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Abstract

Excessive levels of firing costs have been consistently blamed for the relatively weak employment performance in Europe, yet the conclusions to be drawn from the literature are somewhat ambiguous. This paper re-examines the impact of adjustment costs under uncertainty. It is shown that the interaction between the level of adjustment costs and the type of uncertainty can have important ramifications for employment dynamics. More specifically, we find that allowing for the possibility of transitory economic conditions results in a considerable increase in employment persistence. We conclude the analysis with a number of simulation exercises to illustrate that allowing for changes in the economic environment in which firms have operated over the past two decades can considerably enhance our understanding of the evolution of employment within Europe.

Adjustment Costs, Uncertainty and Employment Inertia:

1 Introduction:

The presence of an overly regulated labour market has been consistently cited as one of the principle factors behind the relatively weaker performance of European Community labour markets and the subsequent persistence of unemployment. The results of both applied and theoretical research on this issue is however somewhat ambiguous. Despite this, we feel that statistical evidence suggests that demand side factors play an important role in any explanation of the time series behaviour of employment (and thus unemployment). In this paper therefore, we extend the analysis of Bentolila and Bertola (1990) which applies stochastic control techniques recently developed in the Investment literature to the firm's labour demand decision, in order to assess the impact of changes in a number of factors on the demand side of the labour market, which we feel may be more relevant to the employment debate than the present literature gives credit for. More specifically, we argue that: a) uncertainty, and especially the increases in the degree of uncertainty with respect to the nature of prevailing economic conditions that have occurred over the past twenty years; and b) the increases in hiring costs that occurred in the wake of the second oil price shock have, in the presence of adjustment costs, had an important impact on a firm's optimal employment strategy and thus on the dynamic behaviour of employment.

The remainder of this paper is organised as follows. In section two we briefly assess the argument that a significant amount of unemployment within the European Community can be considered to be a consequence of insufficient employment growth. In section three we model the firm's employment decision in the presence of adjustment costs and uncertainty, extending the work of Bentolila and Bertola to allow for the possibility of transitory economic conditions. Section four offers a comparative static analysis of the effects of changes in a number of factors we feel to have had an increasing influence on a firm's employment decision in the 1980's. We conclude in section five by discussing the implications of our results for the employment protection debate and offer suggestions for future research.

2 European Unemployment: a hiring problem?

An analysis of labour force and employment statistics over the last two decades would tend to suggest that much of the unemployment problem in Europe is due to its inability to generate sufficient employment. For in contrast to other OECD countries, most notably the US and Japan, EC members states have over the past 15 years been, regardless of the prevailing economic conditions, characterised by relatively weaker employment growth, with the majority of member states experiencing a stagnation in employment levels from 1973 onwards[1].

This view of European Unemployment as being largely a "hiring problem" receives further support from unemployment flow and duration data, which illustrate that the increases in the stock of unemployment in the larger Community countries have been associated with: a) a substantial decline in the number of outflows from unemployment; and b) a subsequent build up in unemployment duration. For, although the risk of becoming unemployed is much lower within the Community than for example, the US, the fact that unemployment outflow rates in Europe are significantly lower than those of the US implies that once an individual becomes unemployed in Europe it is much more difficult for him to find another job. In other words, the high levels of unemployment which prevail within the EC appear to be primarily associated with a reduction in the ability of unemployed workers to find a job, rather than an increase in the likelihood of an employed individual losing his job. Such observations beg the question as to why the European employment performance has been so weak?

Whilst this stagnation of employment growth in Europe, as in other OECD countries can, after the first oil shock, be primarily attributed to a combination of depressed domestic and international demand conditions, with the subsequent slow down in the rate of output growth and decline in productivity levels aggravating the situation firther. Employment growth in Europe remained at relatively low levels for much of the 1980's, despite the fact that from 1985 onwards the macroeconomic environment in the majority of the member states was in fact much more conducive to job creation. It is not until the end of the 1980's, a period of relatively low economic growth, that we observe record levels of employment growth within the majority of Community states. The US on the other hand has, despite an overvalued dollar and the subsequent substantial loss in export markets during the first half of the 1980's, experienced a rapid growth in employment throughout the 1980's. Thus they were to all intents and purposes able to offset the large increases in unemployment which occurred in the wake of the second oil price shock and the recession of the early 1980's.

