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ABSTRACT

Spanish Treasury bond auctions have two distinctive features. First, the format used is a hybrid system of discriminatory and uniform price auctions. Second, there is uncertainty about the amount to be issued, since the announced target volume is not compulsory and it is established jointly for two different bonds that are auctioned separately.

This paper explores Spanish Treasury bond auctions both from a theoretical and an empirical perspective. In the theoretical analysis we present a model to explore the revenue efficiency of Spanish Treasury bond auctions. Given the complexity of the Spanish auction game, the model abstracts from many features of the market, but it captures the two distinctive features of the Spanish auction: the format used and the uncertainty about the amount to be issued. The main result is that there exists for the Spanish auction format a pure strategy Nash equilibrium that maximizes the seller's revenue, which is unique in the sense that each of the equilibria gives the same utility to all players. This result suggests that both the discriminatory and the Spanish auction format behave in a similar way in the model proposed.

The empirical analysis uses data of Spanish bond auctions between 1993 and 1997 to test the predictions of the model and to establish the main characteristics of Spanish bond auctions. The main results are as follows. First, as predicted by the model, auction discounts are small in size and not statistically significant for auctions with volume announcements where the bond auctioned is identical to an existing one traded on the secondary market, which we think that the model better characterizes. Second, both participation and competition are significant determinants of the size of auction discounts: participation has a positive effect, since as the number of bidders increases the winner's curse is more severe, and they bid less aggressively. Competition, measured by the cover ratio (volume bid over volume accepted), has a negative effect, as competition reduces the probability of winning and induces bidders to bid more aggressively. Third, variables measuring price and quantity uncertainty faced by bidders also have a positive effect on the size of auction discounts. Although the analysis presented here does not allow for the separation of the effect of price uncertainty from the effect of quantity uncertainty, there is some indication that both sorts of uncertainty matter.

1. INTRODUCTION

Auctions are among the oldest mechanism of price discovery, and nowadays are a common form of organizing trade. They are used to allocate art objects, fish, oil drilling rights, as well as financial securities.

One of the most important auction markets in the world is the market for government debt. Treasuries apply mainly two auction formats: discriminatory and uniform price auctions. The majority of the Treasuries around the world use discriminatory auctions. In a discriminatory auction, winning bidders pay their bid price. A few Treasuries use uniform price auctions, where all winning bidders pay the same price for each unit, the minimum accepted price. But the Spanish Treasury is the only one that uses a hybrid system of discriminatory and uniform price auctions: winning bidders pay their bid price if it is lower than the weighted average price of winning bids, while all other winning bidders pay the weighted average of winning bids. With the Spanish format, the price that a bidder has to pay depends on the bids of all other winning bidders, including his own bids. This fact increases the players' strategic considerations with respect to discriminatory and uniform auctions, even in the more simple models.

A principal aim of auction theory is to identify the auction mechanism that maximizes the expected revenue of the seller. In environments that consider the auction of a single, indivisible good, theoretical models are able to order different auction formats according to expected revenue for the seller. Settings with multiple identical items, where each bidder demand only one unit yield similar results. However, in government debt auctions bidders usually make multiple bids, where a bid is a price-quantity pair. In environments with multiple units and bidders that may buy more than one unit, general results even for the most common auction forms, discriminatory and uniform auctions, remain elusive. The reason is that the game becomes very complicated, since with multiple units, bidders have a very large strategy space. Theoretical studies use a lot of simplifying assumptions, given the complexity of the game, and results are ambiguous. Empirical analyses present also mixed results. For example, the U.S. Treasury traditionally used discriminatory auctions, but now is using also the uniform format for certain issues.

Compared to the overwhelming amount of work about uniform and discriminatory auctions, very little has been said about the Spanish auction format. To our knowledge, the properties of the Spanish auction mechanism have been study only by Salinas (90) and Martínez Méndez (96). Salinas presents a model where demand is restricted to one unit per bidder, and each bidder's reservation price for the good is an independent draw for the same distribution. He uses the results of Maskin and Riley (89) to argue that the Spanish mechanism generates the same expected revenue than uniform and discriminatory auctions. But these two assumptions are not appropriate for Treasury auctions: bidders usually bid for more than one unit and the value of the good is assumed to be common but unknown for all bidders, given the existence of a secondary market. Martínez Méndez (96) offers a detailed technical description of both primary and secondary market for government securities in Spain and discusses the principal aspects of the Spanish auction.

Besides the auction format, Spain (and Hungary for bills of certain maturity), is the only country that has opted "for the extreme solution of not even announcing the amount of bills to be issued", as noted by Bartolini and Cottarelli (94). The authors mention that all other 41 countries in the sample they use, announce the volume of the auction, even if most of them maintain downward flexibility on the issue. This practice has changed since 1995, when the Spanish Treasury started to announce a maximum amount and a target to be issued. But there is still uncertainty on the amount auctioned, both because the target is not compulsory, and because both the maximum and the target figures are announced jointly for two different bonds, that are auctioned separately.

The general director of the Spanish Treasury, Jaime Caruana, mentioned recently that "the adoption of the euro will establish a more efficient market, in which the Spanish debt will have to compete with other countries' debt on interest rates, credit quality and calendar" (El País, April 14, 1998). Although he did not mention that it will have to compete with a different auction mechanism, his statement calls attention on the fact that competition will increase after 1999. Thus, it is important to establish the characteristics of the Spanish auction mechanism, both from the point of view of the seller and the buyers.

This paper explores the revenue efficiency of the Spanish Treasury auctions. First, we present a stylized game theoretical model that captures the two distinctive features of the Spanish auction: the hybrid system of uniform and discriminatory auctions used; and the uncertainty about the amount to be issued. We find the pure strategy Nash equilibria, and compare it with the equilibrium of a discriminatory auction in the same model. We show that, under the assumptions of the model, the auction format used in Spain is equivalent it terms of revenue to the seller to the discriminatory format, and that both formats maximize the seller's revenue. Second, we present an empirical analysis, using data of Spanish bond auctions between 1993 and 1997, to test the predictions of the model and to establish the main characteristics of the Spanish bond auctions. We present summary statistics for the data, that evidence the good functioning of the market, and the relatively low price differentials paid by accepted bids. We test revenue efficiency and study the determinants of the price differential of auction prices with secondary market prices.

Our paper proceeds as follows. In section 2 we describe the auction rules for the Spanish auction. In section 3 we present the theoretical model: 3.1 surveys the theoretical literature, 3.2 presents the model, 3.3 presents the results and 3.4 concludes the theoretical part stating the implications of the model for the empirical analysis. In section 4 we present the empirical analysis: 4.1 describes the data and 4.2 presents the empirical analysis results. We conclude in section 5.

2. AUCTION RULES

Spanish Treasury bond auctions follow a regular calendar that is announced annually, usually in February. At present, bonds are issued monthly except for 30-years bonds that are auctioned every two months. Auctions for 3- and 5-years bonds take place, separately, on the Tuesday following the last Monday in each month, and auctions for 5- and 15-years bonds take place, also separately, on the next day. Auctions for 30-years bonds take place the same day that those for 3- and 10-years bonds. Since February 1998 the settlement date for issued securities is 3 days after auction, but before that date settlement was around 10 days after auction.

Government bond issues are reopened through successive auctions; that is, bonds with identical coupon, maturity and coupon payment dates are successively auctioned until the volume outstanding reaches a certain size. Therefore, each issue remains open a variable number of auctions, depending on demand. The main objective of this policy (used by other countries such as France, UK, Italy and Japan) is to encourage development of the secondary market and to avoid attempts at cornering the market.

Since each issue remains open for a number of auctions not known when the annual calendar is set, the announcement of the nominal features of the bonds to be auctioned are made some two weeks before auction takes place. The announcement does not include the volume to be issued. This practice, which is a singular feature of the Spanish system, was partially abandoned three years ago. Thus, since July 1995 the Treasury establishes a (compulsory) minimum amount to be issued: 30 billion Ptas. in each auction for 3, 5 and 10 years bonds, and 15 billion Ptas. for 15 year bonds. Besides, the Friday before the auction takes place, the Treasury announces, after consultation with market makers, a maximum and a target amount offered. Nevertheless, there is still uncertainty on the volume to be issued, first because the target amount is not compulsory and second because both maximum and target figures are set jointly for auctions of 3 and 10 years bonds on one hand and for auctions of 5 and 15 years bonds on the other.

Any investor, whether resident or non-resident, can participate in the market submitting competitive or non-competitive bids. Participation may be direct or through a member of the public debt market[1].

Non-competitive bids are subject to a limit of 25 million Ptas. per bidder, which is small compared to the average size of a trade in the secondary market among members, which is about 760 million. Competitive bids are subject to a (low) minimum limit of 500,000 Ptas. but do not have a maximum limit and the number of bids submitted by any single investor is unrestricted.

Direct bids by a non-member of the public debt market have to be submitted up to one day in advance of auction day and are subject to a disbursement of 2% of the bid. In contrast, bids by members of the public debt market do not require disbursement and have to be submitted between 8.30 and 10.30 a.m. on auction day.

Auction resolution takes place before noon and determines the accepted volume and the minimum accepted price and the weighted average price of accepted bids (hereafter, stop out price and WAP respectively). Non-competitive bids are fully awarded at the WAP. Competitive bids below the WAP are awarded at the bid price and bids above it pay the WAP. As mentioned earlier, this auction format is only used by the Spanish Treasury. All other Treasuries use either discriminatory or uniform formats.

Immediately after resolution, auction results are made public. Information includes total volume submitted, total volume accepted, non-competitive volume submitted; WAP and yield of accepted bids; stop out price and yield; nominal value placed at the stop out price; first non-accepted price and quantity bid at that price. Some days later, the official government gazette publishes the same information plus the total amount placed at the WAP and the amounts placed at each particular price between the WAP and the stopout price. There is no public information on the number of bids or bidders.

Immediately after auction results are published, a second-round auction may take place where only market makers can participate. Thus, if no volume is pre-announced for the ordinary auction, or if a volume is announced and fully covered, the Treasury opens a mandatory second-round auction[2]. In this round, each market maker may (voluntarily) submit up to

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[1]Almost any financial institution can be a member of the public debt market. For a description of Spanish bond market organization see Pellicer(92).

[2] If a volume was pre-announced for the first round auction and less than 70% of it is placed, the second-round auction is not mandatory. This has never been the case in the period under study even since July 1995, when the Treasury started to announce a maximum and a target amount to be issued, since it has been interpreted that these figures do not qualify as a formal announced volume.

three bids at prices higher or equal to the WAP prevailing in the first-round auction. Accepted bids pay the bid price, so these auctions follow a discriminatory format. Provided that there is enough demand, the Treasury is obliged to place at least a specific amount[3].

