

BANCO DE ESPAÑA

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AN EMPIRICAL ANALYSIS

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María Pérez Jurado and Juan Luis Vega (\*)

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## I. INTRODUCTION AND BACKGROUND

Purchasing power parity (PPP) is an equilibrium condition often assumed in both theoretical and practical economic analysis. Empirical testing of PPP has not, however, provided clear evidence that justifies its broad application -- on the contrary, numerous studies have reached negative conclusions regarding its validity. This paper attempts to reconcile the broad use of purchasing power parity and the empirical evidence, by using more appropriate methods to test the hypothesis, within a multivariate and multi-country setting for Spain, Germany, the United Kingdom, the United States, France and Italy in the period 1970-1992.

In its absolute version, the purchasing power parity theory establishes that the price levels of two countries should be equal when expressed in the same currency. Thus,

$$P = SP^*$$

where S is the nominal exchange rate of the currency of country A expressed in terms of the currency of country B, and P and P\* the price levels of countries A and B, respectively. This version of PPP implies, therefore, that the logarithm of the real exchange rate is constant and equal to zero.

The relative -- and less restrictive -- version of PPP has, however, elicited more attention, since it allows the exchange rate and relative prices to be differentiated by a constant factor K:

$$P = KSP^*$$

whereby the logarithm of the real exchange rate (q) remains constant, but not necessarily equal to zero. In other words, if we denote logarithms with small letters:

$$q = p - s - p^* = k$$

Many attempts have been made to verify this theory empirically because of its decisive role both in theoretical macroeconomic models and in economic policy-making. In more traditional exchange rate models, under which trade flows were understood to be the fundamental determinant, PPP was considered a theory of exchange rate determination. Subsequently, in monetary and portfolio balance models, it has generally played a very important role as an equilibrium condition, although no specific hypotheses have been established regarding the direction of causality. In dynamic exchange rate models, it usually appears as a long-run equilibrium condition. Thus, the justifications behind PPP theory lie in goods market arbitrage and the neutrality of money.

In addition, real exchange rate indices are normally interpreted as yardsticks of the competitiveness of the countries in question and, in many macroeconomic models, the sustainable long-run equilibrium level of the current account balance conforms to a single real exchange rate level. Moreover, in practice, in associating the external balance with a real exchange rate equilibrium level, the PPP exchange rate is very often used as the normative benchmark to judge whether exchange rates are overvalued or undervalued, thereby influencing the design of certain economic policy measures.

In this sense, empirical analysis of purchasing power parity is of prime importance in evaluating to what extent the application of models using this relationship and the reaction of economic policy to deviations from PPP (whatever their size) are justified.

The traditional methodology of empirical PPP analysis tested whether this condition was met in the short run, estimating the equation:

$$s_t = \alpha + \beta (p - p^*)_t + u_t$$

or:

$$\Delta s_t = \gamma + \delta \Delta(p-p^*)_t + u_t$$

and testing the hypothesis:  $H_0: (\alpha, \beta) = (0, 1)$  for absolute PPP and  $H_0: \beta = 1$  or  $H_0: (\gamma, \delta) = (0, 1)$  for the relative version, generally obtaining negative results (see Frenkel, 1981). For the peseta, Reig (1988) estimates both equations with exchange rates vis-à-vis the dollar, the German mark, the British pound and the French franc for the period July 1973-September 1985, using both consumer and wholesale price indices. The empirical evidence is only favourable for the absolute version in two cases -- peseta/dollar using the CPI and peseta/pound using the WPI -- and for the relative version in one case -- peseta/franc using the WPI. But the estimation is imprecise, since there are cases where neither PPP nor the non-significance of relative prices can be rejected<sup>1</sup>. In Reig's paper, other references are cited where conclusions contrary to PPP for the Spanish case are also reached.

A number of studies have taken the alternative route of directly analysing the behaviour of the real exchange rate ( $q$ ) and, more concretely, of testing the hypothesis that this variable follows a random walk. In this area, among the studies frequently cited are Roll (1979) and Adler and Lehmann (1983), who conclude that shocks to the real exchange rate are never reversed and are totally unpredictable.

These results led to a revision of the purchasing power parity theory, whereby an alternative version was established: ex-ante PPP or the market efficiency version, versus the arbitrage version. In this way, by emphasising intertemporal -- rather than spatial -- arbitrage, an explanation is found for the aforementioned findings on the process followed by the real exchange rate -- a random walk -- since:

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<sup>1</sup> An attempt is made to overcome this inefficiency through a multi-country model with four simultaneous equations, under which there is an improvement in the results. However, even if the restriction of equality of relative price ratios is imposed, it remains at 0.641, far from unity.

$$E(\Delta e) = E \left[ \Delta(p - p^*) \right]$$

or:

$$\Delta e = \Delta(p - p^*) + u$$

where  $u$  is an independently distributed error term. This condition, together with uncovered interest rate parity, would imply the equality of real interest rates. In Mishkin (1984) both conditions are jointly tested without obtaining positive results<sup>2</sup>.

An alternative explanation of the negative empirical findings on purchasing power parity found in the literature is that, over time, other fundamental variables cause changes in the equilibrium towards which real exchange rates tend. Thus, Blundell-Wignall and Thomas (1987) show, in the case of the Australian dollar, that whereas the random walk hypothesis cannot be rejected when the alternative is a constant mean reversion, it is rejected when other variables are taken into consideration. Thus, deviations from PPP would be partly predictable due to the information contained in these variables. This alternative would, therefore, require establishing a model for the real exchange rate (beyond the scope of this paper).

There are, however, a number of problems arising in empirical tests of PPP that should be re-examined before it can be definitively accepted that there is no real exchange rate equilibrium level. Several alternatives can be considered in an attempt to reconcile the negative empirical evidence with the broad use of PPP in theoretical models and

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<sup>2</sup> See also Haldane and Pradhan (1992). This version of PPP was in turn reconsidered by Moore (1992), who analyses ex-ante PPP in the general case of risk aversion and shows that, taking the latter into account, the real exchange rate will follow a random walk only under certain conditions. Using consumer price indices for G-7 countries, there is some evidence in favour of this new version of PPP.

its application in studies of international economic trends and competitiveness and even in designing economic policy. First, the methods of measuring price levels generally differ from one country to another, since they incorporate different weightings for different goods. Second, these baskets include non-tradable goods which are not directly affected by the arbitrage mechanism that would otherwise ensure the adjustment to PPP. In this sense, changes in the relative prices of tradables and non-tradables in different countries -- caused, for example, by changes in capital flows, consumer preferences or technology -- would give rise to deviations from PPP.

Third, even in the case of widely tradable goods, there are information and transaction costs and market imperfections that limit spatial arbitrage. By taking advantage of such costs, monopolistic or oligopolistic practices can further weaken the relationship between exchange rates and relative prices. Lastly, even if there were no trade restrictions and the baskets used in price indices were similar, the substitutability of apparently homogeneous goods may not be very great due to differences in quality, after-sales services, etc.

In general, these problems have led to a broad consensus that neither the absolute version nor the short-run relative version of the purchasing power parity theory holds true. I.e. it seems entirely plausible that deviations from PPP observed in reality are strong and persistent in the short run, but should be offset in the long run. Moreover, the lack of consensus as to the most appropriate price index underscores the importance of using several alternative measures, particularly those that take into account manufacturing and industrial sectors which, broadly speaking, better proxy to tradable goods.

As a result, empirical analysis of PPP has tended to be framed in long-run settings and to be based on more powerful tests, since it is reasonably assumed that a distinction must be made between, on the one hand, a situation in which the exchange rate and relative prices tend to converge in the long term but experience very slow adjustment



and strong divergence in the short run, and another in which they do not converge even in the long run<sup>3</sup>.

In this sense, a study of real exchange rate trends that tests whether this variable tends towards an equilibrium level in the long run amounts to testing whether it is stationary around a mean level which, for relative PPP, is not necessarily zero. To this end, recent studies have used tests that check the existence of unit roots in real exchange rate series. But, even though these tests are more appropriate than the traditional ones, the probability of rejecting the null hypothesis when the autoregressive parameter is nearly unity is still very small. Bleaney (1991) runs the augmented Dickey-Fuller and Dickey-Said tests, among others, on the bilateral real exchange rates of five EMS countries in the period 1979-1988, obtaining stationarity only for the French franc/German mark. Taylor (1990) uses the first of these tests and is unable to reject the existence of a unit root for the real exchange rates of the dollar versus five currencies, in the period January 1973-December 1985. Abuaf and Jorion (1990) manage to increase the tests' power by applying Dickey-Fuller tests to a system of autoregressions of real exchange rates, jointly estimated by generalised least squares. Thus, when estimations are made individually, the study is only able to reject the null hypothesis by using a very long sample period (1901-1972) and annual data. But, by jointly estimating and imposing equality of parameters to the different countries, they are able to reject the existence of unit roots both for the previous case and for a monthly frequency in the period 1973-1987; although, as could be expected, the mean reversion is very slow.

In the alternative approach (less restrictive than the previous one) of studying the long-run relationship between the exchange rate and relative prices, cointegration analysis is clearly very appropriate.

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<sup>3</sup> In the aforementioned study by Reig, a partial adjustment model is used to estimate exchange rate elasticities with respect to relative prices in the short and long run. Elasticity in the short run is very low and, in certain cases, is not significant. Elasticity in the long run is greater, but only the case of the peseta versus the D-mark, using wholesale prices, is unable to reject that it is unity.

