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Abstract

Motivated by a strong degree of hysteresis in the stock of monetization observed after the end of hyperinflations, I provide a cash-and-credit model in which the use of money exhibits some persistence because individuals can establish long-lasting credit relationships. This feature helps to account for the main stylized facts of extreme hyperinflations and reconcile some conflicting views on their causes, development and end without departing from rational expectations. Unlike the existing literature, I show that when hysteresis is possible, an orthodox fiscal-monetary reform that successfully stops a speculative hyperinflation may not be sufficient to prevent it.

Keywords: Hyperinflation; Fiscal-Monetary Reform; Multiple Equilibria; Hysteresis

JEL classification: E31; E41; E63

1 Introduction

In this paper I provide a theoretical explanation for the main stylized facts observed during hyperinflationary processes as those experienced by some European, Latin-American and transition countries in the 1920's, 80's and 90's, respectively. These episodes often display a common set of facts, yet much of the previous literature has followed a partial approach concentrating only on a reduced number of stylized features, as stressed by Bental and Eckstein (1990) and, more recently, by Marcet and Nicolini (2003). Indeed, some of the most influential partial explanations, when combined together, yield a picture that does not resemble what we actually observe in real economies. A central contribution aimed here is to present a simple rational-expectations general equilibrium model that potentially accommodates three of the most influential partial approaches -the *fiscal root*, the *bubble explanation* and the *orthodox reform*- (described below), making them consistent with each other and with the empirical facts.

First, weak fiscal conditions that force the government to finance part of its expenses via money injections (seigniorage) are usually thought to be an important factor behind hyperinflations (the *fiscal root*). On the one hand, explosive hyperinflations often take place in countries in which seigniorage is relatively high and, on the other, a drastic reduction in this source of funding is observed after every successful stabilization. For instance, regarding the fiscal stance of some European countries in the 1920's, Sargent (1986; p. 45) observes that "*the governments of these countries (Hungary, Austria, Poland, and Germany) resorted to the printing of new unbacked money to finance government deficits. This was done on such a scale that it led to a depreciation of the currencies of spectacular proportions.*" (parentheses added). Although the exact intertemporal mechanisms linking inflation and seigniorage are likely to be rather complex, as shown by Sargent and Wallace (1981), the common proposition that associates high inflation to a high degree of fiscal-dominance in the implementation of monetary polices remains basically uncontested.[1]

Second, for some students of these episodes, the lack of strong correlation between inflation and seigniorage over the course of some hyperinflations suggests that they may well be driven just by expectations that, in equilibrium, are self-confirmed (the *bubble explanation*). This approach has received a formal treatment under the rational expectations paradigm, among others, by Sargent and Wallace (1987), and Bruno and Fischer (1990). In both cases, it is assumed a demand for real balances *à la* Cagan (1956), which implies the existence of a hump-shaped inflation-tax Laffer curve with two stationary inflation rates (one "low" and one "high") associated with a unique volume of seigniorage, and a hyperinflation is interpreted as the economy moving from the vicinity of the low inflation rate to the high one. According to this view seigniorage is not the sole explanation for high inflation, instead, private expectations are thought to be the prime cause. However, in explaining the occurrence of a hyperinflation as a bubble equilibrium along a standard Laffer curve, the degree of fiscal pressure does not play any

[1] Although there are some divergent views on this issue. For example, Loyo (2001) provides an explanation for the hyperinflationary bursts in Brazil in the 1980's and 90's using a cash-less model in which the volume of seigniorage is negligible.

significant role. More on the contrary, as the economy moves towards the decreasing arm of the Laffer curve, higher levels of seigniorage will result in lower inflation rates. Still, such a possibility has also attracted much attention in the empirical literature.[2] For example, Imrohoroglu (1993), provides empirical support for Sargent and Wallace's multiple-equilibria model using data from the German hyperinflation.

Third, for a wide class of rational-expectations economic models, including those in the tradition of Cagan mentioned above, Obstfeld and Rogoff (1983), and Nicolini (1996) have shown that the prospect of a fiscal-monetary reform aimed at stabilizing the real value of the currency through a restrictive monetary policy (the *orthodox reform*) is itself a sufficient condition to preclude a speculative hyperinflation, thus making it difficult to reconcile the idea of a hyperinflation as a rational-bubble equilibrium. Moreover, once the possibility of a reform is introduced in a model with rational expectations and a Cagan-style demand for money what one learns is that higher seigniorage is uniformly associated with more severe *disinflationary* processes. Yet, if anything is unambiguously true about hyperinflations is that they are always stopped. There is little doubt either that this is the result of the government's determination to reform the fiscal-monetary mix, i.e. to implement "*deliberate and drastic fiscal and monetary measures taken to end the hyperinflations*" (Sargent 1986; p. 44).

Clearly, when combined together, these three partial views do not fit each other. Yet, there have been few attempts to provide a *general* explanation of hyperinflations, including their causes, dynamics and end. To the best of my knowledge, most of them consider models in which the preventive role of an eventual reform, as the one outlined above, is not explicitly considered.[3] Marcet and Nicolini (2003) consider a baseline model similar to the one studied by Sargent and Wallace (1987) but assume that individuals follow a backward-looking rule for forming expectations (*quasi-rational learning*). They show that under some conditions on the learning rule the model is able to match most of the stylized facts. Sargent *et al.* (2005) have recently provided econometric support for the ability of Marcet and Nicolini's model to explain the time series of inflation using data from several Latin-American countries. However, as in this class of models individuals do not internalize the possibility of a future change in the policy-regime when forming their expectations, the prospect of a reform is itself irrelevant for the dynamics of the model.

Using the same Cagan-type model, Kiguel (1989) argues that a volume of seigniorage above the maximum one in a steady-state inflation-tax Laffer curve will unchain an explosive hyperinflation if the money market does not clear instantaneously.[4] Although he explicitly considers the implementation of a reform as a device to stop the hyperinflation, the expectation of a reform does not have any effect on individuals decisions before its implementation either.

[2] Since, at least, the influential work of Flood and Garber (1980), a large number of authors have tried to detect bubble-components using data from hyperinflationary economies. The results are mixed and the comparison across different studies is not always easy, as there is a wide heterogeneity in the specification of the money demand function, the money supply process, etc. For some recent contributions on this area see e.g. Blackburn and Sola (1996), and Hooker (2000), and the references therein.

[3] Drazen and Helpman (1990) provide an exception since they explicitly model the effects of an anticipated change of regime. However, in order for their model to generate a hyperinflationary path, an upward seigniorage-path is required. Yet, as discussed below, this is not a common feature in hyperinflationary episodes.

[4] Romer (2001) presents a simplified version of this model.

The two models just mentioned provide a rationale for the *fiscal root* view, however at the cost of neglecting the government's policy path followed beyond the current date. A different view is taken by Bental and Eckstein (1990) and Paal (2000), since in these models the anticipation of a reform is thought to be the cause of the hyperinflation. Bental and Eckstein explain the relatively low level of monetization (i.e. the stock of real balances) observed for a long time after the end of a hyperinflation as a consequence of a negative wealth effect arising from the increase in income-taxes accompanying a fiscal-monetary reform. Then they interpret a hyperinflation as the economy moving from a scenario of a high demand for real money to a stationary one in which the stock of real balances remains low, in line with a lower level of private wealth. Paal (2000), referring to the hyperinflationary episode in Hungary in the 1940's, argues that a reform based on the stabilization of the inflation rate without directly controlling for a monetary aggregate may actually leave the economy prone to purely expectational-driven equilibria, some of them hyperinflationary, some others exhibiting a pronounced disinflation. Anticipation of such a change in the monetary rule rather than a weak government's fiscal condition could have been, according to this model, the true cause of the Hungarian hyperinflation.[5]

The approach followed here may be seen as a complement to the *general* explanations listed above, for it shares some basic features with them, yet takes a different route. I do not impose *ex ante* any special assumption on the current effects of a future fiscal-monetary reform. Rather, I am concerned with the conditions under which the prospect of an eventual reform exerts a preventive effect. In this sense, I build up a framework within which the effectiveness of a reform to preclude the occurrence of a hyperinflation arises as an endogenous outcome. To this aim, I formulate a rational-expectations model whose cornerstone is its potential ability to display one of the most recurrent empirical facts in hyperinflationary economies, namely, a post-reform hysteresis-effect in the demand for (national currency) real balances. Referring to this phenomenon, Calvo and Végh (1992; p. 11) note that "*high inflation forces a gradual development of new financial instruments [...] Creating new financial products is costly and requires a learning process. Once this investment has taken place, the public will continue using these new financial instruments even if inflation falls*".[6]

Along this line of reasoning, the model developed later features a *gradual development* of a non-monetary financial instrument whose initial adoption is *costly* but can be used *even if inflation falls.* There are several theoretical models in the previous literature designed to account for some of these facts. For example, Ireland (1995), Sachs (1995) and Uribe (1997) all provide examples of economies

[5] This argument resembles the *bubble explanation* above, however with an important difference. For instance, in Sargent and Wallace (1987), the government directly sets a target for seigniorage *at every period.* Paal, on the contrary assumes that at the time of the reform the government, by committing to a non-contingent inflation target, leaves both the volume of seigniorage and the supply of money indeterminate, very much like what we would observe when a central bank switches from a rule based on the control of a monetary aggregate to one based on the control of the short-term interest rate, as described in Sargent and Wallace (1975). Then, she associates the occurrence of an explosive hyperinflation to a self-confirmed high volume of seigniorage at the precise time of the reform.

[6] A similar observation can be found in Dornbusch *et al.* (1990; p. 23-24): "*The distinction between movements along the real money demand schedule and shifts in real money demand helps explain the actual dynamics of real balances during an inflation. The distinction may also explain why there are hysteresis effects. Once a new financial product is in use, it will continue to be used even if inflation declines. [...] Financial institutions are not ready-made to accommodate the flight from money. In the short run, households may hold more real balances than they will after alternative vehicles become available or better known. In practice, this will look like adjustment lags, but it also reflects the adjustments of financial institutions to the increased inflation.*"

in which a form of financial innovation exhibits long-lasting effects. However, to my knowledge, no model of financial innovation or currency substitution has been previously exploited to account for the stylized effects we are after, including an explosive inflation-path and a sudden change of policy-regime. Thus, a methodological contribution of the paper consists in bringing together these two strands of the literature, a natural step after recognizing that the incentives to invest in financial products not directly subject to the inflation-tax are probably highest when inflation reaches several-digits values, and that the most pronounced episodes of rapid demonetization and slow remonetization are observed during hyperinflationary episodes.

Specifically, I model the demand for monetary balances as a version of Ireland (1995) cash- and credit-goods environment. Households may acquire their desired consumption basket using either cash or credit. However, credit requires the prior investment of some valuable resources for building up long-lasting producer-shopper relationships which can be exploited for some time to avoid the inflationary cost of carrying cash.[7] On the side of monetary supply, I follow the standard assumption of a government aiming at collecting an exogenous and constant amount of seigniorage over a *fiscal-dominance* regime which is possibly reversed at some future date by the implementation of a monetary-fiscal reform conducive to a low-inflation *monetary-dominance* regime.[8] Private agents are assumed to be aware of the eventual change of regime.

This simple formal set up provides a natural interpretation of the links among the fiscal roots of hyperinflations, their dynamics and termination and the relative role of expectations and fundamentals. In particular, provided some mild conditions hold, the model exhibits up to three general classes of potential equilibria over the fiscal-dominance regime. First, a unique low-inflation equilibrium that arises when the public expects a "responsible" government policy in the form of a combination of a low level of deficit-monetization and an early and/or restrictive reform. Second, a unique high- and increasing-inflation path that obtains in the opposite policy scenario (i.e. high seigniorage and late/insufficiently restrictive reform). Third, for some structural parameters and government policy choices lying in between the two previous extreme cases, the model displays multiple equilibria, some of them hyperinflationary, whose realization hinges crucially on private expectations, this being a reflection of the fact that a seigniorage-based monetary policy allows for individual complementarities in the decision of investing in the credit-technology.

Overall the main insights of the paper are two. First, on the theory side, the paper argues that the practical relevance of the arguments put forward by Obstfeld and Rogoff (1983), concerning the

[7] I stick at the cash- *versus* credit-transactions parable for simplicity and without loss of generality. The basic arguments can be extended to alternative forms of carrying out transactions that do not rely on the use of domestic money (e.g. dollarization, barter, etc.)

[8] As further clarified later, this is the standard assumption followed in the "general" approaches commented above, in which no attempt to explain the government behavior (beyond that of a mechanic monetization of a portion of the deficit) is made. In another branch of the literature (see e.g. Albanesi 2006 and the references therein) the degree of fiscal pressure on monetary policy is modelled explicitly as the outcome of a game between economic authorities and/or groups of interest. Within this political-economy branch, Mondino *et al.* (1996) provide a model with some similarities to the one developed here, since they allow for a mechanism of financial innovation with log-lasting effects. Yet, the questions they pose and their methodology are different from the ones pursued here.

effectiveness of a future reform to rule out a hyperinflation, must be qualified in view of the empirical evidence, for the presence of hysteresis suggests that the sufficient condition for the validity of their arguments (i.e. the lack of causality running from past to future states along an equilibrium in which the prospect of a reform is internalized by the households) may not hold in real economies. Indeed, in an economy in which hysteresis is a true possibility, the conditions for an *ex ante* effective reform are likely to be much more complex than in the history-independent economy studied by those authors. This observation will allow us to reconcile the three influential views on hyperinflationary processes described above.

Second, by examining the necessary and sufficient conditions for a hyperinflation, the model offers some insights relevant for policy judgment. In particular, fiscal conditions are likely to play an important role, as stressed by Marcet and Nicolini (2003) and Sargent *et al.* (2005). However, using a genuine rational-expectations dynamic framework we learn that seigniorage over the fiscal-dominance regime, while important, does not contain all the relevant fiscal-information, e.g. the horizon over which that regime is implemented and the fiscal-monetary conditions prevailing after the reform are also likely to bear a direct effect on the conditions that determine whether a hyperinflation is a feasible outcome or not.

The rest of the paper is organized as follows. Section 2 describes the main empirical facts observed in a variety of hyperinflations. Section 3 contains the model. Section 4 characterizes an environment in which the prospect of a reform is *ex ante* irrelevant for the equilibrium path. Section 5 discusses conditions under which expectations-driven hyperinflations are possible equilibrium outcomes under the assumption that the government commits to implement the reform at a pre-announced future date, and section 6 extends the analysis to the case of a state-contingent reform. Section 7 summarizes the main results of the paper.

2 Empirical motivation: Stylized facts

As pointed out by Marcet and Nicolini (2003) many hyperinflations, even if distant in time and/or geographical location, display a number of common facts. Below, I briefly describe the most recurrent facts.

- Hyperinflations usually occur in countries experiencing important fiscal imbalances, in the sense that the resources that the government is able to collect from regular taxes and borrowing from financial markets are on average significantly below the level of its expenditures, so that the resulting difference is funded by printing money. Fischer *et al.* (2002) report a strong positive relationship between seigniorage, deficits and inflation using annual observations corresponding to a sample of 24 high-inflation countries over the period 1960-1995. A similar overall picture can be found in Catão and Terrones (2005). Also, many country-specific studies also stress the role of seigniorage as a potential cause of hyperinflationary bursts.[9]

- When inflation reaches its highest values within the hyperinflationary episode there is a lack of strong contemporaneous correlation between seigniorage and inflation. Indeed, seigniorage may well decrease significantly while inflation still grows at increasing rates.[10]

- Stopping a hyperinflationary process usually involves a mix of fiscal and monetary reforms whose aim is to set limits to the ability of the government to monetize its deficit by e.g. cutting public expenditure, fixing the exchange rate to a foreign currency with a good reputation in terms of inflation, establishing legally independent central banks with limitations to the amount of funds let to the government, implementing rules to keep the growth of the money supply under limits (e.g. through currency boards), etc.[11] The process of inflation-stabilization, from the peak of the inflation series to the post-reform low rate, takes a short period (a few months at most).