Whilst the substantial increase in fixed employment costs relative to wage rates which occurred in the European Community from the beginning of the 1970's has undoubtedly had an adverse impact on employment growth, it has to be remembered that such increases were experienced, although to varying

[1] It is interesting to note that in general this period of stagnation coincided with both a slower rate of labour force growth than that experienced for example, by the US, and a long term decline in the proportion of the population in employment. Consequently, the increases that occurred in unemployment could be argued to be much lower than one might of expected, given the poor employment performance.

degrees, throughout the OECD. Moreover, attributing the stagnation in employment solely to increased labour costs fails to explain why European firms appear to have continued to substitute capital for labour during the 1980's: despite a substantial decline in labour costs, a relatively slack labour market, a high real rate of interest and a fall in the relative cost of labour. This continual substitution of capital for labour has, given these developments, been repeatedly attributed to the fact that by the mid-1980's labour had become an increasingly fixed factor of production in Europe. Subsequently, the adverse impact on employment growth of the more regulated European Community markets has, during the latter half of the 1980's, been particularly prevalent in the US versus EC labour market performance debate. The general consensus appearing to have been that a more appropriate focus for research in the '*quest*' for an explanation of the behaviour of employment within US and the European Economies would be to focus on the changes in and the differences between employment protection legislation within the two regions.

The appropriateness of explanations for the relatively weaker employment performance of the European Community which focus on either firing costs per se or on the speed of employment adjustment would appear to be drawn into question however, by: a) the superior performance of the EFTA countries, where firing costs are at least equivalent, if not higher; and b) the fact that comparative work on the speed of adjustment does not indicate any significant difference between the two regions. Employment protection does however, appear to have a fundamental impact on the dynamics of employment. For, an analysis of times series data on employment levels indicates that during so-called normal business cycles the time path of employment is in fact relatively smoother in those economies in which higher firing costs prevail. Suggesting then, that firms located in such environments tend to make relatively less adjustments to their work force to changes in demand conditions than those firms operating in low cost environments. During periods of more prolonged recessions however, both high and low adjustment cost countries tend to experience a significant reduction in employment levels, with differences between these two regimes only appearing to emerge in the aftermath of such recessions. For, one would expect to (and one does in fact) observe a significantly smaller increase in employment levels during the ensuring upturn in economic activity in those countries in which higher adjustment costs prevail, with this trend being reflected in particularly low outflow rates from unemployment. This phenomenon appears to be especially true of European Community labour markets during the upturn in economic activity of the early 1980's.

Focusing solely on the impact of employment protection and in particular on the impact of relatively high levels of firing costs on a firm's labour demand may therefore be somewhat too simplistic an approach to take. For these observations tend to suggest that what may be more relevant to an explanation of the dynamic behaviour of employment then the level of adjustment costs per se, is the interaction of the existence of such a regulated regime with other factors affecting the firm's labour demand decision. In particular, whether such interactions take place in low or high cost environments or during very turbulent economic conditions. It would appear necessary then, when trying to explain recent trends in employment to incorporate the significant changes that have occurred over the past decade in the degree of uncertainty with respect to the nature of the prevailing economic conditions into the firm's employment decision to determine what role, if any, it has had on employers hiring and firing decisions when the firm is faced with considerable restraints on employment adjustment.

In the following section we therefore, adopt the methodology of Bentolila and Bertola in order to examine the impact of: a) increases in the degree of uncertainty that firm's face with regards to general economic conditions; and b) changes in the level of adjustment costs and in particular changes in the level of hiring costs. Factors which we argue to have become more relevant to the employment decision over the 1980's, but which have been somewhat overlooked in the literature. Extending this earlier analysis to allow for the possibility of transitory economic conditions, a modification which we feel to be more reflective of the characteristics of the general economic environment in which firms have found themselves operating in over the past 15 years.

3 The Firm's Employment Decision under Uncertainty:

Following Bentolila and Bertola (1990) we consider a partial equilibrium model of firm's employment decision in which the firm is assumed to:

a) have a linear production function, in which homogenous labour, L, is the only input.

$$Q_\tau \le A_\tau L_\tau, \qquad 1)$$

where Q_τ represents production or sales and A_τ, labour productivity, which is assumed to grow at the deterministic exponential rate, ϑ_A, and τ is time elapsed from a starting date t; and

b) face the following constant elasticity demand finction:

$$Q_\tau = Z_\tau P_\tau^{-1/1-\mu} \qquad 0 < \mu < 1, \qquad 2)$$

where Z denotes the firm's demand; P, the price of the firm's product and μ is the inverse of the firm's mark up factor.