Numerous studies have recently applied the methodology proposed by Engle and Granger to test the long-run PPP hypothesis. Although many of these have resulted in negative findings, there is some evidence in favour of purchasing power parity. Such is the case of Kim (1990), who uses annual data for the period 1900-1987, coinciding with the above-mentioned results of Abuaf and Jorion. Heri and Theurillat (1990) find that in 50% of the cases studied -- the German mark vis-à-vis 17 countries -- cointegration exists, thus indicating a certain long-run relationship between exchange rates and relative prices; in several of these cases, the parameters that relate both variables are very nearly unity. For the peseta versus the dollar and the German mark, using both consumer and wholesale prices, Ngama and Sosvilla-Rivero (1991) find cointegration only between the Pta/DM exchange rate and the corresponding relative wholesale price. In this case, a test on whether the cointegration vector was in accordance with PPP was rejected although, in line with other results discussed, the parameter is very nearly unity.

This methodology still shows certain weaknesses with respect to the PPP test which can, nonetheless, be overcome. First, it should be borne in mind that none of the relevant variables in the analysis of purchasing power parity can be considered exogenous (Ngama and Sosvilla-Rivera find evidence of simultaneity). Second, the simultaneous determination of these variables in the different countries should also be taken into account. In this sense, as set forth in section 4 of this paper, the methodology proposed by Johansen (1988) is particularly appropriate for checking the existence of cointegration relationships in a multivariate setting.

The paper is structured as follows: Section 2 discusses the data used for the empirical test of PPP and presents the findings of the unit root tests for exchange rates and relative prices. These results determine the order of integration of these variables, a prerequisite for applying Johansen's methodology, which is presented in section 4 along with the results obtained. As a preliminary step, findings based on a univariate methodology are presented in section 3, thus providing a point of comparison for the results based on multivariate methods.

Conclusions are given in the final section. Lastly, the paper includes two appendices. The first exhibits the tables with the results of the unit root tests, and the second explains in greater detail the multivariate methodology used.

## II. DATA AND UNIT ROOT TEST

As noted above, given the lack of consensus on the most appropriate price index, a range of indices was used, including consumer and industrial prices, unit labour costs in the manufacturing sector and export prices. In this way, we took into account the indices most frequently used both in the literature on PPP and in analyses of international economic trends and competitiveness. The countries studied are Spain and its five main competitors: the United States, Germany, the United Kingdom, France and Italy. The data bank of the Research Department of the Banco de España was the source for information related to Spain and exchange rates, and all other data were drawn from the OECD's "Main Economic Indicators". Since the study is framed in a long-run setting, a sufficiently long time span was used, i.e. quarterly data for 1970:I-1992:IV, with the exception of unit labour costs, where a similar sample was not available and the period used was 1972:I-1992:IV<sup>4</sup>.

Before applying the methodology presented in section 4, tests must be run on unit roots in order to determine whether the exchange rate and relative price variables, whose cointegration is to be studied, are integrated of the same order<sup>5</sup>.

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<sup>4</sup> Since data prior to 1976 are not available for Italian industrial prices, figures were estimated on the basis of wholesale price trends. In the case of pre-1974 data for Spanish wholesale prices, a similar estimate was made.

<sup>5</sup> Instead of considering the variables  $P$  and  $P^*$  separately, relative prices are used in order to avoid analysing variables integrated of order two,  $I(2)$ .

For this reason, the procedure proposed by Phillips-Perron (1988) was used, since it is more robust than other unit root tests in that it allows residuals to follow a fairly general process by employing a non-parametric correction of the hypothesis tests and, therefore, does not reduce the effective number of observations. This procedure begins with the estimation of the alternative models:

$$Y_t = \tilde{\mu} + \tilde{\beta} \left( t - \frac{T}{2} \right) + \tilde{\alpha} Y_{t-1} + \tilde{\varepsilon}_t \quad \text{Modelo 3}$$

$$Y_t = \mu^* + \alpha^* Y_{t-1} + \varepsilon_t^* \quad \text{Modelo 2}$$

$$Y_t = \hat{\alpha} Y_{t-1} + \hat{\varepsilon}_t \quad \text{Modelo 1}$$

where T is the sample size.

A series of tests is then run on the estimations of the three models in order to ascertain which one is appropriate and to determine whether this model has a unit root. The tests<sup>6</sup> used are outlined in Table 1.

The inference on the existence of a unit root should begin by using the statistics derived from model 3. The reason is that the statistics from model 2 are unable to distinguish a stationary process around a trend from a process with a unit root. Moreover, on the basis

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<sup>6</sup> For a detailed description of the tests and the testing strategy, see Perron (1988).

Table 1

MODEL 3		MODEL 2		MODEL 1	
TEST	HYPOTHESIS	TEST	HYPOTHESIS	TEST	HYPOTHESIS
$Z(\phi_3)$	$H_0: (\tilde{\mu}, \tilde{\beta}, \tilde{\alpha}) = (\tilde{\mu}, 0, 1)$	$Z(\phi_1)$	$H_0: (\mu^*, \alpha^*) = (0, 1)$	$Z(t_{\tilde{\alpha}})$	$H_0: \tilde{\alpha} = 1$
$Z(\phi_2)$	$H_0: (\tilde{\mu}, \tilde{\beta}, \tilde{\alpha}) = (0, 0, 1)$	$Z(t_{\alpha^*})$	$H_0: \alpha^* = 1$		
$Z(t_{\tilde{\alpha}})$	$H_0: \tilde{\alpha} = 1$	$Z(t_{\mu^*})$	$H_0: \mu^* = 0$		
$Z(t_{\tilde{\beta}})$	$H_0: \tilde{\beta} = 0$				

of model 2, the null hypothesis (existence of a unit root) is not likely to be rejected if the series is stationary around a linear trend, and this becomes impossible when the size of the sample increases. Likewise, model 1 is inappropriate if stationarity around a mean other than zero is a plausible alternative.

Therefore, the most general model is the starting point, jointly using the tests  $Z(\phi_3)$ ,  $Z(\phi_2)$  and  $Z(t_{\tilde{\beta}})$ , to determine whether model 3 is the most appropriate -- the use of  $Z(\phi_3)$ ,  $Z(\phi_2)$  is crucial, since the asymptotic distribution of  $Z(t_{\tilde{\alpha}})$  is not invariate with respect to  $\beta$  under the null hypothesis, and that of  $Z(t_{\tilde{\beta}})$  is not invariate with respect to  $\mu$ . If the result is affirmative,  $Z(t_{\tilde{\alpha}})$  will be used to test the existence of a unit root, and there will be no need to continue analysing the tests derived from models 2 and 1. In the event of a negative result,  $Z(\phi_1)$  and  $Z(t_{\mu^*})$  will be considered jointly to determine whether model 2 is the most appropriate. If this proves to be the case,  $Z(t_{\alpha^*})$  will indicate whether the unit root is rejected and, if it is not,  $Z(t_{\tilde{\alpha}})$  will be used.

The results of the tests both on exchange rates and relative prices, in logarithms and using the dollar as the numeraire, are presented in

Tables 1 and 2 of Appendix I<sup>7</sup>. The numbers of the relative prices and exchange rates indicate the following order: Germany, United Kingdom, France, Italy and Spain. In Table A.2, the tests are calculated for the series in first differences, i.e. a test is run on the null hypothesis that the variables are integrated of order two, with integration of order one being the alternative. Table A.1 presents the results for the null hypothesis  $H_0: I(1)$  with respect to  $H_1: I(0)$ .

In line with the above-mentioned strategy, we have presented only those statistics that were necessary for reaching a conclusion. In the case of consumer prices (see Table A.1), for example, since  $Z(\phi_2)$  is clearly significant, the test was run on model 3. Thus, there is no need to take the tests on models 2 and 1 into account, and they are not shown in the tables.

As seen in Table A.2, the existence of two unit roots is rejected at a level of significance of 1% in all cases, except for relative consumer prices in Germany, where rejection occurs at 5%.

With respect to the existence of one unit root, in no case can it be rejected that the variables are  $I(1)$ . As shown in Table A.1, in the case of exchange rates, relative prices, industrial prices for France, export prices for the United Kingdom and unit labour costs for Germany, all statistics must be taken into consideration until reaching model 1, and the existence of a unit root cannot be rejected since  $Z(t_{\hat{\alpha}})$  is not significant at a level of confidence of 95%<sup>8</sup>. For all other relative export prices and for German industrial prices, model 2 is sufficient to conclude that they have a unit root, since  $Z(\phi_1)$  is significant. Finally, for the rest of the variables the existence of a unit root cannot be rejected, and only model 3 must be taken into account, given the clear significance of  $Z(\phi_2)$ .

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<sup>7</sup> For relative prices, seasonal dummy variables were included in the aforementioned models except in the case of unit labour costs, whose original series are already seasonally adjusted.

<sup>8</sup> The level of confidence need only be fixed at 99% to prevent rejection of the unit root in the case of relative export prices in the United Kingdom.

In general, the results concur with those of other studies (see, for example, Ngama and Sosvilla-Rivero, 1990 and Heri and Theurillat, 1989) on PPP and with the findings in the many studies of foreign exchange market efficiency.