3. THEORETICAL MODEL

3.1 Survey of the literature

As mentioned above, a principal aim of auction theory is to identify the auction mechanism that maximizes the expected revenue of sellers. Most of the theory refers to auctions of indivisible goods, and compares uniform price auctions (second-price auctions) with discriminatory auctions (first-price auctions). Milgrom and Weber (82) show that if the good is indivisible, the bidders are risk-neutral and bidders' valuations are affiliated[4], uniform price auctions yield at least as large revenue as discriminatory auctions. Revenue is equal for both auction formats if independent private values are assumed, while uniform price auctions yield more revenue than discriminatory auctions if common values are assumed. The idea is that the winner's curse[5] is less severe in a uniform price auction, and bidders bid more aggressively. In general, Treasury auctions are considered common value auctions. But as Ranjan Das and Sundaram (97) note, "the one clear conclusion to have come out of recent theoretical studies is that no useful lesson on Treasury auction format can be gained from the study of auctions of indivisible goods": the assumption on the indivisibility of the good being auctioned is critical in the Milgrom and Weber result.

Back and Zender (93), Wang and Zender (98) and Asubel and Cramton (98) address the issue

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[3]10% if accepted competitive bids are, in nominal terms, higher than 50% of the total quantity bid, and 20% otherwise.

[4]Affiliation implies that the bidders' valuations are positively correlated, and includes the two most usual assumptions about valuations, i.e. independent private values and common values, as special cases. In an independent private value model, bidders' valuations are independent, while in a common value model, the value of the item to be auctioned is common but unknown.

[5] The winner's curse can arise in common value models. In such models, bidders base their bid on their estimate of the item's value; this raises the possibility that winning is bad news: a bidder wins if all other bidders estimated the common value to be lower.

of ranking uniform and discriminatory auctions in terms of seller revenues, in the case of divisible goods, with smooth demand schedules. Wang and Zender (97) obtain an analytical solution and fully characterize the set of equilibria under risk neutrality and constant absolute risk aversion utility. They use the common value assumption: the good being sold has an unknown value; this is the usual assumption for Treasury security auctions, given the existence of a secondary market. In their model, and assuming that the noncompetitive demand is uniformly distributed, if bidders are risk-neutral, the expected revenue in a uniform auction is smaller than in a discriminatory auction in almost all equilibria of a uniform-price auction. If bidders are risk-averse, the result is ambiguous. This result follows because they obtain a continuum of equilibria for the uniform price auction. In some of the equilibria, the ability to submit very steep demand curves provides the bidders with an important strategic advantage. Asubel and Cramton (98) also establish that the ranking of uniform and discriminatory auctions is ambiguous: they are able to construct reasonable specifications of demand where the discriminatoy auction dominates the uniform auction on expected revenue for the seller, and equally-reasonable specifications of demand where the reverse ranking holds. Thus, they conclude that the choice between auction formats ought to be viewed as an empirical question that depends on the actual nature of demands.

Menezes (95) considers a discriminatory auction with supply uncertainty and shows that there is a unique pure Nash equilibrium that maximizes the sellers' revenue. He assumes that a bid is one price-quantity pair, and captures supply uncertainty introducing a positive probability that the bidders with the lowest price bid may not receive an allocation.

We adapt Menezes' (95) model to represent the Spanish auction format. Therefore, each bidder submits one price-quantity pair, and supply uncertainty is introduced as a positive probability of receiving a zero award. In the Spanish Treasury´s auctions, bidders use multiple bids, and therefore the first assumption must be considered only as an initial approximation. The introduction of uncertainty as a positive probability of receiving a zero award seems appropriate for the Spanish case, where the Treasury has the option of cutting the announced supply objective, and awarding zero to the bidders with the lowest price bid, even if the announced target quantity is not sold.

3.2. The model

We adapt Menezes' (95) model to the Spanish auction format. The auction rules are as follows. The auctioneer announces a minimum price p^0 and an objective for the quantity he wants to sell, Y. Without loss of generality, we assume that $p^0 = 0$ and that there are two risk-neutral competitive bidders, denoted by 1 and 2. Their bids have two components: a price and a quantity, stating the price they are willing to pay for the specified amount of the securities auctioned.

Let x and y be, respectively, the quantities demanded by bidders 1 and 2. The auctioneer orders the bids according to the price, starting from the highest, until the total amount that he wants to sell, Y, is awarded. Following Menezes, we assume that if there is a tie, that is, if both bidders submit the same price-bid, there are two possible cases: (i) if the sum of their bids is less than or equal to Y, each bidder receives the amount requested; (ii) if the sum of their bids is greater than Y, each bidder receives a quantity proportional to his bid. Define the cut-off price, p, as the highest price at which aggregate demand is equal to the quantity offered, or the lowest submitted price if $\{p/x+y \geq Y\} = \varnothing$.

Each bidder i, $i = 1,2$, is characterized by a demand function $D_i(p)$, which specifies for each price $p \in [0, p^+]$ the desired quantity. We assume that $D_i(p^+) = 0$ for $i = 1,2$, and that $\Sigma_{i=1}{}^2 D_i(0) > 0$.

We assume that the demand functions are continuous and strictly decreasing. These demand functions are common knowledge for the bidders, but not for the auctioneer[6]. Refer to p^* as the price such that market clears. Therefore $D_i\ (p^*)$ denotes bidder i´s demand at the market clearing price. Given our assumptions, if there is a market clearing price, it is unique.

Competitive bidders have a positive probability of having a zero award: if they bid the lowest price, there is a positive probability of receiving zero. Let $\xi(p):[p^0,p^+]\rightarrow[0,1]$ be the probability function that determines, for each cut-off price p, the probability that the player submitting this bid will not receive his award. We assume that $\xi(p)$ is continuous and decreasing in p, with $\xi(0)=1$. That is, the player that bids the lowest price has a positive probability (and higher the lower his price bid is), of having a zero award. And if both price bids are equal, both players face this probability. Following Menezes, we assume that $\xi(p) = 0$ for $p \in [p^*, p^+]$[7].

A bidding strategy for bidder i, $b_i(D_i(p), D_j(p), \xi(p))$, $j \neq i$, is a mapping from i's information set into his set of actions. Hence a strategy for player 1, $b_1(.)$, is a pair (p,x), representing a price and a quantity demanded at that price. Denote by (q,y) the price and

[6]This assuption is in Menezes' model. The auctioneer does not know the demand functions, because otherwise he/she would use a take-it-or-leave-it type of mechanism.

[7]If we consider p^* as the expected price in the secondary market, known to all participants in the market, it could be argued that the Treasury would supply the announced quantity Y if bid prices are equal or greater than p^*, but, with a positive probability, would reduce Y if bid prices are below p^*.

quantity bid by player 2.

Let $\pi_i: B_1 x B_2 \rightarrow R$ denote the payoff function for player i, where B_i denotes the feasible bid for player i. Thus, given the auction rules, player 1's payoff function, $\pi_1[p,x,q,y]$, follows from the following considerations:

(i) If $p>q$, player 1 receives with probability 1 the amount of his bid, and the cut-off price is q. His payoff is equal to the area under the demand curve minus the price that he pays, which depends on the price-quantity bid of player 2: with probability $[1-\xi(q)]$ player 2 receives at price q a positive quantity of the good, and player 1 pays the average price, which varies depending on whether $x+y \leq Y$, when player 2 receives quantity y, or $x+y>Y$, when player 2 receives (Y-x); and with probability $\xi(q)$, player 2 receives 0, and the price player 1 pays is p.

(ii) If $p \leq q$, player 1 pays p, the cut-off price is p, and therefore player 1 has a probability $[1-\xi(p)]$ of receiving a positive amount of the good, which will be equal to $[x/(x+y)]Y$ if $p=q$ and $x+y>Y$; equal to $(Y-y)$ if $p<q$ and $x+y>Y$; and equal to x if $p \leq q$ and $x+y \leq Y$.

Therefore, $\pi_1[p,x,q,y]$ is defined as follows (with a similar definition for player 2):

$$\begin{array}{ll}
\int_0^x D_1^{-1}(w)\,dw - [1-\xi(q)]\dfrac{px+qy}{x+y}\,x - \xi(q)px & \text{if } p>q \wedge x+y\leq Y \\
\int_0^x D_1^{-1}(w)\,dw - [1-\xi(q)]\dfrac{px+q(Y-x)}{Y}\,x - \xi(q)px & \text{if } p>q \wedge x+y>Y \\
[\int_0^{\frac{x}{x+y}Y} D_1^{-1}(w)\,dw - p\dfrac{x}{x+y}Y]\,[1-\xi(p)] & \text{if } p=q \wedge x+y>Y \\
[\int_0^{Y-y} D_1^{-1}(w)\,dw - p(Y-y)]\,[1-\xi(p)] & \text{if } p<q \wedge x+y>Y \\
[\int_0^{x} D_1^{-1}(w)\,dw - px]\,[1-\xi(p)] & \text{if } p\leq q \wedge x+y\leq Y
\end{array}$$

Thus, the auction game is defined by the set of competitive players, $i=1,2$, their strategy space, B_i and their payoff function π_i. To simplify notation, let $b \equiv (b_1, b_2)$, and $B \equiv B_1 x B_2$. Define $R_i(b_j)$, for $i, j = 1,2, i \neq j$, as $R_i(b_j) = \{b_i \in B_i \,/\, \pi_i(b_i, b_j) = \max_{b_i' \in B_i} \pi_i(b_i', b_j)\}$. And define a pure strategy Nash equilibrium of the auction game as a pair of vectors b^* such that $b^* \in R_1(b_2^*) x R_2(b_1^*)$.

3.3 The equilibria

Our main result is to find a Nash equilibrium for the Spanish auction.

Proposition 1: The profile $b^* = (p^*, D_1(p^*), p^*, D_2(p^*))$ is a Nash equilibrium of the ξ-auction game, where p^* is such that $D_1(p^*) + D_2(p^*) = Y$, if the following condition is satisfied for any price $p \in [p^0, p^*]$ and a fixed amount z:

$$\frac{\left| \dfrac{\partial\left(\int_{0 \le w \le z} D_i^{-1}(w)\, dw - pz\right)}{\partial p} \right|}{\dfrac{\partial(1-\xi(p))}{\partial p}} \le \frac{\int_{0 \le w \le z} D_i^{-1}(w)\, dw - pz}{1-\xi(p)} \qquad i = 1, 2 \qquad (1)$$

The above condition states that the demand functions and the ξ-function are such that, for any fixed amount, bidders prefer to pay a slightly higher price to receive an award z with a slightly higher probability. Note that the proposition is the same as Menezes' (95) result for discriminatory auctions[8]: we prove that the equilibrium he proposes is also an equilibrium for the Spanish auction.

