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Abstract

The aim of this paper is twofold. First, I study how the proportion of fixed and variable-rate mortgages in an economy can affect the way shocks are propagated. Second, I analyze optimal implementable simple monetary policy rules and the welfare implications of this proportion. I develop and solve a New Keynesian dynamics tochastic general equilibrium model that features a housing market and a group of constrained individuals who need housing collateral to obtain loans. A given proportion of constrained households borrows at a variable rate, while the rest borrows at a fixed rate.The model predicts that in an economy with mostly variable-rate mortgages, an exogenous interest rate shock has larger effects on borrowers than in a fixed-rate economy. Aggregate effects are also larger for the variable-rate economy. For plausible parametrizations, differences are muted by wealth effects on labor supply and by the presence of savers. More persistent shocks, such as inflation target and technology shocks, cause larger aggregate differences. From a normative perspective I find that, in the presence of collateral constraints, the optimal Taylor rule is less aggressive against inflation than in the standard sticky-price model. Furthermore, for given monetary policy, a high proportion of fixed-rate mortgages is welfare enhancing.

Keywords: Fixed/Variable-rate mortgages, monetary policy, housing market, collateral constraint.

JEL classification: E32, E44, E52.

"[...]the structure of mortgage contracts may matter for consumption behavior. In countries like the United Kingdom, for example, where most mortgages have adjustable rates, changes in short-term interest rates have an almost immediate effect on household cash flows. [...] *In an economy where most mortgages carry fixed rates, such as the United States, that channel of effect may be more muted. I do not think we know at this point whether, in the case of households, these effects are quantitatively significant in the aggregate. Certainly, these issues seem worthy of further study".* Ben Bernanke, June 15, 2007.

1 Introduction

Mortgage contracts in an economy can be fixed or variable rate. The proportion of variable-rate mortgages varies from country to country. In countries such as the United States, Germany and France, the majority of mortgages are fixed rate. However, the predominant type of mortgages in countries such as the United Kingdom, Australia and Spain is variable.

Mortgage rate changes affect the amount of mortgage interest payments, causing a direct cash-flow effect on consumption. Interest rate changes also affect housing demand and housing prices. If households are using housing as a collateral, the value of this collateral changes, inducing a wealth effect on household behavior and indirectly affecting consumption (ECB (2003), HM Treasury (2003)). Interest rate shocks affect mortgage rates differently depending on whether the mortgage is fixed or variable rate. Variable-rate mortgages are mortgage loans for which the interest rate is adjusted periodically, typically in line with some measured short-term interest rate. Hence, interest rate shocks directly affect variable rates. In contrast, fixed-rate mortgages are mortgage loans for which the interest rate remains constant through the term of the loan. The fixed interest rate is tied to a longer-term interest rate and is less sensitive to changes in the policy rate.

This raises important questions: How does the mortgage rate structure affect the way macroeconomic shocks are propagated? What are the implications in terms of monetary policy and welfare? These questions are of academic and policy interest. To give an illustrative example, the United Kingdom Treasury explicitly mentions the difference in mortgage structures as an important reason not to join the euro area. In the UK, the vast majority of borrowers have variable-rate mortgages, as opposed to the large countries of the euro area. According to the UK Treasury, British households are more exposed to monetary policy changes than, say, German households (HM Treasury (2003), Miles (2004)).

To address these questions, I build a New Keynesian dynamic stochastic general equilibrium model with housing and collateral constraints to explore how shocks are propagated in the presence of mortgage heterogeneity. I introduce fixed and variable-rate mortgages in the model. For the proportion of variable-rate mortgages to matter via the direct, cash-flow effect of mortgage interest payments on consumption, borrowers and savers are needed. Then, the effect of interest rate changes on borrowing does not cancel out by the presence of a representative consumer. For the indirect, wealth effect to appear, one needs non-durable consumption to be related to house prices. The introduction of collateral constraints tied to housing value for one type of consumers solves both problems since it motivates the presence of borrowers and savers and relates housing prices to consumption. In this model, monetary policy has real effects that are comparable with other sticky-price models. Furthermore, since the model is microfounded it allows me to study optimal monetary policy and welfare.[1]

It is not the aim of this paper to explain how the decision between fixed and variable-rate mortgages is made.[2] For simplicity, I hold the proportion of fixed and variable-rate borrowers constant and exogenous. Although this proportion can vary in reality, there is evidence that it fluctuates around a constant mean which is higher or lower depending on the country.[3] We could think of these cross-country differences as due to institutional, historical or cultural factors, out of the scope of this model.[4]

I use the model to compute impulse responses to interest rate, inflation target and technology shocks. I consider two extreme cases; one in which the economy is composed by variable-rate borrowers and one where the fixed rate is the predominant type of mortgage.

Results show that interest rate shocks affect more strongly those borrowers that have variable-rate mortgages. Given an increase in the interest rate set by the central bank, variable-rate borrowers reduce their consumption and housing demand by more than fixed-rate borrowers. The intuition is as follows: After a monetary policy shock (increase in the interest rate), fixed and variable-rate consumers differ in the real interest rate they face. Consider the most extreme case in which the variable rate changes one for one with the interest rate set by the central bank and the fixed rate is constant. After the shock, the nominal mortgage rate increases for the variable-rate individuals and inflation decreases. For the fixed-rate borrowers, the nominal mortgage interest rate does not react, but inflation is still decreasing

[1] The analysis of optimal monetary policy is restricted to optimization over parameters of a simple implementable Taylor rule.

[2] See Miles (2004) or Campbell and Cocco (2003) for studies that cover this from a microeconomic perspective.

[3] See Appendix 1 for evidence for the UK and the US.

[4] The European Mortgage Federation (EMF) highlights that cultural differences play an important role for the predominant type of mortgage contract in a country. They are linked to real estate law, borrowers' risk aversion, funding system or frequency of house moves.

because the economy is contracting. As a result, real rates increase by more if the mortgage is variable rate. In real terms, payments are increasing by more for variable-rate consumers, and their consumption and housing decrease by more (this is a pure cash-flow effect). A second, wealth effect comes through the collateral constraint. Banks are willing to lend as long as debt repayments do not exceed a fixed proportion of the value of the house collateral. For borrowers with variable-rate mortgages the value of their collateral has been reduced by more since they are demanding less houses. These effects make consumption decrease more strongly for variable-rate borrowers.

Aggregate consumption also declines by more after a monetary policy shock when the economy is mainly borrowing at a variable rate. However, aggregate differences are more muted due to the behavior of savers. In equilibrium, borrowing and saving must be equal. If borrowing decreases, saving must also decrease. Savers are the owners of financial intermediaries in the model, so any loss for the borrowers is a gain for the savers. These manage to offset part of the decrease in consumption following a positive interest rate shock. Results for monetary policy shocks are very robust to different model specifications.

Some aggregate differences arise because the borrowers' marginal propensity to consume is larger than the savers'.[5] However, aggregate differences are not larger due to several reasons; First, results are sensitive to the borrowers ´share. Increasing the size of this group would amplify the differences. Second, income effects on labor supply are important in this model. With the type of preferences used in standard real business cycle models, labor effort is determined together with the intertemporal consumption choice. When consumption is reduced, individuals tend to work more to compensate and smooth consumption. Variable-rate consumers increase their labor to compensate for the extra reduction in consumption they suffer when there is an interest rate shock. Using preferences as in Greenwood, Hercowitz and Huffman (1988)(GHH henceforth), this effect is eliminated. In this case, the channels that are important for the purposes of this paper are emphasized and aggregate effects are larger. Finally, persistence is also a key element here. Interest rate shocks are not very persistent, more persistent shocks such as inflation target or technology shocks amplify the differences.[6]

When the inflation target increases, output responds by more when variable rates are predominant. Real interest rates fall persistently and house prices increase by less than with fixed mortgage rates. Variable-rate borrowers increase their nondurable consumption by more. Since house prices do not

[5] In this model borrowers face collateral constraints and are more impatient than savers. This makes their consumption respond by more to changes in wealth.

[6] Assenmacher-Wesche K., Gerlach, S., (2008) also find in an empirical paper that the financial structure does not have large effects for monetary policy shocks. However, there is no evidence for other type of shocks.

increase as much in the variable-rate case, also savers can consume more nondurables.

With respect to technology shocks, a favorable technology shock increases output and lowers prices. Monetary policy responds in a persistent way and real rates increase. Variable-rate borrowers consume less because the real rate increase affects them and dampens the positive effects of the technology shock for them. The increase in real rates does not affect fixed-rate consumers as much and they can consume more. Output increases by more when fixed rates are predominant.

I also study welfare and optimal monetary policy in the context of fixed and variable-rate mortgages. In particular, I search over parameters of a simple, implementable interest rate rule so that welfare is maximized. I find that, in the presence of collateral constraints, a social welfare maximizing central bank should respond to inflation less aggressively than in the absence of collateral constraints. Results also show that when the central bank focuses only on the savers' welfare, thus ignoring the collateral constraint, the optimized inflation parameter in the Taylor rule is higher. However, when borrowers are taken into account, the central bank optimally responds less to inflation. The central bank faces a trade-off between the borrowers and savers' welfare because on the one hand, low inflation corrects the sticky-price distortion but, on the other hand, inflation relaxes the collateral constraint and improves borrowers' welfare. Comparing welfare across mortgage rate scenarios for given policy shows that this inflation channel is more effective the higher the proportion of fixed-rate mortgages in the economy. Therefore, borrowers are better off with fixed-rate mortgages although this comes at the cost of lower welfare for savers. For aggregate welfare, I find that predominantly fixed-rate contracts are welfare enhancing.

This paper relates to different strands of literature. First, it contributes to the literature on New Keynesian general equilibrium models with housing and collateral constraints such as Aoki et al. (2004) and Iacoviello (2005), who do not consider heterogeneous mortgage contracts. Second, it is also related to a literature that studies fixed and variable-rate mortgages. Campbell and Cocco (2003) and Miles (2004) study the fixed versus variable rate choice from a partial equilibrium perspective. Graham and Wright (2007) develop a model in which some households face binding credit constraints and debt contracts can be fixed or variable rate. However, they do not include a housing market and thus the constraint is not tied to housing stock and housing prices, eliminating the wealth channel. Calza et al. (2007) study how institutional factors, including mortgage contracts, can affect the monetary transmission mechanism. In my model, I focus on fixed versus variable rate mortgages. My results on monetary policy shocks are comparable to theirs under some parameter specifications. Relative to them, I do not only study the exogenous component of monetary policy but also the systematic response to other shocks. The existent literature is silent about how mortgage heterogeneity affects the way shocks such as changes in inflation target or technology are propagated. Finally the paper contributes to the literature on optimal monetary policy with heterogeneous consumers and collateral constraints. See for instance Monacelli (2006) or Mendicino and Pescatori (2007). However, none of these papers studies welfare and optimal monetary policy in the context of different mortgage contracts.

Section II explains the basic model I build for the analysis. Section III shows the results and dynamics and business cycles of the model. Section IV analyzes optimal monetary policy. Section V presents the conclusions. Appendix 1 contains graphs and tables on the empirical evidence mentioned above. Appendix 2 shows model derivations.

2 The Baseline Model

I consider an infinite-horizon economy in which households consume, work and demand real estate. There is a representative financial intermediary that provides mortgages and accepts deposits from consumers. Firms set prices subject to Calvo (1983)-Yun (1996) nominal rigidity. The monetary authority sets interest rates endogenously, in response to inflation and output, following a Taylor rule.

2.1 The Consumer's Problem

There are three types of consumers: unconstrained consumers, constrained consumers who borrow at a variable rate, and constrained consumers who borrow at a fixed rate. Constrained individuals need to collateralize their debt repayments in order to borrow from the financial intermediary. Interest payments for both mortgages and loans cannot exceed a proportion of the future value of the current house stock. In this way, the financial intermediary ensures that borrowers are going to be able to fulfill their debt obligations next period. As in Iacoviello (2005), I assume that constrained consumers are more impatient than unconstrained ones. This assumption ensures that the borrowing constraint is binding, so that constrained individuals do not save and wait until they have the funds to self-finance their consumption. This generates an economy in which households divide into borrowers and savers. Furthermore, borrowers are divided into two groups, those who borrow at a fixed rate and those who borrow at a variable rate. The proportion of each type of borrower is fixed and exogenous. All households derive utility from consumption, housing services assumed proportional to the housing stock and leisure.[7]

2.1.1 Unconstrained Consumers (Savers)

Unconstrained consumers maximize:

$$\max\ E_0 \sum_{t=0}^{\infty} \beta^t \left(\ln C_t^u + j \ln H_t^u - \frac{(L_t^u)^\eta}{\eta} \right), \tag{1}$$

where the superscript u stands for "unconstrained", E_0 is the expectation operator, $\beta \in (0,1)$ is the discount factor, and C_t^u, H_t^u and L_t^u are consumption at t, the stock of housing and hours worked, respectively; $1/(\eta-1)$ is the labor supply elasticity, $\eta > 0$ and $j > 0$ represents the weight of housing in the utility function.