- The stock of real balances falls as inflation increases over the course of a hyperinflation and remains low but exhibiting an upward trend for a long period after the stabilization. In recent times, this observation is closely related to the phenomenon of dollarization, i.e. the quasi-permanent substitution of a foreign currency for the local one. For some recent empirical work on this question see e.g. Kamin and Ericsson (2003) and Reinhart *et al.* (2003). This hysteresis-effect is behind the well documented observation that, just after the stabilization, (nominal) money supply increases at moderate-high rates without causing noticeable inflationary pressure.

Figure 1 contains some evidence on several hyperinflationary episodes recorded in Germany, Argentina, Bolivia and Peru. These data generally confirm the standard facts above. In all cases, the rise in inflation goes parallel to the fall in real balances. The peaks in the inflation rate coincide with the lowest value of the stock of real money recorded for the entire period. At that point the stock of real balances falls bellow the 40% of its value a couple of years before. Although average seigniorage is clearly higher in the pre-reform period than after the collapse of the hyperinflationary path for the four countries, in some cases inflation reaches its highest value while seigniorage is decreasing (as in Germany 10:1923 and Bolivia 12:1984-2:1985) while in others seigniorage remains high and increasing (as in Argentina) or fairly constant (as in Peru). In all cases, after the stabilization, the volume of seigniorage was drastically cut down, inflation remained low and stable, and the stock of real balances exhibited a very slow recovery, thus confirming a strong hysteresis-effect.

[9] See for instance (in italics the country under analysis) Sachs (1987; *Bolivia*), Eckstein and Leiderman (1992; *Israel*), Kiguel and Neumeyer (1995; *Argentina*), and Petrovic *et al.* (1999; *Yusgoslavia*).

[10] The absence of a marked trend in seigniorage during hyperinflationary periods has led many researchers to model this variable as a constant (or with a constant mean, in stochastic environments). This is the strategy followed by e.g. Bental and Eckstein (1990), Bruno and Fischer (1990), Kiguel (1989), Marcet and Nicolini (2003), Paal (2000), Sargent and Wallace (1987), and Sargent *et al.* (2005). The model presented here uses this simplifying assumption too.

[11] See, e.g. Bruno *et al.* (1991) for a comprehensive review of several inflation stabilization processes.

3 The model

The model in this section is inspired in the cash-and-credit transactions economy studied by Ireland (1995), who extends Lucas and Stokey's (1987) framework to allow for persistent effects in the use of credit. Households are assumed to exchange their consumption endowments by using government-provided money or interpersonal credit. The latter requires the existence of a credit-link between the intervening parts whose creation involves some initial fixed and irreversible cost. However an existing link can be used at not further cost at any subsequent date at which it remains operative. Transactions made using money do not carry over any explicit cost beyond that associated with inflation.

The main difference with respect to Ireland's model is intended to keep the subsequent dynamic analysis as simple as possible.[12] In particular, I assume that the volume of investment in the credit technology in any period is bounded above, as the opportunities for investment arrive at a finite rate. This artifact allows me to focus on monetary regimes along which the government is able to extract an exogenously set amount of seigniorage over a non-trivial number of periods.

3.1 The households

Consider a discrete-time economy with N (≥ 3) types of goods and N types of households of population size $1/N$ each. The household of type j produces good j and consumes good $j+1$ (except the one with type N, who consumes good 1). Each household consists of a continuum of members of unit mass. Half of the members within a household are producers, who can obtain their goods up to a given capacity y without cost. The other half are shoppers, without such a production capacity. All members in a household share consumption equally.

In every period t, a fraction n_t^j of household-j's members, producers and shoppers, are linked to the credit system, while the remaining fraction $1-n_t^j$ are not. Each linked shopper has an established bilateral relationship with a producer to buy his consumption good through a credit-link, $c_t^{c,j}$, subject to the capacity constraint $c_t^{c,j} \leq y$. Symmetrically, each linked producer has a bilateral relationship with a shopper to sell her product on credit, $C_t^{c,j}$, subject to the constraint $C_t^{c,j} \leq y$. All non-linked shoppers (producers) go to a centralized market to buy (sell) goods with (in exchange for) cash as anonymous members. Money received by non-linked producers can not be passed to shoppers within the same household until the beginning of the following period.

The timing of events is as follows. At the beginning of every period $t \geq 1$, households convene in the centralized financial market to settle outstanding debts and to accumulate government–issued money and bonds. Both government and private debts mature one period after issued and pay the same nominal risk-free rate. Household-j enters this market with the cash obtained from selling a part of

[12] Previous formal applications of models with financial innovation are either restricted to economies with exogenously determined inflation, as in Ireland (1995) and Uribe (1997), or to one-period or steady state analyses, as in Chang (1994) and Sachs (1995), respectively. The approach taken here imposes the necessity of some simplifying assumptions as I am dealing with explosive endogenous inflation paths and sudden changes of regime.

the common endowment at $t-1$, denoted by $\widetilde{M}_{t-1}^{j}$, with some maturing government bonds, inclusive of interest, $(1+i_{t-1}) B_{t-1}^{j}$, some invoices over other households, corresponding to the amount of goods sold on credit during last period, $(1+i_{t-1}) P_{t-1} C_{t-1}^{c,j}$ and, symmetrically, some debts corresponding to last period's credit-financed purchases, $(1+i_{t-1}) P_{t-1} c_{t-1}^{c,j}$. The terms B_{t-1}^{j}, i_{t-1} and P_{t-1} are, respectively, bond holdings, the nominal interest rate and the general price level.[13] The government participates in this market redeeming maturing bonds, collecting seigniorage (i.e. money injections by the central bank) and taxes (assumed to be paid in cash), and issuing new bonds. The household's constraint in this market is

$$M_t^j + B_t^j + P_t \tau_t \leq \widetilde{M}_{t-1}^{j} + (1+i_{t-1}) \left(B_{t-1}^{j} + C_{t-1}^{c,j} - c_{t-1}^{c,j} \right) \equiv W_t^j \tag{1}$$

where τ_t is the lump-sum tax and $\widetilde{M}_{t-1}^{j}$ is the household's stock of monetary balances at the end of the previous period and, hence, it satisfies $\widetilde{M}_t^j \geq \left(1 - n_t^j\right) P_t y$. Households are precluded from issuing money, i.e.

$$M_t^j \geq 0 \tag{2}$$

Once households rearrange their financial portfolios, non-linked producers engage in a process of finding shoppers for their goods to establish a credit-link. A free producer finds an opportunity to establish a credit link with a shopper with a probability $\gamma < 1$. In such an event the producer may chose to invest in the link by paying a utility cost of θ.[14] In that case, the shopper commits to redeem the resulting debt plus the interest rate in exchange for money in the financial market during the following period. Clearly, imposing $\gamma < 1$ tantamounts to a limit on the speed of investment in the credit technology which, as seen later in detail, translates into an upper bound to the speed of demonetization.[15]

Symmetrically, as in Ireland (1995), I assume that the households' credit capacity is subject to a form of depreciation. In particular, links have on average a finite life, as they are subject to a constant probability of termination at the beginning of each period equal to $1-\delta$.[16] This assumption is critical for the arguments developed later. In practical terms, it implies that current individual decisions on the mix of money and credit may have effects beyond the current date and, hence, it works as a simple device aimed at capturing the presence of hysteresis in the demand for real balances.

While there is uncertainty at the individual agent level (shopper and producer), the assumption of a continuum of agents within each household implies that at the household level there is no uncertainty on the total measure of the flows of new and existing links. Thus, the law of motion governing the measure

[13] I am already assuming that all goods are traded at the same relative value. Given the symmetry imposed in endowments and preferences this turns out to be true in the competitive equilibria analyzed later.

[14] A credit-link will generally create some monopolistic rents for the counterparts. As the interest here is in the evolution of the aggregate stock of real balances and, in turn, inflation, I consider a very simple (and extreme) rule governing the sharing of the benefits arising from a match: I assume that the producer pays the fixed utility-cost and leaves the shopper with an infinitesimal share of the monopolistic pecuniary rents of the match. As participating in a this sort of credit contract is costless for the shopper, he is happy to accept any strictly positive payment. For notational simplification, I omit that negligible transfer in the forthcoming expressions.

[15] The assumptions of a constant probability of arrival of a credit-link opportunity and the symmetric proportion of linked producers and shoppers are compatible with a random matching framework in which each individual, shopper or producer, can only commit to maintain a single credit-link and the matching function exhibits constant returns to scale in the number of non-linked agents on both sides of the market and a linear time-invariant total-efficiency parameter γ.

[16] Similarly, Uribe (1997) assumes that the stock of "social experience" in transacting with dollars, rather than with the local currency, depreciates at an exogenous constant rate.

of credit-linked producers within household-j can be expressed as:

$$n_t^j = \gamma \left[1 - \delta n_{t-1}^j\right] \lambda_t^j + \delta n_{t-1}^j \tag{3}$$

where $\lambda_t^j \in [0,1]$ captures the proportion of producers that take advantage of an opportunity to create a credit-link. Notice that in writing (3), it is assumed that the set of non-linked producers who are eligible to meet a free shopper includes those whose link has just vanished at the beginning of this period.

Then, the centralized market opens. In this market producers who do not deliver their stock of goods on credit sell their production in exchange for money. Here, the following standard cash-in-advance constraint applies,

$$c_t^{m,j} \leq \frac{M_t^j}{P_t} \tag{4}$$

where $c_t^{m,j}$ refers to the total measure of goods consumed by household-j that are purchased with cash. Finally, the members of the household join to consume the shoppers' purchases. In making its choices at t, the household must respect the following flow of funds constraint

$$\frac{W_{t+1}^j + i_t \widetilde{M}_t^j}{1 + i_t} + P_t \left(c_t^{m,j} + c_t^{c,j}\right) \leq P_t \left(y - \tau_t\right) + W_t^j \tag{5}$$

and a borrowing constraint that rules out games *à la* Ponzi

$$\lim_{T \to \infty} W_T^j / \prod_{t=1}^{T-1} \left(1 + i_t\right) \geq 0 \tag{6}$$

All households enter period 1 with the same financial wealth, in an amount exogenously given: $W_1^j = W_1$. Also, the measure of credit-linked producers at the beginning of period 1 (just before the destruction of some of these links) is given by history and identical across households: $n_0^j = n_0$.

Subject to the set of constraints in (1), (2), (3), (4), (5), and (6), household-j chooses a path for $\left\{c_s^{m,j}, c_s^{c,j}, M_s^j, \widetilde{M}_s^j, B_s^j, n_s^j, \lambda_s^j\right\}_{s=t}^{\infty}$ in order to maximize

$$U_t = \sum_{s=t}^{\infty} \beta^{s-t} \left\{\ln c_s^j - \theta \gamma \left[1 - \delta n_{s-1}^j\right] \lambda_s^j\right\} \tag{7}$$

where $c_s^j = c_s^{m,j} + c_s^{c,j}$.

The necessary and sufficient conditions associated with the household's optimization problem are (1), (5), and (6) holding with equality and,

1. The intertemporal allocation of consumption satisfies the Euler equation

$$\frac{c_{t+1}^j}{c_t^j} = \beta \left(1 + i_t\right) \frac{P_t}{P_{t+1}} \tag{8}$$

2. When facing an opportunity, a producer decides to establish a credit-link and pay the fixed utility-cost if $\Psi_t > \theta$, she is indifferent if $\Psi_t = \theta$ and forgoes the opportunity otherwise. The shadow-value function Ψ_t captures the discounted sum of the expected savings, in terms of the inflationary tax,

delivered by a credit-link and is given by

$$\Psi_t = \frac{\sum_{s=1}^{\infty} \delta^{s-1} \phi_t x_{t+s}}{\prod_{l=0}^{s-1} (1 + i_{t+l}) \frac{P_{t+l}}{P_{t+l+1}}} \tag{9}$$

where x_{t+1} is the inflation rate between t and $t+1$, i.e. $\frac{P_{t+1}}{P_t} - 1$, and ϕ_t is an indicator function capturing whether the producer is using that credit-link. It may take two values: 1 if the producers use the links and 0 otherwise. Clearly, the producer sets $\phi_t = 1$ if $x_{t+1} > 0$, is indifferent between transacting using the credit facility or attending the decentralized market if $x_{t+1} = 0$ and decides to trade his endowments in exchange for cash if $x_{t+1} < 0$ ($\phi_t = 0$).

3. Since the nominal interest rate is positive in all the scenarios that I consider later, the household optimally minimizes its end-of-period money holdings, i.e. $\frac{M_t^j}{P_t} = c_t^{m,j}$ and $\frac{\widetilde{M}_t^j}{P_t} = (1 - n_{t-1})\, y$.

3.2 The government

The government in this economy collects taxes, issues and redeems bonds and provides the entire stock of money. There is no government consumption. When choosing a combination of policy-instruments, it is restricted by the two following constraints. First, a period-by-period flow of funds constraint,

$$B_t + M_t + P_t \tau_t = (1 + i_{t-1})\, B_{t-1} + M_{t-1} \tag{10}$$

where M_t and B_t represent the government supply of money and bonds at time t, respectively. Second, an intertemporal constraint given by

$$(1 + i_{t-1})\, B_{t-1} = \sum_{s=0}^{\infty} \frac{P_{t+s}\tau_{t+s} + M_{t+s} - M_{t+s-1}}{\prod_{l=1}^{s} (1 + i_{t+l})} \tag{11}$$

where the initial stock of nominal government-issued assets, B_0 and M_0, is taken as given.

Regarding monetary policy, I consider two alternative policy regimes:

Fiscal-dominance regime. During this first regime, operating from period 1 to $T > 2$ (including both), the government sets an exogenous target for seigniorage. Money supply is then adjusted to satisfy a seigniorage requirement, denoted by α, according to the following rule

$$M_t = P_t \alpha + M_{t-1} \quad (1 \leq t \leq T) \tag{12}$$

Monetary-dominance regime. This regime is implemented at $T+1$ (henceforth referred as the date of reform). From that period on, the government targets a unique sequence of inflation rates by adjusting money supply accordingly. For notational simplicity and without loss of generality, I assume that the targeted level of inflation under this regime is constant and denote it by x^L.