### III. UNIVARIATE EVIDENCE OF PPP

As a preliminary step, this section summarises the results of the application of unit root tests on both bilateral and multilateral real exchange rates, thus providing a point of comparison for the results of the multivariate analysis.

#### Univariate analysis of bilateral PPP

As noted in the introduction, purchasing power parity between two countries implies that the corresponding bilateral real exchange rate is constant. Therefore, if the purpose is to test whether PPP holds in the long run, the equivalent condition is the stationarity of the real exchange rate. Taking this variable in logarithms, it should be stationary around a null mean if absolute PPP holds, and, in the case of the relative version, this mean can take any value. In contrast, if the real exchange rate follows a process that incorporates a unit root, deviations from PPP are accumulated over time, and, consequently, this condition would not tend to hold in the long run.

To test the null hypothesis of the existence of a unit root with respect to the alternatives of stationarity around a constant, whether or not it is zero, the Phillips-Perron test was run, using models 1 and 2 (with seasonal dummies) and following the strategy summarised in the previous section. Results are presented in Tables A.3 and A.4. The augmented Dickey-Fuller test (see table A.5) was also run, in those cases where the Phillips-Perron rejects the null hypothesis of non-stationarity. This was done because, as demonstrated in Perron (1987), the possible existence of moving average components with negative parameters and

high absolute values would significantly distort the size of the unit root tests (especially that of Phillips-Perron) .

The results are summarised in Table 2:

**Table 2**

UNIVARIANTE ANALYSIS. STATIONARY BILATERAL REAL EXCHANGE RATES		
	PHILLIPS-PERRON	AUGMENTED DICKEY-FULLER
CPI	None	None
WPI	DM/FF **	
	DM/LIT *	DM/LIT **
	DM/PTA *	DM/PTA **
	FF/LIT **	FF/LIT **
	FF/PTA *	
XPI	FF/PTA **	FF/PTA **
ULC	DM/LIT **	
	FF/LIT ***	FF/LIT *
	LIT/PTA **	LIT/PTA *

Thus, univariate evidence suggests that bilateral PPP does not hold in the long run. Even using industrial price indices, which provide the largest number of stationary real exchange rates, PPP is only verified in three of the 15 cases, if we consider those cases where the findings of the two tests coincide. Furthermore, the results pose several problems of inconsistency. Any exchange rate that can be obtained through a linear combination of the three cases mentioned should also be accepted as stationary. By way of example, we can cite DM/LIT and DM/PTA, both of which are stationary; however, according to these results, the LIT/PTA exchange rate, which can be obtained as a linear combination of the previous cases, is not stationary.

Lastly, it should be noted that, when stationarity is obtained, the estimations reflect that mean reversion occurs very slowly and that this



mean is significantly different from zero. Thus, for these cases, relative long-run PPP is accepted, although its absolute version is not.

### Univariate analysis of multilateral PPP

In the terminology of Nessén (1992), multilateral PPP is equal to the stationarity of the effective real exchange rate of a country with respect to the rest of the countries considered. This exchange rate is a weighted average of the corresponding bilateral rates, and there are several alternative methods of calculating these weights.

This paper takes into account the double weighting system currently used by the Banco de España and a number of international organisations, such as the IMF, the EC and the OECD. Compared with the version based solely on bilateral trade, this system has the advantage of considering not only the relative importance of each market in the country's trade but also the relative importance of third countries in each market. It takes into account, for example, that Spanish exports compete with French products not only in France -- with France's domestic supply -- but also in third markets where both Spanish and French exports are sold, even in markets not included in the group of countries studied but where the two compete between themselves (see Navascués, 1988)<sup>9</sup>. The resulting weightings are presented in Table 3.

Before discussing the results, it is worth noting that only five of the six effective real exchange rates are linearly independent; at most, four of the five multilateral rates can be stationary, since results gave no stationary bilateral real exchange rates except in a few specific cases.

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<sup>9</sup> The consideration of these markets is particularly relevant, since the structure of the weightings was calculated for a group of countries chosen for their importance in Spanish trade, but not in the trade of the others.

Table 3

EFFECTIVE EXCHANGE RATE INDICES: WEIGHTING VECTORS						
	U.S	Germany	U.K	France	Italy	Spain
U.S	-	.3004	.3612	.2011	.2382	.1868
Spain	.0509	.0522	.0411	.0731	.0382	-
Germany	.3496	-	.2876	.3508	.3428	.2988
U.K	.2656	.1709	-	.1663	.1214	.1239
France	.1802	.2778	.1802	-	.2594	.2733
Italy	.1537	.1987	.1299	.2087	-	.1172
	1.000	1.000	1.000	1.000	1.000	1.000

Source: Banco de España. Oficina de Estadística, Unidad del Sector Exterior.

However, as can be deduced from Table A.6, the existence of a unit root is only rejected in the following cases: the effective real exchange rate of the French franc using industrial prices and unit labour costs, at 90%, and that of the Italian lira using unit labour costs, at 99%. Moreover, the stationarity of these variables is not supported by the augmented Dickey-Fuller test.

In general, this evidence against PPP concurs with the findings of other studies discussed above that use this methodology to test purchasing power parity. Nonetheless, as shown in Banerjee et al. (1992), the tests developed in this section would impose a common factor restriction, which may not be valid. The result could be a loss of relative power with respect to other tests, such as that developed in the next section, which do not impose such a restriction.

#### IV. MULTIVARIATE EVIDENCE OF PPP

This section applies the procedure proposed by Johansen (1988) to a vector of variables  $X_t = (s_t^1, s_t^2, s_t^3, s_t^4, s_t^5, p_t^1, p_t^2, p_t^3, p_t^4, p_t^5)$ , consisting of five bilateral exchange rates vis-à-vis the dollar (mark, pound, franc, lira and peseta) and their corresponding five relative prices. The analysis is repeated for the four alternative definitions of price indices.

Appendix II develops the technical details of the procedure and the tests used in this section, which we will only briefly outline here to allow a fluid reading of the paper. Basically, the procedure permits the maximum likelihood estimation of  $r$  cointegrating vectors between  $n$  integrated variables of order one,  $I(1)$ , which can be represented by a finite multivariate model  $VAR(k)$ . When cointegration exists, the  $VAR(k)$  model can be reparametrised in the form of an error correction mechanism model (ECM):

$$\Delta x_t = \sum_{i=1}^{k-1} A_i^* \Delta x_{t-i} - \Pi x_{t-k} + \mu + \epsilon_t \quad (1)$$

and the hypothesis of the existence of  $r$  cointegrating vectors can be formulated as the hypothesis of reduced rank of the matrix  $\Pi$ :

$$H_0 : \Pi = \alpha\beta', \text{ where } \alpha \text{ and } \beta \text{ are } n \times r \text{ matrices} \quad (2)$$

where the  $\beta$  columns are the cointegrating vectors and the  $\alpha$  rows the weights with which each cointegrating vector enters a given equation.

Johansen (1988) solves the problem of the maximum likelihood estimation of the model formed by equations (1) and (2) and proposes a likelihood ratio test for the existence of  $r$  cointegrating vectors. More concretely, this paper uses the "trace test", where the null hypothesis

$H_0: r \leq r_1$  is tested against the alternative  $H_a: r = n$ . The resulting statistic does not have a standard distribution, and the critical values of the test are tabulated in Osterwald-Lenum (1992), depending on whether or not there is a constant term in (1) and how this term enters the model. We will call T the trace test when  $\mu \neq \alpha\beta_0$  in (1) and  $T^*$  when  $\mu = \alpha\beta_0$ . The latter two hypotheses are associated, respectively, with the existence or not of linear trends in the non-stationary variables  $x_t$ .

One difficulty of the procedure stems from the fact that it does not allow a unique estimate of  $\beta$ , i.e.  $\beta$  is not identified in the econometric sense. This can be easily seen by defining any non-singular matrix  $w$  of the order  $r \times r$  so that in (2):

$$\Pi = \alpha\beta' = \alpha w'^{-1} w'\beta' = \alpha^*\beta^* \quad (3)$$

In other words, the problem lies in the fact that any linear combination of stationary vectors is also stationary. Thus, the procedure does not guarantee that the estimators  $\hat{\beta} = (\hat{\beta}_1, \dots, \hat{\beta}_r)$  correspond to the relationships of interest<sup>10</sup>. In fact, what is estimated (except in the trivial case of  $r=1$ ) is a base of the vectoral subspace generated by the  $\beta$  columns, and, consequently, tests on several structural hypotheses must be run in order to identify  $\beta$  and ensure that it makes economic sense.

This paper uses one of the mentioned tests that restricts a subset of  $r_1$  cointegrating vectors, freely estimating the remaining  $r_2 = r - r_1$ <sup>11</sup>. The derivation of this test is found in Johansen and Juselius (1991), and its distribution is  $\chi^2$  with  $(n-s-r)r$  degrees of freedom, where  $n-s$  is the number of restrictions imposed.

Model (1) was estimated for the different price indices and for  $k=2$ , including seasonal dummies. This allows us to obtain non-autocorrelated

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<sup>10</sup> For a discussion of this aspect, see Johansen and Juselius (1992).