Proof:
Suppose that player 2 submits a bid $(p^*, D_2(p^*))$. Player 1 can:

i) Bid $(p<p^*, x)$. Player 2 gets $D_2(p^*)$ with probability 1, given that player 1 bids the lower price, and player 1 pays price p and receives at most $D_1(p^*)$. Since $p<p^*$ and demand is strictly decreasing, $x = D_1(p^*)$ maximizes his payoff function [9].

ii) Bid $(p=p^*, x)$. He pays p^*. There are two cases: If $x \le D_1(p^*)$, he receives x with probability 1, and therefore it is optimal to bid $D_1(p^*)$. If $x \ge D_1(p^*)$, the optimal x solves the following problem:

[8] Menezes' proposition establishes that the equilibrium is unique, which is not true in the Spanish case, and assumes that demand functions are identical for both players.

[9] Note that any $x > D_1(p^*)$ also maximizes his payoff function for $p<p^*$.

$$\max_{x \geq 0} \int_0^{\frac{x}{x+D_2(p^*)}Y} D_1^{-1}(w)dw - p^* \frac{x}{x+D_2(p^*)}Y$$

$$s.t.\ x \geq D_1(p^*)$$

Ignoring the restriction, first order conditions imply that it is optimal to bid x = $D_1(p^*)$ if p=p*. Since the restriction holds, we conclude that x = $D_1(p^*)$ if p=p* is the solution to the problem.

iii) Bid (p>p*,x). He receives x with probability 1, and the price he pays varies. If x $>D_1(p^*)$, $x+D_2(p^*)>Y$, and he pays [px+p*(Y-x)]/Y][10]. If x $\leq D_1(p^*)$, $x+D_2(p^*)\leq Y$, and he pays [px+p*D(p*)]/[x+D(p*)].

Given p>p*, which quantity maximizes his payoff?

- A bid x > $D_1(p^*)$) is not optimal, since player 1 receives x with probability 1, pays a price greater than p* and demand is strictly decreasing.

- Consider a bid x ≤ $D_1(p^*)$). Player 1 chooses x to solve the following problem

$$\max_{x \geq 0} \int_0^x D_1^{-1}(w)\, dw - \frac{px + p^*D_2(p^*)}{x + D_2(p^*)}x$$

$$s.t.\ x \leq D_1(p*)$$

Ignoring the restriction, first order conditions imply

$$D_1^{-1}(x) - \frac{px + p^*D_2(p^*)}{x + D_2(p^*)} = (p-\frac{px + p^*D_2(p^*)}{x + D_2(p^*)})\frac{x}{x + D_2(p^*)}$$

[10] Note that since player 2 bids at price p*, he gets a positive amount with probabilty 1.

The right-hand-side term is positive, since $p > p^*$ and $x > 0$, and it follows that the optimal x for player 1 is lower than $D_1([px + p^*D_2(p^*)]/[x + D_2(p^*)])$, and therefore lower than $D_1(p^*)$. Notice that the fact that player 1 bids for $x < D_1([px + p^*D_2(p^*)]/[x + D_2(p^*)])$ is a particularity of the Spanish case: since a higher x increases the price he has to pay, player 1 lowers his quantity bid below the quantity demanded at the price he has to pay.

Since the restriction holds, we conclude that if $p > p^*$, $x < D_1(p^*)$.

But bids i) and iii) are dominated by ii):

- Bid i), $(p < p^*, D_1(p^*))$ is dominated by bid ii), $(p^*, D_1(p^*))$. This result is identical to that of Menezes, and follows from the assumption of the proposition. The result follows because the assumption implies that bidders prefer to pay a slightly higher price to receive an award with a slightly higher probability.

- Bid iii), $(p > p^*, x < D_1(p^*))$ is dominated by bid ii), $(p^*, D_1(p^*))$. The payoff that player 1 gets with bid $(p^*, D_1(p^*))$ is given by

$$\pi_1(p^*, D_1(p^*), p^*, D_2(p^*)) = \int_0^{D_1(p^*)} D_1^{-1}(w)\, dw - p^* D_1(p^*)$$

It is possible to rewrite the above expression as

$$\begin{aligned}
&\left[\int_0^x D_1^{-1}(w)dw - \frac{px + p^*D_2(p^*)}{x + D_2(p^*)}\, x\right] + \\
&+ \left[\int_x^{D_1(p^*)} D_1^{-1}(w)dw - p^*(D_1(p^*) - x)\right] + \\
&\quad + \left[\left(\frac{px + p^*D_2(p^*)}{x + D_2(p^*)} - p^*\right) x\right]
\end{aligned}$$

The first bracket is player 1 payoff when he bids $(p > p^*, x < D_1(p^*))$, $\pi_1(p, x, p^*, D_2(p^*))$; since the second and the third brackets are positive, it follows that

$$\pi_1(p^*, D_1(p^*), p^*, D_2(p^*)) > \pi_1(p, x, p^*, D_2(p^*)) \quad \textit{for } p>p^*,\ x<D_1(p^*)$$

We have shown that (p^*, $D_1(p^*)$) is a best response to (p^*, $D_2(p^*)$). For reasons of symmetry, the converse is also true, and therefore we have shown that $b^* = (p^*, D_1(p^*), p^*, D_2(p^*))$ is a pure strategy Nash equilibrium.

Corollary 1: Profiles $b^*=(p^*, x, p^*, y)$, *for* x *and* y *such that* $x+y \geq Y$, $D_1(p^*) = (xY)/(x+y)$ *and* $D_2(p^*) = (yY)/(x+y)$, *are pure strategy Nash equilibria.*

Note that in all the equilibria of Corollary 1, each bidder receives the same amount, $D_1(p^*)$ and $D_2(p^*)$, respectively, pays the same price, p^*, and the seller gets the same revenue. Therefore utilities for all players are equal in the set of proposed equilibria. The proof of Corollary 1 is similar to the proof of Proposition 1, and is in the Appendix 1. Corollary 2 establishes that there are not other equilibria.

Corollary 2: There are not other pure strategy Nash equilibria than the set proposed in Corollary 1.

Proof of Corollary 2:
First, we show that in any Nash equilibrium, both bidders bid the same price. Suppose that there is a Nash equilibrium where player 1 bids (p, x) and player 2 bids (q, y), for $p>q$. Bidder 1 can increase his profits bidding ($q+\epsilon$, x'), for $\epsilon>0$ and x' such that this maximizes his profits given the price he has to pay. He receives x' with probability one and pays a lower price.

Next, we show that there is not a Nash equilibrium where both bidders bid $p>p^*$. If that is the case, both of them have incentives to lower their price bids. Suppose that there is a Nash equilibrium where player 1 bids (p, x) and player 2 bids (p, y), for $p>p^*$. Note that in any such equilibrium, $x+y \leq Y$, and therefore both players have an incentive to lower p to p^*, since they receive their quantity bid with probability one and pay a lower price.

Finally, suppose there is a Nash equilibrium where both bidders bid $p<p^*$. Both of them have an incentive to raise their price, given (1). This concludes the proof of the Corollary.

3.4 Testable implications of the model

Our model predicts that the Spanish auction has a pure strategy Nash equilibrium, in which the seller's revenue is maximized.

Given that we only observe data from one type of auction, the Spanish one, it is not possible to test one auction mechanism versus the other in terms of revenue efficiency. To test the prediction of the model, we follow the usual practice in the empirical literature, and calculate the difference between when-issued or secondary market prices and auction prices, i.e. the auction discount. Since the Treasury's revenues cannot increase by more than the bidders' current profits, i.e. the auction discount, if the discount is small and statistically insignificant, it would support the model, in the sense that it maximizes the seller's revenue.

Since in the model bidders know the demand of rivals and the market clearing price, it seems that the model is more appropriate for auctions where the bond being issued has been auctioned in previous months (hereafter, reopening auctions) than for auctions where the bond is issued for the first time (hereafter, initial auctions). This is because in reopening auctions bidders have, probably, more price information than in initial auctions, given that an identical bond to that being auctioned is trading in the secondary market. Also, in the model the seller announces the quantity that he wants to sell, but there is supply uncertainty, since bidders face a positive probability of receiving a zero award if they bid the lowest price even if the final volume issued is lower than the one announced. Therefore, we consider that the model is more appropiate for auctions with volume announcement, since the target volume announced is not compulsory.

Summarizing, the model predicts that auction discounts in reopening auctions with non-compulsory volume announcement are small in size and statistically no significant.

4. EMPIRICAL ANALYSIS

Empirical investigation of auctions is motivated mainly by one reason: auctions are very complicated games, so that auctions models, such as the one explained in section 3, abstract from many characteristics of real auctions and the predictions of the theoretical model need to be tested to see if they hold in more complicated environments.

Most empirical studies try to test whether the auction format used is revenue-efficient, from the seller's point of view, by examining the *auction discount* defined as the price differential between secondary market prices and auction prices[11], where secondary market prices are used as a proxy for the true value of the bond. If the bond has not been auctioned before, when-issued market prices are used, and if the auction is a reopening, so that there is an identical bond being traded in the secondary market, spot secondary market prices are used. If the auction format is revenue-efficient the auction discount should not be significantly different from zero. In table 1, empirical results found in the literature are summarized. Results are mixed for discriminatory auctions. On the contrary, and with some exception, a statistically non-significant auction discount is found for uniform auctions.

Many of the empirical studies also examine determinants of auction discount. Examples of such analysis are: Berg (96), Breedom and Ganley (96), Cammack (91), Hamao and Jegadeesh (97), Scalia (97), Spindt and Stolz (92) and Umlauf (93). These studies have in common that they look, essentially, for two possible determinants of auction discount, namely uncertainty about the true value of the security and the level of competition among bidders, although they differ in the proxies used for such variables, in the inclusion of other explanatory variables and in the results they obtain. Nevertheless, since there is not a generally accepted model for multiple-unit and multiple-bid auctions with a resale market, empirical models are somehow *ad hoc*. Then, the inclusion of variables as regressors for the auction discount relies on the grounds of having some weight in the determination of the auction price in some of the existing simple theoretical models. And most of the studies rely on the predictions of one unit auctions theory.