As the model admits multiplicity of equilibria when the government operates by choosing the amount of money supplied in the financial market in the latter regime, some of them hyperinflationary, it is also assumed that the government is willing to respond to upwards deviations from the inflation target by backing the stock of money according to the following strategy. Taking P_T and the aggregate equilibrium stock of money at that date, M_T, as given, the government announces a sequence of money supplies $\{M_t\}_{t=T+1}^{\infty}$. Such a sequence of money supplies will map into a unique sequence of equilibrium inflation rates if and only if the demand for real balances and, hence, the price level follow a unique path from $T+1$ on. Let us denote this sequence of prices as $\left\{P_t^G\right\}_{t=T+1}^{\infty}$, whose exact elements are understood to depend on $\{M_t\}_{t=T+1}^{\infty}$, M_T and P_T, and satisfy $\frac{P_{T+1}^G}{P_T} = \frac{P_{T+s+1}^G}{P_{T+s}^G} = 1 + x^L$, for $s = 1, ..., \infty$. Then, in view of an arbitrary price $P_t > P_t^G$ at any $t \geq T+1$, the government reacts by offering the households the possibility of redeeming each monetary unit in exchange for $\frac{1}{\widetilde{P}_t}$ bonds, where $P_t^G \leq \widetilde{P}_t < P_t$. Obviously, such a deal creates an arbitrage opportunity which is not compatible with individual optimization and, hence, any P_t above the one consistent with the government's inflation target can not be part of an equilibrium.[17]

Notice that the credibility of this backing-scheme hinges on the households' expectation about a current and/or future fiscal adjustment consistent with the endogenous higher supply of government bonds. Specifically, credibility of this government's contingent strategy requires the commitment to increase the discounted stream of taxes by an amount equal to $\frac{M_t}{\widetilde{P}_t}$.

As shown later, in most cases after the reform there will be a upward adjustment in the demand for real balances. Thus, a constant targeted x^L over this regime will require money injections, i.e. $M_t > M_{t-1}$ for $t \geq T+1$, which in turn implies that the government will earn some seigniorage whose amount is (endogenously) driven by the path followed by the stock of real balances. Therefore, in this regime the supply of bonds and the sequence of taxes must be adjusted to satisfy (10) and (11).

3.3 Equilibrium

In the remaining of the paper, I restrict attention to symmetric competitive equilibria defined as

Definition 1 *A perfect-foresight symmetric competitive equilibrium in this economy is a collection of sequences* $\left\{c_s^m, c_s^c, M_s, \widetilde{M}_s, B_s, n_s, \lambda_s, \phi_s, P_s, i_s\right\}_{s=t}^{\infty}$ *and a government policy such that the following conditions are satisfied:*

1. Households maximize their utility subject to the initial conditions, W_1 *and* n_0*, and the constraints (1)-(6), taking as given the price and interest rate sequences and the government policy.*

2. The government satisfies its budget constraints (10) and (11).

3. All markets clear at each date.

(where I have dropped for simplicity the household's superscript j). Note that this definition is sufficiently broad so as to apply to the two fiscal-monetary regimes described in section 3.2.

[17] This backing scheme resembles the one considered by Obstfeld and Rogoff (1983) and Nicolini (1996).

4 Fiscal-monetary reform (in)effectiveness

In what follows I check the ability of the model developed before to (qualitatively) account for the empirical facts listed earlier and to provide an answer to the following questions:

First. *When is a credible fiscal-monetary reform effective to prevent a hyperinflation?*

Second. *What are the factors that may explain the occurrence of a hyperinflation as a self-fulfilled prophecy?*

I order to make the above queries explicit within the model, it is convenient to give a meaning to the terms "hyperinflation" and "self-fulfilled hyperinflation" using the own language of the model. For this purpose, an important part of the following discussion is referred to two possible (extreme) equilibrium paths: the *high inflation path* and the *low inflation path* (HIP and LIP, henceforth). Using these two almost self-explanatory inflationary paths (whose formal definition is given below), I will henceforth use the term "hyperinflation" as the occurrence of the HIP and "self-fulfilled" or "speculative" hyperinflation as the occurrence of the HIP whenever the LIP is simultaneously a possible equilibrium outcome. On the one hand, this particular convention sacrifices some generality. First, because for a class of reforms (non-contingent or fixed-date reforms) it might be difficult to empirically distinguish the HIP from other possible equilibria that yield inflation paths almost as high as the HIP. Second, because other paths may coexist as potential equilibria with the LIP and/or the HIP, so that could equally be labelled as self-fulfilled prophecies. On the other hand, by focusing on the conditions under which the HIP may occur we will draw an important and interesting conceptual line, namely that one separating an *ex ante effective* fiscal-monetary reform (to be clarified later) from an *ineffective* one. Nonetheless, the formal discussion that follows also devotes some attention to other non-extreme inflationary paths, in part because it is shown that under some conditions, non-extreme paths are the only possible equilibrium outcome.

In the remaining of this section, I first describe some conditions on the inflation rate targeted after the reform and the volume of seigniorage collected over the fiscal-dominance regime that are needed to characterize a meaningful hyperinflationary scenario and then provide an answer to the first question above: *when is a reform effective to prevent a hyperinflation?* I carry out this task here under the assumption that the government credibly commits to reform its fiscal-monetary plan at some future date $T+1$, regardless of the state of the economy at that date.[18] In section 5 I focus on the case of self-fulfilled hyperinflations and in section 6 I consider an alternative (state-contingent) rule for abandoning the fiscal-dominance regime showing that the key properties of the analysis conducted under the benchmark case of a fixed-date reform continue to hold under this alternative specification of government policy.

[18] The assumption that the public knows with certainty the date of the reform is followed by Bental and Eckstein (1990) and Paal (2000).

4.1 Parameter assumptions

Let us consider the two extreme cases in which the households either exploit the possibility to establish a credit-link whenever possible (thus setting $\lambda_t = 1$) or refuse to invest in the credit technology at all ($\lambda_t = 0$) over the fiscal-dominance regime. Assuming the former case, the economy-wide measure of producers with a credit-link at t, according to (3), is given by

$$n_t = \gamma + (1-\gamma)\,\delta n_{t-1} \tag{13}$$

Thus, denoting by m_t the proportion of producers that sell their goods in exchange for money, i.e. $m_t = 1 - n_t$, we can write

$$m_t = (1-\gamma)\left[1 - \delta\left(1 - m_{t-1}\right)\right] \equiv \Gamma^I\left(m_{t-1}\right) \tag{14}$$

After applying the normalization $y = 1$, from the equilibrium condition $M_t = \widetilde{M}_t$, it follows that m_t above represents the end-of-period stock of real balances in the economy, which may take values in the unit interval as long as $i_t > 0$.

Equation (14) captures the law of motion of m conditional on every producer investing in the credit technology at the highest pace. Conversely, when no new credit-links are made, it can be readily verified that the degree of monetization evolves according to

$$m_t = 1 - \delta\left(1 - m_{t-1}\right) \equiv \Gamma^N\left(m_{t-1}\right) \tag{15}$$

Next, I introduce the following general assumptions which will be maintained throughout unless otherwise noted.

Assumption 1 (Ex post effective reform). The inflation rate targeted in the post-reform monetary-dominance regime, x^L, is bounded above by

$$x^L \equiv x_{T+s} < \min\left\{x_T,\quad \frac{\theta}{\sum_{s=1}^{\infty}\beta^s\delta^{s-1}}\right\},\qquad s \geq 1 \tag{16}$$

The bounds above imply, first, that the change of regime always leads to a fall in inflation at the time of the reform, regardless of the history up to that date (i.e. $x^L < x_T$) and, second, that the reform is always successful in stopping an eventual "flight from money" from the time of its implementation on (i.e. $x^L < \theta\left\{\sum_{s=1}^{\infty}\beta^s\delta^{s-1}\right\}^{-1}$).[19] Thus, in equilibrium $\lambda_s = 0$, $s \geq T$. Both features are clearly consistent with the evidence described before.

Assumption 2 (Feasibility of the seigniorage target) The level of seigniorage collected over the fiscal-dominance regime, α, is bounded above by

$$\alpha < \alpha^{\max} \equiv (1-\delta)(1-\gamma)\,\frac{1-\eta^{T-1}}{1-\eta} + \eta^{T-1}m_0 \tag{17}$$

[19] In writing the term in the right side of this inquality, we note that the real return on debts, r, is constant over time, $1 + r \equiv (1+i_t)\,\frac{P_t}{P_{t+1}} = \beta^{-1}$, a result that follows from the goods-market clearing condition and the Euler equation (8).

where $\eta \equiv \delta(1-\gamma)$. When this inequality holds, the amount of seigniorage targeted by the government before the implementation of the reform is feasible even when every household invests in the credit technology in every period $t = 1, ..., T-1$, so that m_{T-1} takes its minimum possible value (given by the right hand side of (17)). Implicit in this argument is the fact that households do not invest in the credit technology at T. This is consistent with the class of reforms considered in assumption 1.

Assumption 3 (Dynamic negative association between real balances and inflation). For $\delta > 0$, the targeted level of seigniorage satisfies

$$\alpha > 1 - \delta \tag{18}$$

This condition implies that, regardless of the evolution of the degree of monetization over the fiscal-dominance regime, i.e. both if m_t follows (14) or (15), the resulting equilibrium inflation rate, x_t, can be expressed as decreasing function of the last-period degree of monetization, m_{t-1}. To see this, let us write (12) as

$$x_t = \frac{m_{t-1}}{m_t - \alpha} - 1, \qquad 2 \leq t \leq T \tag{19}$$

Then, substituting m_t for its two extreme values given in (14), $\Gamma^I(m_{t-1})$, and (15), $\Gamma^N(m_{t-1})$, and taking a simple derivative in (19), we can write

$$\begin{aligned} \left.\frac{\partial x_t}{\partial m_{t-1}}\right|_{m_t=\Gamma^I(m_{t-1})} &< 0 \Leftrightarrow \alpha > (1-\delta)(1-\gamma) \\ \left.\frac{\partial x_t}{\partial m_{t-1}}\right|_{m_t=\Gamma^N(m_{t-1})} &< 0 \Leftrightarrow \alpha > 1-\delta \end{aligned}$$

Thus, as $\gamma < 1$, the inequality in (18) is a sufficient condition for the inflation rate to be negatively related to m_{t-1}, regardless of the law of motion of m_t. The following result is a direct consequence of the inflation-generation process (19) and the specification of the Γ-functions, (14) and (15). Given $\delta > 0$, if assumption 3 holds then the following inequalities are satisfied

$$x_t^I > x_t^N \quad \text{for } 2 \leq t \leq T \tag{20}$$

$$x_{t+1}^I \geq x_t^I \quad \text{and} \quad x_{t+1}^N \leq x_t^N \quad \text{for } 2 \leq t \leq T-1 \tag{21}$$

where x_t^I is the inflation rate satisfying (19), conditional on m_s satisfying $m_s = \Gamma^I(n_{s-1})$, for $s = 2, ..., t$. Analogously, x_t^N is defined as the inflation rate at t conditional on no credit-link being established at any date from 1 up to t, both inclusive, i.e. $m_s = \Gamma^N(m_{s-1})$, for $s = 2, ..., t$. Thus, it follows that, for a sufficiently high level of seigniorage, the inflation rate associated with a sequence of degrees of monetization satisfying $m_s = \Gamma^I(m_{s-1})$ is always higher than the corresponding to a sequence obeying $m_s = \Gamma^N(m_{s-1})$. Also from the above inequalities, we learn that for a sufficiently high level of seigniorage a decreasing m-sequence is associated with an increasing sequence of inflation rates and vice versa, i.e. increasing m over time leads to lower inflation rates.

The basic content of the above result is a very simple one: by restricting the volume of seigniorage to satisfy (18), we focus on fiscal-dominance regimes along which a higher degree of monetization is associated with a lower inflation rate. This idea resembles the one underlying the analyses based on the interpretation of seigniorage as a tax paid by money holders, along the increasing arm of an inflation-tax Laffer curve, in the sense that a higher tax base (real balances or degree of monetization) needs a lower tax rate (inflation) to yield the same total revenue (seigniorage). As (18) imposes a lower bound on seigniorage or, alternatively, a lower bound on the degree of persistence of the credit technology, δ, it is not at odds with the common view of high levels of seigniorage being, at least in part, responsible for hyperinflationary processes and with the empirical evidence presented in section 2 regarding the slow recovery of the degree of monetization observed after a hyperinflation.

4.2 The case of an *ex ante* ineffective reform

Next I define the two extreme paths (HIP and LIP) using the laws of motion for the degree of monetization under the assumption that λ_t takes one of the extreme values, 1 or 0, obtained before in (14) and (15), respectively.

Definition 2 *The high inflation path (HIP) is the sequence* $\left\{x_t^I\right\}_{t=2}^T$ *that satisfies (19) given that the sequence* $\{m_t\}_{t=2}^{T-1}$ *satisfies* $m_t = \Gamma^I(m_{t-1})$. *The low inflation path (LIP) is the sequence* $\left\{x_t^N\right\}_{t=2}^T$ *that satisfies (19) given that the sequence* $\{m_t\}_{t=2}^{T-1}$ *satisfies* $m_t = \Gamma^N(m_{t-1})$.

Using the shadow-value function (9) together with the equilibrium conditions $\frac{1+i_t}{1+x_{t+1}} = \beta^{-1}$ and $c_t = 1$, we can define the time-t (unique) shadow-value function associated with the HIP as

$$\Psi_t^I = \sum_{s=1}^{T-t} \beta^s \delta^{s-1} x_{t+s}^I + \frac{\beta^{T-t+1}\delta^{T-t}}{1-\beta\delta}\phi_t^L x^L \tag{22}$$

where $\phi_t^L \in \{0,1\}$ denotes whether pre-existing credit-links are used after the reform. According to definition 2, x_t^I is given by

$$x_t^I = \frac{(1-\delta)(1-\gamma)\frac{1-\eta^{t-1}}{1-\eta} + \eta^{t-1}m_0}{(1-\delta)(1-\gamma)\frac{1-\eta^t}{1-\eta} + \eta^t m_0 - \alpha} \tag{23}$$

where $m_0 \equiv 1 - n_0$. Thus, equation (22) captures the (gross) profit given by a credit link when all households exploit every opportunity for investment in the credit technology in each period within the pre-reform period. As the households follow this pattern, the resulting equilibrium sequence of inflation rates is the highest possible one. Then, denoting $\inf\left\{\Psi_t^I\right\}_{t=1}^{T-1}$ by $\underline{\Psi}^I$, and according to (9), the following condition is necessary and sufficient for the feasibility of the HIP as an equilibrium path

$$\underline{\Psi}^I \geq \theta \tag{24}$$

By simple inspection of (22) and (23) the qualitative effect of most parameters and policy variables on condition (24) becomes evident. Higher values of β increase the total return of the investment in a credit-link, thus raising Ψ_t^I, and, hence, $\underline{\Psi}^I$, thus tending to make the HIP a feasible outcome. A similar effect comes from δ, as higher δ means a longer average life for a link. Also, from (23), higher γ contributes to a more rapid demonetization and, thus, to a higher sequence of future inflation rates and a higher value for the credit-technology at any t. This latter effect is also caused by a lower initial condition, m_0. From (22) and (24), it is also clear that a lower value for the cost of establishing a credit link, θ, tends to favor the occurrence of the HIP.

Regarding the policy instruments, α, T and x^L, their influence on condition (24) is also very intuitive. High seigniorage goes in hand with high inflation rates, given a path for m, thus pushing $\underline{\Psi}^I$ up. A restrictive monetary supply over the post-reform period contributes, given every thing else, to keep inflation low also in the pre-reform period, since low values for x^L reduce the expected return gained from each credit-link. Also, an early reform (low T) reduces the incentives to invest in the credit technology, since it shortens the period over which that investment yields high inflation-savings (the pre-reform period). Finally, the effect of time on Ψ_t^I is likely to be non-monotonic due to a discount-effect. Indeed, all the numerical simulations discussed later yield a hump-shaped Ψ_t^I function, i.e. the lowest values are located at the beginning and at the end of the pre-reform period.