Uncertainty enters the model through changes in demand strength, which are assumed to follow a stochastic process. In contrast to the Bentolila and Bertola model where variations in demand strength, Z, are assumed to evolve according to a geometric Brownian motion process, we make what we feel to be the more realistic assumption, given events of the past twenty years, that in addition to these smooth fluctuations in demand there exists the possibility that aggregate demand strength can be subject to discrete jumps. For analytical tractability the possibility of a jump in demand strength is modelled via the following Poisson jump process:

$$\frac{dZ_\tau}{Z_\tau} = \frac{1}{1-\mu}\left(\frac{\eta_0}{\mu A_\tau{}^{\mu} L_\tau{}^{\mu-1} Z_\tau{}^{1-\mu}} - 1\right) dq,$$

where: η_0 is a given constant to be defined later; $\mu A_\tau{}^{\mu} L_\tau{}^{\mu-1} Z_\tau{}^{1-\mu}$ is the marginal revenue product of labour (MRPL); and dq is a jump process, the arrival rate of which follows a Poisson distribution. If we define ρ as the mean arrival rate of a jump during an infinitesimal time period dt, then the probability that a jump will occur is given by ρdt and the probability that no jump occurs is given by $1-\rho dt$. Hence:

$dq = 0$ *with probability* $1-\rho dt$ and

$dq = 1$ *with probability* ρdt.

Formulating the evolution of demand in this manner enables us therefore, to establish a link between the size of the jump in demand and the distance of the MRPL from a given point η_0. More specifically, it will be argued that η_0 can be interpreted as an equilibrium value for η_τ. Thus the size of the jump in demand increases as the firm moves further away from the equilibrium level η_0.

In specifying the jump process in this manner we are trying to capture the idea that 'good' times tend to be followed by 'bad' times. Thus although in the present model the firm should in principle (under Marshallian conditions) stay at the equilibrium level of the MRPL, if changes in $\sigma_z dw$ cause a variation in demand so that the MRPL deviates from η_0, the firm thinks there is possibility that a jump in demand will occur, which will take it back to the equilibrium level, η_0. Such a specification implies then that there is a level of demand $\bar{Z}_\tau$, at which no jump occurs, where $\bar{Z}_\tau$ is given by:

$$\bar{Z}_\tau = \left(\frac{\eta_0}{\mu \cdot A_\tau^{\mu}}\right)^{\frac{1}{1-\mu}} L_\tau .$$

One can think of this level as being an 'equilibrium' level of demand. It is straight forward to verify that: a) dZ=0 when either Z=0 or Z is at its long run level; and b) that the jump in demand is positive if Z is below its equilibrium level and is negative if Z is above.

Under such an assumption demand strength evolves according to the following process mixed Brownian-Jump process:

$$\frac{dZ_\tau}{Z_\tau} = \vartheta_Z dt + \sigma_Z dW_\tau + \frac{1}{1-\mu}\left(\frac{\eta_0}{\mu A_\tau{}^{\mu} L_\tau{}^{\mu-1} Z_\tau{}^{1-\mu}} - 1\right) dq. \qquad 3)$$

where: $\{W_t\}$ is a standard Wiener process, i.e. the continuous time equivalent of a random walk, whose increments have instantaneous variance 1 and drift equal to zero and dq and dW are independent, thus $E(dqdW) = 0$.

In other words, demand is assumed to fluctuate continuously according to a geometric Brownian motion process, but during each time interval dt there exists the probability ρdt that a jump in demand will occur and that demand will thereafter continue to fluctuate according to the Brownian motion process until another jump occurs. As a Poisson jump does not occur very often, most of the time variance in demand will, within a short period of time, be determined by the Brownian motion component, $\sigma^2 dt$. When a jump does occur however, it leads to a significant deviation in demand thus it is essential that this possibility is incorporated into any calculation of the expected variance of demand made at time t. From equation 3, the expected rate of change of $\{Z\}$ in our analysis will therefore be equal to:

$$\frac{E\{dZ_\tau\}}{Z_\tau}\frac{1}{d\tau} = \vartheta_Z + \frac{\rho}{1-\mu}\left(\frac{\eta_0}{\mu A_\tau^{\mu} L_\tau^{\mu-1} Z_\tau^{1-\mu}} - 1\right).$$

Thus in contrast to the Bentolila and Bertola model, where Z grows at an expected constant rate equal to the drift parameter in the Brownian motion process, the expected rate of growth of Z in our model actually decreases with Z. To obtain the expected future path of Z we simply solve the above differential equation for a given level of productivity, A, and labour, L. At time τ, the expected value of Z will therefore be equal to:

$$E\{Z_\tau\} = \left[Z_t^{1-\mu} e^{(\vartheta_Z(1-\mu)-\rho)(\tau-t)} + K\left(1 - e^{(\vartheta_Z(1-\mu)-\rho)(\tau-t)}\right)\right]^{\frac{1}{1-\mu}},$$

where K is a constant defined as:

$$K \equiv \frac{\eta_0 \rho}{\mu A^{\mu} L^{\mu-1}((\rho - \vartheta_Z)(1-\mu))}.$$

Thus when ρ=0 the expected value of demand at time τ will, as in Bentolila and Bertola analysis, be given by $E_t\{Z_\tau\} = Z_t e^{\vartheta_Z(\tau-t)}$. For clarity, the expected value of Z under the two alternative assumptions (ours and Bentolila and Bertola's) as to the stochastic process that demand follows is illustrated in figure 3.1. It should be evident that the size of the jump which takes the economy back to its equilibrium value increases in magnitude the further demand moves away from its long run equilibrium level. Thus the size of the expected jump will, despite the constant hazard rate ρ, also increase the further away demand is from its equilibrium value. As is evident from figure 3.2, this will result in a lower expected rate of growth of Z in our analysis than that obtained by Bentolila and Bertola.