4.1 Data description

The data sample consists of individual bids for 3, 5, 10 and 15-year bond auctions held between January 1993 and August 1997[12]. For each bid the data include identification code of the bidder, quantity and price bid, quantity accepted, price to be paid (if accepted) and date when bid is made. The sample covers data for 192 auctions, 29 of which are initial auctions, (the bond is issued for the first time), and 163 are reopenings (auctions of bonds with the same coupon and maturity as a previously issued bond).

Secondary market data includes two sets of prices: the first set are quoted prices (average of bid and ask quotes). The second set consists of individual traded prices in the secondary

[11] Some studies perform the analysis by examining yields diferentials, auction yield minus secondary market yields, instead of price diferentials.

[12] These data have been provided by the Domestic Operations Department of the Banco de España.

market among members. In both cases, secondary market prices are adjusted to the settlement date of the auctioned bond using repo rates[13]. For initial auctions, secondary market prices correspond to the *when-issued* market[14].

4.2 Empirical analysis results

The empirical analysis of the Spanish auctions of government bonds presented in this paper is divided into three parts: examination of general features of the functioning of Spanish auctions; test of revenue efficiency; and analysis of possible determinants of the auction discount.

4.2.1 Summary characteristics of auctions results

Column 1 of table 2.A and table A.1 in the Appendix 2, summarize the results of Spanish government bond auctions. In brief, the principal features are as follows:

1. In terms of volume, non-competitive bids are insignificant. On average non-competitive bids represent only 0.7% of total volume bid and 1.4% of total volume issued. In practice, then, auctions are mostly competitive.

2. Competitive participation is, in relative terms, high. On average, there are 31 bidders submitting at least one competitive bid. Although this figure is small compared with other government bond auctions like the Italian case, where there are about 60 bidders in each auction, it is high if we take into consideration that participation in the Spanish bond secondary market for all outstanding bonds averages about 50 participants daily.

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[13] That is: if secondary market price at t (P^t_{Ts}) corresponds to settlement date in Ts days and settlement of the auctioned bond is in Ta days ($T_s <= T_a$), the secondary market adjusted price (P^t_{Ta}) will be calculated as:

$$P^t_{Ta}+cc_{Ta}=(P^t_{Ts}+cc_{Ts})(1+r^t\frac{Ta-Ts}{360})-C(1+r^t\frac{Ta-Tc}{360})$$

where:
cc= acrued interest;
r= repo rate at t for (Ta-Ts) days;
C= coupon, if Ta<Tc<Ts, 0 otherwise

[14] In many bond markets bonds are traded before the bond is issued. This market is called *when-issued* and it is a forward market with settlement date on the date the bond is to be issued.

3. On average there are 31 non-competitive bids, 14 of which correspond to bidders who also submitt competitive bids. Nevertheless, the small size of the maximum amount allowed for non-competitive bids (25 million Ptas. versus the 760 million Ptas. of the average trade in the secondary market among members) suggests that non-competitive bids are submitted in order to cover compromises with clients more than to avoid the competitive game.

4. Bidders submit, in average, 2.7 competitive bids. In practice, then, auctions are multiple bid games.

5. Competition level, measured by the ratio of volume submitted in competitive bids to volume accepted (cover ratio) is comparable to competition level in other bond auctions markets. Thus, for the Spanish bond auctions, the average cover ratio is 3.1, while it is 2.04 for Italian bond auctions and 3.9 for Japanese bond auctions. In terms of number of bids, competition is lower: the ratio of total competitive bids to accepted bids is 2.3.

6. On average, most volume awarded is at the bid price (57.9%) while most bids are awarded at the weighted average price.

7. Price differential paid by accepted bids (WAP minus stop-out price) is relatively low: 0.10 on average, smaller than the bid-ask spread in the secondary market that averages 0.18 for the sample used, and to other bond auction markets like the Japanese where this figure is 0.12. Also, standard deviation of prices paid by winning bids is low (0.036). On the contrary, the bid price range of accepted bids is quite high 3.5. That is, meanwhile most bids come at very similar prices, there are, generally, a few bids at prices far above. This suggests that there may be a few bidders, who are in fact playing as non-competitive bidders[15]: by bidding at very high prices they insure themselves against not getting the security, avoiding the quantity limit imposed in non-competitive bids and paying the same price (the WAP), albeit with a cost, since bidding very aggressively increases the average price.

These general comments are for the whole sample. However, as mentioned before, the sample includes two different types of auctions: the initial auctions and the reopening auctions. The only difference between them is an important one: for the reopenings there is

[15] Although on some occasions the wide range of accepted bid prices is due to errors at the time of submitting the bid, the persistence of this wide range suggests something else.

a secondary market that provides an alternative market for the same security and, particularly, price information, but for the initial auctions, although there exists a *when-issued* market that could play the same informational role, it is not very liquid in practice. For instance, only 9 of the 29 initial auctions covered by the sample have when-issued operations the day before auction. Hence, the theoretical model developed in section 3, albeit with limitations, replicates better the reopening auctions than the initial ones, for which price information is, probably, poorer.

Columns 2 and 3 of table 2.A summarize general features of the functioning of Spanish bond auctions for the initial and reopening auctions, respectively. On average, demand is bigger for initial auctions. Thus, average bid volume, number of bids and number of bidders per auction are higher in initial auctions. Potentially, the practice of reopening auctions may have either a positive or a negative effect on demand: on the one hand, demand can be reduced because there is an alternative market for the very same security; and on the other, better information provided by the secondary market may encourage participation. It seems that in average, the negative effect overcomes the positive. The other side of the market, the supply side, also seems bigger for initial auctions: average volume issued and target amount to be issued are higher for them. The reason for a bigger supply in initial auctions may be as that for using the practice of reopening, that is, an issue with a small size is more easily cornered, and the Treasury wishes to avoid that.

Nevertheless, cover ratios in terms of volume (bid volume/awarded volume) and in terms of number of bids (number of bids/number of winning bids) are higher for reopenings. This suggests that competition is higher for reopenings, so that the decrease in demand from initial auctions to reopenings is, in relative terms, smaller than the decrease in supply[16]. On the other hand, range prices (WAP minus stop-out price, maximum bid price minus stop-out price and maximum bid price minus minimum bid price) are smaller for reopenings, which is consistent with the notion that reopenings benefit from better information through the secondary market.

As mentioned before, one peculiar feature of the Spanish auctions is that up to 1995 there was no announcement about offered volume. The practice changed in July 1995, and although there is still quantity uncertainty, the change can be considered an important one and may have had an effect on auction results. To this end, tables 2.B and A.1 in the Appendix 2 summarize auction results distinguishing between auctions with no announcement (the ones before July 1995) from the ones with one (from July 1995 on). The main

[16] A higher cover ratio could also be explained by a poorer information of bidders, but this explanation seems less reasonable than that given in the text, since the secondary market for reopenings provides information that initial auctions lack.

conclusion of this analysis is that price dispersion of bids is lower for auctions with an announcement, which is consistent with the idea that volume announcement decreases uncertainty faced by bidders. Nevertheless, the lower dispersion could be also due to a less volatile financial environment from 1995 onwards than during 1993-1995.

4.2.2 Testing revenue efficiency

As mentioned earlier, auction revenue efficiency is empirically tested by examining auction discount defined as the difference between secondary market price and auction price. If the auction is revenue-efficient, auction discount should not be statistically different from zero.

In order to calculate auction discounts, for auction price we use the weighted average of paid prices[17] (hereafter, WAPT). For secondary market prices two data sets are used: average of quoted prices and average traded prices. Neither is problem-free since calculation of the relevant auction discount should use secondary market prices at the time bids are submitted (before 10.30 a.m. on the auction day). Unfortunately, such prices are not available. Quoted prices available correspond to 5.00 p.m. and traded prices are averages of traded prices during the day[18]. With these limitations, the most appropriate comparison between auction prices and secondary market prices is, probably, to use quoted prices the day before the auction (bearing in mind that prices could have changed between 5.00 p.m. and the next morning) or, as a second best, to use auction day average traded prices (taking into account that they could have been affected by any price change after the auction). In any event, in order to be more confident about results, auction discounts have been calculated using quoted and traded prices for the day before the auction and the auction day.

Table 3 reports summary statistics for auction discounts. When all auctions are taken into consideration the auction discount is positive and statistically different from zero. Nevertheless, the auction discount is small in size, their mean value being smaller than the secondary market bid-ask spread, which averages 0.18 for the sample.

However, since initial and reopening auctions are different, it seems appropriate to separate them when analyzing auction discounts. The results of such analysis are reported in rows 3

[17] Notice that in discriminatory auctions the weighted average price paid to the Treasury is the WAP. For the Spanish case the WAP is the maximum price paid to the Treasury but the average price paid to the Treasury is the WAPT, where WAPT $\leq$ WAP.

[18] We do have information about each individual trade, but not about the time of the day the trade is made.

to 6 of table 3. For initial auctions the discount is statistically significant[19] and with a mean value higher than the secondary market bid-ask spread, which for the corresponding observations averages 0.26. For reopening auctions, the discount is statistically significant but with a mean value that is smaller than the secondary market bid-ask spread, that averages 0.16 for the corresponding observations, and much smaller than the mean value for initial auctions. Hence, results indicate that when there is less uncertainty about prices, the auction discount is lower. This result is consistent with predictions of the single-unit auction theory: the less uncertainty about the value of the good to be auctioned, the lower the winners' curse, and therefore bidders bid more aggressively so that the selling price increases. Nevertheless, it should be mentioned that the positive and higher auction discount found for initial auctions compared to reopenings could be explained not only by poorer price information but also by greater supply: the Treasury may be willing to issue at a discount with respect to the secondary market in order to avoid launching an issue of a small size that could be easily cornered.

Next, the auction discount analysis is performed separating not only initial auctions from reopenings but also auctions without volume announcement from those with it. The corresponding results are reported in rows 7 to 14 of table 3. As before, for initial auctions the auction discount is statistically significant with a positive sign and a high mean value in both cases, i.e. whithout and with volume annuoncement (rows 7 to 10 of table 3).

With respect to reopening auctions, auction discount is positive and statistically significant for auctions without volume announcement (rows 11 and 12 of table 3), although they are smaller in value than the corresponding auction discount for intitial auctions. For auctions with volume announcement, i.e. for reopening auctions from July 1995 on, the auction discount is statistically non-different from zero. This is consistent with predictions of the model presented in section 3. Notice, first that the model has volume announcement although there is uncertainty about the final amount issued. This is the case for the Spanish auctions since July 1995: the Treasury announces a non-compulsory target amount to be issued. Second, in the model bidders know the demand of rivals and the market clearing price. Therefore, the model probably characterizes better reopening auctions during a period with less market volatility as is the case after July 1995. Hence, we conclude that the results are consistent with the model.