The budget constraint is:

$$C_t^u + q_t H_t^u + b_t^u \leq q_t H_{t-1}^u + w_t^u L_t^u + \frac{R_{t-1} b_{t-1}^u}{\pi_t} + F_t^v + S_t^v, \tag{2}$$

where q_t is the real housing price and w_t^u is the real wage for unconstrained consumers. These can buy houses or sell them at the current price q_t. I assume zero housing depreciation for simplicity. As we will see, this group will choose not to borrow at all; they are the savers in this economy. b_t^u is the amount they save. They receive interest R_{t-1} for their savings. π_t is inflation in period t. S_t and F_t are lump-sum profits received from the firms and the financial intermediary, respectively. We can think of these consumers as the wealthy agents in the economy, who own the firms and the financial intermediary.

The first-order conditions for this unconstrained group are:

$$\frac{1}{C_t^u} = \beta E_t \left(\frac{R_t}{\pi_{t+1} C_{t+1}^u} \right), \tag{3}$$

$$w_t^u = (L_t^u)^{\eta-1} C_t^u, \tag{4}$$

$$\frac{j}{H_t^u} = \frac{1}{C_t^u} q_t - \beta E_t \frac{1}{C_{t+1}^u} q_{t+1}. \tag{5}$$

[7] I do not allow for renting. This is needed to generate borrowers and savers in the economy. If renting were allowed, borrowers could use renting to save and the wealth effect would disappear. Furthermore, in the US, homeownerships have been quite high in the last years (about 65 percent, according to the US Census Bureau).

Equation (3) is the Euler equation for consumption, equation (4) is the labor-supply condition, and equation (5) is the Euler equation for housing. This states that the benefits from consuming housing must be equal to the costs at the margin.

2.1.2 Constrained Consumers (Borrowers)

Constrained consumers can be of two types: those who borrow at a variable rate and those who do it at a fixed rate. The difference between them is simply the interest rate they face. The fixed-rate borrower faces $\overline{R}_t$, set by the financial intermediary, whereas the variable-rate counterpart faces R_t, set by the central bank. The proportion of variable-rate consumers is fixed and exogenous and equal to $\alpha \in [0,1]$.

Constrained and unconstrained consumers are different in the way they discount the future. Constrained consumers are more impatient than unconstrained ones. I assume that constrained consumers face a limit on the debt they can acquire. The maximum amount they can borrow is proportional to the value of their collateral, in this case the stock of housing. That is, the debt repayment next period cannot exceed a proportion of tomorrow's value of today's stock of housing:

$$E_t \frac{R_t}{\pi_{t+1}} b_t^{cv} \leq k E_t q_{t+1} H_t^{cv}, \tag{6}$$

$$E_t \frac{\overline{R}_t}{\pi_{t+1}} b_t^{cf} \leq k E_t q_{t+1} H_t^{cf}, \tag{7}$$

where (6) represents the collateral constraint for the variable-rate constrained consumer and (7) is the constraint for the fixed-rate one.[8]

Without loss of generality, I present the problem for the variable-rate borrower, since the one for the fixed-rate is symmetric. Variable-rate borrowers maximize their lifetime utility function subject to the budget constraint and the collateral constraint:

$$\max \; E_0 \sum_{t=0}^{\infty} \widetilde{\beta}^t \left(\ln C_t^{cv} + j \ln H_t^{cv} - \frac{(L_t^{cv})^\eta}{\eta} \right), \tag{8}$$

subject to:

$$C_t^{cv} + q_t H_t^{cv} + \frac{R_{t-1} b_{t-1}^{cv}}{\pi_t} \leq q_t H_{t-1}^{cv} + w_t^{cv} L_t^{cv} + b_t^{cv}, \tag{9}$$

[8] The superscript cv stands for "constrained variable" while cf stands for "constrained fixed"

and (6).[9]

As noted above, constrained consumers are more impatient than unconstrained ones, so that $\widetilde{\beta} < \beta$. This assumption is crucial for the borrowing constraint to be binding and therefore, for there to be both borrowers and savers in the economy.

The first-order conditions for variable-rate constrained consumers are:

$$\frac{1}{C_t^{cv}} = \widetilde{\beta} E_t \left(\frac{R_t}{\pi_{t+1} C_{t+1}^{ci}} \right) + \lambda_t^{cv} R_t, \tag{10}$$

$$w_t^{cv} = \left(L_t^{cv}\right)^{\eta-1} C_t^{cv}, \tag{11}$$

$$\frac{j}{H_t^{cv}} = \frac{1}{C_t^{cv}} q_t - \widetilde{\beta} E_t \frac{1}{C_{t+1}^{cv}} q_{t+1} - \lambda_t^{cv} k E_t q_{t+1} \pi_{t+1}. \tag{12}$$

These first-order conditions differ from those of the unconstrained individuals. In the case of constrained consumers, the Lagrange multiplier on the borrowing constraint (λ_t^{cv}) appears in equations (10) and (12). From the Euler equations for consumption of unconstrained consumers, we know that $R = 1/\beta$ in steady state. If we combine this result with the Euler equation for consumption of constrained individuals we have that $\lambda^{cv} = \left(\beta - \widetilde{\beta}\right)/C^{cv} > 0$ in steady state. This means that the borrowing constraint holds with equality in steady state. Since we log-linearize the model around the steady state and assume that uncertainty is low, we can generalize this result to off-steady-state dynamics. Then, the borrowing constraint is always binding, so that constrained individuals are going to borrow the maximum amount they are allowed to and unconstrained consumers are never in debt.[10]

Given the borrowing amount implied by (6) at equality, consumption for variable-rate constrained individuals can be determined by their flow of funds:

$$C_t^{cv} = w_t^{cv} L_t^{cv} + b_t^{cv} + q_t \left(H_{t-1}^{cv} - H_t^{cv}\right) - \frac{R_{t-1} b_{t-1}^{cv}}{\pi_t}, \tag{13}$$

and the first-order condition for housing becomes:

$$\frac{j}{H_t^{cv}} = \frac{1}{C_t^{cv}} \left(q_t - \frac{k E_t q_{t+1} \pi_{t+1}}{R_t} \right) - \widetilde{\beta} E_t \frac{1}{C_{t+1}^{cv}} (1-k)\, q_{t+1}. \tag{14}$$

[9] We will see from the firm's problem that $w_t^{cv} = w_t^{cf} = w_t^c$.

[10] This is a typical assumption for this kind of models. See Iacoviello (2005), Appendix C for a detailed analysis of when do constraints bind.

2.1.3 Aggregate Variables

Given the fraction α of variable-rate borrowers, we can define aggregates across constrained consumers as $C_t^c \equiv \alpha C_t^{cv} + (1-\alpha) C_t^{cf}, L_t^c \equiv \alpha L_t^{cv} + (1-\alpha) L_t^{cf}, H_t^c \equiv \alpha H_t^{cv} + (1-\alpha) H_t^{cf}, b_t^c \equiv \alpha b_t^{cv} + (1-\alpha) b_t^{cf}$.

Therefore, economy-wide aggregates are: $C_t \equiv C_t^u + C_t^c, L_t \equiv L_t^u + L, H_t \equiv H_t^u + H_t^c$. In this model, aggregate supply of housing is fixed, so that market clearing requires[11]: $H_t = H_t^u + H_t^c = H$.

2.2 The Financial Intermediary

The financial intermediary accepts deposits from savers, and extends both fixed and variable-rate loans to borrowers. The profits of the financial intermediary are:

$$F_t = \alpha R_{t-1} b_{t-1}^{cv} + (1-\alpha) \overline{R}_{t-1} b_{t-1}^{cf} - R_{t-1} b_{t-1}^u. \tag{15}$$

To simplify, since the typical time horizon of a mortgage is large, I consider the maturity of mortgages to be infinite, although this assumption is not crucial for the dynamics of the problem.

In equilibrium, aggregate borrowing and saving must be equal, that is,

$$b_t^c = b_t^u. \tag{16}$$

Substituting (16) into (15) ,we obtain,

$$F_t = (1-\alpha) b_{t-1}^{cf} \left(\overline{R}_{t-1} - R_{t-1} \right). \tag{17}$$

I assume that the financial intermediary operates under perfect competition. Therefore, the optimality condition for the financial intermediary implies that at each point in time τ, the intermediary is indifferent between lending at a variable or fixed rate. Hence, the expected discounted profits that the intermediary obtains by lending new debt in a given period at a fixed interest rate must be equal to the expected discounted profits the intermediary would obtain by lending it at variable rate:

$$E_\tau \sum_{i=\tau+1}^{\infty} \beta^{i-\tau} \Lambda_{\tau,i} \overline{R}_\tau^* \left(b_\tau^{cf} - b_{\tau-1}^{cf} \right) = E_\tau \sum_{i=\tau+1}^{\infty} \beta^{i-\tau} \Lambda_{\tau,i} R_{i-1} \left(b_\tau^{cf} - b_{\tau-1}^{cf} \right), \tag{18}$$

[11] This assumption provides an easy way to specify the supply of housing and have variable prices. A two-sector model with production of housing does not generate significatively different results (see Appendix 2).

where $\Lambda_{t,i} = \left(\dfrac{C_t^u}{C_{t+i}^u}\right)$ is the unconstrained consumer relevant discount factor. Since the financial intermediary is owned by the savers, their stochastic discount factor is applied to the financial intermediary's problem.

We can obtain the optimal value of the fixed rate in period τ from expression (18) :

$$\overline{R}_\tau^* = \frac{E_\tau \sum\limits_{i=\tau+1}^{\infty} \beta^{i-\tau} \Lambda_{\tau,i} R_{i-1}}{E_\tau \sum\limits_{i=\tau+1}^{\infty} \beta^{i-\tau} \Lambda_{\tau,i}}. \tag{19}$$

Equation (19) states that, for every new debt issued at date τ, there is a different fixed interest rate that has to be equal to a discounted average of future variable interest rates. Notice that this is not a condition on the stock of debt, but on the new amount obtained in a given period. New debt at a given point in time is associated with a different fixed interest rate. Both the fixed interest rate in period τ and the new amount of debt in period τ are fixed for all future periods. However, the fixed interest rate varies with the date the debt was issued, so that in every period there is a new fixed interest rate associated with new debt in this period. If we consider fixed-rate loans to be long-term, the financial intermediary obtains interest payments every period from the whole stock of debt, not only from the new ones. Hence, we can define an aggregate fixed interest rate that is the one the financial intermediary effectively charges every period. This aggregate fixed interest rate is composed of all past fixed interest rates and past debt, together with the current period optimal fixed interest rate and new amount of debt. Therefore, the effective fixed interest rate that the financial intermediary charges for the stock of fixed-rate debt every period is:

$$\overline{R}_t = \frac{\overline{R}_{t-1} b_{t-1}^{cf} + \overline{R}_t^* \left(b_t^{cf} - b_{t-1}^{cf}\right)}{b_t^{cf}}. \tag{20}$$

Equation (20) states that the fixed interest rate that the financial intermediary is actually charging today is an average of what it charged last period for the previous stock of mortgages and what it charges this period for the new amount. Importantly, this assumption is not crucial for results. Both $\overline{R}_\tau^*$ and $\overline{R}_t$ are practically unaffected by interest rate shocks. This assumption is a way to reconcile the model with the fact that fixed-rate loans are not one-period assets but longer term ones.

As noted above, if any, profits from financial intermediation are rebated to the unconstrained consumers every period. Even if the financial intermediary is competitive and it does not make profits in absence of shocks, if there is a shock at a given point in time, the fact that only the variable interest rate is affected can generate non-zero profits.