In sum, an equilibrium along which the prospect of an orthodox fiscal-monetary reform does not have any positive effect on the demand for real balances and, hence, on the inflation rate except just at the moment of its implementation can not always be ruled out. It is worth noticing the different nature of the reform-ineffectiveness result in this environment and in those studied in some earlier works. In Bental and Eckstein (1990) and Paal (2000) the prospect of a reform is the underlying determinant of an increasing inflation path and, hence, in both cases, the absence of such a reform would preclude such an outcome. In contrast, in the economy studied here, the prospect of a reform may or not preclude the occurrence of a hyperinflation, but its sole expectation never causes it.

Also, the fact that the reform considered here may exert an *ex ante* anti-inflationary effect, reversing the inequality in (24), distinguishes this economy from the ones studied by Kiguel (1989) and Marcet and Nicolini (2003), since in both cases the possibility of a reform is never acknowledged by the public. However, in this economy, when the reform takes place and how it is implemented, i.e. T and x^L, respectively, matters.

5 Speculative hyperinflations

5.1 Coexistence conditions

In this section I extend the analysis to the case of speculative or self-fulfilled hyperinflations. As commented before, for the sake of the clarity, I concentrate the discussion on the conditions under which the coexistence of the two extreme inflationary paths in definition 2 is possible. For the moment, I retain the exogenous reform-date assumption. Let us define the time-t shadow-value function associated with the LIP as

$$\Psi_t^N = \sum_{s=1}^{T-t} \beta^s \delta^{s-1} x_{t+s}^N + \frac{\beta^{T-t+1}\delta^{T-t}}{1-\beta\delta}\phi_t^L x^L \qquad (25)$$

where x_t^N is given by

$$x_t^N = \frac{1-\delta^{t-1}(1-m_0)}{1-\delta^t(1-m_0)-\alpha} \tag{26}$$

The function Ψ_t^N captures the benefits for a single household from investing in a credit-link conditional on that link being the only one made from that period on, i.e. it captures the individual incentives to invest in the credit technology when it is understood that nobody else will ever invest. As assumptions 1 and 3 hold, we learn that Ψ_t^N is a decreasing function of time and satisfies $\Psi_t^N < \Psi_t^I$ for $t = 1, ..., T-1$. Hence, the following sign condition is necessary and sufficient for the feasibility of the LIP

$$\Psi_1^N \leq \theta \tag{27}$$

Clearly, the influence of the parameters and policy variables on condition (27) is the opposite one with respect to (24), that is, low values of β, α, δ, x^L and T, and high values of m_0 and θ run in favor of the feasibility of the LIP.

Then, combining (24) and (27), we learn that an extreme speculative hyperinflation is possible in this environment if the following condition holds

$$\Psi_1^N \leq \theta \leq \underline{\Psi}^I \tag{28}$$

Next, I analyze the role of some of the parameters of the model regarding the fulfillment of condition (28), restricting most of the analysis to δ, γ and α. Devoting special attention to δ and γ is obliged here, since these parameters are the critical ingredients in this economy. Also, by focusing on seigniorage, we get a net picture of the main differences between this model and some previous approaches based on relating the observed inflation rate to the amount of seigniorage through an inflation-tax Laffer curve.

5.1.1 A dynamic Laffer-curve

In order to investigate how seigniorage over the fiscal-dominance regime relates to the possibility of multiple equilibria, I characterize the threshold conditions (24) and (27) as functions of α, taking everything else as given. The following proposition describes the necessary conditions for the existence of a non empty set of α's for which the HIP and LIP coexist and establishes the boundaries of that set.

Proposition 1 *Let us define the following two limit values for* θ,

$$\begin{aligned} \overline{\theta} &\equiv \lim_{\alpha \nearrow \alpha^{\max}} \Psi_{T-1}^I \\ \underline{\theta} &\equiv \lim_{\alpha \searrow 1-\delta} \Psi_1^N \end{aligned}$$

If $\underline{\theta} \leq \theta \leq \overline{\theta}$, *then there exist a unique* α^I *such that* $\underline{\Psi}_t^I\left(\alpha^I\right) = \theta$ *and a unique* α^N *such that* $\Psi_1^N\left(\alpha^N\right) = \theta$, *and there are multiple equilibria if* $\alpha^I < \alpha^N$ *and* $\alpha \in \left[\alpha^I, \alpha^N\right]$.

Proof. The necessity of a bounded θ is simple. The upper bound $\overline{\theta}$ is defined as the cost of a credit link such that the HIP is feasible when α is set arbitrarily close to its maximum sustainable level, $\alpha^{\max}$, as given in (17). In computing $\overline{\theta}$, notice that $\lim_{\alpha\nearrow\alpha^{\max}} x^I_{T-1} \to \infty$, so for finite T and θ, $\lim_{\alpha\nearrow\alpha^{\max}} \Psi^I_t \to \infty > \theta$ for $1 \leq t \leq T-2$, and, hence, feasibility of the HIP hinges exclusively on the value of $\lim_{\alpha\nearrow\alpha^{\max}} \Psi^I_{T-1}$. Likewise, the lower bound $\underline{\theta}$ implies that θ is always sufficiently high so that individuals optimally give up their opportunities to expand their portfolio of credit-links whenever α is set close to its minimum possible value, $1-\delta$, consistently with assumption 3.

Hence the existence of α^I and α^N amounts to impose a rather mild condition, namely, that we can find a sufficiently high (low) α, within the limits imposed in assumptions 1 and 2, such that the HIP (LIP) is a possible equilibrium outcome.

Uniqueness of the threshold values α^I and α^N and the necessity of an intermediate $\alpha \in \left[\alpha^I, \alpha^N\right]$, both follow from the fact that $\underline{\Psi}^I_t$ and Ψ^N_1 are (strictly) monotonically increasing functions of α. ■

Figure 2 depicts x_T (i.e. the last endogenously determined inflation rate) as a function of α, under the assumption that $\alpha^I < \alpha^N$, for the two extreme paths, HIP and LIP. This figure shares an important feature with a standard hump-shaped inflation-tax Laffer-curve, namely that some levels of seigniorage can be financed at either a stable and low (or moderate) inflation rate or at an unstable rising one. However, in this economy a speculative hyperinflation will not take place for low values of seigniorage ($\alpha < \alpha^I$) since the prospect of a future reform renders the investment in the credit technology no profitable enough. Another important feature of this economy is that along a speculative hyperinflationary path inflation is always positively associated with the level of seigniorage, just the opposite result found in the analyses performed by Sargent and Wallace (1987) and Bruno and Fischer (1990), since, in those models, the fact that a speculative hyperinflation is understood as the economy slipping into the decreasing arm of the Laffer-curve implies that one should expect higher rates of inflation when seigniorage is low.

Figure 2 also makes clear that the model is compatible with the main result obtained in the *money-market partial-adjustment* framework employed by Kiguel (1989) and Romer (2001), namely that "too high" seigniorage, i.e. in this model values of $\alpha > \alpha^N$, will invariably result in a hyperinflationary path. However, while admitting the potential importance of seigniorage in causing a hyperinflation, this model still allows for equilibrium hyperinflationary paths even when the quantitative measure of fiscal-dominance before the reform is thought to be only "moderate", i.e. values of α falling within the interval $\left[\alpha^I, \alpha^N\right]$.

5.1.2 Persistency of the credit technology

The possibility of persistent effects of the credit technology beyond the time of the reform is critical for the co-existence of the two extreme equilibria. There are two scenarios in this economy in which the use of credit over the fiscal-dominance-regime is not maintained after the reform: (i) if the government chooses a post-reform inflation target, x^L, below zero, i.e. the cost of using an existing link, and (ii) if a credit link is only operative one period, i.e. $\delta = 0$. Below I deal with the latter case and postpone to the next section the case of $x^L < 0$.

Proposition 2 *If $\delta = 0$, then there exists a unique equilibrium.*

Proof. Let us assume that the credit technology only lasts for one period, i.e. each credit-link is only effective in the period of its creation. Then (9), in equilibrium, simplifies to

$$\Psi_t = \beta \phi_t x_{t+1} \tag{29}$$

where ϕ_t now simply indicates whether the credit-link is established ($\phi_t = 1$) or not ($\phi_t = 0$). Since the reform, by assumption (see (16)), precludes the creation of new links, i.e. $x^L < \frac{\theta}{\beta}$, we learn that in equilibrium $\phi_T = 0$ and, according to (15), $m_T = 1$ for any value of m_{T-1}. Let us consider the following two values of m_{T-1}, each associated with an extreme λ_{T-1}: 0 and 1, and the corresponding (according to (19)) inflation rates at T:

$$m_{T-1} = \begin{cases} m^I_{T-1} = 1-\gamma \\ m^N_{T-1} = 1 \end{cases} \qquad x_T = \begin{cases} x^I_T = \frac{1-\gamma}{1-\alpha} - 1 \\ x^N_T = \frac{1}{1-\alpha} - 1 \end{cases} \tag{30}$$

Thus, as $\gamma < 1$, we learn that $x^I_T < x^N_T$, i.e. just before the implementation of the reform, there is a negative relationship between the last period's degree of monetization, m_{T-1}, and the current period's inflation rate, x_T.[20] Next, let us consider three alternative cases, depending on the sign of the following differences: $x^I_T - \widehat{\theta}$ and $x^N_T - \widehat{\theta}$, where $\widehat{\theta} \equiv \frac{\theta}{\beta}$.

Case 1. $x^N_T - \widehat{\theta} \leq 0$. As $x^I_T < x^N_T$, it follows that $x^I_T - \widehat{\theta} < 0$. Then, the optimal households' decision at $T-1$ is not to invest in the credit technology, i.e. there is no $\lambda_{T-1} \in (0,1]$ such that $\frac{1-\lambda_{T-1}\gamma}{1-\alpha} - 1 - \widehat{\theta} > 0$. Thus, in equilibrium, $m_{T-1} = m_T = 1$. The same argument clearly applies to any period $t \in [1, T-1]$, so, from (19), we learn that the equilibrium inflation rate is also uniquely determined (and constant) over the pre-reform regime, i.e.

$$x^{(1)}_t = \frac{1}{1-\alpha} - 1 \quad \text{for} \quad 2 \leq t \leq T.$$

Case 2. $x^I_T - \widehat{\theta} \geq 0$. In this case $x^N_T - \widehat{\theta} > 0$ must hold and the households optimally decide to exhaust their opportunities to invest in the credit technology at $T-1$, so that $\lambda_{T-1} = 1$ and $m_{T-1} = 1-\gamma$. Then, the lowest possible inflation at $T-1$ is $\frac{1-\gamma}{1-\gamma-\alpha} - 1 > x^I_T > \widehat{\theta}$, i.e. the rate that would prevail in a situation in which the households set $\lambda_{T-2} = 1$. Thus, the households optimally set $\lambda_{T-2} = 1$, and, applying this argument backwards, we find that in any equilibrium $\lambda_t = 1$ and $m_t = 1-\gamma$ for $t = 1, ..., T-1$. Then, given a unique equilibrium sequence $\{m_t\}_{t=1}^{T-1}$, it follows that there is also a unique equilibrium sequence of inflation rates satisfying

$$x^{(2)}_t = \begin{cases} \frac{1-\gamma}{1-\gamma-\alpha} - 1, & 2 \leq t \leq T-1 \\ \frac{1-\gamma}{1-\alpha} - 1 & t = T \end{cases} \tag{31}$$

Case 3. $x^N_T - \widehat{\theta} > 0$ and $x^I_T - \widehat{\theta} < 0$. This case is clearly incompatible with the limiting values of λ_{T-1}, 0 and 1. First, $\lambda_{T-1} = 0$ can not be optimal since the inflation rate resulting from the highest level of

[20] Notice that this result is directly driven by the assumption $\delta = 0$, which is itself inconsistent with assumption 2. To see this, notice that the sign condition in (18) cannot be satisfied as $\delta = 0$ and, thus, the chains of implications following that condition do not longer hold.

monetization, $m^N_{T-1}=1$, is higher than the unit cost of a credit-link. Symmetrically, $\lambda_{T-1}=1$ can not be optimal either, for the inflation rate consistent with $m^I_{T-1}=1-\gamma$ falls below the cost of a link. Thus, in a symmetric equilibrium, the household chooses an intermediate $\lambda_{T-1}\in(0,1)$, i.e. some producers within the household invest and some others, while having the opportunity, do not. But this implies that at the margin the household must be indifferent between establishing a link or not. Therefore, the (unique) equilibrium inflation rate at T satisfies $x_T=\widehat{\theta}$ and $x_T\in\left(x^I_T,x^N_T\right)$. Then, we can use (19) to solve for the unique equilibrium λ_{T-1} :

$$\widehat{\theta}=\frac{1-\lambda_{T-1}\gamma}{1-\alpha}-1$$

Given a unique $m_{T-1}=1-\lambda_{T-1}\gamma$, we learn that $x^N_{T-1}\equiv\frac{1}{1-\lambda_{T-1}\gamma-\alpha}-1>x^N_T>\widehat{\theta}$, so that the relevant condition governing the households' choice for λ_{T-2} is given by the sign of $x^I_{T-1}-\widehat{\theta}$, where $x^I_{T-1}\equiv\frac{1-\gamma}{1-\lambda_{T-1}\gamma-\alpha}-1$. If $x^I_{T-1}-\widehat{\theta}<0$, following the same argument above, the equilibrium inflation rate at $T-1$ is equal to $\widehat{\theta}$ and (19) can be solved for the unique equilibrium λ_{T-2}. This iterative process continues up to a period t^* at which the following inequality holds,

$$x^I_{t^*+1}\equiv\frac{1-\gamma}{1-\lambda_{t^*+1}\gamma-\alpha}-1\geq\widehat{\theta}\tag{32}$$

with $\lambda_{t^*+1}<1$, in which case, we learn that individual optimality calls for $\lambda_{t^*}=\lambda_{t^*-1}=...=\lambda_1=1$. Combining (32) for two consecutive dates, $s-1$ and s, we can write the following dynamic equation

$$\lambda_{s-1}=\frac{1-\widehat{\theta}\,(1-\alpha)}{\gamma}+\widehat{\theta}\lambda_s$$

which, for a terminal condition λ_{T-1} given in (32), can be solved for a unique equilibrium sequence $\{\lambda_s\}^{t^*+1}_{s=T-1}$. Further, since $\widehat{\theta},\gamma$, and λ_s all fall below 1, the elements in this sequence satisfy $\lambda_{s-1}>\lambda_s$ and, hence, $m_{s-1}<m_s$. The degree of monetization for all $t=1,..,t^*$ is constant and equal to $\frac{1-\gamma}{1-\gamma-\alpha}$. Then, given a unique equilibrium sequence $\{m_t\}^T_{t=1}$, we can solve for the unique equilibrium sequence of inflation rates $\{x_t\}^T_{t=2}$,

$$x^{(3)}_t=\begin{cases}\frac{1-\gamma}{1-\gamma-\alpha}-1, & 2\leq t\leq t^*\\ \frac{1-\gamma}{1-\lambda_{t^*+1}\gamma-\alpha}-1 & t=t^*+1\\ \widehat{\theta} & t=t^*+2,...,T\end{cases}$$

Using the terminology introduced above, Case 1 corresponds to the LIP and Case 2 to the HIP. We can then solve for the two critical α-thresholds, such that both extreme cases are feasible, to find the following step-function relating seigniorage to the equilibrium inflation sequence over the pre-reform period

$$\underset{2\leq t\leq T}{x_t}=\begin{cases}x^{(1)}_t & \text{for } \alpha^{(1)}\leq\alpha^N\equiv\frac{\widehat{\theta}}{1+\widehat{\theta}}\\ x^{(2)}_t & \text{for } \alpha^{(2)}\geq\alpha^I\equiv\frac{\gamma+\widehat{\theta}}{1+\widehat{\theta}}\\ x^{(3)}_t & \text{for } \alpha^{(3)}\in\left(\alpha^L,\alpha^H\right)\end{cases}\tag{33}$$

The three possible equilibrium inflation sequences, $x^{(1)}_t$, $x^{(2)}_t$ and $x^{(3)}_t$ are plotted in figure 3. ■

This proposition is intuitive. As $\delta=0$, the reform considered here leads to a unique optimal means-of-transaction portfolio decision over the monetary-dominance regime, which tantamounts to say that the households' optimal actions from T on are completely independent of their actions at any earlier date.