<sup>11</sup> Concretely, this involves testing hypotheses of the type:  $H_0: \beta(H\phi, \psi)$ , where  $H$  is a known  $n \times s$  matrix.

residuals in (1), although the Bera-Jarque statistics detect problems of non-normality in several of the exchange rate equations, most notably in the case of the peseta, the pound and the lira. These problems are generally explained by the presence in the sample of a number of abrupt movements in several of the exchange rates, often associated with devaluations of the currencies but whose treatment is not obvious in the setting developed here. We decided to maintain the VAR model with no intervention, even though the problem of the non-normality of the residuals could be solved through the ad-hoc inclusion of dummy variables which take into account several of the most important devaluations.

Tables 4.A and 4.B exhibit the trace tests on the number of cointegrating vectors ( $r$ ) for the models estimated with different price indices and in the cases where there is a restricted ( $T^*$ ) and unrestricted ( $T$ ) constant.

According to Johansen (1992), both tables must be read from top to bottom and from left to right, stopping at the first non-rejection of the null hypothesis. Under this criterion, we are unable to reject the hypotheses of five cointegrating vectors in the VAR models with export prices and unit labour costs, six in the case of consumer prices and nine in the case of industrial prices. Furthermore, in all cases, the presence of a restricted constant in the model would be accepted, i.e. the absence of linear trends in exchange rate and relative price data. The estimated cointegrating vectors are not reported since, as noted above, they do not have a direct structural interpretation. Nonetheless, in the rest of this section we will analyse, in the same multivariate setting, the empirical evidence for several economic hypotheses of interest.

In the first place, a test is run on whether bilateral real exchange rates between the six countries are stationary, i.e. bilateral PPP following the terminology proposed by Nessén (1992). These tests, conditioned by the previous choices of  $r$ , are presented in Tables 5.A and 5.B. Rejections of the null hypotheses at 1%, 5% and 10% levels of significance are indicated by three, two and one asterisks, respectively.

Table 4.A

n-r	r	Consumer prices		Industrial prices	
		T*	T	T*	T
10	0	371.0*	353.3*	412.8*	395.2*
9	1	274.8*	265.6*	306.7*	293.1*
8	2	212.6*	204.0*	228.0*	216.1*
7	3	161.1*	152.6*	171.7*	160.8*
6	4	118.6*	111.7*	121.7*	117.4*
5	5	78.5*	71.6*	85.8*	81.6*
4	6	50.5	44.0	58.1*	54.1*
3	7	30.1	23.7	38.1*	35.1*
2	8	15.5	9.7	20.8*	17.9*
1	9	5.4	4.3	8.3	6.4*

Table 4.B

n-r	r	Export prices		Unit labour costs unitarios	
		T*	T	T*	T
10	0	347.3*	330.5*	351.4*	320.8*
9	1	261.9*	254.8*	267.4*	243.8*
8	2	199.4*	192.5*	197.1*	180.3*
7	3	145.1*	138.4*	149.5*	136.3*
6	4	102.8*	99.7*	105.7*	95.9*
5	5	67.8	64.7	73.8	65.9
4	6	40.2	37.4	46.9	40.6
3	7	22.6	20.2	28.9	25.5
2	8	14.6	9.1	13.9	12.6
1	9	5.2	0.1	6.6	5.4

Table 5.A

Bilateral PPP with consumer prices (upper matrix, $\chi_4^2$ ) and with industrial prices (lower triangular matrix, $\chi_1^2$ ).						
	U.S	Spain	Germany	U.K	France	Italy
U.S	-	12.72**	16.12***	12.98**	12.91**	13.07**
Spain	0.43	-	16.06***	12.26**	16.19***	14.36***
Germany	1.22	2.30	-	14.46***	13.03**	8.13*
U.K	0.33	3.75*	4.04**	-	10.82**	13.47***
France	2.59	2.02	0.32	2.86*	-	10.83**
Italy	1.94	4.15**	1.39	4.27**	0.04	-

Table 5.B

Bilateral PPP with export prices (upper triangular matrix, $\chi_5^2$ ) and with unit labour costs (lower triangular matrix, $\chi_5^2$ ).						
	U.S	Spain	Germany	U.K	France	Italy
U.S	-	14.09***	15.73***	18.02***	10.53*	19.17***
Spain	13.17**	-	28.02***	20.00***	26.46***	23.81***
Germany	16.48***	13.03**	-	9.92*	26.43***	12.00**
U.	14.01***	8.61	15.75***	-	9.61*	20.28***
France	14.48***	10.70*	20.80***	3.08	-	18.41***
Italy	13.52***	15.23***	10.55*	14.31***	16.62***	-

For two given countries, this hypothesis is tested by imposing restrictions on  $r_1 = 1$  of the cointegrating vectors and freely estimating the remaining  $r_2 = r - 1$ . Thus, to test that real peseta-dollar and peseta-mark exchange rates are stationary, we will formulate the hypotheses of whether the vector  $\beta_1 = (0, 0, 0, 0, 1, 0, 0, 0, 0, -1)'$  or whether  $\beta_2 = (-1, 0, 0, 0, 1, 1, 0, 0, 0, -1)'$  belong to the cointegration space, respectively. In general, the bilateral PPP hypothesis imposes nine restrictions on one of the cointegrating vectors, whereby the distribution of the statistics contained in Table 5.A and 5.B will be  $\chi_4^2$  in the case of consumer prices,  $\chi_1^2$  for industrial prices and  $\chi_5^2$  for export prices and unit labour costs.

Even though multiple cointegration relationships were found between the different exchange rates and relative prices, the restrictions imposed by bilateral PPP -- i.e. the stationarity of the bilateral real exchange rates -- are rejected in all cases when consumer and export prices are used, and in most cases when unit labour costs are used. Non-rejection in the majority of cases is only possible when industrial prices are used, although there would still be several problems, which seem to centre on the case of the United Kingdom vis-à-vis the rest of the countries except for the United States.

Also, when the simultaneous presence in the cointegration space of five bilateral real exchange rates -- the maximum number of linearly independent rates -- is tested<sup>12</sup>, cointegration is again strongly rejected when consumer prices (  $\chi_{20}^2 = 78.07$  ), export prices (  $\chi_{25}^2 = 103.8$  ) and unit labour costs (  $\chi_{25}^2 = 88.94$  ) are used, but only marginally when industrial prices are used (  $\chi_5^2 = 12.25$  ).

To summarise, the evidence strongly rejects the bilateral PPP hypothesis in the case of consumer prices, export prices and unit labour costs. In contrast, a reading of Tables 5.A and 5.B tends not to reject bilateral PPP when industrial prices are used. In the latter case, several problems in the interpretation of the results persist, since the non-

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<sup>12</sup> This hypothesis imposes five restrictions on  $r_1 = 5$  of the cointegrating vectors, leaving the remaining  $r - 5$  free.



rejection of the stationarity hypothesis for five bilateral real exchange rates, in the event that these are linearly independent, should, in theory, signify non-rejection for the rest, given that the latter could be expressed as a linear combination of the previous ones. Thus, the rejection of the stationarity of the bilateral real pound-mark exchange rate is contradicted by the non-rejection in the case of real dollar-pound and dollar-mark exchange rates. Nevertheless the rejections are always marginal, favouring the interpretation of non-rejection of bilateral PPP when industrial prices are used. Additional evidence presented below supports this interpretation.

The multilateral PPP hypothesis is also tested, under the terms proposed by Nessén (1992), i.e. whether the effective real exchange rate for a given country is stationary with respect to the other five countries. Thus, for example, in the case of the United States, it will be tested whether the vector  $\beta_1 = (.3496, .2656, .1802, .1537, .0509, -.3496, -.2656, -.1802, -.1537, -.0509)$  belongs to the cointegration space, and, for the case of Spain, the vector  $\beta_2 = (-.2988, -.1239, -.2733, -.1172, 1, .2988, .1239, .2733, .1172, -1)$  will be considered. The distribution of the resulting statistics is  $\chi_4^2$  in the case of consumer prices,  $\chi_1^2$  for industrial prices and  $\chi_5^2$  for export prices and unit labour costs.

Before analysing the results obtained, several points should be noted. On the one hand, only five of the six possible effective real exchange rates are linearly independent, since only five bilateral real exchange rates are linearly independent. Nonetheless, tests are run on the stationarity of all six in order to offer as much empirical evidence as possible and to allow its comparison with the findings of studies not cast in a multi-country setting. On the other hand, these tests are not independent of the previous ones on the stationarity of bilateral exchange rates. Therefore, if the stationarity of the latter has not been rejected, then it should trivially follow that any index constructed as a linear combination of the latter (regardless of the weightings used) is stationary. This would be the case for indices constructed with industrial prices. In contrast, in cases where it is rejected that bilateral real exchange rates are  $I(0)$ , stationary effective real exchange rates can only be obtained if there is cointegration between the first ones. In this case,

we would have a maximum of four stationary effective real exchange rates, although the economic interpretation of this hypothesis is unclear.

Table 6 presents the results of testing multilateral PPP. In each of the rows, the hypothesis to be tested is whether the effective real exchange rate of a given country with respect to the other five is stationary.