2.3 Firms

2.3.1 Final Goods Producers

There is a continuum of identical final goods producers that aggregate intermediate goods according to the production function

$$Y_t = \left[\int_0^1 Y_t(z)^{\frac{\varepsilon-1}{\varepsilon}} dz \right]^{\frac{\varepsilon}{\varepsilon-1}}, \tag{21}$$

where $\varepsilon > 1$ is the elasticity of substitution between intermediate goods. The final good firm chooses $Y_t(z)$ to minimize its costs, resulting in demand of intermediate good z:

$$Y_t(z) = \left(\frac{P_t(z)}{P_t} \right)^{-\varepsilon} Y_t. \tag{22}$$

The price index is then given by:

$$P_t = \left[\int_0^1 P_t(z)^{1-\varepsilon} dz \right]^{\frac{1}{\varepsilon-1}}. \tag{23}$$

Market clearing for the final good requires:

$$Y_t = C_t = C_t^u + C_t^c.$$

2.3.2 Intermediate Goods Producers

The intermediate goods market is monopolistically competitive. Following Iacoviello (2005), intermediate goods are produced according to the production function:

$$Y_t(z) = A_t L_t^u(z)^{\gamma} L_t^c(z)^{(1-\gamma)}, \tag{24}$$

where $\gamma \in [0, 1]$ measures the relative size of each group in terms of labor. This Cobb-Douglas production function implies that labor efforts of constrained and unconstrained consumers are not perfect substitutes. This specification is analytically tractable and allows for closed form solutions for the steady state of the model. This assumption can be economically justified by the fact that savers are the managers of the firms and their wage is higher than the one of the borrowers.[12] Experimenting with a production function in which hours are substitutes leads to very similar results in terms of model dynamics. In the appendix, I derive the model under this alternative assumption and show that both are comparable. Under the Cobb-Douglas specification each household has mass one. γ is a constant that represents the labor income share of the patient household and L_t^u are total hours worked by the patient household. In the alternative specification derived in the appendix, ωL_t^u represents the total hours worked by the patient household while ω is the fraction of savers in the population. Therefore, both specifications are very similar but, while γ represents the economic size of savers, ω is its absolute size.

A_t represents technology and it follows the following autorregressive process:

$$\log(A_t) = \rho_A \log(A_{t-1}) + u_{At}, \tag{25}$$

where ρ_A is the autorregressive coefficient and u_{At} is a normally distributed shock to technology.

Labor demand is determined by:

$$w_t^u = \frac{1}{X_t}\gamma\frac{Y_t}{L_t^u}, \tag{26}$$

$$w_t^c = \frac{1}{X_t}(1-\gamma)\frac{Y_t}{L_t^c}, \tag{27}$$

where X_t is the markup, or the inverse of marginal cost.[13]

The price-setting problem for the intermediate good producers is a standard Calvo-Yun setting. An intermediate good producer sells its good at price $P_t(z)$, and $1-\theta, \in [0,1]$, is the probability of being able to change the sale price in every period. The optimal reset price $P_t^*(z)$ solves:

$$\sum_{k=0}^{\infty}(\theta\beta)^k E_t\left\{\Lambda_{t,k}\left[\frac{P_t^*(z)}{P_{t+k}} - \frac{\varepsilon/(\varepsilon-1)}{X_{t+k}}\right]Y_{t+k}^*(z)\right\} = 0. \tag{28}$$

The aggregate price level is then given by:

$$P_t = \left[\theta P_{t-1}^{\varepsilon} + (1-\theta)(P_t^*)^{1-\varepsilon}\right]^{1/(1-\varepsilon)}. \tag{29}$$

[12] It could also be interpreted as the savers being older than the borrowers, therefore more experienced.
[13] Symmetry across firms allows us to write the demands without the index z.

Using (28) and (29), and log-linearizing, we can obtain a standard forward-looking New Keynesian Phillips curve which is presented in the Appendix.

2.4 Monetary Policy

The model is closed with a Taylor Rule with interest rate smoothing, to describe the conduct of monetary policy by the central bank:[14]

$$R_t = (R_{t-1})^{\rho} \left[\left(\frac{\pi_t}{\pi_t^*} \right)^{(1+\phi_\pi)} R \right]^{1-\rho} \varepsilon_{Rt}, \tag{30}$$

where $0 \leq \rho \leq 1$ is the parameter associated with interest-rate inertia, and $\phi_\pi > 0$ measures the response of interest rates to current inflation. R is the steady-state values of the interest rate. ε_{Rt} is a white noise shock with zero mean and variance σ_ε^2. π_t^* is the inflation target that evolves according to:

$$\log(\pi_t^*) = \rho_\pi \log\left(\pi_{t-1}^*\right) + \varepsilon_{\pi t}, \tag{31}$$

where $\varepsilon_{\pi t}$ is normally distributed with variance σ_π^2.

3 Shock Transmission and Business Cycles

I linearize the equilibrium equations around the steady state. Details are shown in Appendix 2. For calibration, I consider the following parameter values: The discount factor, β, is set to 0.99 so that the annual interest rate is 4% in the steady state. The discount factor for borrowers, $\widetilde{\beta}$, is set to 0.98. Lawrance (1991) estimates discount factors for poor consumers between 0.95 and 0.98 at quarterly frequency. Results are not sensitive to different values within this range. This value of $\widetilde{\beta}$ is low enough to endogenously divide the economy into borrowers and savers. The weight of housing on the utility function, j, is set to 0.1 in order for the ratio of housing wealth to GDP in the steady state to be consistent with the data. This value of j implies a ratio of approximately 1.40, in line with the Flow

[14] This is a realistic policy benchmark for most of the industrialized countries. A more realistic rule would also include output but it complicates building intuition about the workings of the model. Furthermore, estimations deliver a small response to the output gap in the last two decades (See Clarida, Gali and Gertler (2000)). Nevertheless, robustness checks to this specification will be performed.

of Funds data.[15] I set $\eta = 2$, implying a value of the labor supply elasticity of 1.[16] For the loan-to-value ratio, I pick $\kappa = 0.9$, consistent with the evidence that in the last years borrowing constrained consumers borrowed on average more than 90% of the value of their house.[17] The labor income share of unconstrained consumers, γ, is set to 0.64, following the estimate in Iacoviello (2005). I pick a value of 6 for ε, the elasticity of substitution between intermediate goods. This value implies a steady state markup of 1.2. The probability of not changing prices, θ, is set to 0.75, implying that prices change every four quarters. For the Taylor Rule parameters I use $\rho = 0.8$, $\phi_\pi = 0.5$. The first value reflects a realistic degree of interest-rate smoothing.[18] The second one, is consistent with the original parameter proposed by Taylor in 1993. For α, I consider two polar cases for comparison. In the first case, the proportion of variable-rate mortgages in the economy is 0, that is, all constrained consumers in the economy borrow at a fixed rate. In the second case, the proportion of variable-rate mortgages is 1. Table 1 shows a summary of the parameter values.

[15] See Table B.100. In this model, consumption is the only component of GDP. To make the ratio comparable with the data I multiply it by 0.6, which is approximately what nondurable consumption and services account for in the GDP, according to the data in the NIPA tables.

[16] Microeconomic estimates usually suggest values in the range of 0 and 0.5 (for males). Domeij and Flodén (2006) show that in the presence of borrowing constraints this estimates could have a downward bias of 50%.

[17] We can identify constrained consumers with those that borrow more than 80% of their home. In the US, among those borrowers, the average LTV ratio exceeds 90% for the period 1973-2006. See the data from the Federal Housing Finance Board.

[18] See McCallum (2001).

Parameter Values		
β	.99	Discount Factor for Savers
$\widetilde{\beta}$	.98	Discount Factor for Borrowers
j	.1	Weight of Housing in Utility Function
η	2	Parameter associated with labor elasticity
k	.9	Loan-to-value ratio
γ	.64	Labor share for Savers
α	0/1	Proportion of variable-rate borrowers
X	1.2	Steady-state markup
θ	.75	Probability of not changing prices
ρ_π	.975	Inflation target persistence
ρ_A	.9	Technology persistence
ρ	.8	Interest-Rate-Smoothing Parameter in Taylor Rule
ϕ_π	.5	Inflation Parameter in Taylor Rule

Table 1: Parameter Values

3.1 Impulse Responses

3.1.1 Monetary Policy Shock

Impulse responses to a one standard deviation (0.29 percent) increase of the interest rate are presented in Figure 1.[19] We can see that when the economy is mainly composed by individuals indebted at a variable-rate, the effects of monetary policy on consumption for the borrowers are stronger than in the fixed-rate case. Borrowers' housing demand, initially, also decreases more strongly after a monetary policy shock if the predominant type of mortgages in the economy is variable rate. These findings show that the proportion of variable-rate mortgages matters for the monetary transmission mechanism. When the proportion of variable-rate borrowers is very high, a monetary policy shock affects more strongly those individuals who are constrained and need to borrow.

In the aggregate, output in the variable-rate economy also decreases more strongly (See Figure 2). There is a redistribution between borrowers and savers but we can still find aggregate differences because

[19] Iacoviello (2005) estimates a Taylor Rule for the US economy and finds a 0.29 percent standard deviation on a quarterly basis. I use this number as an empirically plausible one-standard deviation increase in the interest rate.

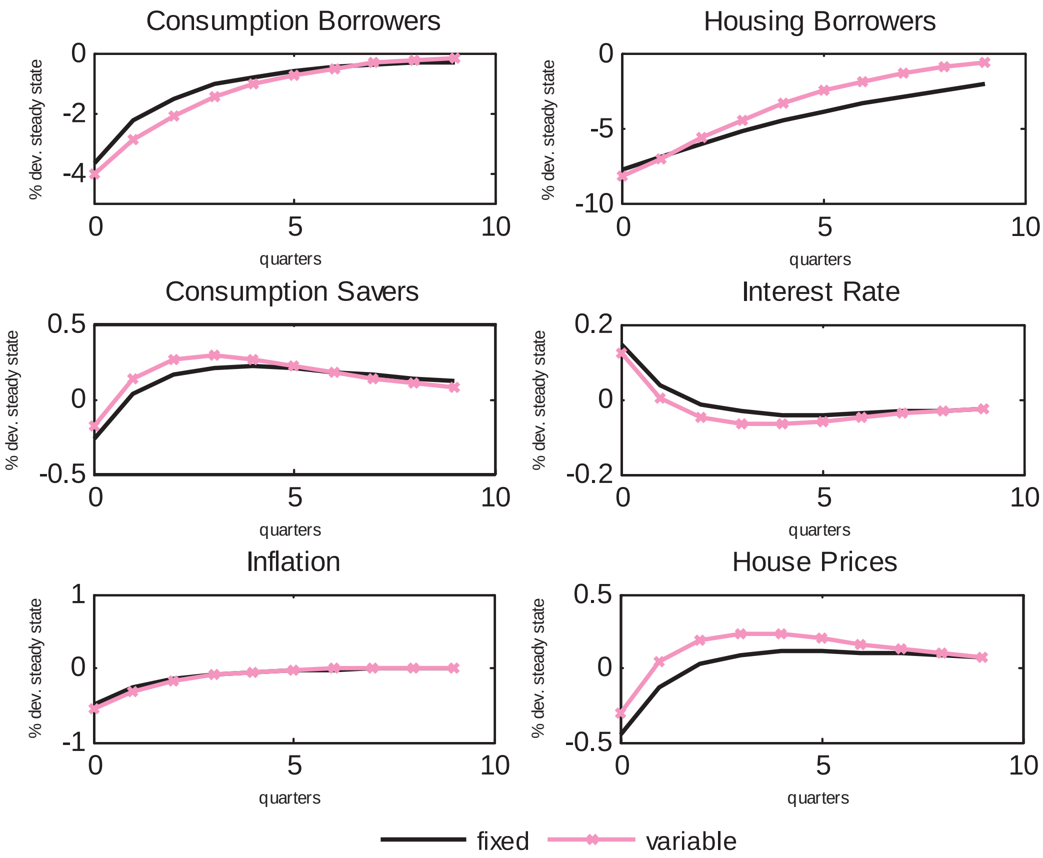

Figure 1: Impulse Responses to a Monetary Policy Shock. Baseline Specification.

borrowers are more sensitive to changes in wealth (they are more impatient and use housing wealth as collateral).[20]

Results for monetary policy shocks with standard preferences are very robust to alternative model specifications. We can introduce capital in the basic model or assume nonseparability between housing and consumption in the utility function. The basic results for the variables of interest are maintained.[21]

Relative Sizes and Income Effects on Labor Supply Differences between the two scenarios are not larger for several reasons; First, results are sensitive to the wage share of unconstrained individuals in the economy.[22] Figure 3 shows that by decreasing the size of the savers aggregate differences are

[20] Using an alternative Taylor rule that also considers interest rate responding to output delivers very similar impulse responses. However, on impact aggregate differences between scenarios are slightly larger under the alternative interest rate rule.

