For Per Capita Output growth = $(y_t/y_{t-1})(N_{t-1}/N_t)$ we show the Change relative to the initial BGP. For Share of Automated Sector - $y_{m,t}$ and Consumption - C_t/y_t we show the percentage change relative to the initial BGP.

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