In plain words, "the future is independent of past". Then, the ability of the government to implement a monetary rule consistent with an exogenously fixed level of seigniorage implies, in view of (19), that for the unique "possible future" there is a unique "possible past".

By imposing $\delta = 0$, we get a version of the model that replicates one of the salient features of the Cagan-type demand for real balances function, namely, that the only relevant endogenous variable for the individual's decision of how much money to hold is the next period expected inflation, as made clear by (29). Hence, in this critical aspect the no-hysteresis version of our economy falls within the general class examined Obstfeld and Rogoff (1983) and Nicolini (1996) and not surprisingly for a given path of government policy choices there is only space for one set of equilibrium beliefs.

On the other hand, when $\delta > 0$, the previous conclusion does not necessarily hold and there is a chance for the coexistence condition (28) to hold. A successful post-reform policy aimed at stabilizing the rate of inflation at a low level induces a unique optimal private behavior from the time of the reform on but it may not be sufficient to induce a unique optimal behavior before its implementation and, hence, private expectations may play a role in selecting a particular equilibrium.

5.1.3 The speed of demonetization

In some models of hyperinflations the existence of an upper bound in the velocity of demonetization (i.e. the fall in m between two consecutive periods) is a necessary condition to generate a hyperinflationary path when seigniorage is high, such that violations of that threshold lead to a reversal of the dynamics of the model. For example, in the Kiguel-Romer framework with partial adjustment in the money market, a sufficiently high speed of adjustment may invert the sign of the correlation between seigniorage and inflation, yielding disinflationary paths when seigniorage is above the maximum level dictated by the stationary inflation-tax Laffer-curve. A similar effect is also found in some models with backward looking expectations (crude adaptive or quasi-rational expectations as in the modern literature on learning mechanisms).[21] To put it in simple terms, such a feature implies that should individuals be able to "fly from money" at a sufficiently fast rate, say because there are not important obstacles to access to alternative means of payment (i.e. high γ), we would observe very severe hyperinflations when seigniorage is low and pronounced disinflations with high volumes of seigniorage.

Such a counterintuitive feature is also shared by the version of the model analyzed before when $\delta = 0$. The following example illustrates this point. Let us assume an initial situation with a level of seigniorage satisfying $\alpha > \frac{\gamma^0+\widehat{\theta}}{1+\widehat{\theta}}$. According to (33), individual optimality implies that equilibrium inflation in the pre-reform regime is given by (31), with $\gamma = \gamma^0$. Now, let us suppose that γ rises up to $\gamma' > \gamma^0$, such that, for the same α, now $\alpha < \frac{\gamma'+\widehat{\theta}}{1+\widehat{\theta}}$. This upward adjustment in γ moves the economy into the intermediate

[21] For a discussion of this point in the context of a mechanism of crude adaptive expectations see, e.g. Bruno and Fischer (1990). Lettau and van Zandt (2003) and Adam, Evans and Honkapojha (2005) investigate how the usage of current versus past inflation rates in forming expectations affect the dynamic properties of a standard model of seigniorage under alternative learning rules, showing that under some widely used specifications for the learning rule, the use of updated, rather than lagged, information tends to make an explosive hyperinflation a more unlikely result.

region in (33), thus lowering the inflation rate at T (since $\frac{1-\gamma^0}{1-\alpha} - 1 > \widehat{\theta}$) and at every date sufficiently close to the reform, at which $x = \widehat{\theta}$ for $\gamma = \gamma'$. Further, for sufficiently high γ (or low T) the equilibrium inflation sequence is uniformly lower under γ' than under γ^0, that is, a more effective mechanism for bringing together unlinked shoppers and producers (i.e. higher γ) results in a lower degree of usage of credit and, thus, at some dates, in lower inflation.

The previous outcome vanishes as soon as the parameters allow for the possibility of hysteresis after the reform, as formally stated in the following proposition.

Proposition 3 *If $\delta > 0$ and $x^L > 0$, then higher γ implies (i) a uniformly higher inflation rate along the HIP, and (ii) a wider range of coexistence of extreme equilibria.*

Proof. (i) Straightforward. From the definition of x_t^I (see (23)), we learn that $\frac{dx_t^I}{d\gamma} > 0$.

(ii) By totally differentiating (22) and using the definition of α^I in proposition 1, we obtain that $\frac{d\alpha^I}{d\gamma} < 0$. Similarly, using (25) and (26), we learn that $\frac{d\alpha^N}{d\gamma} = 0$. Hence, it trivially follows that $\frac{d\left(\alpha^N - \alpha^I\right)}{d\gamma} > 0$. ■

Thus, when the private sector can easily substitute credit for money as an alternative means of carrying out transactions *and* individuals internalize the persistence of their decisions, a simple and intuitive proposition applies: extreme speculative *hyperinflations* are more likely outcomes.[22]

Finally, when the velocity of demonetization or its degree of persistence are sufficiently low, given everything else, the necessary condition for the existence of an extreme speculative hyperinflation, $\alpha^I \leq \alpha^N$, is violated, so that there is never an overlap between the HIP and LIP, even if (18) holds. When this is the case, it can be shown that for intermediate levels of seigniorage $\alpha \in \left(\alpha^N, \alpha^I\right)$ there is always, at least, one equilibrium in which every household exhausts its opportunities to invest in the credit technology up to some period t, for $2 \leq t \leq T-2$ and stop investing from $t+1$ on, so that the equilibrium path for m exhibits a ∪-shape over time. Further, over this range of α's, this class of m-paths are the only admissible ones, thus implying that the reform only exerts a positive effect on the demand for real balances when the date of the reform is perceived as sufficiently close.[23] A complete proof is provided in the Appendix.

5.2 Some numerical illustrations

Figures N1 to N7 contain the results of some numerical simulations of the model. In all cases considered here, the minimum value of the shadow-value function Ψ_t^I over the period running from $t = 1$ to $T-1$ was found at $T-1$, this reflecting, among other things, that the prospect of a reform supporting a low inflation rate (x^L was set in most numerical exercises at 0.1) tends to reduce the incentives for investing in

[22] In his classic study on inflationary finance, Nichols (1974) formulates an intuitive principle to avoid hyperinflations, or at least to keep inflation under some control, in face of a given target for seigniorage: restrict the public's access to those assets that may be seen as close substitutes for money. Chang (1994) also points out that policies tending to decrease the usage of foreign currency should reduce inflation. Proposition 3 shows that those reflections are plainly compatible with this model.

[23] In general, this class of paths may also exist when $\alpha^I < \alpha^N$.

the credit technology specially in the period preceding its implementation. Figures N1 and N2 depict the size of the overlap between the two extreme inflation paths (i.e. the the difference $\alpha^N - \alpha^I$) as a function of the degree of persistence, δ, and the speed of demonetization, γ, respectively, for several pre-reform time-horizons. For both parameters, the size of the α-set such that extreme speculative hyperinflations are possible is positively related to the value of the relevant parameter regardless of the length of the pre-reform period. For low values of these parameters the overlap disappears (i.e. $\alpha^N - \alpha^I$ takes negative values).

Figure N3 is the numerical counterpart of the time T Laffer curve in figure 2. The parameters are chosen so that the overlap $\alpha^N - \alpha^I$ is positive, thus both the LIP and the HIP may coexist. In addition to these two extreme paths, this figure plots the time T inflation rates arising under non-monotonic paths along which the equilibrium level of monetization, m, decreases up to some period $i-1$ and then grows up uniformly from i on. The discontinuous effect of seigniorage on the time T inflation rate (and, hence, on the entire inflation-sequence) and the scope for multiple, and quantitatively very different, equilibrium paths become apparent.

Figure N4, on the other hand, depicts a situation in which δ, while positive, is so low that $\alpha^N < \alpha^I$. The emerging picture is then very different: the effect of seigniorage on the optimal portfolio decision and the resulting inflation sequence exhibits a high degree of smoothness, very much as the suggested by the conventional arguments based on the upward sloped arm in a static inflation-tax Laffer curve, in the sense that high volumes of seigniorage tend to uniformly cause high inflation.

Figures N5 to N7 represent the two extreme shadow-value functions together with those associated with $\cup$-shaped m-paths,[24] the time-invariant cost function (left column) and the corresponding sequences of the inflation rate and the degree of monetization (right column) for $T = 10$. As in every case considered here assumption 2 holds, the shadow value function is uniformly higher as individuals postpone the time for stopping their investments in credit-links. In the three cases depicted in figures N5 to N7, x^L is equal to 0.1, i.e. hysteresis in m after the reform is possible, and $\alpha^N > \alpha^I$, so that there is an overlap between the LIP and the HIP. The case depicted in the first row (figures N5A and N5B) corresponds to a level of seigniorage sufficiently low so that the only feasible equilibrium corresponds to the LIP. Hence, for a given initial condition for m_0 below 1, m grows and x decreases over the pre-reform period. Figures N6A and N6B represent a situation with an intermediate level of seigniorage falling within the interval $\left(\alpha^I, \alpha^N\right)$. Thus, both the LIP and the HIP are possible equilibrium outcomes. Also, a path along which the households coordinate their actions to invest in credit-links up to $t = 8$ and to stop investing from $t = 9$ on is possible (at the same time, no other $\cup$-shaped m-path can be an equilibrium). In the case depicted in Figures N7A and N7B, seigniorage is above α^I and it is high enough so as to preclude any path different from the HIP: the prospect of a reform does not help to prevent an all-the-way flight from money. It is worth noticing that the required increase in α to move the economy from the situation in

[24] The function $\widetilde{\Psi}_t$ is also depicted (see the Appendix for its definition) to facilitate the graphic inspection of the fulfillment of the necessary conditions for the occurence of $\cup$-shaped m-paths.

figure 6 to the one in figure 7 may be only a marginal one. The consequences, however, can be dramatic in terms of the potential differences in the observed inflation paths.

Also notice that the inflation rate in the HIP just before the reform, x_T, is in all cases below the one observed in the preceding period, so that the inflation rate sequence is not a monotonic function of time along that path. This attenuating effect is driven by the positive (negative) effect of the reform on m_T (x_T). However this feature of the model can not be understood as being incompatible with the empirical evidence, as a short period of downward adjustment in the inflation rate following its peak and just before the implementation of the reformed were observed in some of the most explosive hyperinflations, as those experienced by Argentina and Germany (see figure 1).

6 State-contingent reforms

Although the discussion up to here has referred to the case in which the government commits to implement a fiscal-monetary reform at some time T regardless of the history of the economy up to that date, most of the previous key results regarding the (in)effectiveness of an anticipated reform can be easily extended to account for the possibility that the government will only react if things get sufficiently bad, i.e. if the inflation rate threatens to go beyond some limits. The case of a state-contingent reform is interesting in its own as in fact this is the assumption followed by Obstfeld and Rogoff (1983), Nicolini (1996) and Marcet and Nicolini (2003). All these papers consider a simple rule followed by the government for deciding whether to reform or not: to implement a real-backing mechanism at some date t if, absent the threat of such a reform, the inflation rate would violate some finite threshold value, $\overline{x}$ (*reaction point*).[25]

As the interest here is to analyze the effects of such a commitment on the occurrence of purely expectations-driven hyperinflations, the natural context to frame this question is one in which, for a given fiscal-monetary pre-reform regime, there is a non-hyperinflationary path (a *no-crisis* equilibrium). This general picture may be embedded in the present model imposing the following conditions

$$\begin{array}{l}\text{Reform at } t \text{ if } x_t^{NR} \geq \overline{x} \text{ and } \left\{x_t^{NR}\right\}_{s=2}^{t-1} < \overline{x} \\ \text{Do not reform at } t \text{ if } \left\{x_t^{NR}\right\}_{s=2}^{t} < \overline{x}\end{array}$$

$$\sum_{s=1}^{\infty} \beta^s p^{s-1} \frac{1-\delta^s(1-m_0)}{1-\delta^{s+1}(1-m_0)-\alpha} \leq \theta \tag{34}$$

$$\overline{x} < \left[1 - \frac{(1-\delta)(1-\gamma)}{1-\eta}\alpha\right]^{-1} \tag{35}$$

The first lines describe the rule followed by the government to switch from a regime of targeted seigniorage to one of targeted (low) inflation. The government abandons the former regime at the first period at which the inflation rate within that regime, x_t^{NR}, would be equal or above some limit $\overline{x}$, exogenously chosen. It also assumed that (16) still holds, so the reform is always *ex post* effective.

[25] Yet, they differ in the particular real-backing mechanism at work. While Obstfeld and Rogoff (1983) assume that the government reacts by offering the possibility of redeeming money giving some productive capital in exchange, in Nicolini (1996) and Marcet and Nicolini (2003) it is assumed that the government operates a fixed-exchange rate to end the crisis.

The inequality in (34) imposes an implicit upper bound to the volume of seigniorage targeted in the first regime, such that there is, at least, one no-crisis equilibrium (the LIP) that is consistent with the seigniorage target, α, being implemented forever, i.e. a scenario in which the threat of the reform is never executed.

The left side of that inequality corresponds to Ψ_1^N in (25) when this function is defined over an infinite horizon. Notice that, provided (18) holds, this is the relevant measure of the long-run sustainability of the LIP as a potential equilibrium since x_t^N and Ψ_t^N are both decaying over time. The right side in the inequality in (35) is the (limiting) constant inflation rate attained when the degree of monetization is at its minimum possible value, i.e. the stationary state with zero net flow of new credit links. Thus, that inequality rules out the possibility that the threshold value $\overline{x}$ is set at a sufficiently high level so as to always preclude the implementation of the reform. Notice that this last sign condition is only relevant if α is low enough to be maintained in the stationary state in which m reaches its minimum possible value, i.e. $\frac{(1-\delta)(1-\gamma)}{1-\eta}$. But nothing forces the government to choose a α that is sustainable in every contingency, e.g. a particular α may not be implementable by the government over a long period *if* individuals find optimal to fly from money massively.[26]

Now, in deriving a necessary and sufficient condition for the feasibility of the HIP similar to (24), we must take into account that the time of the eventual reform is determined endogenously. Apart from this observation, the problem is similar in its basic aspects to the case with exogenous T. I collect the main insights of this case in the following proposition.

Proposition 4 *Assume (34) and (35) hold. Then, the reform is never executed if (i) the reaction point* $\overline{x}$ *is sufficiently low, or (ii) there is no post-reform hysteresis. Otherwise, the HIP, that triggers the reform, is possible.*

Proof. (i) In deriving the counterpart of (24), it is helpful to consider the following two steps. First, compute the inflation rate sequence associated with the HIP using (23), from $t = 2$ up to the first date at which the inflation threshold would be violated in the absence of a reform and denote that date by $T^* + 1$. The solution for T^* is implicitly given by the following correspondence

$$\frac{m_{T^*-1}}{m_{T^*} - \alpha} - 1 < \overline{x} \leq \frac{m_{T^*}}{m_{T^*+1} - \alpha} - 1, \quad \left[m_t = \Gamma^I\left(m_{t-1}\right) \text{ for } t = 2, ..., T^* + 1\right]$$

where $\frac{m_{T^*}}{m_{T^*+1}-\alpha}$, by construction, is an increasing function of T^*.