[19] The number of observations for the initial auctions is very small since the when-issued market is not liquid, so t-statistic should be analyzed carefully. However, a large majority of observations yields a positive auction discount which gives some support to the sign of the mean value.

4.2.3 Auction discount determinants

As mentioned earlier, the absence of a generally accepted model for multiple-bid multiple-unit auctions implies that the empirical analysis of auction discount determinants contains ad-hoc elements. The approach taken for most of the empirical work on the subject is to test if predictions of single-unit single-bid auction models hold in bond auctions. In such theoretical models, auction participation, auction competition and uncertainty about the true value of the security are factors that affect auction discounts. Therefore, proxies for these variables are generally used as explanatory variables of auction discounts.

In single-unit single-bid auction models with a finite number of players, the effect of an increase in participation, i.e. the effect of an increase in the number of bidders, has an ambiguous effect on auction discounts (see, for example, the explanation given in Umlauf(1993)). On one hand, when the number of bidders increases the winner's curse[20] is more severe, inducing lower bidding, and hence increasing auction discounts. On the other, in single-unit single-bid models, an increase in the number of bidders implies an increase in competition[21], reducing the probability of winning, and therefore inducing higher bidding and hence decreasing auction discounts. Wilson (1988) argues that for most plausible examples the increase in competition is the stronger of the two effects.

However, in multiple-unit multiple-bid auctions an increase in participation does not necessarily imply an increase in competition, since the offered volume may vary as well. For this reason, in the empirical analysis presented here we use as regressors both participation and competition. Participation is measured by the number of bidders submitting at least one competitive bid (*BIDDERS*) and it is expected to have a positive effect on auction discounts. Competition is proxied by the cover ratio (*COVERC*), defined as the competitive volume of bids over competitive volume accepted, and it is expected to have a negative effect on auction discounts.

As concerns the inclusion of price uncertainty as an explanatory variable of the auction discount, it is argued (for example by Umlauf(1993), Berg(1997), Hamao and Jegadeesh (1997)) that the marginal probability of losing an auction by lowering the bid by a given amount decreases with pricing risk. Hence, price uncertainty is expected to have a negative effect on auction price and, therefore, a positive effect on auction discount. Price uncertainty,

[20] For an explanation of the winner's curse see footnote 5.

[21] Measuring competition by volume bid over volume offered, in single-unit single-bid models competition takes a value equal to the number of bidders. Hence, in such models competition and participation take the same value.

which is not observable, is proxied by bond market volatility (*VOLATILITY*)[22].

But for Spanish bond auctions, bidders face not only price uncertainty but also quantity uncertainty, which could also affect auction discount. Unfortunately, quantity uncertainty is difficult to measure. As mentioned several times, in July 1995 the Spanish Treasury started to make (non-compulsory) quantity announcements. Thus, a tentative solution to measure the effect of quantity information could be to use as a regressor a dummy variable (*DANNOUNCE*, taking value 0 up to July 1995 and 1 since then). However, such a dummy could capture another effect: the decrease in bond market volatility observed since mid-1995[23]. Because of that, in order to capture quantity uncertainty effects we use as an explanatory variable the variance of submitted prices (*BIDVAR*). We conjecture that *BIDVAR* would decrease with an increase in the information set available to bidders. Therefore, a positive sign is expected for the coefficient of BIDVAR. Nevertheless, it should be kept in mind that this variable will capture not only quantity uncertainty effects but also price uncertainty effects, so that we will not be able to analyze separately the effect of quantity uncertainty.

The sample we use includes auctions for 3, 5, 10 and 15-year bonds, and auction discounts may have different sizes across bonds. For that reason, we allow the constant coefficients to differ across bonds by using as regressors a dummy for each type of bond (*DB3*, *DB5*, *DB10* and *DB15*). Besides, for further auction discount differences across bonds we use as an additional regressor the variable *MATURITY*, measured as the period of time between auction settlement date and redemption date, which is expected to have a positive sign since a greater maturity is generally associated with higher volatility.

For many of the auctions included in the sample, a bond with the same original maturity as the one being auctioned was to mature shortly. Since this fact could affect both the supply and the demand side of the auction, and therefore auction prices, the variable *REDEMPTIONS* is used as a regressor. *REDEMPTIONS* is the nominal amount maturing in the auction month in bonds with original maturity similar to that auctioned. This variable can have either a positive or a negative effect on auction discount: bigger redemptions may imply higher pressure on the Treasury to issue a bigger quantity, and then a positive sign on auction discount could be expected. But on the other hand, this variable gives some quantity information to bidders, and then a negative effect on auction discount could be expected.

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[22] Measured by the standard deviation of the last 20 daily price differentials.

[23] In fact, performing an equal means Wald test for January 1993-July 1995 and for July 1995-August 1997 bond market volatility the null hypothesis of equal means is rejected.

Finally, to take into account differences between initial auctions and reopening auctions we used *DNEW*, which is a dummy variable with value 1 if the auction is an initial one and 0 otherwise. According to the results presented in the previous section, a positive sign is expected for this variable.

Summary statistics and the correlation matrix for explanatory variables are reported in table 4. It should be noted that the variable *COVERC* is jointly determined with the dependent variable, the auction discount. For that reason, we use the cover ratio observed in the adjacent auction of an other bond as an instrument for *COVERC*[24]. Results of 2SLS regressions of the auction discount on the aforementioned explanatory variables are showed in column 1 of table 5. Results can be summarized as follows:

1. Participation and competition, measured by number of bidders and cover ratio respectively, have a statistically significant effect with the expected sign, i.e. a positive sign for participation and a negative one for competition. It may be argued that these two variables should not be included in the same regression equation since an increase in the number of bidders implies an increase in competition. As discussed before, in multiple-unit auctions this may not hold, and an example is the Spanish case. In effect, table 4 shows that the variables *BIDDERS* and *COVERC* have a correlation coefficient of (-.17). Besides, dropping any of the two variables from the regression yields a coefficient for the variable included similar to the regression including both of them, and a statistically significant one. In other empirical work (Spindt and Soltz (92), Scalia (97), Berg(97)) a negative sign is also obtained for the cover ratio while results for number of bidders are mixed (positive in Umlauf (93) and Berg (97) and negative in Scalia (97)).

2. Variables measuring price and quantity uncertainty (*BIDBAR*, *VOLATILITY* and *MATURITY*) have, as hypothesized, a positive effect. This result is compatible with the conjecture that quantity information affects the size of auction discount.

   Regressions displayed in columns 2 and 3 of table 5 show that using a dummy variable to measure the effect of an increase in quantity information (*DANNOUNCE*) may capture instead the effect of a decrease in market volatility. Thus, when replacing *VOLATILITY* by *DANNOUNCE*, the coefficients for *BIDVAR* and *DANNOUNCE* are statistically significant. However, when replacing *BIDVAR* by *DANNOUNCE*, the coefficient for *VOLATILITY* is not statistically significant while

[24] More precisely: for auctions on 3-year, 5-year and 15-year bonds the instrument used is the cover ratio of the adjacent 10-year bond auction; for the 10-year bond auction the instrument used is the cover ratio of the adjacent 3-year bond auction.

that for *DANNOUNCE* is.

3. The variable *REDEMPTIONS* seems to have no effect on the auction discount. Its coefficient has a negative sign but it is not statistically significant. Dropping this variable from the regression (see column 6 of table 5) yields very similar results in terms of R^2 and of the coefficients of all other explanatory variables.

4. The coefficient of the dummy variable used to separate initial auctions from reopening auctions (*DNEW*) has, as expected, a positive sign, but it is not statistically significant. A possible explanation for it is that the number of observations for initial auctions is very small compared to the observations for reopening auctions (7 and 155 respectively). Running a regression that includes only observations for reopening auctions yields very similar results (not displayed here) that those shown in column 1 of table 5.

5. The adjusted R^2 is small, (.25), but similar or higher to that obtained in other empirical work, which ranges 0.12-.22.

5. CONCLUSIONS

This paper explores Spanish Treasury auctions. First, we present a model that considers both the hybrid system of uniform and discriminatory auctions used in Spain and the uncertainty about the volume to be issued. The main result is that for the Spanish auction format, there exists a pure strategy Nash equilibrium that maximizes the seller's revenue, which is unique in the sense that each of the equilibria gives the same utility to all players. This result gives the idea that both the discriminatory and the Spanish auction format behave in a similar way in the model proposed. The model we use is a stylized version of government bond auctions, and it abstracts from many features of the market. Note that auctions of multiple units are very complicated games, and that the Spanish auction format is even more complex, since the price winning bidders pay if their bid is above the weighted average of winning bids depends on all other winning bids; this fact makes the model very difficult to solve.

Second, we present an empirical analysis, using data of Spanish bond auctions between 1993 and 1997, to test the predictions of the model and to establish the main characteristics of Spanish bond auctions. The main results of the empirical analysis are as follows: First, as predicted by the model, auction discounts are small in size and not statistically significant for auctions with volume announcements where the bond auctioned is identical to an existing one traded in the secondary market (reopening auctions) which we think that the model better characterizes. Second, both participation and competition are significant determinants of the size of auction discounts: participation has a positive effect, since as the number of bidders increases the winner's curse is more severe, and they bid less aggressively. Competition, measured by the cover ratio, has a negative effect, as competition reduces the probability of winning and induces bidders to bid more aggressively. Third, variables measuring uncertainty faced by bidders seem also to have a positive effect on the size of auction discounts. Although in the analysis presented here it is not possible to separate the effect of price uncertainty from the effect of quantity uncertainty there is some indication that both sorts of uncertainty matter.

We think that several policy implications can be drawn from the results presented here. First, Spanish auction format proves to be non-prejudicial to the Treasury, at least for reopening auctions, since auction discounts are, on average, close to zero and, in any case, generally no bigger than the observed bid-ask spread in the secondary market. Second, reopenings are a beneficial practice to the Treasury, since in these auctions price uncertainty faced by bidders is lower through the price discovery function of the secondary market, so that average auction discounts are considerably smaller in size for reopenings than for initial auctions. Note that reopenings are not widely used, and some authors have expressed doubts about their desirability. And third, as predicted by single-unit auctions models, the Treasury's announcement of target amounts, even as a joint figure, seems also to be a beneficial practice to the Treasury, since there is some indication that auction discounts are smaller the better the participants are quantity informed. The obvious implication is that it would be even more beneficial to the Treasury to make more specific and committed volume announcements.