The empirical evidence in this table rejects stationarity in the case of indices constructed with consumer prices, export prices and unit labour costs. In contrast, the stationarity of effective real exchange rates measured in terms of industrial prices is not rejected, as could be expected in view of the findings of the bilateral PPP tests<sup>13</sup>. Lastly, to

**Table 6**

<b>MULTILATERAL PPP</b>				
	consumer prices ( $\chi_4^2$ )	industrial prices ( $\chi_1^2$ )	export prices ( $\chi_5^2$ )	unit labour costs ( $\chi_5^2$ )
<b>U.S</b>	14.90 <sup>***</sup>	1.15	16.49 <sup>***</sup>	14.71 <sup>**</sup>
<b>Spain</b>	14.00 <sup>***</sup>	1.05	23.47 <sup>***</sup>	12.12 <sup>**</sup>
<b>Germany</b>	14.84 <sup>***</sup>	0.50	15.11 <sup>***</sup>	21.64 <sup>***</sup>
<b>U.K</b>	11.66 <sup>**</sup>	2.46	17.00 <sup>***</sup>	12.97 <sup>**</sup>
<b>France</b>	11.53 <sup>**</sup>	1.78	9.39 <sup>*</sup>	15.99 <sup>***</sup>
<b>Italy</b>	10.47 <sup>**</sup>	2.89	20.61 <sup>***</sup>	9.44 <sup>*</sup>

<sup>13</sup> The results with respect to multilateral PPP are sounder for the Spanish case than for the other five countries in the study, since the weightings depend on the group that was chosen from the viewpoint of Spain. However, as noted, the double weighting system minimises this problem.

give further support to the conclusion in favour of PPP using wholesale prices, it is worth pointing out that the joint presence of five effective real exchange rates -- notice that the sixth would be redundant -- in the cointegration space is not rejected ( $\chi_{10}^2 = 14.93$ ).

Let us briefly recap on the methodology employed in this section and the results obtained. To test the PPP hypothesis, measured in terms of different price indices and in a multivariate and multi-country setting, we used the maximum likelihood estimation procedure proposed in Johansen (1988). Under this methodology, several problems that have traditionally arisen in empirical PPP tests can be avoided. First, a coherent statistical model is formulated which takes into account the properties of the data (in particular, the non-stationarity of exchange rates and relative prices) and investigates the existence of cointegration capable of being interpreted as long-run equilibrium relationships, this being the framework in which PPP should be understood. Second, it avoids using untested assumptions regarding the exogeneity of prices or exchange rates and the direction of the causal relationship between the two. Lastly, in a multi-country setting, cross relationships between the different bilateral exchange rates can also be taken into account. Thus, for example, this methodology would encompass situations in which the mark-dollar and the franc-dollar exchange rate may not be stationary, but where cointegration may be such that the franc-mark exchange rate would be stationary.

On this basis, the empirical evidence analysed here rejects the PPP hypothesis in both its bilateral and multilateral versions, when indices of consumer prices, export prices or unit labour costs are used. It is unable to reject PPP only when industrial prices are used. This can be interpreted as evidence supporting PPP as a long-run equilibrium condition solely in the tradable goods sector, where the industrial price index is a good approximation of prices<sup>14</sup>.

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<sup>14</sup> Regarding export prices, the methodological problems associated with the construction of unit value indices seem to be the most reasonable explanation for the results obtained. However, it was not the purpose of this paper to analyse the characteristics of the different price indices in order to determine their greater or lesser appropriateness to PPP, but

## V. CONCLUSIONS

The existence of an equilibrium real exchange rate is an assumption often used in both theoretical and practical economic analysis. On the one hand, several theoretical macroeconomic models assume purchasing power parity as an equilibrium condition, at least in the long run. On the other, studies of international economic trends and competitiveness frequently judge whether a currency is overvalued or undervalued on the basis of the deviation of its real exchange rate from a supposed equilibrium -- hence assuming that PPP holds true, with the resulting impact on the design and evaluation of certain economic policy measures. Finally, from the perspective of the European Monetary System, it is generally accepted that one of the advantages for a smaller member country, for which foreign prices can be considered as given, is that its prices can be adjusted to those of countries with the lowest price levels, either by fixing its exchange rate to such currencies or by adjusting it less than indicated by relative price trends. This implicitly assumes that whatever the adjustment mechanisms are, they will act in such a way that PPP conditions will be satisfied in the long run, and, therefore, the maintenance of the exchange rate fluctuation band forces adjustments to be reflected in prices.

The empirical testing of purchasing power parity has not, however, resulted in a clear evidence that justifies its broad application, while, in contrast, numerous studies have reached negative conclusions regarding its validity. Due to this, research has taken a number of different routes. On the one hand, attempts have been made to reformulate PPP in order to justify the absence of real exchange rate equilibrium. On the other hand, the existence of a variable equilibrium has been studied in terms of other economic variables.

A third alternative route has sought to improve the methodology applied to empirical PPP research by resolving past shortcomings and

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rather it started from the basis that these are the most frequently used indices in drawing certain conclusions which, in the light of the results, may have to be re-examined.

assuming that, at best, this condition will be met in relative terms in the long run, but will have strong and persistent deviations in the short run. This translates into a need for more appropriate tests capable of distinguishing between a very slow adjustment process towards equilibrium and another in which there is no equilibrium even in the long run.

The latter route was used in this paper in an attempt to reconcile the broad use of purchasing power parity and the empirical evidence. Under the methodology used, we were able to frame the test adequately in a long-term setting; to avoid choosing a priori between the absolute and relative versions of the PPP doctrine; to make no assumptions regarding the direction of causality, considering the possible simultaneous determination of exchange rates and relative prices, and, finally, to consider the issue within a multi-country setting, taking into account the simultaneous determination of these variables in the countries in question.

Thus, this paper presents evidence about the validity of purchasing power parity in the long run for Spain and its five main competitors, taking into account the price indices most frequently used both in the empirical literature on PPP and in studies of international economic trends. To this end, we first applied a univariate methodology, which led to ambiguous results, and we then employed the methodology proposed by Johansen, especially designed for cointegration analysis in a multivariate framework, which led to much more conclusive results.

Purchasing power parity is shown not to hold when indices of consumer prices, export prices and unit labour costs are used. However, there is evidence that in the long run industrial prices in Spain, Italy, France, the United Kingdom, Germany and the United States, expressed in the same currency, tend to converge in such a way that the bilateral and multilateral real exchange rates of these countries follow processes that tend towards a constant long-run equilibrium. As expected, significant deviations from PPP in the short run and a slow adjustment process are observed. These deviations may be due to the slowness of arbitrage related to imperfections in goods markets and may also be partly

explained by exchange rate targets that pursue price discipline precisely through such deviations. In any event, the slowness of the adjustment process has important implications for economic policy, since it indicates that misalignments caused by nominal shocks can have strong and persistent effects (costs) in real terms.

However, even though long-run PPP was found, it is difficult to conclude whether this is the result of the afore mentioned effect on prices or whether, despite the establishment of an exchange rate target, this target is altered in the long run because prices fail to adjust. This distinction is decisive, since the interpretation of the results using industrial prices would be radically different in one case or another -- for example, in deciding whether to believe that exchange rate agreements can truly foster adjustment mechanisms aimed at nominal convergence. In this sense, it would be interesting to continue this line of investigation by studying the dynamics and causal relationships in the short run, once long-run equilibrium is found to exist.

These findings do, however, seem to lead to one conclusion for a country of Spain's size and structure, which takes foreign prices as given and which adheres to an exchange rate target: namely, that an adjustment of industrial prices to those of its competitors could be relatively feasible but, in contrast, an adjustment based on labour costs and consumer prices in general would be much more difficult. This seems to suggest not only the existence of an important non-tradable goods sector unaffected by foreign influences, but also that the burden of adjustment -- i.e. pressure from international competition -- in sectors affected by external factors does not weigh on labour costs, but rather on other costs and/or company margins.

Thus, the evidence presented favours considering the wholesale price index as a comparatively good approximation for measuring price trends of tradable goods. In contrast, it does not support drawing conclusions regarding a currency's overvaluation or undervaluation on the basis of observed deviations of the real exchange rate from a past equilibrium level, no matter what method of calculation of this equilibrium is used (an average of the real exchange rate over a long period of time

or that observed in a period of nil external balance are frequently used), if the conclusion rests on any of the indices studied here -- other than the industrial price index -- since a constant long-run equilibrium level cannot be empirically verified.

## REFERENCES

- Abuaf, N. and P. Jorion, 1990: "Purchasing Power Parity in the Long Run". *The Journal of Finance*, vol. 45, n<sup>o</sup> 1, March, 157-174.
- Adler, M. and B. Lehmann, 1983: "Deviations from Purchasing Power Parity in the Long Run". *The Journal of Finance*, vol. 38, n<sup>o</sup> 5, December.
- Banerjee, A., J. Dolado and R. Mestre, 1993: "On Some Simple Tests for Cointegration: The Cost of Simplicity". Working Document 9302, Research Department, Banco de España.
- Bleaney, M., 1992: "Does Long-Run Purchasing Power Parity Hold within the European Monetary System?" *Journal of Economic Studies*, vol. 19, n<sup>o</sup> 3, 66-72.
- Blundell-Wignall, A. and M. Thomas, 1987: "Deviations from Purchasing Power Parity: The Australian Case". Reserve Bank of Australia, Research Discussion Papers.
- Frenkel, J.A., 1981: "The Collapse of Purchasing Power Parities during the 1970s". *European Economic Review*, 145-165.
- Haldane, A.G. and M. Pradhan, 1992: "Testing Real Interest Rate Parity in the European Monetary System". Discussion Paper n<sup>o</sup> 2, Bank of England.
- Heri, E.W. and M.J. Theurillat, 1990: "Purchasing Power Parities for the DM: A Cointegration Exercise". *Kredit und Kapital Heft 3 23 Jahrgang*, 333-350.
- Johansen, S., 1988: "Statistical Analysis of Cointegration Vectors". *Journal of Economic Dynamics and Control*, vol. 12, 231-254.