Second, use T^* in (22) to compute the sign of $\inf\left\{\Psi_t^I\right\}_{t=1}^{T^*-1} - \theta$. If it is positive, then a speculative hyperinflationary path that unchains the reform is possible. Since $\inf\left\{\Psi_t^I\right\}_{t=1}^{T^*-1}$ is an increasing function

[26] This illustrates an important feature of this model. For some parameter-configuration, we can find a non-empty set of α's for which the *status quo* equilibrium is a possible equilibrium while the HIP necessarily triggers a reform since, otherwise, the seigniorage-target at some point becomes unsustainable, which in turn means that inflation can reach arbitrarily high values just before the change of regime. Such a set is $\left[\alpha^N, \frac{(1-\delta)(1-\gamma)}{1-\eta}\right]$. For some authors, see e.g. Marcet and Nicolini (2003), and Gutiérrez and Vázquez (2004), this is a desirable property for a model of hyperinflations, since the empirical evidence suggests that these processes may be better understood as being inherently unstable along which inflation explodes without bound at an ever increasing rate.

of T^*, and T^* increases (discretely) with $\overline{x}$, we learn that there is a minimum $\overline{x}$ such that a HIP is only possible for values of $\overline{x}$ above that lower bound.

(ii) Imagine that $\delta = 0$ or $x^L < 0$ hold. The latter case implies that in equilibrium $\phi_s = 0$ for $s \geq T^*$. In either case, we can write $\Psi^I_{T^*-1} = \beta x^I_{T^*}$. The shadow-value function associated with the no-crisis equilibrium LIP, $\Psi^N_{T^*-1}$ (computed over an infinite period), satisfies $\Psi^N_{T^*-1} \leq \Psi^N_1 \leq \theta$, with $x^N_s > 0$ for $s \geq T^* + 1$. Define $x^N_{T^*} = \frac{m^{LIP}_{T^*-1}}{m^{LIP}_{T^*} - \alpha} - 1$ and $x^I_{T^*} = \frac{m^{HIP}_{T^*-1}}{1-\alpha} - 1$, where $m^{LIP}_{T^*-1}$ and $m^{HIP}_{T^*-1}$ are the time $T^* - 1$ stocks of monetization associated with the respective extreme path. Then, it trivially follows that $x^N_{T^*} > x^I_{T^*}$, which implies that $\Psi^I_{T^*-1} < \Psi^N_{T^*-1} \leq \theta$, and hence $\inf \left\{\Psi^I_t\right\}_{t=1}^{T^*-1} - \theta < 0$. Thus, the reform is never implemented. ■

The intuition behind these results is simple. A credible commitment by the government to switch from a fiscal-dominance to an orthodox monetary-dominance one when the inflation rate threatens to reach a sufficiently high value will only prevent a speculative hyperinflation and, hence, the regime switch will not take place in equilibrium, if the public understands that the government will not let the inflation rate to reach very high values before intervening or if it is able to implement a sufficiently restrictive monetary policy so that the economy jumps to the full-level of monetization at the time of the reform. Yet, as emphasized in section 2, these two conditions are widely violated in real hyperinflationary episodes.

Figure 4 depicts three alternative scenarios. In the first one, with $x^L > 0$, the threshold $\overline{x}$ is set at a sufficiently low level, x^a, so that the reform is never implemented, i.e. $\inf \left\{\Psi^I_t\right\}_{t=1}^{T^*(x^a)-1} < \theta$. In the second one, maintaining $x^L > 0$, the government is assumed to react only when inflation is sufficiently high, x^b ($x^b > x^a$) so that the eventual reform is effective in stopping the hyperinflation but not in preventing it, i.e. $\inf \left\{\Psi^I_t\right\}_{t=1}^{T^*(x^b)-1} > \theta$. In the last one, the government commits to a negative post-reform rate, $x^L < 0$, so that regardless the particular $\overline{x}$ (x^c in the figure), the reform is never implemented.

One can also follow a similar strategy of treating a certain parameter of interest as exogenously given, like $\overline{x}$ above, to shed light on a different channel through which fiscal conditions may affect monetary movements beyond that of an exogenous volume of seigniorage over the pre-reform period. Regarding the different nature in the fiscal-monetary regime before and after the reforms implemented in several European countries in the interwar period, Sargent (1986; p. 45-46) writes the following:

> The hyperinflations were each ended by restoring or virtually restoring covertibility to the dollar or equivalently to gold. For this reason it is good to keep in mind the nature of the restrictions that the adherence to the gold standard imposed on a government. Under the gold standard, a government issued demand notes and longer-term debt it promised to convert into gold under certain specified conditions, that is, on demand, for notes [...]. More important in practice, since usually a government did not hold 100 percent reserves of gold, a government's notes and debts were backed by the commitment of the government to levy taxes in sufficient amounts, given its expenditures, to make good on its debt [...]. According to this view, what mattered was not the current government deficit but the present value of current and prospective future government deficits.

The money-backing mechanism described in section 3.2 clearly resembles the one highlighted by Sargent and, hence, its success in stopping a continuos fall in the real value of money rests on the same factor: the degree of government's fiscal solvency over the post-reform period. Thus a similar argument to the one exploited before would follow: the government threat to implement a backing-scheme conditional on a speculative flight from money will be effective in precluding it if the public understands that the volume of resources that the government can seize to back its currency is not too low.[27]

7 Concluding remarks

The model presented in this paper aims to reconcile several long-standing views on the causes and dynamics of extreme hyperinflationary processes with the objective of providing a simple theoretical explanation of the main stylized facts observed during those episodes.

A central theme in the paper is how to make three of the most influential stories in the literature compatible with each other and, of course, with the empirical facts we are after. First, a high level of fiscal pressure leading to relatively high levels of deficit monetization is usually identified as a leading cause of hyperinflations. Indeed, stopping a hyperinflation usually involves, among other things, a drastic reduction in seigniorage. Second, it is well understood that a seigniorage-based monetary rule may be compatible with a multiplicity of inflation equilibrium-paths, as this rule leaves the monetary aggregate indeterminate. Private expectations are frequently thought as of being an important mechanism behind hyperinflationary processes, an argument that seems specially attractive to account for one of the most robust stylized facts: the lack of a strong correlation between seigniorage (i.e. the fiscal "fundamental") and inflation over the course of a hyperinflation. Third, for a large class of models widely studied in the previous theoretical and empirical literature (namely, models incorporating a demand for real balances *à la* Cagan), it is difficult to accept an explanation for a hyperinflation based on rational self-fulfilled prophecies, for the government commitment to eventually implement a regime-reform, abandoning the fiscal-dominance regime, and to back its currency with taxes implies, first, that rational speculative paths are not possible, as shown by Obstfeld and Rogoff (1983) and Nicolini (1996), and, second, that higher volumes of seigniorage over that regime would be associated with more severe *disinflationary* processes.

The mutual incompatibility of the above arguments with the hypothesis of rational expectations and with the empirical evidence is not a new result. The solutions given to this problem in the previous literature vary to a great extent. In some cases the rational expectations assumption is abandoned (e.g. Marcet and Nicolini 2003, and Sargent *et al.* 2005) while in others the possibility of a fiscal-monetary reform is left out of the picture (e.g. Bruno and Fischer 1990) or, even, the anticipation of such a reform is blamed for being the cause of a hyperinflation (e.g. Bental and Eckstein 1990, and Paal 2000). The approach taken in this paper follows a different route

[27] Marcet and Nicolini (2003) also devote some attention to this point, as they argue that the drastic fall in the stock of real balances at the end of the hyperinflationary episodes in the late 80's and early 90's, indeed, could have enabled the Argentinian government to peg its currency to the dollar by reducing the required volume of reserves to do so.

Based on some previous works on *dynamic* processes of financial innovation, I study how the persistence in the use of alternative means of transactions affects the effectiveness of an equilibrium selection device similar to the one studied by Obstfeld and Rogoff (1983) and Nicolini (1996). This is a natural question in this context, for the experience of those countries which suffered extreme hyperinflations reveals a high degree of hysteresis in the demand for real money. Using a simple cash-and-credit model that allows for persistence in the usage of credit, it is argued that the robustness, and the rather extreme implications, of an argument *à la* Obstfeld and Rogoff's follow directly from the assumption of instantaneous adjustment in the private demand for real money (i.e. no hysteresis). Once this assumption is relaxed, the effectiveness of the prospect of a future orthodox reform for ruling out hyperinflationary paths, speculative or "fundamental", is an endogenous outcome. In particular, whether the government's commitment to reform exerts any anti-inflationary effect before the time of its implementation, hinges on a wide array of structural factors and policy choices. For example, a high volume of seigniorage collected over the pre-reform period, a long period of government inactivity and a weak fiscal position after the reform are likely to set down the necessary conditions for a hyperinflation. Further, when those variables reach "too high" values the promise of a low-inflation future may well be totally disregarded by the public: no credible threat to reform will preclude a continuous flight from money and a hyperinflation will be the unique possible outcome. When, in addition, individuals do not face important barriers to access to the credit-technology and the effects of the investments in this technology extend over a longer horizon, an extreme speculative hyperinflation happens to be a true possibility, even if individuals rationally expect a future drastic reform.

While the model offers a theoretical basis compatible with the view of a hyperinflation as a bubble phenomenon, it also provides some useful guidance in identifying the economic-policy conditions that may lead to such a painful experience, for the conditions under which such paths are possible are not arbitrary, as in some of the previous literature. Namely, only countries that, for some reason, are advocated to rely on seigniorage in a significant amount and/or for a sufficiently prolonged period are likely to put themselves on the knife-edge. On the theoretical side, the model offers a simple resolution to the incompatibility problem among the three popular approaches aforementioned in a way that renders it a useful tool to understand the empirical evidence. And it does so by including an ingredient which can hardly be labelled as unrealistic or empirically irrelevant: the existence of hysteresis in the degree of monetization following the end of the hyperinflation.

Appendix

Proof of the existence of ∪-shaped m-paths when $\alpha \in \left(\alpha^N, \alpha^I\right)$

Let us define $\widetilde{\Psi}_t$ as the time t shadow-value function associated with the following m-sequence

$$m_s = \begin{cases} \Gamma^I\left(m_{s-1}\right) & \text{for } s = 2, ..., t-1 \\ \Gamma^N\left(m_{s-1}\right) & \text{for } s = t, ..., \infty \end{cases}$$

It can be readily verified that (18) is sufficient for $\widetilde{\Psi}_t$ to be bounded by Ψ_t^N and Ψ_t^I, i.e. $\Psi_t^N < \widetilde{\Psi}_t < \Psi_t^I$, for $t = 2, ..., T-1$. To see this notice that for any t within this interval, $\widetilde{\Psi}_t$ and Ψ_t^N look similarly, as both functions are defined under the assumption that no household is investing in the credit technology from date (inclusive) on. However, as m_{t-1} under $\widetilde{\Psi}_t$ is lower than under Ψ_t^N, (18) implies that every element in the equilibrium inflation sequence $\{x_s\}_{s=t+1}^T$ is higher for $\widetilde{\Psi}_t$. When comparing $\widetilde{\Psi}_t$ and Ψ_t^I, notice that m_{t-1} is common for both functions, but m_s for $s \geq t$ is lower under Ψ_t^I, so the corresponding equilibrium inflation sequence $\{x_s\}_{s=t+1}^T$ is higher for Ψ_t^I than for $\widetilde{\Psi}_t$ and, hence, $\widetilde{\Psi}_t < \Psi_t^I$.

Let us first consider the case in which $\widetilde{\Psi}_2 \leq \theta$. As $\alpha \in \left(\alpha^N, \alpha^I\right)$, we learn that $\Psi_1^I > \Psi_1^N > \theta$ must hold, where the first inequality follows from (18) and the second one holds by construction. Consider a m-path such that $m_t = \Gamma^N\left(m_{t-1}\right)$ for $t \geq 2$. Clearly such a path is an equilibrium one. First, $\widetilde{\Psi}_2 \leq \theta$ implies, by definition, that having invested in the credit technology at $t = 1$,no investing from period 2 on is optimal. Second, the actual shadow-value function at $t = 1$ conditional on investing in that period and no investing from that period on, Ψ_1, must satisfy $\Psi_1^I > \Psi_1 > \Psi_1^N > \theta$, so it is indeed optimal to invest in the first period. Then, consider the case in which $\widetilde{\Psi}_2 > \theta$, so that $\Psi_2^I > \theta$, as well. As $\underline{\Psi}_t^I < \theta$, there must exist, at least, one date $t_1 \geq 3$ such that $\Psi_{t_1=1}^I \geq \theta > \Psi_{t_1}^I$. Let us denote the lowest possible t_1 as $t_1^{\min}$. As $\widetilde{\Psi}_t < \Psi_t^I$, there exists also, at least, one date t_2, with $3 \leq t_2 \leq t_1^{\min}$, such that $\Psi_{t_1=1}^I \geq \theta > \Psi_{t_1}^I$. Let us denote the lowest possible t_2 as $t_2^{\min}$. Then, by looking forward, not investing from period $t_2^{\min}$ on is optimal, conditional on having invested in every period $t = 1, ..., t_2^{\min} - 1$. Looking backwards, as $\widetilde{\Psi}_t > \theta$ for $t = 1, ..., t_2^{\min} - 1$, we learn that the actual Ψ_t satisfies $\Psi_t > \widetilde{\Psi}_t > \theta$ for $t = 1, ..., t_2^{\min} - 1$, so it is optimal to invest over that interval when it is anticipated that no new investments will be made from $t_2^{\min}$ on. Thus, there is always, at least, one equilibrium path for any $\alpha \in \left(\alpha^N, \alpha^I\right)$ along which the time-path of m has a ∪-shape and, hence, the equilibrium inflation path depicts a ∩-shape over the pre-reform period.