[21] The details of the model are presented in Appendix 2. The parameter values used for the calibration are 0.025 for capital depreciation and 10 for capital adjustment costs. The elasticity of substitution between non-durable consumption goods and housing of 0.5. The rest of the parameter values are the same as in the baseline model.

[22] This parameter represents the relative economic size of each group in the economy.

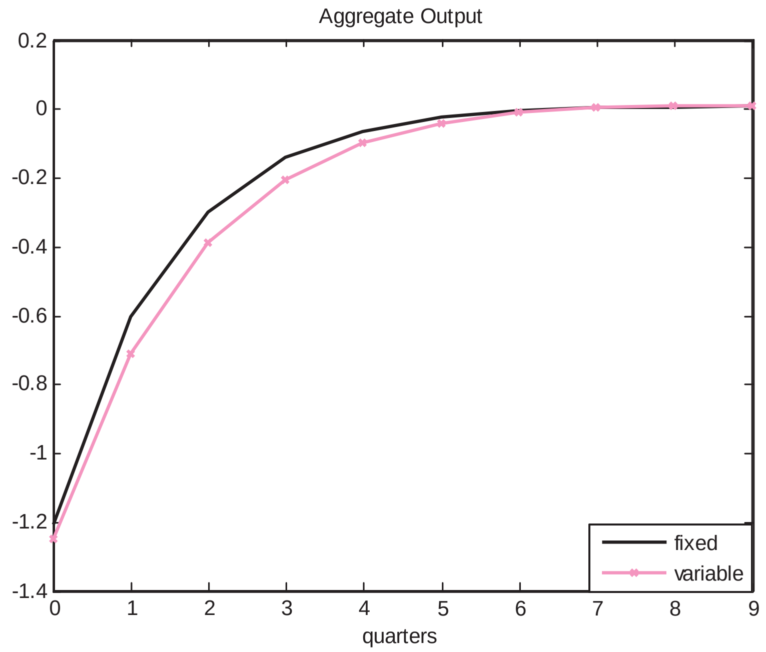

Figure 2: Aggregate Output Response to a Monetary Policy Shock. Baseline Specification.

amplified.

Second, in this model income effects on the labor supply decision are important. In the baseline model preferences are separable in consumption and labor. In this case, the labor supply decision depends on the level of consumption. Given a negative shock to the economy, labor supply moves both in response to a substitution and an income effect. On the one hand, lower wages make consumers want to work less. On the other hand, lower consumption generates an income effect that makes consumers want to work more. Income effects can partly offset aggregate differences. GHH preferences have the property of shutting down the income effect on the labor supply decision. In this preferences, labor and consumption are non-separable. This makes labor effort to be determined independently from the intertemporal consumption-savings choice.[23] There an extensive literature that has also used these preferences to emphasize other channels that are partially offset by this income effects.[24] Impulse responses, in line with other studies that use GHH preferences, show how consumption responses are stronger and aggregate differences are amplified (See Figure 4).

Finally, differences across scenarios are not larger because monetary policy shocks do not show high persistence. More persistent shocks such as inflation target or technology shocks are able to amplify those

[23] See Appendix 2 for details on GHH preferences and derivations.
[24] See for example Raffo (2006) and references therein.

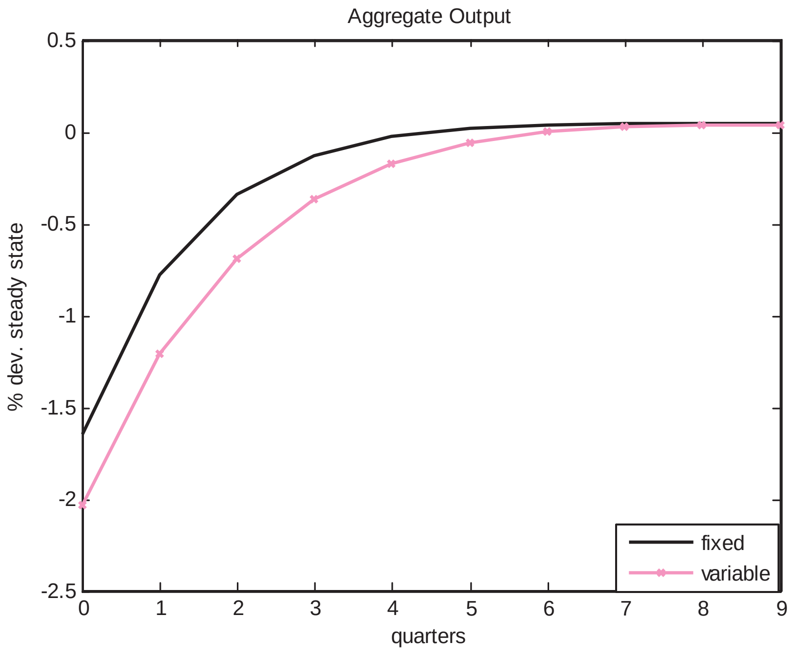

Figure 3: Aggregate Output Response to a Monetary Policy Shock. Increasing the share of borrowers to 60%.

differences. The next two subsections present impulse responses for these two other types of shocks.

3.1.2 Inflation Target Shock

Instead of a shock to the interest rate, we can also consider a more persistent monetary policy disturbance such as a shock to the inflation target. Figure 5 shows the responses of the variables of interest to an increase in the inflation target of 0.1 percent, with 0.975 persistence.[25]

Aggregate differences are amplified with this type of shock.[26] In particular, output increases by more in the variable-rate case. Shock persistence is a key issue in the analysis. In the case of exogenous interest rate shocks we have seen that unless we unrealistically increase the borrowers share or use GHH preferences, there is no much difference, at the aggregate level, between the two mortgage contracts. A change in the interest rate that does not last long creates an initial wedge between the rates that the two different borrowers face but it fades away very fast. However, we also have to take into account the endogenous component of monetary policy. Interest rates also respond to other shocks to the economy.

[25] In line with Adolfson et al. (2007) or Iacoviello and Neri (2008).

[26] Since the degree of amplication of inflation shocks may depend in the type of monetary rule adopted, I have repeated the exercise assuming that the interest rate also responds to the output gap with a coefficient of 0.5. I find that the dynamics are very similar although the amplification in the differences is slightly stronger in this case. However, for comparison with the rest of the cases I present the results for the baseline Taylor rule.

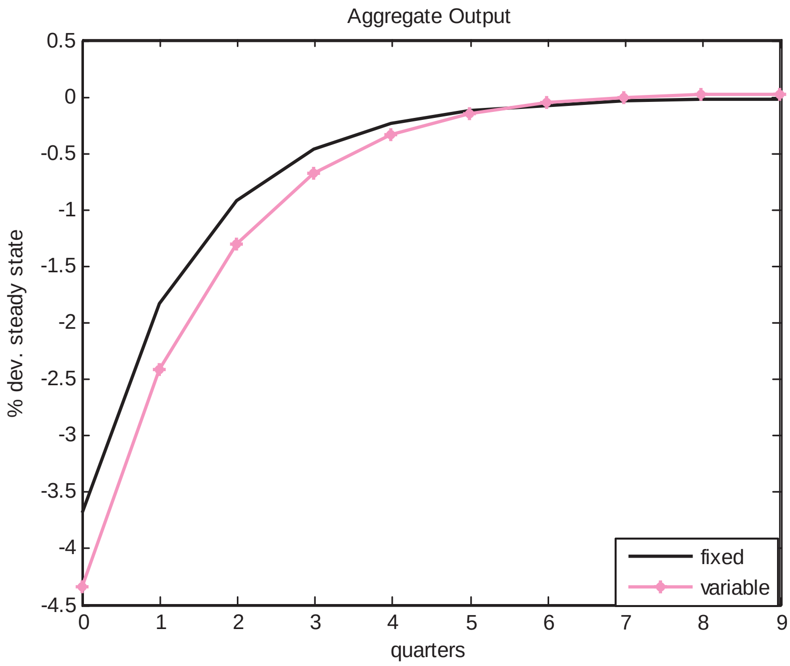

Figure 4: Aggregate Output Response to a Monetary Policy Shock. GHH Preferences.

And the more persistent are these other shocks, the more persistent is the response of monetary policy. A persistent change in the policy interest rate makes the wedge between the two mortgage rates also persist and it amplifies the differences.

When there is an inflation target shock, interest rates respond systematically to the shock in a very persistent way. In particular, an increase in the inflation target, since it is an expansionary measure, makes inflation and output increase. The nominal interest rate increases persistently but real rates fall. House prices increase by less in the variable-rate economy. Variable-rate borrowers increase by more their nondurable consumption because real rates fall. Since house prices do not increase that much in the variable-rate case, also savers can consume more nondurables. In the aggregate, the combination of these two elements makes the variable-rate economy respond more strongly to the shock.

3.1.3 Technology Shock

A shock to technology may also have different effects on the economy depending on whether individuals are mainly borrowing at variable or fixed rate. Impulse responses to a 1 percent positive shock to technology with 0.9 persistence are showed in Figure 6.[27] We see that the economy responds more

[27] This high value of persistence is consistent with estimates in the literature. See for instance Iacoviello and Neri (2008).

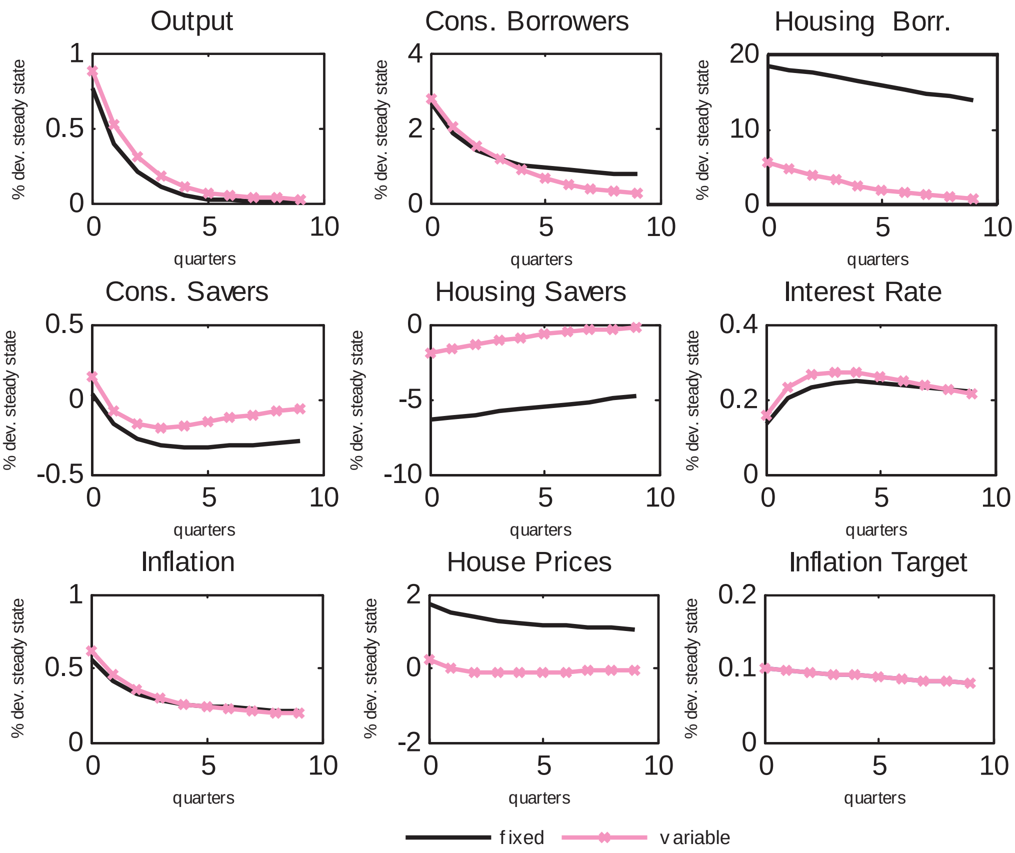

Figure 5: Impulse Responses to an Inflation Target Shock. Baseline Specification.

strongly after a technology shock when the majority of its borrowers have a fixed-rate mortgage and the differences across scenarios are larger than in the exogenous monetary policy disturbance case.[28] The argument is similar to the inflation target shock. The systematic part of monetary policy responds to the technology shock and, since this is a persistent shock, this creates a persistent wedge between fixed and variable rates.