- Johansen, S., 1991: "Estimation and Hypothesis Testing of Cointegration Vectors in Gaussian Vector Autoregressive Models". *Econometrica*, vol. 59, nº 6.
- Johansen, S., 1992: "Determination of Cointegration Rank in the Presence of a Linear Trend". *Oxford Bulletin of Economics and Statistics*, 54, 3, 383-397.
- Johansen, S. and K. Juselius, 1990: "Maximum Likelihood Estimation and Inference on Cointegration with Application to the Demand for Money". *Oxford Bulletin of Economics and Statistics*, 52, 2, 169-210.
- Johansen, S. and K. Juselius, 1991: "Testing Structural Hypotheses in a Multivariate Cointegration Analysis of the PPP and UIP Relations between Denmark and Germany". *Journal of Econometrics*.
- Johansen, S. and K. Juselius, 1992: "Identification of the Long-Run and Short-Run Structure, an Application to the IS-LM Model". Discussion Paper, University of Copenhagen.
- Kim, Y., 1990: "PPP in the Long Run: A Cointegration Approach". *Journal of Money, Credit and Banking*, vol. 22, nº 4.
- Krugman, P.R., 1991: "International Economics: Theory and Policy". Harper Collins, New York.
- Mishkin, F.S., 1984: "Are Real Interest Rates Equal Across Countries? An Empirical Investigation of International Parity Conditions". *The Journal of Finance*, vol. 39, nº 5, 1345-1357.
- Moore, M.J., 1992: "Covered Purchasing Power Parity, ex-ante PPP and Risk Aversion". Centre for Economic Policy Research, Discussion Paper nº 635, April.

- Navascués, M. , 1988: "Un índice de la posición efectiva de la peseta, con la consideración de la competencia en terceros mercados (sistema de doble ponderación)". Banco de España, Boletín Económico, September, 23-36.
- Nessén, M. , 1992: "Common Trends in Prices and Exchange Rates, Tests of Long-Run Purchasing Power Parity". Paper presented at the Meeting of the Econometric Society, Brussels, 1992.
- Ngama, Y.L. and S. Sosvilla-Rivero, 1991: "An Empirical Examination of Absolute Purchasing Power Parity". Revista Española de Economía, Instituto de Estudios Fiscales, vol. 8, nº 2, 285-311.
- Osterwald-Lenum, M., 1992: "A Note with Quantiles of the Asymptotic Distribution of the Maximum Likelihood Cointegration Rank Statistics". Oxford Bulletin of Economics and Statistics, 54, 3, 461-472.
- Perron, P. , 1988: "Trends and Random Walks in Macroeconomic Time Series". Journal of Economic Dynamics and Control, 12, 297-332, North-Holland.
- Phillips, P.C.B. and P. Perron, 1988: "Testing for Unit Roots in Time Series Regression". Biometrika, vol. 75, 335-346.
- Pippenger, J., 1986: "Arbitrage and Efficient Markets Interpretations of Purchasing Power Parity: Theory and Evidence". Economic Review, Federal Reserve Bank of San Francisco, nº 1, Winter, 31-47.
- Reig, E. , 1988: "Tipo de cambio y paridad del poder adquisitivo: una contrastación para el caso español". In J. Velarde, J.L. García Delgado and A. Pedreño, eds.: "El sector exterior de la economía española", 303-342.

Roll, R., 1979: "Violations of Purchasing Power Parity and their Implications for Efficient International Commodity Markets". In M. Sarnat and G.P. Szego, eds. "International Finance and Trade", vol. 1, Chap. 6, Cambridge, Mass., Ballinger Publishing Company.

Taylor, M.P., 1990: "On Unit Roots and Real Exchange Rates: Empirical Evidence and Monte Carlo Analysis". *Applied Economics*, 22, 1311-1321.

**APPENDIX I**

**UNIT ROOT TESTS**

TABLE A.1.A. UNIT ROOT TESTS. H <sub>0</sub> : I (1) (A)															
TEST	CONSUMER PRICES					INDUSTRIAL PRICES					EXPORT PRICES				
	$\frac{P_1}{P^*}$	$\frac{P_2}{P^*}$	$\frac{P_3}{P^*}$	$\frac{P_4}{P^*}$	$\frac{P_5}{P^*}$	$\frac{P_1}{P^*}$	$\frac{P_2}{P^*}$	$\frac{P_3}{P^*}$	$\frac{P_4}{P^*}$	$\frac{P_5}{P^*}$	$\frac{P_1}{P^*}$	$\frac{P_2}{P^*}$	$\frac{P_3}{P^*}$	$\frac{P_4}{P^*}$	$\frac{P_5}{P^*}$
$Z(\phi_3)$	.65	3.38	2.45	.82	1.91	.56	2.36	3.42	-.98	.88	2.03	1.10	.78	.82	2.16
$Z(\phi_2)$	5.46	10.85	5.07	15.83	17.19	4.01	9.11	2.28	5.34	5.92	3.81	1.97	.61	4.51	1.79
$Z(t_{\alpha})$	-.50	-.47	1.02	.77	.66	-.18	-.73	-1.59	.14	.46	-.45	-.66	-.71	-.32	-1.71
$Z(t_{\mu})$	-.67	1.77	1.87	2.07	2.23	-1.42	1.00	-1.41	1.02	1.37	-1.84	-.05	-.49	.48	-1.32
$Z(t_{\beta})$	-.49	-.39	-1.74	-1.03	-1.06	.13	.20	1.23	-.49	-.82	.44	.27	.47	.05	1.76
$\hat{\alpha}$	.98	.99	1.02	1.02	1.02	1.00	.98	.90	1.00	1.01	.99	.98	.98	.99	.94
TEST	UNIT LABOUR COSTS					EXCHANGE RATES					CRITICAL VALUES				
	$\frac{P_1}{P^*}$	$\frac{P_2}{P^*}$	$\frac{P_3}{P^*}$	$\frac{P_4}{P^*}$	$\frac{P_5}{P^*}$	$e_1$	$e_2$	$e_3$	$e_4$	$e_5$	1%	5%	10%		
$Z(\phi_3)$	1.45	3.13	10.81	1.27	3.45	1.46	1.54	.65	.60	.92	8.79	6.51	5.48		
$Z(\phi_2)$	1.17	17.42	54.37	7.02	40.05	2.00	1.26	.44	1.28	.78	6.55	4.91	4.17		
$Z(t_{\alpha})$	-1.69	-1.41	.11	-1.51	-.30	-1.66	-1.73	-1.11	-.89	-1.34	-4.06	-3.46	-3.15		
$Z(t_{\mu})$	1.83	.55	2.34	-.43	2.79	1.72	1.70	1.11	.83	1.31	3.79	3.11	2.74		
$Z(t_{\beta})$	-.71	.97	-.92	1.39	-.28	1.01	-1.01	-.31	-.39	-.69	3.79	2.79	2.38		
$\hat{\alpha}$	.95	.96	1.00	.93	1.00	.94	.93	.97	.98	.98					

(A) Relative prices and exchange rates are both given in logarithms relative to the United States (P\*) and correspond to Germany, the United Kingdom, France, Italy and Spain, in this order.

TABLE A.1.B. UNIT ROOT TESTS. H0: I (1) (A)																
TEST	INDUSTRIAL PRICES		EXPORT PRICES					ULC	EXCHANGE RATES					CRITICAL VALUES		
	$\frac{P_1}{P^*}$	$\frac{P_3}{P^*}$	$\frac{P_1}{P^*}$	$\frac{P_2}{P^*}$	$\frac{P_3}{P^*}$	$\frac{P_4}{P^*}$	$\frac{P_5}{P^*}$	$\frac{P_1}{P^*}$	$\theta_1$	$\theta_2$	$\theta_3$	$\theta_4$	$\theta_5$	1%	5%	10%
MODEL 2																
$Z(\hat{\phi}_1)$	6.14	1.97	5.82	2.81	.67	6.85	-.63	1.48	2.49	1.38	.61	1.86	.77	6.74	4.73	3.87
$Z(\hat{\tau}_{\alpha^*})$	-1.13	-1.97	-2.06	-1.35	-1.02	-1.09	-.45	-1.52	-1.37	-1.42	-1.09	-1.02	-1.02	-3.51	-2.90	-2.58
$Z(\hat{\tau}_{\mu^*})$	-2.32	-1.59	-1.88	-.11	-.64	1.49	.23	1.68	1.45	1.39	1.10	-.91	-.97	3.23	2.54	2.17
$\hat{\alpha}^*$	.99	.95	.98	.98	.99	.99	1.00	.95	.97	.96	.97	.98	.99			
MODEL 1																
$Z(\hat{\tau}_{\alpha})$		-1.15		-2.36	-.96		-1.10	-1.10	1.67	-.89	.07	-1.69	-.77	-2.60	-1.95	-1.61
$\hat{\alpha}$		.98		.98	.99		.99	.99	1.00	1.00	1.00	1.00	1.00			

(A) See note to Table 3.A.