We have still a lot of work to do. In the theoretical model, we want to relax the assumption that demands are known to the bidders, and see whether the results of the model hold in a game where incomplete information is not only because of supply uncertainty. Also, we want to allow more than one bid per bidder. In the empirical analysis, we have analyzed only average behaviour, but we plan to characterize individual bidder behaviour.

TABLE 1

SURVEY OF EMPIRICAL WORK

AUTHOR	DATA AND SAMPLE	DISCOUNT MEASURE	DISCOUNT SIZE (PRICES)	DISCOUNT SIZE (YIELDS)
Cammack (1991)	3 months US Tbills 1973-1984 **discriminatory**	Quoted price at auction day - WAP	4 bp significant	
Spindt & Stolz (1992)	3 months US Tbills (1982-1988) **discriminatory**	When-issued quoted price 30 minutes before auction -WAP	1.3 bp significant	
Cherebuni et al. (1993)	Italian BTPs (1990-1991) **uniform**	Log (average traded price on acution day/stopout price)	14 bp significant	5.2 bp
Umlauf (1993)	1month Mexican Tbills (1986-91) **discriminatory and uniform**	Average resale price/WAP	1.7 bp significant	
Bikhchandani et all (1994)	1&3 months US Tbills (1990-91) **discriminatory**	When-issued price quoted at time of auction - WAP	1 bp non-signific.	
Simon (1994)	US T-notes (1990-91) **discriminatory**	Average auction rate-when-issued rate at auction time		0.37 bp significant
Buttiglione and Drudi, 1994	Italian BTPs, CCTs and CTOs (1989-92) **uniform**	Average traded price on auction day-stop out price	7 bp no test	
Malvey, Archibald and Flynn, 1996	US T-Notes (1992-95) **discriminatory and uniform**	WAP-When-issued rate at auction time		uniform:-0.22 bp, non-signific. discrim: 0.69 bp, signific.
Nyborg and Sundaresan (1996)	US Tbills, notes and bonds (1992-1993) **discriminatory and uniform**	Average auction rate- when-issued rate 30 minutes before auction		uniform:-.2 bp, no signif. discrim: 0.4 bp no significant
Druddi and Massa (1997)	Italian BTPs and CCTs **uniform**	Average traded price just before auction-stop out price	4 bp non-signifc.	
Scalia (1997)	Italian BTPs and CCTs (1995-96) **uniform**	Average traded price before auction- stop out price	4.2 bp non-signifc.	
Breedon and Ganley (1996)	UK Gilts (1988-1996) **discriminatory**	When-issued quoted price just before auction- WAP	10.9 bp non-signifi.	
Hamao and Jegadeesh (1997)	Japanese Bonds (1989-95) **discriminatory**	Average auction rate-Secondary market price day after auction		2.8 bp, no significant
Berg (1997)	Central Bank of Norway certificates (1993-1995) **discriminatory**	Average auction rate-reference secondary market rate day after auction		5.7 bp no test

Note: bp= basis points (1 bp=0.01 of 1 percentage point)

This table is taken from Scalia (1997) except for the last four references

TABLE 2.A

GENERAL CHARACTERISTICS OF SPANISH TREASURY BOND AUCTIONS

	ALL AUCTIONS	INITIAL AUCTIONS	REOPENING AUCTIONS
Number of auctions analyzed	192	29	163
	mean	mean	mean
Competitive volume submitted	254.0	355.7	236.3
Total volume issued	124.6	194.2	112.5
Vol. of competitive bids accepted	122.9	193.0	110.7
Num. bids (c)	83.27	121.3	76.62
Num competitive bidders	30.86	41.34	29.02
Num non-competitve bids	23.82	21.34	24.25
Num bidders with only competitive bids	16.65	24.66	15.25
Num bids per bidder	2.162	2.541	2.096
vol. bid per bid	4.245	3.543	4.368
Vol. bid per bidder	7.075	7.087	7.073
Cover ratio (bid vol/accepted vol) (c)	3.099	2.029	3.286
%winning bids (c)	43.8	51.8	42.4
%winners (c)	61.7	71.3	60.1
%vol awarded at WAP (c)	42.1	38.1	42.7
%bids paying WAP (c)	61.5	61.4	61.6
%winners at WAP(c)	62.8	61.01	63.2
%vol paying stop-out price	46.6	38.1	48.0
WAP-stop out price	0.103	0.163	0.092
Max bid-stop out price	3.499	3.928	3.425
Max bid price-Min bid price	5.842	5.944	5.824
std. of bid prices	0.795	0.609	0.827
std. of paid prices	0.036	0.058	0.032
Target vol for 3+10-year bonds	279.7	333.9	271.3
Target vol for 5+15-year bonds	228.7	280.4	220.6

Volumes are in billions of pesetas.
Bids by non-members are aggregated by price.
Non-competitive bids by non-members are counted as 1 bid.

(c) refers to competitive bids

TABLE 2.B

GENERAL CHARACTERISTICS OF SPANISH TREASURY BOND AUCTIONS

	INITIAL AUCTIONS		REOPENING AUCTIONS	
	without[1]	with[1]	without[1]	with[1]
Number of auctions analyzed	15	14	73	90
	mean	mean	mean	mean
Competitive volume submitted	369.2	341.2	212.0	256.8
Volume issued	218.1	168.7	112.1	112.8
Competitive volume accepted	216.7	167.6	110.1	111.2
Num. bids (c)	138.1	103.4	80.89	73.01
Num competitive bidders	49.47	32.64	31.03	27.33
Num non-competitive bidders	19.4	23.43	25.29	23.37
Num bidders w. only comp. bids	31.27	17.57	17.25	13.57
Num bids per bidder	2.532	2.55	2.011	2.167
Vol. bid per bid	3.243	3.865	2.904	5.604
Vol. bid per bidder	6.302	7.928	4.974	8.847
Cover ratio (c)	1.875	2.195	4.201	2.514
%winning bids (c)	57.8	45.3	47.4	38.3
%winners (c)	75.7	66.5	62.8	57.7
%vol awardet at WAP (c)	29.6	45.4	34.7	48.0
%bids paying WAP (c)	67.0	52.9	63.1	60.3
%winners at WAP(c)	66.1	53.4	64.2	62.3
%vol paying stop out price	42.7	33.1	55.6	41.6
WAP-stop out price	0.207	0.117	0.093	0.091
Max bid-stop out price	4.103	3.739	3.906	3.018
Max bid-Min bid	6.668	5.168	6.037	5.644
std. of bid prices	0.655	0.56	1.011	0.672
std. of paid prices	0.073	0.041	0.032	0.033
Target vol for 3+10-year bonds		333.9		271.3
Target vol for 5+15-year bonds		280.4		220.6

Volumes are in billions of pesetas.
Bids by non-members are aggregated by price.
Non-competitive bids by non-members are counted as 1 bid.

(c) refers to competitive bids.
(1) with and without refers to *with volume announcement* and *without volume announcement*, respectively.

TABLE 3

AUCTION DISCOUNT ANALYSIS

DATA SET USED	USING SECONDARY MARKET DATA AT T-1									USING SECONDARY MARKET DATA AT T								
	num obs	mean	t-st	num obs >0	max	90% perc.	median	10% perc.	min	num obs	mean	t-st	num obs>0	max	90% perc	median	10% perc	min
ALL AUCTIONS																		
QUOTED PRICES	164	0.10	3.74	101	1.42	0.51	0.08	-0.30	-0.88	177	0.08	3.71	109	1.59	0.40	0.04	-0.21	-0.75
TRADED PRICES	166	0.13	2.50	108	2.20	0.96	0.14	-0.79	-2.03	189	0.08	2.21	116	2.24	0.55	0.09	-0.47	-2.28
INITIAL AUCTIONS																		
QUOTED PRICES	7	0.33	6.55	7	0.46	0.46	0.36	0.15	0.15	16	0.26	5.31	15	0.67	0.49	0.21	0.01	-0.04
TRADED PRICES	9	0.61	2.95	9	1.86	1.86	0.55	0.01	0.01	23	0.13	2.55	16	1.08	0.36	0.05	-0.01	-0.05
REOPENING AUCTIONS																		
QUOTED PRICES	157	0.09	3.23	94	1.42	0.52	0.06	-0.33	-0.88	161	0.06	2.72	94	1.59	0.34	0.03	-0.22	-0.75
TRADED PRICES	157	0.10	1.92	99	2.20	0.95	0.12	-0.79	-2.03	166	0.07	1.78	100	2.24	0.55	0.12	-0.51	-2.28
INITIAL AUCTIONS WITHOUT VOLUME ANNOUNCEMENT																		
QUOTED PRICES	7	0.33	6.55	7	0.46	0.46	0.36	0.15	0.15	9	0.33	5.13	9	0.67	0.67	0.32	0.10	0.10
TRADED PRICES	8	0.46	2.94	8	1.24	1.24	0.35	0.01	0.01	9	0.28	2.50	7	1.08	1.08	0.18	-0.05	-0.05
INITIAL AUCTIONS WITH VOLUME ANNOUNCEMENT																		
QUOTED PRICES										7	0.18	2.52	6	0.49	0.49	0.13	-0.04	-0.04
TRADED PRICES	1	1.86		1	1.86	1.86	1.86	1.86	1.86	14	0.03	2.77	9	0.11	0.08	0.02	-0.01	-0.02
REOPENING AUCTIONS WITHOUT VOLUME ANNOUNCEMENT																		
QUOTED PRICES	75	0.19	5.13	57	1.42	0.60	0.15	-0.18	-0.50	72	0.08	1.94	41	1.16	0.59	0.03	-0.27	-0.75
TRADED PRICES	76	0.23	3.47	56	2.20	0.96	0.20	-0.40	-1.24	76	0.18	3.46	54	2.24	0.65	0.19	-0.29	-1.12
REOPENING AUCTIONS WITH VOLUME ANNOUNCEMENT																		
QUOTED PRICES	82	-0.01	-0.21	37	0.74	0.37	-0.01	-0.40	-0.88	89	0.05	1.91	53	1.59	0.29	0.02	-0.18	-0.49
TRADED PRICES	81	-0.02	-0.28	43	1.27	0.92	0.04	-0.98	-2.03	90	-0.02	-0.39	46	1.22	0.54	0.01	-0.61	-2.28

TABLE 4

SUMMARY STATISTICS AND CORRELATION MATRIX FOR REGRESSORS

	auction discount	coverc	coveradj	bidders	bidvar	volat.	Redemp.	Maturity
MEAN	9.56	3.33	3.48	30.52	1.22	2.7	0.08	7.58
STD	32.76	3.89	4.06	15.23	2.19	1.13	0.2	4.26
N	162	162	162	162	162	162	162	162
			CORRELATION MATRIX					
AUCTION DISCOUNT	1							
COVERC	-0.26	1						
COVERADJ	-0.23	0.72	1					
BIDERSC	0.31	-0.19	-0.15	1				
BIDVAR	0.13	0.13	0.1	-0.26	1			
VOLATILITY	0.12	0.19	0.24	-0.02	0.25	1		
REDEMPTIONS	-0.07	-0.09	0.01	0.02	-0.08	0.08	1	
MATURITY	0.06	-0.04	-0.08	-0.25	0.07	-0.07	-0.29	1

AUCTION DISCOUNT= (Secondary market quoted price at t-1 - weighted average of paid prices)x100. **COVERC** is volume bid over volume accepted. **COVERADJ** is the instrument used for **COVERC**. For observations of 3, 5 and 15-year bond auctions, **COVERADJ** is the cover ratio of the 10-years bond auction taking place in the same day (or the day before). For observations of 10-year bond auctions, **COVERADJ** is the cover ratio of the 3-year bond auction taking place on the same day. **BIDERSC** is the number of competitive bidders. **BIDVAR** is the variance of bid submitted. **VOLATILITY** is secondary market volatility for 3-year bond benchmark. **MATURITY** is the period of time, in years, between settlement auction date and redemption date of the bond being auctioned. For observations of 3 and 5-year bond auctions, **REDEMPTIONS** is the volume of bonds with an original maturity of 3 and 5 years maturing in auction month. For observations of 10 and 15-year bond auctions, **REDEMPTION** is the volume of bonds with an original maturity of 10 years maturing in auction month.