In particular, we can see in the figure that a positive technology shock increases output and lowers prices. As a reaction, nominal rates decrease persistently but real interest rates increase. Variable-rate borrowers consume less because increase in real rate affects them negatively. However, fixed-rate consumers are better off in comparison and they can consume more. Responses for savers do not differ much across scenarios and then, the redistribution effect between borrowers and savers is not as strong as in this case of monetary policy shocks. As a result, output increases by more for fixed-rate consumers.

[28] Robustness checks for the Taylor rule specification have also been performed in this case. A Taylor rule that also includes an output term, with a coefficient of 0.5 delivers similar results for the aggregate differences between scenarios. The output response, however, is hump–shaped.

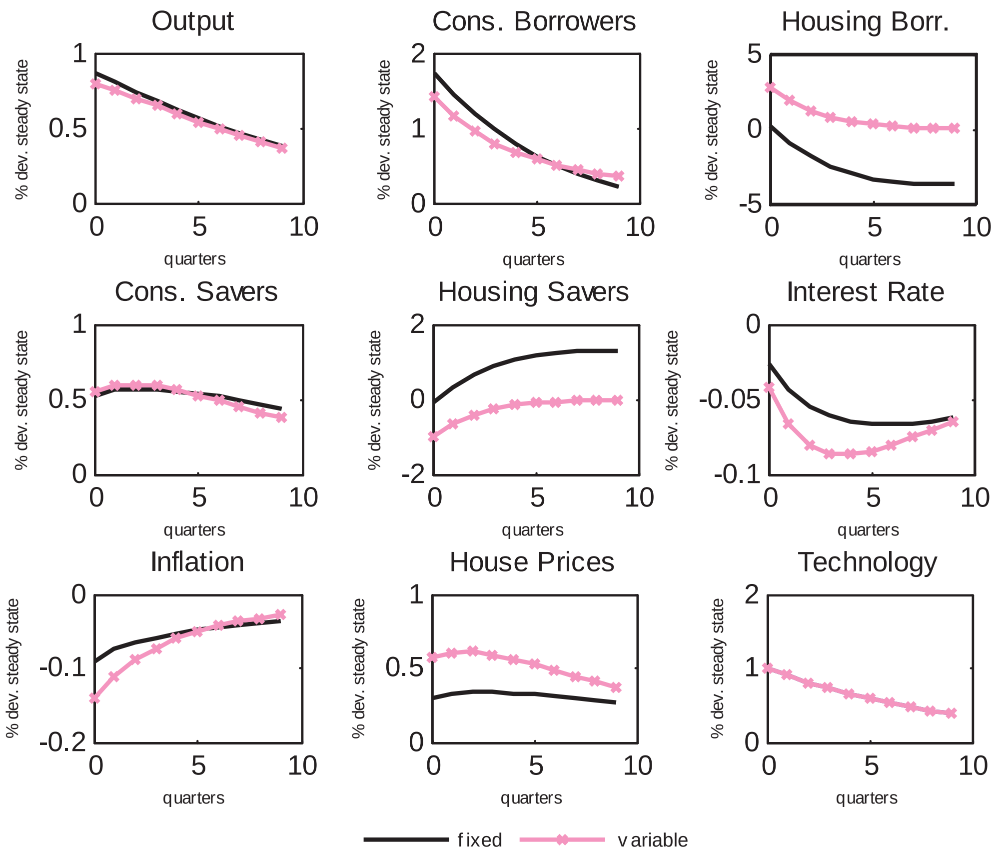

Figure 6: Impulse Responses to a Technology Shock. Baseline Model.

Comparing across shocks, it is interesting to note that monetary policy shocks or inflation target shocks cause the real interest rate to vary countercyclically, which is why flexible-rate mortgages amplify the effects those shocks. Technology shocks, by contrast, cause the real interest rate to vary procyclically: it rises when output rises, which is why flexible-rate mortgages dampen the effects of those shocks.

3.2 Second Moments

Table 2 shows the standard deviations of the main variable both from the model and the data.[29] The model generates a standard deviation of GDP of 2.0127 for the variable-rate case and 2.126 for the fixed-rate economy. This is slightly smaller but close to the data (2.26), especially for the fixed-rate economy.[30]The volatility of consumption and housing demand is always greater for those individuals that are constrained but smaller in the case of variable rates. The volatility of inflation and house prices

[29]Theoretical moments calculated for technology shocks. Standard deviations from the data taken from Davis and Heathcote (2005).

[30]Davis and Heathcote (2005) also find smaller output volatility.

is smaller in the model than in the data while the correlation between output and house prices is greater.

Business Cycle Properties				
% SD Rel. to GDP		**Data**	**Model (Fixed Rates)**	**Model (Variable Rates)**
	$\mathbf{y}$	2.26	2.126	2.013
	$\mathbf{c}^u$		0.931	0.904
	$\mathbf{c}^c$		1.413	1.304
	$\mathbf{h}^u$		2.276	0.646
	$\mathbf{h}^c$		6.525	1.852
	π	0.78	0.094	0.121
	$\mathbf{q}$	1.37	0.552	0.911
Correlations	$\mathbf{y}, \mathbf{q}$	0.65	0.960	0.993

Table 2: Business Cycle Properties.

4 Welfare and Optimal Monetary Policy

So far, I have described the model dynamics under the two types of mortgage contracts. However, this does not allow to answer question such as "what is better for the economy, fixed or variable-rate mortgages?", or "is the optimal monetary policy different depending on the predominant type of mortgage in the economy?". In this section, I complement the previous analysis with some normative assessment. In particular, I compare different simple monetary policy rules based on welfare evaluations, both for the whole economy and for different types of consumers. I also compare welfare for the two scenarios for given policy rule.

The individual welfare for savers and borrowers respectively is defined as follows:[31]

$$V_{u,t} \equiv \; E_t \sum_{m=0}^{\infty} \beta^m \left(\ln C_{t+m}^u + j \ln H_{t+m}^u - \frac{\left(L_{t+m}^u\right)^\eta}{\eta} \right), \tag{32}$$

$$V_{ci,t} \equiv \; E_t \sum_{m=0}^{\infty} \widetilde{\beta}^m \left(\ln C_{t+m}^{ci} + j \ln H_{t+m}^{ci} - \frac{\left(L_{t+m}^{ci}\right)^\eta}{\eta} \right), \tag{33}$$

[31] I numerically compute the second order approximation of the utility function as a measure of welfare.

Following Mendicino and Pescatori (2007), I define social welfare as a weighted sum of individual welfare for the different types of households:

$$V_t = (1-\beta) V_{u,t} + \left(1-\widetilde{\beta}\right) \left[\alpha V_{cv,t} + (1-\alpha) V_{cf,t}\right]. \tag{34}$$

Borrowers and savers' welfare are weighted by $\left(1-\widetilde{\beta}\right)$ and $(1-\beta)$ respectively, so that the two groups receive the same level of utility from a constant consumption stream. As in Mendicino and Pescatori (2007), I take this approach to be able to evaluate the welfare of the three types of agents separately.[32]

To begin, I evaluate the welfare achieved under the ad-hoc Taylor rule used in the baseline model. Results are presented in Table 3:

Ad-hoc Taylor Rule: $\rho = 0.8, \phi_\pi = 0.5$		
	Variable Rate	**Fixed Rate**
Social Welfare	-6.0693	-4.7097
Savers Welfare	21.5748	-822.6597
Borrowers Welfare	-314.2514	175.8456
$\boldsymbol{\sigma}(\pi)$	0.2436	0.1999

Table 3: Welfare comparison. Ad-hoc Taylor Rule.

The economy with fixed-rate mortgages achieves a higher level of welfare than the variable-rate economy. Notice as well that there is a trade-off between savers and borrowers' welfare: Although a larger fraction of fixed-rate borrowers raises aggregate welfare, this comes at the cost of lower welfare for savers.

Figure (7) shows how the welfare level varies with the proportion of variable rate mortgages in the economy.[33] This figure clearly illustrates this trade-off between borrowers and savers. When mortgages are at a fixed rate, savers, who own the financial intermediary, bear all the risk associated with interest rate changes and therefore their welfare is lower. Borrowers, are however insured against interest-rate risk. Furthermore, when mortgage rates are fixed the collateral constraint is relaxed and then less distorting for the economy. This enhances welfare. If we look at the loglinearized collateral constraint (see equation (46) in Appendix 2), we can observe that, at a given level of inflation, in real terms, mortgage payments are lower, the lower the value of α is. As a result of this trade-off between borrowers

[32] See Monacelli (2006) for an example of the Ramsey approach in a model with heterogeneous consumers.
[33] Welfare is rescaled so that it appears in the positive axis. Additionally, borrowers and savers' welfare is divided by 100.

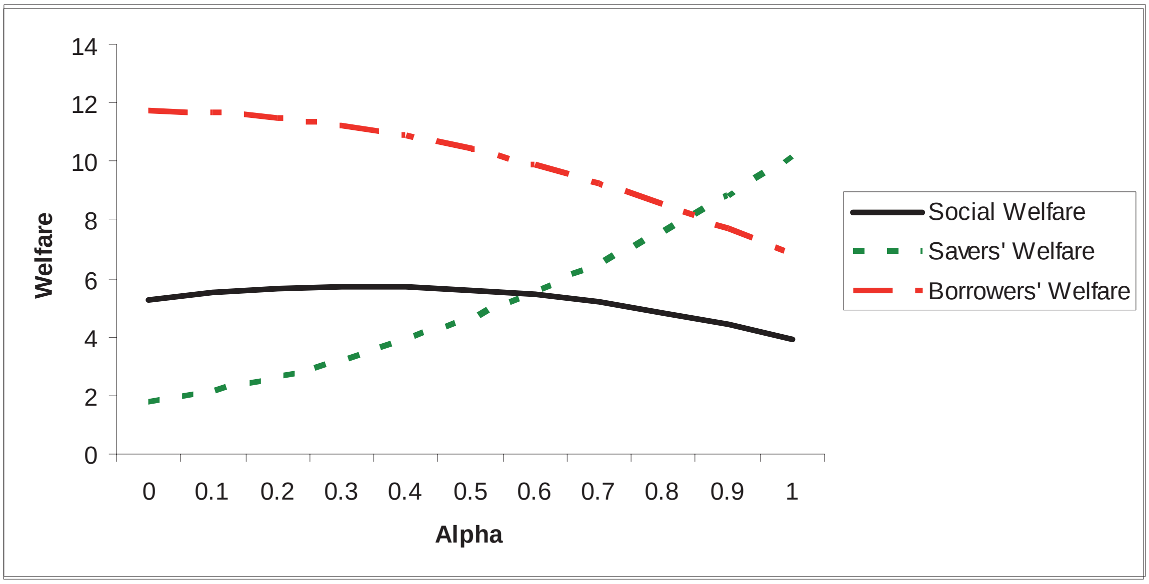

Figure 7: Welfare level for different values of α. Ad-hoc Taylor rule.

and savers, the economy achieves the maximum level of social welfare at around the value of $\alpha = 0.3$, that is, when 70 percent of the mortgages are fixed rate. Then, in this economy in which there are two types of distortions (price rigidities and credit frictions), fixed-rate contracts are welfare enhancing first because they decrease the distorting effects of the borrowing constraint and second because borrowers do not bear the risk associated to the interest rate variability.

Next, I study what is the monetary policy that maximizes welfare. The design of optimal monetary policy in the presence of collateral constraints is more complicated than in the standard sticky-price setting. As I mentioned, there are two types of distortions. On the one hand, the central bank should aim at lowering inflation volatility because, given sticky prices, inflation distorts production decisions. On the other hand, inflation relaxes the borrowing constraints and improves the borrowers' welfare. However, as noticed above, this inflation channel is much more effective when fixed-rate mortgages are predominant. The loglinearized collateral constraint shows that mortgage payments decrease with inflation but increase with the interest rate. Inflation relaxes the collateral constraint for borrowers, as long as the interest rate does not react too much to it. Therefore, the inflation channel for borrower welfare is stronger the less the central bank responds to inflation but also the lower the value of α. In the limit, an economy with just fixed-rate mortgages maximizes the favorable effects of inflation on the collateral constraint.