TABLE A.2. UNIT ROOT TESTS. H0: I (2) (A)															
TEST	CONSUMER PRICES					INDUSTRIAL PRICES					EXPORT PRICES				
	$\frac{P_1}{P^*}$	$\frac{P_2}{P^*}$	$\frac{P_3}{P^*}$	$\frac{P_4}{P^*}$	$\frac{P_5}{P^*}$	$\frac{P_1}{P^*}$	$\frac{P_2}{P^*}$	$\frac{P_3}{P^*}$	$\frac{P_4}{P^*}$	$\frac{P_5}{P^*}$	$\frac{P_1}{P^*}$	$\frac{P_2}{P^*}$	$\frac{P_3}{P^*}$	$\frac{P_4}{P^*}$	$\frac{P_5}{P^*}$
$Z(\phi_3)$	6.48	20.19	15.76	12.27	21.96	17.16	16.46	12.86	11.67	27.21	13.34	14.11	18.87	27.64	36.38
$Z(\phi_2)$	4.35	13.46	10.51	8.18	14.64	11.44	10.98	8.58	7.78	18.14	8.90	9.41	12.58	18.43	24.26
$Z(t_{\alpha})$	-4.02	-6.21	-5.76	-5.18	-6.13	-6.12	-6.26	-4.94	-4.53	-6.84	-5.66	-5.54	-5.90	-7.34	-8.38
$Z(t_{\mu})$	-2.16	3.02	2.77	3.91	4.56	-2.77	3.40	-.01	2.31	3.26	-2.04	1.60	.45	3.07	1.13
$Z(t_{\beta})$	-.07	-2.42	-2.36	-1.60	-2.18	1.04	-1.51	.18	-1.08	-1.26	1.62	-.70	-.07	-.92	.29
$\bar{\alpha}$	.70	.38	.48	.55	.36	.42	.41	.54	.58	.23	.49	.49	.39	.21	.09
TEST	UNIT LABOUR COSTS					EXCHANGE RATES					CRITICAL VALUES				
	$\frac{P_1}{P^*}$	$\frac{P_2}{P^*}$	$\frac{P_3}{P^*}$	$\frac{P_4}{P^*}$	$\frac{P_5}{P^*}$	$\alpha_1$	$\alpha_2$	$\alpha_3$	$\alpha_4$	$\alpha_5$	1%	5%	10%		
$Z(\phi_3)$	45.31	38.14	31.98	69.85	12.90	25.56	25.53	18.57	18.61	18.87	8.80	6.51	5.49		
$Z(\phi_2)$	30.22	25.46	21.35	46.51	8.62	17.05	17.07	12.39	12.46	12.62	6.56	4.91	4.17		
$Z(t_{\alpha})$	-9.47	-8.69	-7.95	-11.68	-5.05	-7.11	-7.08	-6.06	-6.03	-6.08	-4.06	-3.46	-3.15		
$Z(t_{\mu})$	.73	5.01	6.11	5.35	3.69	1.26	-.75	.01	-1.26	-.76	3.79	3.11	2.74		
$Z(t_{\beta})$	-.01	-1.78	-3.41	-1.50	-1.43	-.31	.09	.07	-.25	-.02	3.79	2.79	2.38		
$\bar{\alpha}$	-.02	.07	.16	-.13	.54	.26	.19	.40	.33	.36					

(A) See note to Table 3.A.

**TABLE A.3.A. PHILLIPS-PERRON UNIT ROOT TEST. REAL EXCHANGE RATE WITH CPI**

TEST MODEL 2	\$/DM	\$/£	\$/FF	\$/LIT	\$/PTA	DM/£	DM/FF	DM/LIT	DM/PTA	£/FF	£/LIT	£/PTA	FF/LIT	FF/PTA	LIT/PTA
$Z(\phi_1)$	2.08	2.04	1.47	1.08	2.18	1.20	3.85	1.33	1.27	1.58	1.63	1.81	1.01	1.88	3.45
$Z(t_{\alpha^*})$	-1.91	-1.91	-1.54	-1.33	-1.42	-1.53	-2.75	-1.55	-1.34	-1.76	-1.79	-1.62	-1.38	-1.59	-2.11
$Z(t_{\mu^*})$	1.94	1.93	1.56	1.35	1.49	1.52	2.74	1.54	1.37	1.76	1.79	1.64	1.37	1.62	2.15
$\alpha^*$	.95	.93	.95	.96	.97	.95	.95	.96	.96	.93	.93	.94	.96	.95	.95
<b>MODEL 1</b>															
$Z(t_{\hat{\alpha}})$	.59	.54	.68	.56	1.44	-.26	-.27	-.49	.80	.04	-.16	.93	-.35	1.03	1.46
$\hat{\alpha}$	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00

**TABLE A.3.B. PHILLIPS-PERRON UNIT ROOT TEST. REAL EXCHANGE RATE WITH IPI**

TEST MODEL 2	\$/DM	\$/£	\$/FF	\$/LIT	\$/PTA	DM/£	DM/FF	DM/LIT	DM/PTA	£/FF	£/LIT	£/PTA	FF/LIT	FF/PTA	LIT/PTA
$Z(\phi_1)$	1.95	1.96	1.10	1.20	1.63	1.27	6.15	4.07	4.11	2.27	1.39	1.92	4.62	4.59	2.97
$Z(t_{\alpha^*})$	-1.81	-1.75	-1.47	-1.48	-1.60	-1.58	-3.14	-2.68	-2.84	-1.93	-1.48	-1.93	-2.99	-2.85	-2.33
$Z(t_{\mu^*})$	1.84	1.78	1.47	1.50	1.62	1.58	3.11	2.67	2.84	1.89	1.46	1.93	2.99	2.87	2.34
$\alpha^*$	.95	.94	.95	.95	.95	.94	.89	.84	.81	.94	.96	.92	.85	.89	.89
<b>MODEL 1</b>															
$Z(t_{\hat{\alpha}})$	.69	.83	.05	.38	.77	-.01	-1.50	-.92	-.24	-.93	-.78	-.26	.39	.88	.62
$\hat{\alpha}$	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00

CRITICAL VALUES			
TEST	1%	5%	10%
$Z(\phi_1)$	6.74	4.73	3.87
$Z(t_{\alpha^*})$	-3.51	-2.90	-2.58
$Z(t_{\mu^*})$	3.23	2.54	2.17
$Z(t_{\hat{\alpha}})$	-2.60	-1.95	-1.61



TABLE A.4.A. PHILLIPS-PERRON UNIT ROOT TEST. REAL EXCHANGE RATE WITH XPI (1)															
TEST MODEL 2	\$/DM	\$/£	\$/FF	\$/LIT	\$/PTA	DM/£	DM/FF	DM/LIT	DM/PTA	£/FF	£/LIT	£/PTA	FF/LIT	FF/PTA	LIT/PTA
$Z(\phi_1)$	1.49	2.41	1.32	1.49	1.19	.81	1.71	1.54	3.54	1.13	1.92	1.79	1.71	4.90	2.88
$Z(t_{\alpha})$	-1.60	-2.16	-1.55	-1.61	-1.51	-1.18	-1.68	-1.71	-2.30	-1.49	-1.94	-1.72	-1.83	-2.96	-2.02
$Z(t_{\mu^*})$	1.62	2.16	1.56	1.63	1.50	1.17	1.67	1.70	2.28	1.49	1.94	1.69	1.83	2.94	1.99
$\alpha^*$	.96	.90	.94	.94	.95	.96	.92	.92	.91	.95	.91	.96	.93	.86	.95
MODEL 1															
$Z(t_{\alpha})$	.57	.29	.43	.54	-.30	-.49	-.77	-.39	-1.32	.04	.19	-.82	.16	-.97	-1.29
$\hat{\alpha}$	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00

TABLE A.4.B. PHILLIPS-PERRON UNIT ROOT TEST. REAL EXCHANGE RATE WITH ULC (1)															
TEST MODEL 2	\$/DM	\$/£	\$/FF	\$/LIT	\$/PTA	DM/£	DM/FF	DM/LIT	DM/PTA	£/FF	£/LIT	£/PTA	FF/LIT	FF/PTA	LIT/PTA
$Z(\phi_1)$	2.55	1.99	7.40	.71	4.72	1.62	3.60	4.66	1.33	3.67	2.32	2.06	9.56	1.79	6.80
$Z(t_{\alpha})$	-1.69	-1.52	-1.79	-.71	-1.38	-1.69	-1.49	-3.00	-1.41	-1.72	-2.13	-1.50	-3.93	-1.73	-3.42
$Z(t_{\mu^*})$	1.76	1.58	2.08	.76	1.58	1.67	1.61	3.00	1.44	1.83	2.14	1.55	3.87	1.71	3.46
$\alpha^*$	.96	.96	.98	.98	.98	.94	.98	.83	.96	.96	.89	.96	.87	.94	.87
MODEL 1															
$Z(t_{\alpha})$	1.40	1.19	3.15	.91	2.59	-.64	2.11	-.45	.75	1.96	.17	1.28	-1.83	-.79	1.14
$\hat{\alpha}$	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00

(1) See critical values in previous table.