TABLE 5 DETERMINANTS OF AUCTION DISCOUNTS

Dependent variable: AUCTION DISCOUNT; Estimation procedure: 2SLS; Number of observations= 162

	COL 1	COL 2	COL 3	COL4	COL5
DB3	-51.901	-29.777	-27.709	-22.965	-52.840
t-stat	*-2.95*	*-1.89*	*-1.60*	*-2.68*	*-2.99*
DB5	-72.241	-54.606	-54.390	-20.482	-72.530
t-stat	*-2.57*	*-2.13*	*-2.03*	*-2.53*	*-2.58*
DB10	-132.335	-122.039	-124.031	-22.685	-128.590
t-stat	*-2.34*	*-2.28*	*-2.30*	*-2.38*	*-2.28*
DB15	-174.375	-172.010	-184.028	-12.551	-168.508
t-stat	*-2.08*	*-2.16*	*-2.33*	*-1.46*	*-2.01*
COVERC	-2.642	-3.251	-3.420	-2.540	-2.613
t-stat	*-3.35*	*-4.55*	*-4.96*	*-3.14*	*-3.34*
BIDERSC	0.596	0.442	0.274	0.757	0.613
t-stat	*3.43*	*2.49*	*1.53*	*4.99*	*3.49*
BIDVAR	3.342	2.874		3.549	3.460
t-stat	*2.85*	*2.88*		*3.32*	*3.29*
VOLATILITY	4.042		1.505	4.102	3.824
t-stat	*2.03*		*0.76*	*2.00*	*1.96*
MATURITY	11.642	13.656	14.735		11.223
t-stat	*1.97*	*2.38*	*2.61*		*1.90*
REDEMPTION	-15.793	-5.818	-8.035	-13.817	.
t-stat	*-1.22*	*-0.47*	*-0.62*	*-1.10*	.
DNEW	5.847	-4.213	-3.268	6.777	6.269
t-stat	*0.66*	*-0.44*	*-0.36*	*0.78*	*0.73*
DANNOUNCE.	.	-22.253	-23.431	.	.
t-stat		*-4.67*	*-4.74*		
(adj)R^2	0.252	0.318	0.291	0.237	0.254
RMSE	29.370	28.060	28.604	29.680	29.346
DW d-stat	1.90	2.06	2.09	1.86	1.87
White's general test for heteroscedasticity H_0=homoscedasticity	reject H_0	reject H_0	reject H_0	reject H_0	reject H_0

t-stat are heteroscedasticity consistent.

AUCTION DISCOUNT= (Secondary market quoted price at t-1 - weighted average of paid prices)x1 .
DB3,DB5,DB10,DB15 are dummies for 3, 5, 10 and 15-yearbonds, respectively. **COVERC** is volume bid over volume accepted. **BIDERSC** is the number of competitive bidders. **BIDVAR** is the variance of bids submitted. **VOLATILITY** is secondary market volatility for 3-year bond benchmark. **MATURITY** is the period of time, in years, between settlement auction date and redemption date of the bond being auctioned. For observations of 3 and 5-year bond auctions, **REDEMPTIONS** is the volume of bonds with an original maturity of 3 and 5 years maturing in auction month. For observations of 10 and 15-year bond auctions, **REDEMPTION** is the volume of bonds with an original maturity of 10 years maturing in auction month. **DNEW** is a dummy for initial auctions with value 1 if the observation is of an initial auction and 0 otherwise. **DANNOUNCE** is a dummy for volume announcement which takes value 0 up to July 1995 and 1 thereafter.
For the variable **COVERC** the instrument **COVERADJ** was used. For observations of 3, 5 and 15-year bond auctions, **COVERADJ** is the cover ratio of the 10-year bond auction taking place on the same day (or the day before). For observations of 10-years bond auctions, **COVERADJ** is the cover ratio of the 3-years bond auction taking place in the same day.

APPENDIX 2

Proof of Corollary 1

First, note that $x \geq [x/(x+y)]Y = D_1(p^*)$, and $y \geq [y/(x+y)]Y = D_2(p^*)$.

Assume that player 2 submits a bid (p^*, y). Player 1 can:

i) Bid $(p< p^*, x)$. He receives at most $Y-y \leq Y - D_2(p^*) = D_1(p^*)$. Any x greater or equal to Y-y maximazes his payoff function. Therefore $x = D_1(p^*)$ maximizes his payoff function.

ii) Bid $(p = p^*, x)$. He pays p^*, and therefore it is optimal to bid x such that $D_1(p^*) = [x/(x+y)]Y$.

iii) Bid $(p > p^*, x)$. Since he pays the weighted average price, that is lower than p and higher than p^*, receives x with probability 1, and demand is decreasing. it follows that $x < D_1(p^*)$.

But bids i) and iii) are dominated by bid ii):

- Bid i), $(p< p^*, D_1(p^*))$ is dominated by bid ii). This result follows from the assumption of the proposition.

- Bid iii), $(p > p^*, x < D_1(p^*))$, is dominated by bid ii), since player 1 pays a lower price, p^*, and receives $D_1(p^*)$ with probability 1.

APPENDIX 1

TABLE A.1. SUMMARY STATISTICS FOR SOME AUCTION RESULTS

	ALL AUCTIONS					INITIAL AUCTIONS					REOPENING AUCTIONS				
	mean	max	min	median	std	mean	max	min	median	std	mean	max	min	median	std
Competitive vol submitted	254	867.3	15.36	242.5	174.4	355.7	658.3	115.8	375.2	148.5	236.3	867.3	15.36	224.3	172.9
Volume issued	124.6	516.9	2.28	121.4	97.83	194.2	359.9	43.58	190	84.95	112.5	516.9	2.28	104	95.02
Competitive vol accepted	122.9	516.6	1.112	119.7	97.86	193	357.6	43.32	188.7	84.75	110.7	516.6	1.112	102.7	95.02
Num. bids (c)	83.27	420	11	77	50.45	121.3	231	63	112	39.81	76.62	420	11	70	49.25
Num competitive bidders	30.86	84	6	29	14.58	41.34	72	21	36	13.61	29.02	84	6	27	13.99
Num non-competitive bidders	23.82	48	3	25	10.32	21.34	34	6	23	8.739	24.25	48	3	25	10.53
Num of bidders with only competitive bids	16.65	63	1	14	10.82	24.66	56	11	20	11.95	15.25	63	1	13	10.01
Num bids per bidder	2.162	4.798	1.231	2.132	0.461	2.541	3.348	1.94	2.452	0.359	2.096	4.798	1.231	2.072	0.445
Vol. bid per bid	4.245	32.17	0.086	3.011	4.722	3.543	9.373	1.154	3.327	1.744	4.368	32.17	0.086	2.924	5.06
Vol. bid per bidder	7.075	69.05	0.159	5.702	7.522	7.087	16.19	2.248	6.506	3.003	7.073	69.05	0.159	5.141	8.062
Cover ratio	3.099	29.16	1.041	2.18	3.59	2.029	5.743	1.041	1.847	0.965	3.286	29.16	1.073	2.238	3.842
%winning bids (c)	43.8	90.9	2.4	44.4	19.8	51.8	90.9	14.5	53.5	20.1	42.4	90.2	2.4	43.3	19.4
%winners (c)	61.7	96.3	5.3	66.7	21.2	71.3	96.3	22.7	72.7	20.1	60.1	96.2	5.3	64	21
%vol awarded at WAP (c)	42.0	84.8	0.0	39.1	37.3	38.2	67.2	6.4	35.9	30.3	42.7	84.8	0.0	40.0	38.2
%bids paying WAP (c)	61.6	94.6	7.7	54.5	62.6	61.4	84.7	24.1	56.7	53.0	61.6	94.6	7.7	53.3	63.7
% winnersat WAP(c)	62.9	93.5	11.1	56.3	61.1	61.0	81.2	33.3	56.3	48.0	63.2	93.5	11.1	56.7	62.5
%vol paying stop out price	46.6	100	9.5	43.5	22.1	38.1	93.7	9.7	34	20.2	48	100	9.5	46.5	22.2
WAP-stop out price	0.103	0.875	0	0.081	0.104	0.163	0.875	0	0.125	0.182	0.092	0.5	0	0.074	0.08
Max bid p-stop out price	3.499	29.5	0	2.1	4.311	3.928	16.3	0.375	2.375	3.751	3.425	29.5	0	1.938	4.407
Max bid price-Min bid p	5.842	32	0.45	4.35	5.198	5.944	17.25	1.5	4.5	4.427	5.824	32	0.45	4.25	5.333
std. of bid prices	0.795	3.617	0.102	0.509	0.689	0.609	1.553	0.194	0.45	0.396	0.827	3.617	0.102	0.528	0.724
std. of paid prices	0.036	0.262	0	0.028	0.035	0.058	0.262	0	0.045	0.055	0.032	0.2	0	0.023	0.029
Target vol for 3+10 y bonds	279.7	525	150	275	89.19	333.9	525	150	312.5	112.9	271.3	500	150	275	82.53
Target vol for 5+15 years bonds	228.7	400	125	225	73.53	280.4	375	150	262.5	76.07	220.6	400	125	225	70.18

Volumes are in billions of pesetas.