Monacelli (2006) and Mendicino and Pescatori (2007) perform a full-blown optimal monetary policy analysis in a model with collateral constraints. They find that in the presence of credit frictions, the aggressiveness towards inflation by the central bank is reduced. I perform a simple exercise here that also allows me to distinguish between mortgage contracts. In particular, given a grid of possible parameters for the Taylor rule, I perform a search that maximizes welfare, subject to determinacy requirements. For simplicity, I start by keeping the value of ρ fixed to 0.8 and I search over different values of ϕ_π, the response coefficient to inflation. In this way, I can build intuition about on much the central bank should respond to inflation in different cases for the same degree of interest-rate smoothing. Results are presented in Table 4:

Optimized Taylor Rule (Maximize Social Welfare)		
	Variable Rate	**Fixed Rate**
	$\boldsymbol{\rho = 0.8, \phi_\pi^* = 0.1}$	$\boldsymbol{\rho = 0.8, \phi_\pi^* = 0.1}$
Social Welfare	-3.0619	-0.9131
Savers Welfare	-256.1426	-3398.0803
Borrowers Welfare	-25.0259	1653.3866
$\boldsymbol{\sigma}(\pi)$	0.5611	0.5233
Optimized Taylor Rule (Maximize Savers Welfare)		
	Variable Rate	**Fixed Rate**
	$\boldsymbol{\rho = 0.8, \phi_\pi^* = 3.85}$	$\boldsymbol{\rho = 0.8, \phi_\pi^* = 20}$
Social Welfare	-10.4962	-8.8568
Savers Welfare	146.6523	-4.4361
Borrowers Welfare	-598.1339	-440.6205
$\boldsymbol{\sigma}(\pi)$	0.0353	0.0058
Optimized Taylor Rule (Standard Sticky Price)		
	$\boldsymbol{\rho = 0.8, \phi_\pi^* = 20}$	
Social Welfare	-51.0065	
$\boldsymbol{\sigma}(\pi)$	0.0182	

Table 4: Welfare Values for Optimized Taylor Rule

For the model with collateral constraints, I consider two cases: a central bank that is a social welfare maximizer and a central bank that neglects the borrowers' welfare. Within each case, mortgage contracts are either fixed or variable rate. Then, I compare the results with a model without collateral constraints. As in Monacelli (2006) and Mendicino and Pescatori (2007), lenders prefer the central bank being aggressive against inflation. However, borrowers obtain welfare gains from a monetary policy that minimizes credit market inefficiencies. The central bank aggressively fights inflation if it considers only the welfare of those not facing credit constraints. However, economies with fixed-rate contracts achieve a higher welfare in all cases because they are less distorted by the collateral constraint. If we compare the results with a model without collateral constraints, we clearly see that the central bank should respond to inflation less aggressively than in the standard sticky-price model, without collateral constraints. In fact, for the model with collateral constraints the optimal value of ϕ_π corresponds to the minimum value allowed in the search while in the absence of collateral constraints it corresponds to the maximum one.

Optimized Taylor Rule (Maximize Social Welfare)		
	Variable Rate	**Fixed Rate**
	$\boldsymbol{\rho^* = 0.9, \phi_\pi^* = 0.35}$	$\boldsymbol{\rho^* = 0.1, \phi_\pi^* = 0.1}$
Social Welfare	-2.9985	20.7278
Savers Welfare	-183.2848	-16768.7875
Borrowers Welfare	-58.2802	9420.7829
$\boldsymbol{\sigma}(\pi)$	0.3490	0.9423
Optimized Taylor Rule (Maximize Savers Welfare)		
	Variable Rate	**Fixed Rate**
	$\boldsymbol{\rho^* = 0.1, \phi_\pi^* = 0.35}$	$\boldsymbol{\rho^* = 0.9, \phi_\pi^* = 20}$
Social Welfare	-13.0488	-8.8798
Savers Welfare	187.7239	-1.2493
Borrowers Welfare	-746.3038	-443.3643
$\boldsymbol{\sigma}(\pi)$	0.3428	0.0077
Optimized Taylor Rule (Standard Sticky Price)		
	$\boldsymbol{\rho^* = 0.1, \phi_\pi^* = 20}$	
Social Welfare	-50.8178	
$\boldsymbol{\sigma}(\pi)$	0.0126	

Table 5: Welfare Values for Optimized Taylor Rule

Table 5 shows results for an optimized Taylor rule in which I search for both the values of ρ and ϕ_π so that welfare is maximized. Again in this case we can clearly see that the optimal response to inflation by the central bank is less aggressive in the presence of collateral constraints. However, intuition in the previous case was more straightforward because ρ was kept fixed.

5 Conclusions

In this paper, I have developed a New Keynesian general equilibrium model with housing and collateral constraints to study first, how the proportion of variable-rate mortgages in the economy can affect the transmission of shocks and then, what the welfare implications of mortgage contracts are. There are unconstrained and constrained individuals that correspond to the savers and borrowers of the economy. I explicitly introduce fixed and variable-rate mortgages, that is, constrained individuals can be of two types: those who borrow at a variable rate and those who borrow at a fixed rate.

Model responses are in line with the intuition. A monetary policy shock affects more strongly those individuals who are borrowing in economies in which the predominant type of mortgages is at variable rate. Consumption and housing demand decrease by more after an interest rate increase if constrained consumers are variable rate. In a general equilibrium framework, the partial equilibrium effects are maintained, but muted by a redistribution between borrowers and savers and strong wealth effects in labor supply decisions. GHH preferences generate larger aggregate differences between the two scenarios considered. Aggregate differences are also amplified when the relative size of the borrowers is increased.

Monetary policy shocks are not persistent. More persistent shocks such as technology or inflation target shocks are able to generate much larger differences in the aggregate economy.[34] Monetary policy responds to these shocks in a very persistent way causing large aggregate differences between the fixed and the variable-rate economy. Inflation target shocks have more effect on output in variable-rate economies. On the contrary, technology shocks increase output by more in those economies mainly borrowing at a fixed rate, due to the procyclicality of real interest rates in this case.

From a normative perspective, I find that the optimal interest-rate response to inflation by the central

[34] This is also consistent with Krusell and Smith (1998) or Gourinchas (2001). They study the effects of the distribution of income and wealth and the implications of precautionary savings and life cycle for the macroeconomy in a general equilibrium framework with heterogeneous agents. Their results are not very different from what one would obtain in a representative agent model, behaviors of different agents practically offset each other in the aggregate when considering realistic parameter specification. They also find that permanent shocks would generate larger effects on the aggregate economy.

bank is weaker when a group of consumers need collateral to obtain loans, as compared to the standard sticky-price model. Inflation relaxes the collateral constraint and therefore reduces the distortions created by this extra friction. However, this channel is stronger the higher the proportion of fixed-rate mortgages in the economy. A high proportion of fixed-rate contracts is welfare enhancing.

The model presented here can set directions for future research. The proportion of fixed and variable-rate mortgages is kept constant. A natural extension would be to endogeneize it by modelling the mortgage choice. For instance, borrowers could be heterogeneous in their risk aversions or market-powered banks could price mortgages charging a spread on fixed-rate mortgages depending on economic conditions. Furthermore, this model is not able to keep track of the new fixed-rate mortgages issued every period. For tractability I assume that the financial intermediary charges an average of the new fixed interest rate and the old interest rate for fixed-rate mortgages every period. An overlapping generations version could solve this issue. It would also be interesting to study shock transmission and monetary policy in international versions of the model with heterogeneous mortgage structures across countries.

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Appendix 1: Tables and Figures

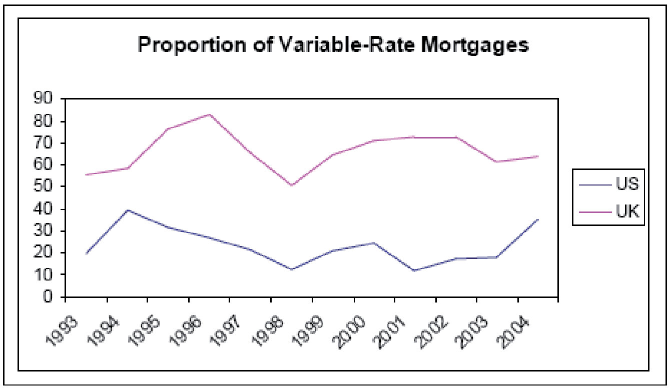

Figure 8: Proportion of Variable-Rate Mortgages in the US and UK. Source: Federal Housing Finance Board and Council of Mortgage Lenders

Residential Debt to GDP Ratio (2005)			
EU15	48.9%	**Italy**	17.2%
Austria	21.9%	**Denmark**	94%
France	29.4%	**Spain**	52.6%
Germany	51.7%	**United Kingdom**	80%
Greece	25.1%	**United States**	71.2%

Table 6: Residential Debt to GDP Ratio. Source: European Mortgage Federation

Predominant Type of Mortgage Interest Rate			
Australia	Variable	**Italy**	Mixed
Austria	Fixed	**Japan**	Mixed
France	Fixed	**Spain**	Variable
Germany	Fixed	**United Kingdom**	Variable
Greece	Variable	**United States**	Fixed

Table 7: Predominant Type of Mortgage Interest Rate. Source: ECB (2003), IMF

Appendix 2: Model Derivations and Alternative Specifications

Steady-State Relationships

Using (3) in the steady state we obtain $R = 1/\beta$. From (19) and (20) we have that $\overline{R}^{*} = \overline{R} = R = 1/\beta$.

From the first order conditions for housing we can obtain the steady-state consumption-to-housing ratio for both constrained and unconstrained consumers:

$$\frac{C^u}{qH^u} = \frac{1}{j}\left(1-\beta\right), \tag{35}$$

$$\frac{C^c}{qH^c} = \frac{1}{j}\left(1-\widetilde{\beta}-k\left(\beta-\widetilde{\beta}\right)\right) = \frac{q}{j}\Phi, \tag{36}$$

where $\Phi \equiv \left(1-\widetilde{\beta}-k\left(\beta-\widetilde{\beta}\right)\right)$. From (13) and (27)we obtain the constrained and unconstrained consumption-to-output ratio in the steady state:

$$\frac{C^c}{Y} = \frac{1-\gamma}{X}\left(\frac{\Phi}{\Phi+jk\left(1-\beta\right)}\right), \tag{37}$$

$$\frac{C^u}{Y} = 1-\frac{C^c}{Y}, \tag{38}$$

where $X = \varepsilon/\left(\varepsilon-1\right)$

The housing-to-output ratio for constrained and unconstrained consumers:

$$\frac{qH^c}{Y} = \frac{\left(1-\gamma\right)j}{X}\left(\frac{1}{\Phi+jk\left(1-\beta\right)}\right), \tag{39}$$

$$\frac{qH^u}{Y} = \frac{Xj\left(\Phi+jk\left(1-\beta\right)\right)-j\left(1-\gamma\right)\Phi}{X\left(\Phi+jk\left(1-\beta\right)\right)\left(1-\beta\right)}. \tag{40}$$

Log-Linearized Model

The model can be reduced to the following linearized system in which all lower-case variables with a hat denote percent changes from the steady state and steady-state levels are denoted by dropping the time index:

Financial intermediary

$$\widehat{\overline{r}}^*_\tau = \frac{(1-\beta)}{\beta} E_\tau \sum_{i=\tau+1}^{\infty} \beta^{i-\tau} \widehat{r}_{i-1}, \tag{41}$$

$$\widehat{\overline{r}}_t = \widehat{\overline{r}}_{t-1} \Rightarrow \widehat{\overline{r}}_t = \widehat{\overline{r}} = 0. \tag{42}$$

Equation (41) is the log-linearized fixed interest rate in each period τ. Using this result we can obtain the log-linearized aggregate fixed interest rate, which is zero in deviations from the steady state (equation (42)), given the initial condition of being at the steady state in the absence of shocks.