TABLE A.5. AUGMENTED DICKEY-FULLER UNIT ROOT TEST. BILATERAL REAL EXCHANGE RATE (1)									
	INDUSTRIAL PRICES					EXPORT PRICES	UNIT LABOUR COSTS		
TEST MODEL 2	DM/FF	DM/LIT	DM/PEA	FF/LIT	FF/PEA	FF/PEA	DM/LIT	FF/LIT	LIT/PEA
$Z(\hat{\phi}_1)$	3.68	5.00	5.28	5.66	2.90	2.68	3.08	6.21	4.90
$Z(\hat{c}_{\alpha^*})$	-2.62	-2.96	-3.23	-3.34	-2.28	-2.18	-2.01	-2.74	-2.64
$Z(\hat{c}_{\mu^*})$	2.61	2.95	3.22	3.34	2.29	.76	2.01	2.70	2.66
$\alpha^*$	.89	.79	.76	.80	.90	.89	.87	.89	.88
MODEL 1									
$Z(\hat{c}_{\alpha})$	-.67	-1.03	-.19	-.13	.66	-2.18	-.04	-1.60	.65
$\hat{\alpha}$	1.00	1.00	1.00	1.00	1.00	.91	1.00	1.00	1.00

(1) See critical values in Table 5.

TABLE A.6.A. PHILLIPS-PERRON UNIT ROOT TEST. EFFECTIVE REAL EXCHANGE RATE (1)												
TEST MODEL 2	CONSUMER PRICES						INDUSTRIAL PRICES					
	\$	DM	£	FF	LIT	PIA	\$	DM	£	FF	LIT	PIA
$Z(\phi_1)$	2.08	2.07	2.23	2.23	1.09	2.11	1.82	2.89	2.02	3.92	2.59	3.82
$Z(\tau_{\alpha^*})$	-1.91	-1.96	-2.09	-2.08	-1.46	-1.54	-1.79	-2.18	-1.88	-2.68	-2.25	-2.69
$Z(\tau_{\mu^*})$	1.93	1.93	-2.09	-2.09	-1.46	-1.58	1.81	2.13	-1.93	-2.65	-2.25	-2.70
$\alpha^*$	.95	.94	.92	.93	.96	.97	.95	.92	.94	.91	.90	.88
MODEL 1												
$Z(\tau_{\hat{\alpha}})$	.82	-.59	.04	.15	-.12	1.30	.58	-1.08	.51	-.83	-.22	.55
$\hat{\alpha}$	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00

TABLE A.6.B. PHILLIPS-PERRON UNIT ROOT TEST. EFFECTIVE REAL EXCHANGE RATE (1)												
TEST MODEL 2	EXPORT PRICES						UNIT LABOR COSTS					
	\$	DM	£	FF	LIT	PIA	\$	DM	£	FF	LIT	PIA
$Z(\phi_1)$	1.79	1.48	1.94	1.97	1.97	3.27	1.39	1.39	2.56	8.62	7.06	2.78
$Z(\tau_{\alpha^*})$	-1.81	-1.58	-1.96	-1.98	-1.96	-2.26	-1.07	-1.63	-2.23	-2.68	-3.71	-2.08
$Z(\tau_{\mu^*})$	1.82	1.54	-1.95	-1.98	-1.96	-2.23	1.13	1.60	-2.21	-2.97	-3.70	-2.11
$\alpha^*$	.94	.96	.92	.91	.91	.93	.98	.95	.92	.76	.89	.94
MODEL 1												
$Z(\tau_{\hat{\alpha}})$	.47	-.74	-.11	-.01	.29	-1.24	1.23	-.39	-.40	2.64	-.48	1.01
$\hat{\alpha}$	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00

(1) See critical values in Table 5.

**APPENDIX II**

**THE JOHANSEN PROCEDURE**

Johansen (1988) considers an n-dimensional process where  $x'_t = (x_{1t}, x_{2t} \dots x_{nt})$ , , integrated of order one, and which allows a finite VAR(k) representation, i.e.:

$$A(L)X_t = \mu + \phi D_t + \varepsilon_t ; \varepsilon_t \sim \text{iid } N(0, \Sigma) \quad (\text{A.1})$$

where  $A(L) = A_1 L - A_2 L^2 \dots - A_k L^k$  is a matrix of lag polynomials.

When the x variables are cointegrated, with a cointegration rank r, the Granger Representation Theorem says that (A.1) is represented in the form of an error correction mechanism:

$$A^*(L)\Delta x_t = -\alpha\beta'x_{t-k} + \mu + \phi D_t + \varepsilon_t \quad (\text{A.2})$$

with  $A^*(0) = I_n$  ;  $\alpha$  and  $\beta$  matrices of (n,r) order;  $A(1)=\alpha\beta'$  and  $\text{rank}[A(1)]=r$ . The  $\beta$  columns are the cointegrating vectors, where the  $\alpha$  rows indicate the weight at which each vector enters a given equation.

The equation (A.2) can be rewritten as:

$$\Delta x_t = \sum_{i=1}^{k-1} A_i^* \Delta x_{t-i} - \Pi x_{t-k} + \mu + \phi D_t + \varepsilon_t \quad (\text{A.3})$$

and the hypothesis that r cointegrating vectors exist can be formulated as:

$$H_0 : \quad \alpha\beta' \quad (\text{A.4})$$

The estimation method proposed by Johansen consists of concentrating the likelihood function with respect to the parameters  $A_1$ , computing the regressions:

$$\Delta x_t / \Delta x_{t-1}, \Delta x_{t-2} \dots \Delta x_{t-(k-1)}, \mu, D_t \quad (\text{A.5})$$

If we use  $R_{0t}$  and  $R_{kt}$ , respectively, to indicate the residuals of

$$x_{t-k} / \Delta x_{t-1}, \Delta x_{t-2} \dots \Delta x_{t-(k-1)}, \mu, D_t \quad (\text{A.6})$$

(A.5) and (A.6), we have:

$$R_{0t} = \Pi R_{kt} + \varepsilon_t \quad (\text{A.7})$$

whose likelihood function is:

$$L(\alpha, \beta, \Lambda) = C |\Lambda|^{T/2} \text{EXP} \left[ -\frac{1}{2} \sum_{t=1}^T (R_{0t} + \alpha \beta' R_{kt})' \Lambda^{-1} (R_{0t} + \alpha \beta' R_{kt}) \right] \quad (\text{A.8})$$

an expression which can be maximised for known  $\beta$  by computing the regression of  $R_{0t}$  on  $-\beta' R_{kt}$ , so that:

$$\hat{\alpha}(\beta) = -S_{0k} \beta (\beta' S_{kk} \beta)^{-1} \quad (\text{A.9})$$

$$\hat{\Lambda}(\beta) = S_{00} - \hat{\alpha}(\beta) \beta' S_{k0} \quad (\text{A.10})$$

where:

$$S_{ij} = \frac{1}{T} \sum_1^T R_{it} R_{jt}' \quad (i, j = 0, k) \quad (\text{A.11})$$

Substituting (A.9) and (A.10) into (A.8), the logarithm of the likelihood function is:

$$\log L(\beta) = -\frac{T}{2} + \log |\Lambda(\beta)| \quad (\text{A.12})$$

The first-order condition of the maximisation problem of (A.12), after establishing the usual normalisation  $\beta' S_{kk} \beta = I$ , is:

$$\left| \lambda S_{kk} - S'_{k0} S_{00}^{-1} S_{0k} \right| = 0 \quad (\text{A.13})$$

or, alternatively:

$$\left| \lambda I - C^{-1} S'_{k0} S_{00}^{-1} S_{0k} C^{-1} \right| = 0 \quad (\text{A.14})$$

where  $C$  is a non-singular triangular matrix defined by the Choleski breakdown of  $S_{kk}$ , i.e.,  $S_{kk} = CC'$ .

The maximum likelihood estimator of  $\beta$  is given by  $\hat{\beta} = (\hat{\beta}_1, \dots, \hat{\beta}_r)$ ,  $\hat{\beta}_i = C^{-1} \hat{v}_i$ , being:  $\hat{v} = (\hat{v}_1, \dots, \hat{v}_r)$ , the  $r$  eigenvectors associated with the largest  $r$  eigenvalues. On the basis of (A.9), the maximum likely estimator of  $a$  will be:  $\hat{a} = S_{0k} \hat{\beta}$

The maximum likelihood function is obtained, under  $H_0$ , for:

$$L^*(H_0) = k - \frac{T}{2} \sum_{i=1}^r \log(1 - \hat{\lambda}_i) \quad (A.15)$$

and a test on the number of cointegration vectors can be easily computed. Johansen proposes two, one being the trace test, in which the null hypothesis  $H_0: r=r_1$  is tested against the alternative  $H_1: r=n$ . The second is the maximum test, where  $H_0$  is tested against  $H_2: r=r_1+1$ .

Starting from (A.15), we can compute  $L^*(H_1)$  and  $L^*(H_2)$ , whereby the likelihood ratio tests will be:

$$LR(H_1, H_0) = 2[L^*(H_1) - L^*(H_0)] = -T \sum_{r_1+1}^n \log(1 - \hat{\lambda}_i) \quad (A.16)$$

$$LR(H_2, H_0) = 2[L^*(H_2) - L^*(H_0)] = -T \log(1 - \hat{\lambda}_{r_1+1}) \quad (A.17)$$

These statistics do not have a standard distribution. The critical values of the test were recently tabulated by Osterwald-Lenum (1992).



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