Also, notice that if $\alpha^I > \alpha^N$ holds, then there can not exist any equilibrium with a ∩-shaped path for m over the pre-reform period, i.e. situations in which the households coordinate to give up every opportunity for investing in the credit technology up to some date between 2 and $T-1$ and to invest from that particular date on. To see this, notice that a necessary condition for not investing in the first period is that $\Psi_1^N \leq \theta$, and, hence, an equilibrium ∩-shaped path for m requires $\alpha \leq \alpha^N$. Also, for investing from some date $\widehat{t}$ on, such that $2 \leq \widehat{t} \leq T-1$, not having invested over the period running from 1 to $\widehat{t} - 1$, the following condition must hold

$$\widehat{\Psi}_t \equiv \sum_{s=1}^{T-t} \beta^s \delta^{s-1} \widehat{x}_{t+s} + \frac{\beta^{T-t+1}\delta^{T-t}}{1-\beta\delta} x^L \geq \theta, \ \ \forall t = \widehat{t}, ..., T-1 \tag{A1}$$

where

$$\widehat{x}_{t+s} = \frac{(1-\delta)(1-\gamma)\frac{1-\eta^{s-1}}{1-\eta} + \eta^{s-1} m_{\widehat{t}}}{(1-\delta)(1-\gamma)\frac{1-\eta^s}{1-\eta} + \eta^s m_{\widehat{t}} - \alpha}, \tag{A2}$$
$$\text{where } \ s = 1, ..., T-\widehat{t} \ \text{ for some } \widehat{t} = 2, ..., T-1$$
$$\text{and } \ m_{\widehat{t}} = 1 - \delta^{\widehat{t}}\left(1 - m_0\right)$$

By comparing (23) and (A2), and exploiting the fact that the shadow-value functions at any date t are inversely related to m_t, regardless of the evolution of m from that date on, we learn that

$$\Psi_t^I > \widehat{\Psi}_t, \ \ \forall t = \widehat{t}, ..., T-1$$

Thus, if the necessary condition for a ∩-shaped m-path (A1) holds and $\underline{\Psi}^I$ is located within the time-interval $\widehat{t}, ..., T-1$, then it must be the case that $\underline{\Psi}^I > \theta$. A stronger result can be obtained for the alternative case in which $\underline{\Psi}^I$ is located before $\widehat{t}$, since, in this case, the positive effect of a lower m on the shadow-value function is reinforced by a positive effect coming from a longer horizon over which the investment in the credit technology is expected to yield positive returns, i.e. strictly positive savings, so that the following inequality must hold

$$\inf \left\{\Psi_t^I\right\}_{t=1}^{\widehat{t}-1} > \sup \left\{\widehat{\Psi}_t\right\}_{t=\widehat{t}}^{T=1} \geq \theta$$

Therefore, regardless of the time-location of $\underline{\Psi}^I$, the existence of a ∩-shaped m-path requires a level of seigniorage α, such that $\alpha^I < \alpha \leq \alpha^N$, and, as a result, when $\alpha^I > \alpha^N$ holds, the only class of non-monotonic equilibrium m-paths are ∪-shaped, i.e., as the time of the reform comes closer, its prospect exerts a positive effect on the demand for real balances. *Q.E.D.*

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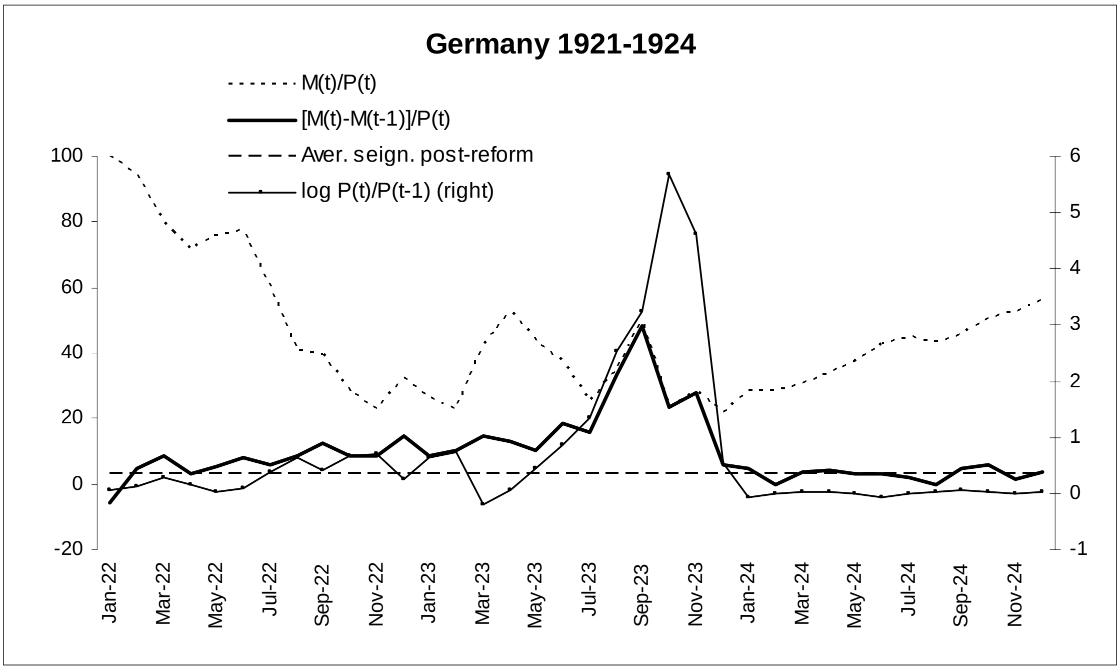
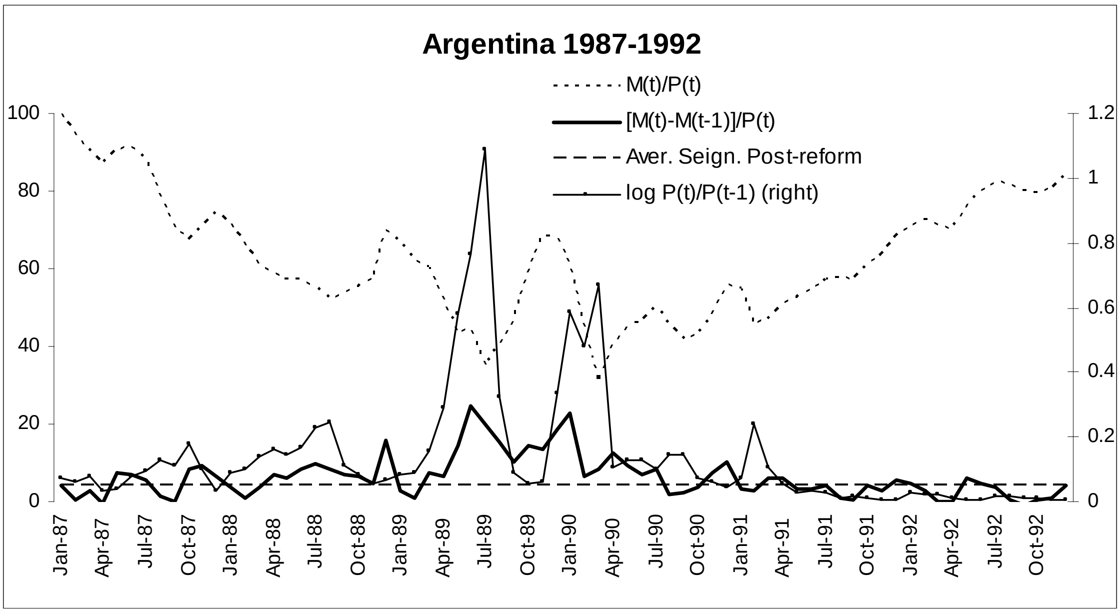
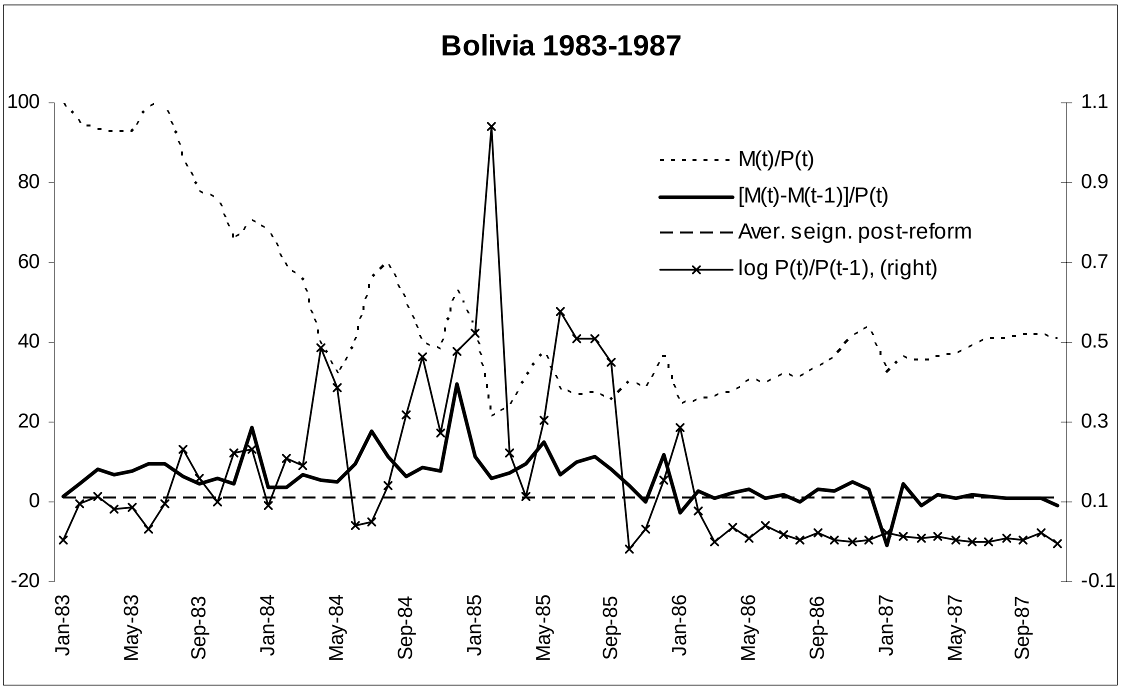
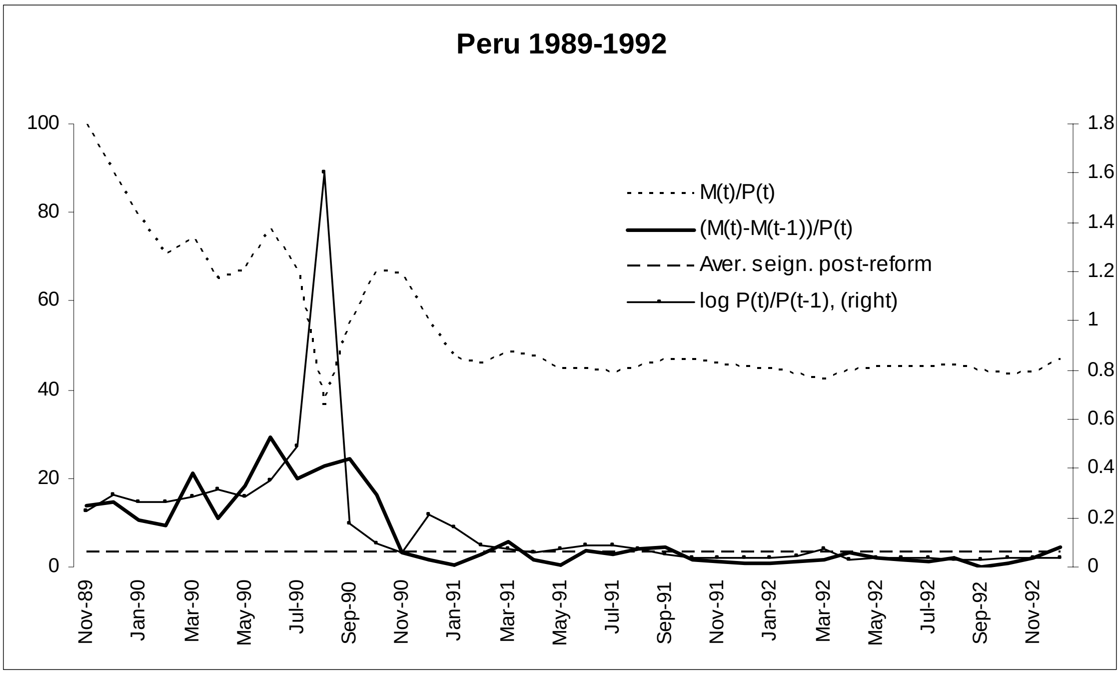
Figure 1: Monetary base, seigniorage and inflation[1]





[1] Left scale: real balances and seigniorage computed using base money (M_0) as percentage of the initial date value for real balances. Average seigniorage in the post-reform regime is computed as the simple mean of seigniorage for the following periods: Germany (Jan 1924 – Dec 1924), Argentina (Apr 1990 - Dec 1992), Bolivia (Mar 1986 – Nov 1987), Peru (Sep 1990 - Dec 1992).
Right scale: natural log of gross inflation computed from wholesale prices (Germany) and CPI (Argentina, Bolivia and Peru).





Sources: Germany: Holtfrerich (1986) and Sargent (1986). Argentina: Boletín Estadístico, Banco Central de la República Argentina (several issues). Bolivia: Memoria, Banco Central de Bolivia (several issues). Perú: Boletín Mensual, Banco Central de Reserva del Perú (several issues).

Figure 2. Dynamic Laffer-curve ($\alpha^I < \alpha^N$)

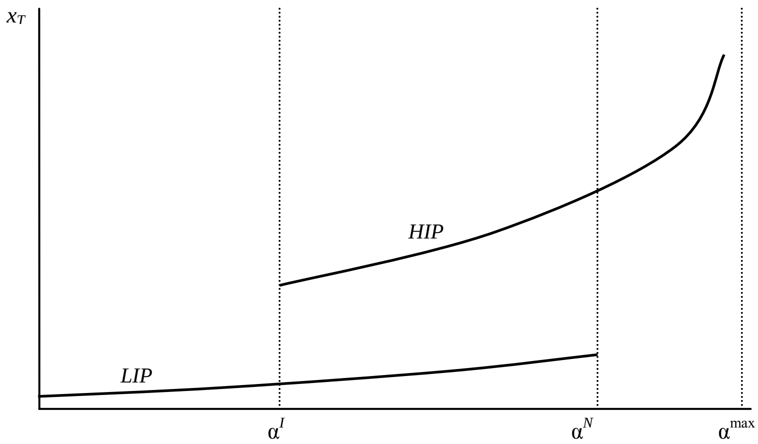
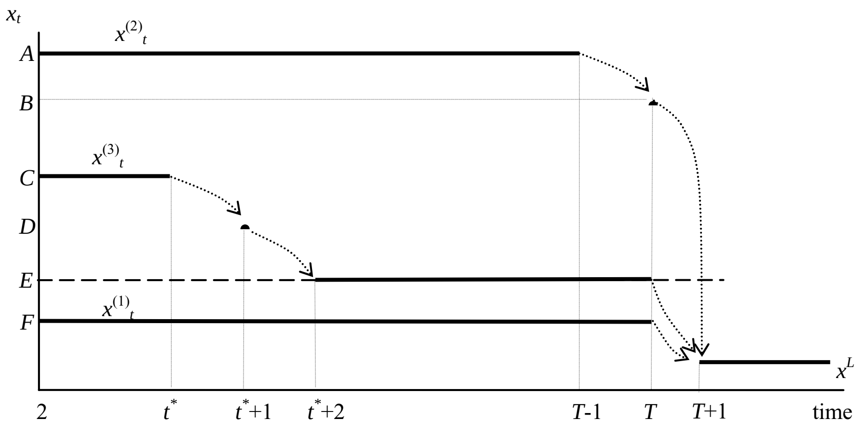
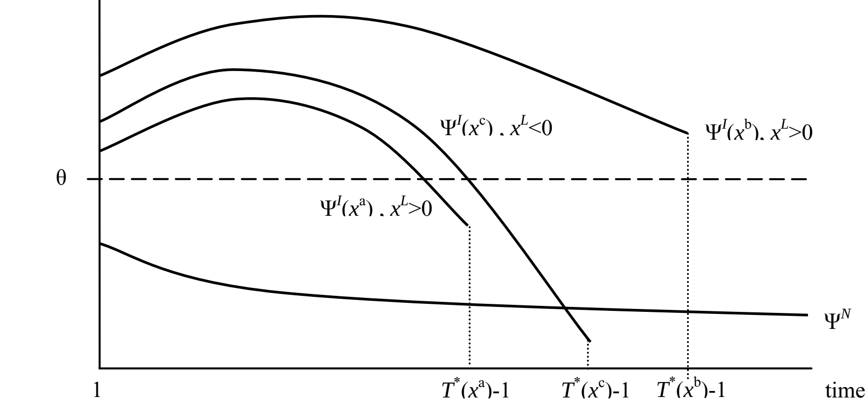