Aggregate Demand

$$\widehat{y}_t = \frac{C^u}{Y}\widehat{c}^u_t + \frac{C^c}{Y}\widehat{c}^c_t, \tag{43}$$

$$\widehat{c}^u_t = E_t\widehat{c}^u_{t+1} - \left(\widehat{r}_t - E_t\widehat{\pi}_{t+1}\right), \tag{44}$$

$$\begin{aligned}\widehat{c}^c_t = &\left(\frac{\Phi + jk\left(1-\beta\right)}{\Phi}\right)\left(\widehat{y}_t - \widehat{x}_t\right) - \frac{j}{\Phi}\left(\widehat{h}^c_t - \widehat{h}^c_{t-1}\right) \\ &+\frac{kj}{\Phi}\left(\beta\widehat{b}^c_t - \widehat{b}^c_{t-1}\right) - kj\left(\alpha\widehat{r}_{t-1} - \widehat{\pi}_t\right),\end{aligned} \tag{45}$$

$$\widehat{b}^c_t = E_t\widehat{q}_{t+1} + \widehat{h}^c_t - \left(\alpha\widehat{r}_t - E_t\widehat{\pi}_{t+1}\right). \tag{46}$$

Equation (43) is the log-linearized goods market clearing condition. Equation (44) is the Euler equation for unconstrained consumption. Equation (45) is the budget constraint for constrained individuals, which determines constrained consumption. Equation (46) is the log-linearized collateral constraint.

Housing Equations

$$\frac{H^u}{Y}\widehat{h}^u_t + \frac{H^c}{Y}\widehat{h}^c_t = 0, \tag{47}$$

$$\widehat{h}_t^u = \frac{1}{1-\beta}\left(\widehat{c}_t^u - \widehat{q}_t\right) - \frac{\beta}{1-\beta}E_t\left(\widehat{c}_{t+1}^u - \widehat{q}_{t+1}\right), \tag{48}$$

$$\widehat{h}_t^c = \frac{1-k\beta}{\Phi}\widehat{c}_t^c - \frac{1}{\Phi}\widehat{q}_t - \frac{k\beta}{\Phi}\left(\alpha\widehat{r}_t - E_t\widehat{\pi}_{t+1}\right) + \frac{\widetilde{\beta}}{\Phi}\widehat{q}_{t+1} - \frac{\widetilde{\beta}\left(1-k\right)}{\Phi}E_t\widehat{c}_{t+1}^c. \tag{49}$$

Equation (47) is the log-linearized market clearing condition for housing. Equation (48) is the housing margin for unconstrained consumers. Equation (49) is the analogous expression for constrained consumers.

Aggregate Supply

$$\widehat{y}_t = -\frac{1}{\eta-1}\left(\gamma\widehat{c}_t^u + (1-\gamma)\,\widehat{c}_t^c + \widehat{x}_t\right), \tag{50}$$

$$\widehat{\pi}_t = \beta E_t\widehat{\pi}_{t+1} - \widetilde{k}\widehat{x}_t + u_{\pi t}. \tag{51}$$

Equation (50) is the production function combined with labor market clearing. Equation (51) is the New Keynesian Phillips curve that relates inflation positively to future inflation and negatively to the markup ($\widetilde{k} \equiv (1-\theta)(1-\beta\theta)/\theta$). $u_{\pi t}$ is a normally distributed cost-push shock.

Monetary Policy

$$\widehat{r}_t = \rho\widehat{r}_{t-1+}\,(1-\rho)\left[(1+\phi_\pi)\left(\widehat{\pi}_t - \widehat{\pi}_t^*\right) + \phi_y\widehat{y}_t\right] + e_t. \tag{52}$$

Alternative Model Specifications

The Model with GHH Preferences

Under GHH preferences, savers maximize:

$$E_0\sum_{t=0}^{\infty}\beta^t\left[\ln\left(C_t^u - \frac{\left(L_t^u\right)^\eta}{\eta}\right) + j\ln H_t^u\right], \tag{53}$$

The first-order conditions are:

$$\frac{1}{C_t^u - \frac{\left(L_t^u\right)^\eta}{\eta}} = \beta E_t\left[\frac{R_t}{\pi_{t+1}\left(C_{t+1}^u - \frac{\left(L_{t+1}^u\right)^\eta}{\eta}\right)}\right], \tag{54}$$

$$w_t^u = (L_t^u)^{\eta-1}\,, \tag{55}$$

$$\frac{j}{H_t^u} = \frac{1}{C_t^u - \frac{(L_t^u)^\eta}{\eta}} q_t - \beta E_t \frac{1}{C_{t+1}^u - \frac{\left(L_{t+1}^u\right)^\eta}{\eta}} q_{t+1}. \tag{56}$$

Note that consumption no longer appears in the labor-supply decision (equation (55)).

Similarly, we can obtain the first-order conditions for variable-rate borrowers:

$$\frac{1}{C_t^{cv} - \frac{(L_t^{cv})^\eta}{\eta}} = \widetilde{\beta} E_t \left[\frac{R_t}{\pi_{t+1}\left(C_{t+1}^{cv} - \frac{\left(L_{t+1}^{cv}\right)^\eta}{\eta}\right)} \right] + \lambda_t^{cv} R_t, \tag{57}$$

$$w_t^{cv} = (L_t^{cv})^{\eta-1}\,, \tag{58}$$

$$\frac{j}{H_t^{cv}} = \frac{1}{C_t^{cv} - \frac{(L_t^{cv})^\eta}{\eta}} q_t - \widetilde{\beta} E_t \frac{1}{C_{t+1}^{cv} - \frac{\left(L_{t+1}^{cv}\right)^\eta}{\eta}} q_{t+1} - \lambda_t^{cv} k E_t q_{t+1} \pi_{t+1}. \tag{59}$$

The Model with Capital

We can add capital to the model so that unconstrained consumers have more saving choices. Since borrowers would not hold capital, the only part of the model that changes is the one of the unconstrained consumers:

Unconstrained consumers maximize their expected lifetime utility function:

$$\max \; E_0 \sum_{t=0}^{\infty} \beta^t \left(\ln C_t^u + j \ln H_t^u - \frac{(L_t^u)^\eta}{\eta} \right), \tag{60}$$

subject to the budget constraint which includes capital:

$$\begin{gathered} C_t^u + q_t H_t^u + K_t - (1-\delta) K_{t-1} + \frac{\phi}{2} \left(\frac{K_t - K_{t-1}}{K_{t-1}} \right)^2 + \frac{R_{t-1} b_{t-1}^u}{\pi_t} \leq q_t H_{t-1}^u \\ + z_t K_{t-1} + w_t^u L_t^u + b_t^u + F_t^v + S_t^v, \end{gathered} \tag{61}$$

So, in this case, savers can buy houses or sell them at the current price q_t and hold bonds. They can also hold capital K_t, whose price is normalized to unity, which they rent to firms at rental price z_t. δ is the depreciation rate of capital. Consumers also have to pay quadratic adjustment costs for capital.

Maximizing (60) subject to (61) , we obtain the first-order conditions:

$$\frac{1}{C_t^u} = \beta E_t \left(\frac{R_t}{\pi_{t+1} C_{t+1}^u} \right), \tag{62}$$

$$w_t^u = \left(L_t^u \right)^{\eta - 1} C_t^u, \tag{63}$$

$$\frac{j}{H_t^u} = \frac{1}{C_t^u} q_t - \beta E_t \frac{1}{C_{t+1}^u} q_{t+1}. \tag{64}$$

$$\frac{1}{C_t^u} \left[1 + \phi \left(\frac{K_t - K_{t-1}}{K_{t-1}} \right) \right] = \beta E_t \frac{1}{C_{t+1}^u} \left(z_t + (1-\delta) + \phi \frac{K_{t+1}}{K_t^2} \left(\frac{K_{t+1} - K_t}{K_t} \right) \right), \tag{65}$$

Now, we have a fourth first order condition, equation (65), which is the first order condition with respect to capital.

Unconstrained individuals are not going to hold capital in equilibrium so their problem remains unchanged.

Intermediate goods are going to be produced according to the following production function (ignoring technology):

$$Y_t = \left(L_t^u \right)^{\mu \gamma} \left(L_t^c \right)^{\mu (1-\gamma)} K_{t-1}^{1-\mu}, \tag{66}$$

where γ measures the relative size of each group in terms of labor and μ is the labor share.

Firms choose employment and capital to

$$\min w_t^u L_t^u + w_t^c L_t^c + z_t K_{t-1},$$

subject to the production function, demand and the constraint imposed by nominal rigidity.

The first-order conditions for labor and capital demand are the following:

$$w_t^u = \frac{1}{X_t} \gamma \mu \frac{Y_t}{L_t^u}, \tag{67}$$

$$w_t^c = \frac{1}{X_t}(1-\gamma)\mu\frac{Y_t}{L_t^c}, \tag{68}$$

$$z_t = \frac{1}{X_t}(1-\mu)\frac{Y_t}{K_{t-1}}, \tag{69}$$

Non Separability between Housing and Non-Durable Consumption in the Utility Function

Unconstrained consumers consume an index of non-durable goods and housing defined as:

$$I_t^u = \left[(1-\nu)^{\frac{1}{\mu}}(C_t^u)^{\frac{\mu-1}{\mu}} + \nu^{\frac{1}{\mu}}(H_t^u)^{\frac{\mu-1}{\mu}}\right]^{\frac{\mu}{\mu-1}}, \tag{70}$$

where ν is the share of housing in the composite consumption index and μ is the elasticity of substitution between non-durable consumption goods and housing.

Unconstrained consumers maximize an expected lifetime utility function with two arguments; the consumption index and labor/leisure.

$$\max\ E_0\sum_{t=0}^{\infty}\beta^t\left(\ln I_t^u - \frac{(L_t^u)^\eta}{\eta}\right), \tag{71}$$

Subject to the budget constraint:

$$C_t^u + q_t H_t^u + \frac{R_{t-1}b_{t-1}^u}{\pi_t} \leq q_t H_{t-1}^u + w_t^u L_t^u + b_t^u + F_t^v + S_t^v, \tag{72}$$

Maximizing (71) subject to (72), we obtain the first-order conditions:

$$\frac{(C_t^u)^{\frac{-1}{\mu}}}{(1-\nu)^{\frac{1}{\mu}}(C_t^u)^{\frac{\mu-1}{\mu}} + \nu^{\frac{1}{\mu}}(H_t^u)^{\frac{\mu-1}{\mu}}} = \beta E_t\left[\left(\frac{R_t}{\pi_{t+1}}\right)\left(\frac{(C_{t+1}^u)^{\frac{-1}{\mu}}}{(1-\nu)^{\frac{1}{\mu}}\left(C_{t+1}^u\right)^{\frac{\mu-1}{\mu}} + \nu^{\frac{1}{\mu}}\left(H_{t+1}^u\right)^{\frac{\mu-1}{\mu}}}\right)\right], \tag{73}$$

$$w_t^u\frac{(1-\nu)^{\frac{1}{\mu}}(C_t^u)^{\frac{-1}{\mu}}}{(1-\nu)^{\frac{1}{\mu}}(C_t^u)^{\frac{\mu-1}{\mu}} + \nu^{\frac{1}{\mu}}(H_t^u)^{\frac{\mu-1}{\mu}}} = (L_t^u)^{\eta-1}, \tag{74}$$

$$\begin{aligned}\frac{\nu^{\frac{1}{\mu}}(H_t^u)^{\frac{-1}{\mu}}}{(1-\nu)^{\frac{1}{\mu}}(C_t^u)^{\frac{\mu-1}{\mu}} + \nu^{\frac{1}{\mu}}(H_t^u)^{\frac{\mu-1}{\mu}}} &= \frac{(1-\nu)^{\frac{1}{\mu}}(C_t^u)^{\frac{-1}{\mu}}}{(1-\nu)^{\frac{1}{\mu}}(C_t^u)^{\frac{\mu-1}{\mu}} + \nu^{\frac{1}{\mu}}(H_t^u)^{\frac{\mu-1}{\mu}}}q_t \\ &-\beta E_t\frac{(1-\nu)^{\frac{1}{\mu}}\left(C_{t+1}^u\right)^{\frac{-1}{\mu}}}{(1-\nu)^{\frac{1}{\mu}}\left(C_{t+1}^u\right)^{\frac{\mu-1}{\mu}} + \nu^{\frac{1}{\mu}}\left(H_{t+1}^u\right)^{\frac{\mu-1}{\mu}}}q_{t+1}\end{aligned} \tag{75}$$

In the same way, we have the problem of the variable-rate constrained consumers.