TABLE A.1. SUMMARY STATISTICS FOR SOME AUCTION RESULTS

	ALL AUCTIONS					INITIAL AUCTIONS					REOPENING AUCTIONS				
	mean	max	min	median	std	mean	max	min	median	std	mean	max	min	median	std
Competitive vol submitted	254	867.3	15.36	242.5	174.4	355.7	658.3	115.8	375.2	148.5	236.3	867.3	15.36	224.3	172.9
Volume issued	124.6	516.9	2.28	121.4	97.83	194.2	359.9	43.58	190	84.95	112.5	516.9	2.28	104	95.02
Competitive vol accepted	122.9	516.6	1.112	119.7	97.86	193	357.6	43.32	188.7	84.75	110.7	516.6	1.112	102.7	95.02
Num. bids (c)	83.27	420	11	77	50.45	121.3	231	63	112	39.81	76.62	420	11	70	49.25
Num competitive bidders	30.86	84	6	29	14.58	41.34	72	21	36	13.61	29.02	84	6	27	13.99
Num non-competitive bidders	23.82	48	3	25	10.32	21.34	34	6	23	8.739	24.25	48	3	25	10.53
Num of bidders with only competitive bids	16.65	63	1	14	10.82	24.66	56	11	20	11.95	15.25	63	1	13	10.01
Num bids per bidder	2.162	4.798	1.231	2.132	0.461	2.541	3.348	1.94	2.452	0.359	2.096	4.798	1.231	2.072	0.445
Vol. bid per bid	4.245	32.17	0.086	3.011	4.722	3.543	9.373	1.154	3.327	1.744	4.368	32.17	0.086	2.924	5.06
Vol. bid per bidder	7.075	69.05	0.159	5.702	7.522	7.087	16.19	2.248	6.506	3.003	7.073	69.05	0.159	5.141	8.062
Cover ratio	3.099	29.16	1.041	2.18	3.59	2.029	5.743	1.041	1.847	0.965	3.286	29.16	1.073	2.238	3.842
%winning bids (c)	43.8	90.9	2.4	44.4	19.8	51.8	90.9	14.5	53.5	20.1	42.4	90.2	2.4	43.3	19.4
%winners (c)	61.7	96.3	5.3	66.7	21.2	71.3	96.3	22.7	72.7	20.1	60.1	96.2	5.3	64	21
%vol awarded at WAP (c)	42.0	84.8	0.0	39.1	37.3	38.2	67.2	6.4	35.9	30.3	42.7	84.8	0.0	40.0	38.2
%bidspaying WAP (c)	61.6	94.6	7.7	54.5	62.6	61.4	84.7	24.1	56.7	53.0	61.6	94.6	7.7	53.3	63.7
%winners at WAP(c)	62.9	93.5	11.1	56.3	61.1	61.0	81.2	33.3	56.3	48.0	63.2	93.5	11.1	56.7	62.5
%vol paying stop out price	46.6	100	9.5	43.5	22.1	38.1	93.7	9.7	34	20.2	48	100	9.5	46.5	22.2
WAP-stop out price	0.103	0.875	0	0.081	0.104	0.163	0.875	0	0.125	0.182	0.092	0.5	0	0.074	0.08
Max bid p-stop out price	3.499	29.5	0	2.1	4.311	3.928	16.3	0.375	2.375	3.751	3.425	29.5	0	1.938	4.407
Max bid price-Min bid p	5.842	32	0.45	4.35	5.198	5.944	17.25	1.5	4.5	4.427	5.824	32	0.45	4.25	5.333
std. of bid prices	0.795	3.617	0.102	0.509	0.689	0.609	1.553	0.194	0.45	0.396	0.827	3.617	0.102	0.528	0.724
std. of paid prices	0.036	0.262	0	0.028	0.035	0.058	0.262	0	0.045	0.055	0.032	0.2	0	0.023	0.029
Target vol for 3+10 y bonds	279.7	525	150	275	89.19	333.9	525	150	312.5	112.9	271.3	500	150	275	82.53
Target vol for 5+15 years bonds	228.7	400	125	225	73.53	280.4	375	150	262.5	76.07	220.6	400	125	225	70.18

Volumes are in billions of pesetas.
Bids by non-members are aggregated by price.
Non-competitive bids by non-members are counted as 1 bid.
(c) refers to competitive bids

TABLE AI (CONT)

	REOPENING AUCTIONS WITHOUT VOLUME ANNOUNCEMENT					REOPENING AUCTIONS WITH VOLUME ANNOUNCEMENT				
	mean	max	min	median	std	mean	max	min	median	std
Competitive volume submitted	212	867.3	15.36	141.9	197.5	256.8	565.1	42.14	250.3	146.9
Volume issued	112.1	516.9	2.28	59.63	122.9	112.8	251.5	15.71	123.6	63.33
Competitive volume accepted	110.1	516.6	1.112	57.04	123.4	111.2	249.9	15.29	121	62.57
Num. bids (c)	80.89	420	11	65.5	68.5	73.01	128	26	73	22.65
Num competitive bidders	31.03	84	6	25.5	19.16	27.33	47	15	27.5	6.871
Num of non-competitive bidders	25.29	48	3	24	11.93	23.37	38	4	26	9.169
Num. of bidders with only competitive bids	17.25	63	1	12	13.96	13.57	28	7	13	3.898
Num bids per bidder	2.011	4.798	1.231	1.872	0.613	2.167	2.759	1.797	2.153	0.2
Vol. bid per bidder	2.904	20.5	0.086	2.39	2.837	5.604	32.17	0.618	3.645	6.108
Num bids per bidder	4.974	41	0.159	3.633	5.483	8.847	69.05	0.995	6.787	9.392
Cover ratio	4.201	29.16	1.073	2.036	5.468	2.514	5.838	1.084	2.328	0.919
%winning bids (c)	47.4	81.8	4.2	48.4	19.5	38.3	90.2	2.4	38.8	18.5
%winners (c)	62.8	95.2	12.5	68.4	19.7	57.7	96.2	5.3	59.2	21.9
%vol awarded at WAP (c)	34.7	67.3	0.0	29.6	30.5	48.0	84.8	0.0	44.2	40.9
%bids paying WAP (c)	63.1	94.6	7.7	50.0	70.5	60.3	82.4	20.0	55.1	48.2
%winners at WAP(c)	64.2	93.5	11.1	50.0	68.1	62.3	90.0	20.0	58.3	53.1
%vol paying stop out price	55.6	100	9.6	52.8	23.2	41.6	100	9.5	39.5	19.1
WAP-stop out price	0.093	0.5	0	0.068	0.094	0.091	0.389	0	0.077	0.066
Max bid p-stop out p	3.906	29.5	0	2.738	4.508	3.018	22.7	0	1.4	4.303
Max bid p-Min bid p	6.037	32	0.65	4.688	4.775	5.644	25	0.45	3.9	5.784
std. of bid prices	1.011	3.617	0.143	0.732	0.807	0.672	3.041	0.102	0.473	0.609
std. of paid prices	0.032	0.2	0	0.021	0.035	0.033	0.142	0	0.027	0.024
Target vol for 3+10 y bonds						271.3	500	150	275	82.53
Target vol for 5+15 years bonds						220.6	400	125	225	70.18

Volumes are in billions of pesetas.
Bids by non-members are aggregated by price.
Non-competitive bids by non-members are counted as 1 bid.
(c) refers to competitive bids

TABLE A1 (CONT)

	INITIAL AUCTIONS WITHOUT VOLUME ANNOUNCEMENT					INITIAL AUCTIONS WITH VOLUME ANNOUNCEMENT				
	mean	max	min	median	std	mean	max	min	median	std
Competitive volume submitted	369.2	658.3	155.1	375.2	167.1	341.2	535.9	115.8	342	130.4
Volume issued	218.1	359.9	83.34	209.4	94.62	168.7	299.4	43.58	184.7	67.39
Competitive volume accepted	216.7	357.6	81.57	208	94.54	167.6	298.4	43.32	183	67.09
Num. bids (c)	138.1	231	63	144	44.66	103.4	144	69	98.5	24.39
Num competitive bidders	49.47	72	21	53	13.99	32.64	41	22	33	5.514
Num of non-competitive bidders	19.4	34	6	18	8.959	23.43	33	6	24	8.309
Num. of bidders with only competitive bids	31.27	56	11	31	13.49	17.57	22	13	17	2.681
Num bids per bidder	2.532	3.348	1.94	2.5	0.402	2.55	3.175	2.066	2.43	0.322
Vol bid per bid	3.243	6.766	1.154	3.184	1.634	3.865	9.373	1.805	3.356	1.861
Vol bid per bidder	6.302	13.24	2.248	5.731	2.885	7.928	16.19	3.61	7.757	2.997
Cover ratio	1.875	5.743	1.041	1.561	1.155	2.195	4.035	1.329	2.048	0.715
%winning bids (c)	57.8	90.9	15.7	56.3	22	45.3	66	14.5	51.1	16.1
%winners (c)	75.7	96.3	30.3	78.3	18.9	66.5	90.9	22.7	71.6	21
%vol awarded at WAP (c)	29.6	60.2	6.4	22.2	26.3	45.4	67.2	32.1	43.1	29.5
%bids paying WAP (c)	67.0	84.7	24.1	67.1	57.0	52.9	70.0	26.3	50.2	38.2
%winners at WAP(c)	66.1	81.2	38.9	63.6	51.6	53.4	71.4	33.3	51.7	33.4
%vol paying stop out price	42.7	93.7	9.7	39.8	24.8	33.1	55.5	13.8	33.5	13.1
WAP-stop out price	0.207	0.875	0	0.125	0.242	0.117	0.211	0.031	0.105	0.058
Max bid p-stopout p	4.103	11.5	0.375	3.125	3.356	3.739	16.3	0.4	2.225	4.254
Max bid price-Min bid p	6.668	17.25	1.5	5.125	4.721	5.168	17.1	1.55	4.475	4.117
std. of bid prices	0.655	1.553	0.219	0.45	0.431	0.56	1.488	0.194	0.451	0.364
std. of paid prices	0.073	0.262	[illegible]	0.052	0.072	0.041	0.069	0.011	0.04	0.019
Target vol for 3 + 10 y bonds						333.9	525	150	312.5	112.9
Target vol for 5 + 15 years bonds						280.4	375	150	262.5	76.07

Volumes are in billions of pesetas.
Bids by non-members are aggregated by price.
Non-competitive bids by non-members are counted as 1 bid.
(c) refers to competitive bids

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