Figure 3. Alternative inflation-paths for δ=0



$$A=(1-\gamma)\big/(1-\gamma-\alpha^{(2)}),\ B=(1-\gamma)\big/(1-\alpha^{(2)}),\ C=(1-\gamma)\big/(1-\gamma-\alpha^{(3)}),\ D=(1-\gamma)\big/(1-\lambda_{t^*-1}\gamma-\alpha^{(3)}),\ E=\hat{\theta},\ A=1\big/(1-\alpha^{(1)})$$

Figure 4. Contingent reforms



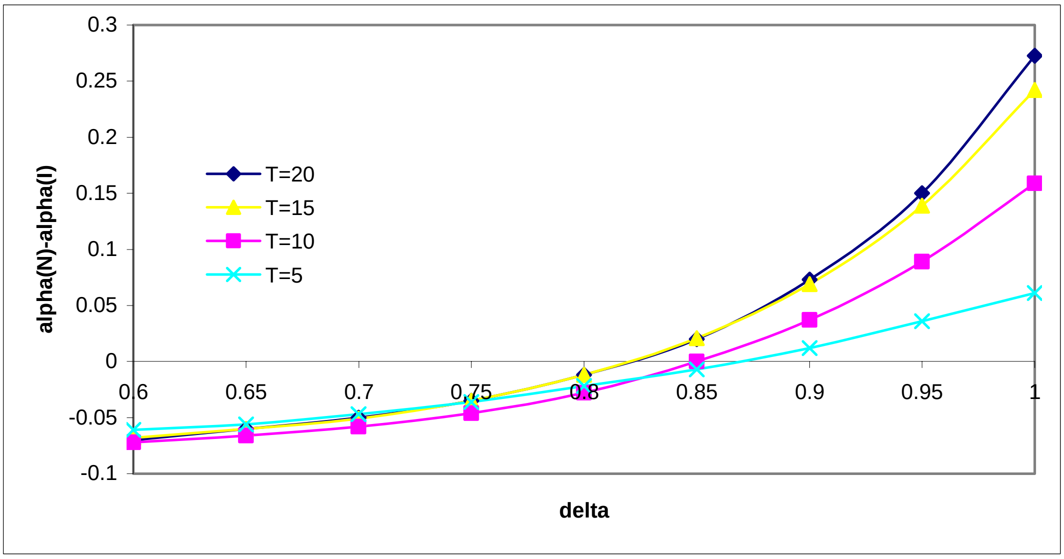


Figure N1: α^N- α^I as a function of δ
Parameter values: $m_0 = 1$, $\theta = 1.35$, $\beta = 0.8$, $x^L = 0.1$, $\gamma = 0.15$

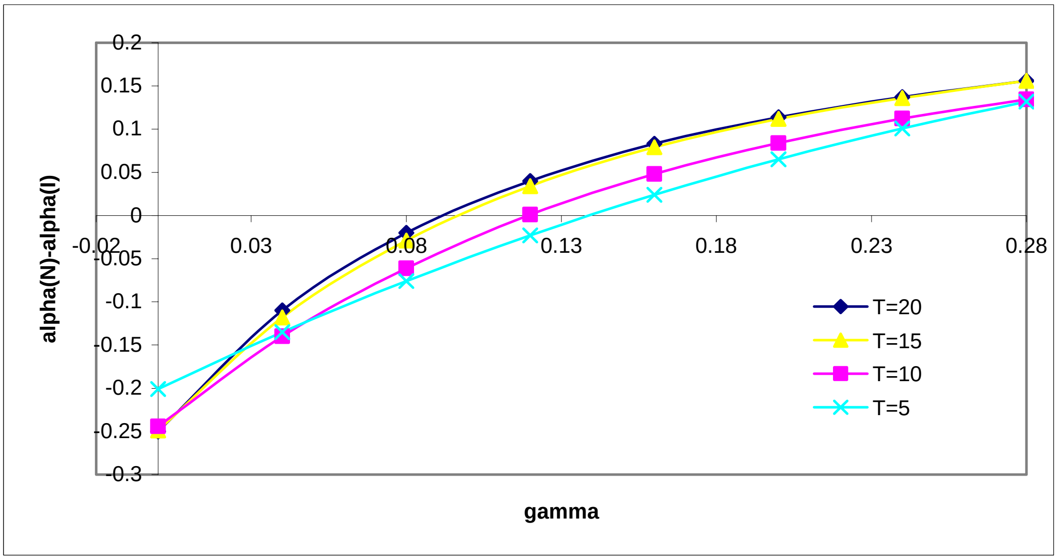


Figure N2: α^N- α^I as a function of γ.
Parameter values: same as in Figure N1 with $\delta = 0.9$

Figure 2. Dynamic Laffer-curve ($\alpha^I < \alpha^N$)

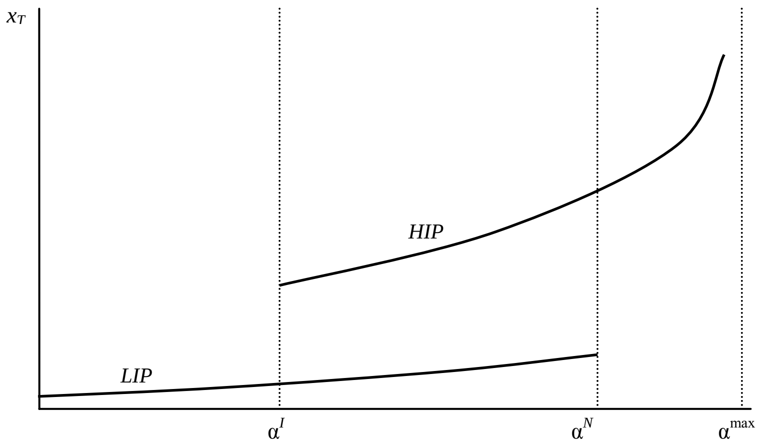
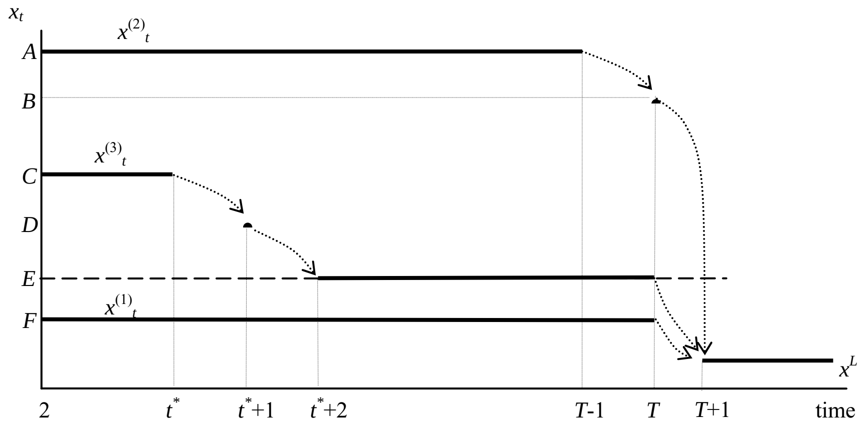
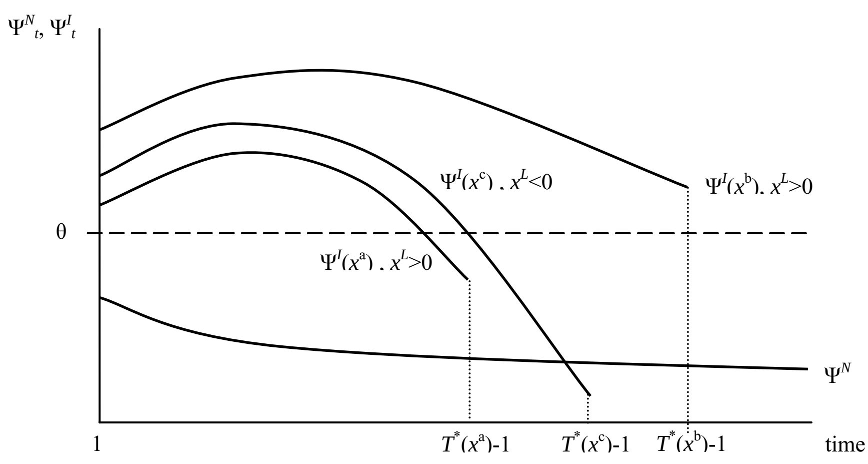


Figure 3. Alternative inflation-paths for δ=0



$A = (1-\gamma)/(1-\gamma-\alpha^{(2)})$, $B = (1-\gamma)/(1-\alpha^{(2)})$, $C = (1-\gamma)/(1-\gamma-\alpha^{(3)})$, $D = (1-\gamma)/(1-\lambda_{t^*-1}\gamma-\alpha^{(3)})$, $E = \hat{\theta}$, $A = 1/(1-\alpha^{(1)})$

Figure 4. Contingent reforms



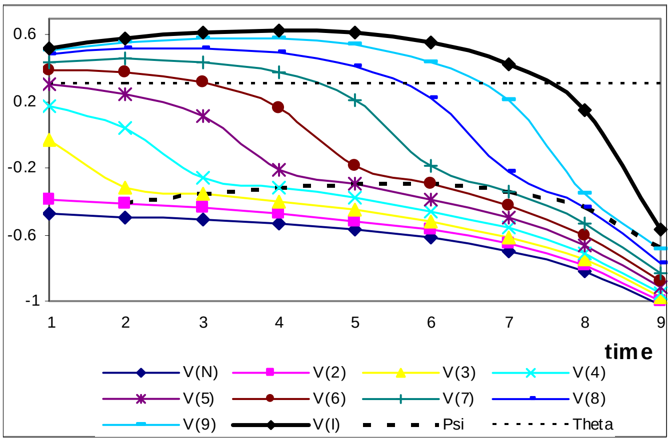


Fig. N5: Shadow-value functions (**α**=0.20)

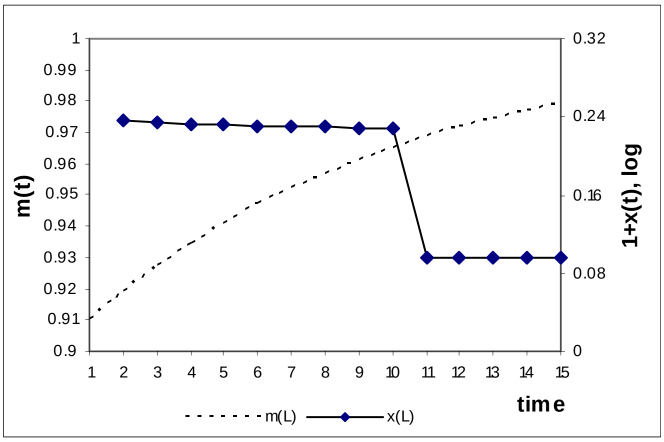


Fig. N5A: LIP-Inflation and real balances

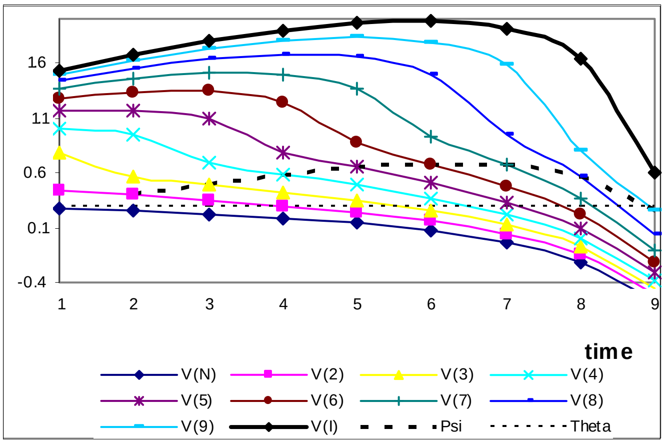


Fig. N6: Shadow-value functions (**α**=0.34)

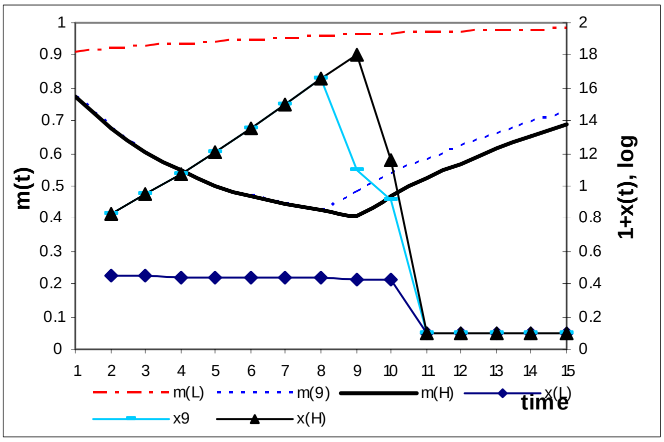


Fig. N6A: Multiple inflation and balances paths

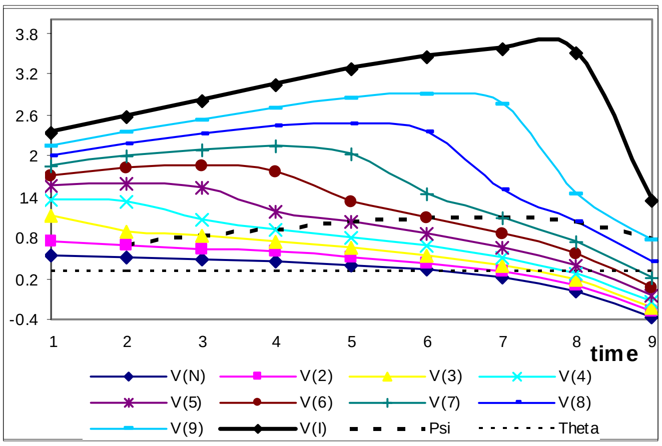


Fig. N7: Shadow-value functions (**α**=0.40)

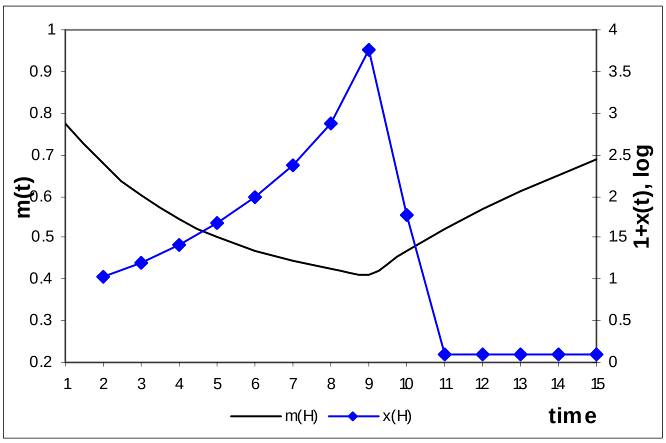


Fig. N7A: HIP-Inflation and real balances

Parameter values (Figs. N5 to N7A): $T = 10$, $m_0 = 0.9$, $\theta = 1.35$, $\beta = 0.8$, $x^L = 0.1$, $\gamma = 0.15$, $\delta = 0.9$. Vertical axis: (natural log of) θ and $V(i)$ which denotes the function Ψ_t associated with a λ-sequence such that $\lambda_t = 1$ for $t = 1,\ldots, i\text{-}1$ and $\lambda_t = 0$ for $t \geq i$. $V(N)$ and $V(I)$ denote, respectively, the shadow-value function associated with the LIP and HIP. *Psi* corresponds to the function $\tilde{\Psi}_t$ defined in the appendix.

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WORKING PAPERS[1]

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