They also consume a consumption index that aggregates non-durable goods and housing:

$$I_t^{cv} = \left[(1-\nu)^{\frac{1}{\mu}} \left(C_t^{cv}\right)^{\frac{\mu-1}{\mu}} + \nu^{\frac{1}{\mu}} \left(H^{cv}\right)^{\frac{\mu-1}{\mu}}\right]^{\frac{\mu}{\mu-1}}, \tag{76}$$

Variable-rate constrained consumers maximize the lifetime utility function subject to the budget constraint and the collateral constraint:

$$\max \ E_0 \sum_{t=0}^{\infty} \widetilde{\beta}^t \left(\ln I_t^{cv} - \frac{\left(L_t^{cv}\right)^{\eta}}{\eta}\right), \tag{77}$$

subject to:

$$C_t^{cv} + q_t H_t^{cv} + \frac{R_{t-1} b_{t-1}^{cv}}{\pi_t} \leq q_t H_{t-1}^{cv} + w_t^{cv} L_t^{cv} + b_t^{cv}, \tag{78}$$

$$E_t \frac{R_t}{\pi_{t+1}} b_t^{cv} \leq k E_t q_{t+1} H_t^{cv}. \tag{79}$$

The first-order conditions for the consumers are:

$$\frac{(1-\nu)^{\frac{1}{\mu}} \left(C_t^{cv}\right)^{\frac{-1}{\mu}}}{(1-\nu)^{\frac{1}{\mu}} \left(C_t^{cv}\right)^{\frac{\mu-1}{\mu}} + \nu^{\frac{1}{\mu}} \left(H_t^{cv}\right)^{\frac{\mu-1}{\mu}}} = \widetilde{\beta} E_t \left[\left(\frac{R_t}{\pi_{t+1}}\right) \left(\frac{(1-\nu)^{\frac{1}{\mu}} \left(C_{t+1}^{cv}\right)^{\frac{-1}{\mu}}}{(1-\nu)^{\frac{1}{\mu}} \left(C_{t+1}^{cv}\right)^{\frac{\mu-1}{\mu}} + \nu^{\frac{1}{\mu}} \left(H_{t+1}^{cv}\right)^{\frac{\mu-1}{\mu}}}\right)\right] + \lambda_t^{cv} R_t, \tag{80}$$

$$w_t^{cv} \frac{(1-\nu)^{\frac{1}{\mu}} \left(C_t^{cv}\right)^{\frac{-1}{\mu}}}{(1-\nu)^{\frac{1}{\mu}} \left(C_t^{cv}\right)^{\frac{\mu-1}{\mu}} + \nu^{\frac{1}{\mu}} \left(H_t^{cv}\right)^{\frac{\mu-1}{\mu}}} = \left(L_t^{cv}\right)^{\eta-1}, \tag{81}$$

$$\frac{\nu^{\frac{1}{\mu}} \left(H_t^{cv}\right)^{\frac{-1}{\mu}}}{(1-\nu)^{\frac{1}{\mu}} \left(C_t^{cv}\right)^{\frac{\mu-1}{\mu}} + \nu^{\frac{1}{\mu}} \left(H_t^{cv}\right)^{\frac{\mu-1}{\mu}}} = \frac{(1-\nu)^{\frac{1}{\mu}} \left(C_t^{cv}\right)^{\frac{-1}{\mu}}}{(1-\nu)^{\frac{1}{\mu}} \left(C_t^{cv}\right)^{\frac{\mu-1}{\mu}} + \nu^{\frac{1}{\mu}} \left(H_t^{cv}\right)^{\frac{\mu-1}{\mu}}} q_t - \widetilde{\beta} E_t \frac{(1-\nu)^{\frac{1}{\mu}} \left(C_{t+1}^{cv}\right)^{\frac{-1}{\mu}}}{(1-\nu)^{\frac{1}{\mu}} \left(C_{t+1}^{cv}\right)^{\frac{\mu-1}{\mu}} + \nu^{\frac{1}{\mu}} \left(H_{t+1}^{cv}\right)^{\frac{\mu-1}{\mu}}} q_{t+1} - \lambda_t^{cv} k E_t q_{t+1} \pi_{t+1}. \tag{82}$$

These first-order conditions differ from those of the unconstrained individuals. In the case of constrained consumers, the Lagrange multiplier on the borrowing constraint (λ_t^{cv}) appears in the equations. From the Euler equations for consumption of the unconstrained consumers, we know that $R = 1/\beta$ in steady state. If we combine this result with the Euler equation for consumption for the constrained individual we have that $\lambda^{cv} = \frac{\left(\beta-\widetilde{\beta}\right)(1-\nu)^{\frac{1}{\mu}}(C^{cv})^{\frac{-1}{\mu}}}{(1-\nu)^{\frac{1}{\mu}}(C^{cv})^{\frac{\mu-1}{\mu}}+\nu^{\frac{1}{\mu}}(H^{cv})^{\frac{\mu-1}{\mu}}} > 0$ in steady state.

The problem of the financial intermediary, the firms and the monetary policy is identical to the baseline model.

A Two-Sector Model

We can relax the assumption that the housing supply is fixed and consider a two-sector model in which consumers can supply labor to the housing sector and the consumption sector.

Unconstrained consumers**:**

$$\max \ E_0 \sum_{t=0}^{\infty} \beta^t \left(\ln C_t^u + j \ln H_t^u - \left(\left(L_{ct}^u\right)^{1-\nu} + \left(L_{ht}^u\right)^{1-\nu} \right)^{\frac{1+\eta}{1-\nu}} \right), \tag{83}$$

subject to the budget constraint:

$$C_t^u + q_t H_t^u + \frac{R_{t-1} b_{t-1}^u}{\pi_t} \leq q_t (1-\delta) H_{t-1}^u + w_{ct}^u L_{ct}^u + w_{ht}^u L_{ht}^u + b_t^u + F_t^v + S_t^v, \tag{84}$$

Maximizing (83) subject to (84), we obtain the first-order conditions:

$$\frac{1}{C_t^u} = \beta E_t \left(\frac{R_t}{\pi_{t+1} C_{t+1}^u} \right), \tag{85}$$

$$w_{ct}^u = (1+\eta) \left(\left(L_{ct}^u\right)^{1-\nu} + \left(L_{ht}^u\right)^{1-\nu} \right)^{\frac{\nu+\eta}{1-\nu}} \left(L_{ct}^u\right)^{-\nu} C_t^u, \tag{86}$$

$$w_{ht}^u = (1+\eta) \left(\left(L_{ct}^u\right)^{1-\nu} + \left(L_{ht}^u\right)^{1-\nu} \right)^{\frac{\nu+\eta}{1-\nu}} \left(L_{ht}^u\right)^{-\nu} C_t^u, \tag{87}$$

$$\frac{j}{H_t^u} = \frac{1}{C_t^u} q_t - \beta E_t \frac{1}{C_{t+1}^u} (1-\delta) q_{t+1}. \tag{88}$$

Equations (85) is the consumption Euler equation. Equations (86) and (87) are the labor-supply condition for the consumption and the housing sector, respectively. Equation (88) is the Euler equation for housing and states that the benefits from consuming housing have to be equal to the costs.

Variable-rate constrained consumers maximize the lifetime utility function subject to the budget constraint and the collateral constraint:

$$\max\ E_0 \sum_{t=0}^{\infty} \widetilde{\beta}^t \left(\ln C_t^{cv} + j \ln H_t^{cv} - \left(\left(L_{ct}^{cv}\right)^{1-\nu} + \left(L_{ht}^{cv}\right)^{1-\nu} \right)^{\frac{1+\eta}{1-\nu}} \right), \tag{89}$$

subject to:

$$C_t^{cv} + q_t H_t^{cv} + \frac{R_{t-1} b_{t-1}^{cv}}{\pi_t} \leq q_t \left(1-\delta\right) H_{t-1}^{cv} + w_{ct}^{cv} L_{ct}^{cv} + w_{ht}^{cv} L_{ht}^{cv} + b_t^{cv}, \tag{90}$$

$$E_t \frac{R_t}{\pi_{t+1}} b_t^{cv} \leq k E_t q_{t+1} H_t^{cv}. \tag{91}$$

The first-order conditions are:

$$\frac{1}{C_t^{cv}} = \widetilde{\beta} E_t \left(\frac{R_t}{\pi_{t+1} C_{t+1}^{cv}} \right) + \lambda_t^{cv} R_t, \tag{92}$$

$$w_{ct}^{cv} = (1+\eta) \left(\left(L_{ct}^{ci}\right)^{1-\nu} + \left(L_{ht}^{ci}\right)^{1-\nu} \right)^{\frac{\nu+\eta}{1-\nu}} \left(L_{ct}^{ci}\right)^{-\nu} C_t^{ci}, \tag{93}$$

$$w_{ht}^{cv} = (1+\eta) \left(\left(L_{ct}^{cv}\right)^{1-\nu} + \left(L_{ht}^{cv}\right)^{1-\nu} \right)^{\frac{\nu+\eta}{1-\nu}} \left(L_{ht}^{cv}\right)^{-\nu} C_t^{cv}, \tag{94}$$

$$\frac{j}{H_t^{cv}} = \frac{1}{C_t^{cv}} q_t - \widetilde{\beta} E_t \frac{1}{C_{t+1}^{cv}} \left(1-\delta\right) q_{t+1} - \lambda_t^{cv} k E_t q_{t+1} \pi_{t+1}. \tag{95}$$

The problem for the financial intermediary and the final good producer is identical to the baseline model. The problem for the intermediate good producer is slightly changed. Intermediate goods are produced according to the following production function:

$$Y_{ct} = \left(L_{ct}^{u}\right)^{\gamma} \left(L_{ct}^{c}\right)^{(1-\gamma)}, \tag{96}$$

where γ measures the relative size of each group in terms of labor.

Analogously, the production function for the housing sector is the following (ignoring technology):

$$Y_{ht} = \left(L_{ht}^{u}\right)^{\gamma} \left(L_{ht}^{c}\right)^{(1-\gamma)}, \tag{97}$$

Firms choose employment to

$$\min w_{ct}^{u} L_{ct}^{u} + w_{ct}^{c} L_{ct}^{c} + w_{ht}^{u} L_{ht}^{u} + w_{ht}^{c} L_{ht}^{c},$$

subject to the production function, demand and the constraint imposed by nominal rigidity.

The first-order conditions for labor demand are the following:

$$w_{ct}^{u} = \frac{1}{X_t} \gamma \frac{Y_{ct}}{L_{ct}^{u}}, \tag{98}$$

$$w_{ct}^{c} = \frac{1}{X_t} (1-\gamma) \frac{Y_{ct}}{L_{ct}^{c}}, \tag{99}$$

$$w_{ht}^{u} = q_t \gamma \frac{Y_{ht}}{L_{ht}^{u}}, \tag{100}$$

$$w_{ht}^{c} = q_t (1-\gamma) \frac{Y_{ht}}{L_{ht}^{c}}, \tag{101}$$

Production function in which labor for savers and labor for borrowers are substitutes

The consumers' and financial intermediary's problem remains unchanged. However, the intermediate goods firm's production function (ignoring technology) turns into the following one:

$$Y_t = \omega L_t^{u} + (1-\omega) \left[\alpha L_t^{cv} + (1-\alpha) L_t^{cf}\right] = \omega L_t^{u} + (1-\omega) L_t^{c}, \tag{102}$$

where ω is the size of the unconstrained group.

Firms choose employment to

$$\min \omega w_t^{u} L_t^{u} + (1-\omega) w_t^{c} L_t^{c},$$

subject to the production function, demand and the constraint imposed by nominal rigidity.

The first-order conditions for labor demand are the following:

$$w_t^u = w_t^c = \frac{1}{X_t}. \tag{103}$$

We see that in this case, the wage paid to each group is the same.

Aggregate variables are defined as follows:

$$C_t \equiv \omega C_t^u + (1-\omega)\, C_t^c. \tag{104}$$

$$L_t \equiv \omega L_t^u + (1-\omega)\, L_t^c. \tag{105}$$

$$H_t \equiv \omega H_t^u + (1-\omega)\, H_t^c. \tag{106}$$

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