The Firm's Optimisation Problem:

In order to compare the impact of hiring and firing costs on a firm's optimal labour demand decision we initially assume that there are no costs associated with employment adjustment. The problem facing the optimising firm is to choose the profit maximising employment policy.

From the production and sales functions at time τ, the firm's revenues, R, at time τ can, using the fact that the marginal revenue from a CES demand function will always be positive, given that equation one holds with equality, be defined as:

$$R_\tau = Z_\tau^{(1-\mu)}(A_\tau L_\tau)^\mu,$$

where productivity is assumed to grow at the deterministic rate:

$$dA_\tau = \vartheta_a A_\tau d\tau,$$

and labour to evolve according to the following process:

$$dL_\tau = -\delta L_\tau d\tau,$$

where δ is the constant rate at which employees are assumed to voluntarily leave their jobs.

Total profits, π, at time τ are therefore given by:

$$\pi_\tau \equiv Z_\tau^{1-\mu}(A_\tau L_\tau)^\mu - wL_\tau, \qquad 4)$$

where the wage rate, w is for analytical tractability assumed to be constant.

Taking wages as given, the firm sets its employment level in order to maximise the expected present value of future revenue over an infinite horizon. Thus it maximises the following value function:

$$V_\tau = \underset{X_\tau}{Max}\, E\left\{\int_t^\infty e^{-r(\tau-t)}\left[\pi_\tau d\tau\right]\right\}, \qquad 5)$$

subject to the labour accumulation constraint:

$$dL_\tau = dX_\tau - \delta L_\tau d\tau,$$

where: r denotes the rate of return; dL, the cumulative labour turnover process; and $\{X_\tau\}$ represents hiring and firing. We also make the standard assumption that the value of the control variable must be chosen using the information available in the current period[2].

It is important to note however, that in determining its optimal employment level the firm has also to consider the fact that its employment level will be affected by the number of voluntary quits. By hypothesis an individual worker can quit with a constant hazard rate δ at each moment in time, thus the probability that the worker remains with the firm over the period t to τ is equal to $e^{-\delta(\tau-t)}$. Future profits of the firm over the time period t to τ will therefore, only be given by the expression on the right hand side of equation five if no quits occur. To allow for the possibility that a worker may quit we need, when calculating expected future profits, to weight the firm's value function in the following manner:

[2] Here then we are simply assuming that all the necessary information incorporated to make a forecast at any future date is in effect already incorporated in today's values.

$$V_t = \underset{[X_t]}{Max}\, E\left\{\int_t^\infty e^{-r(\tau-t)}\left[\pi_\tau d\tau\right]\right\} e^{-\delta(\tau-t)}.$$

The final term on the right hand side of the equation can however, be taken inside the expectations operator, since the parameter δ does not change over time. Thus the function to be maximised becomes:

$$V_t = \underset{[X_t]}{Max}\, E\left\{\int_t^\infty e^{-(r+\delta)(\tau-t)}\left[\pi_\tau d\tau\right]\right\}. \quad 6)$$

Partially differentiating equation six with respect to L in order to determine the profit maximising level of employment yields the following standard first order condition which is satisfied at all points in time when $\eta_\tau = w$:

$$E\left\{\int_t^\infty e^{-(r+\delta)(\tau-t)}(\eta_\tau - w)d\tau\right\} = 0,$$

where to maintain consistency of notation with the Bentolila and Bertola analysis, the marginal revenue product of labour at time τ is defined as being equal to $\eta_\tau \equiv \mu A_\tau^\mu Z_\tau^{1-\mu} L_\tau^{\mu-1}$.

According to the optimal labour demand rule the firm should in the absence of adjustment costs: a) hire when the expected discounted marginal revenue product of labour of the additional worker is greater than the wage the firm has to pay the additional employee discounted back to the current time period; and b) fire the marginal worker when the expected marginal revenue product of this worker is less than the wage the firm has to pay the additional employee discounted back to the current time period.

To determine the profit maximising level of employment we simply equate the discounted expected marginal revenue product of labour to the discounted wage the firm has to pay the worker in each period, i.e. $w \equiv \mu A_\tau^\mu Z_\tau^{1-\mu} L_\tau^{\mu-1}$. Solving this expression for L the optimal employment level in each time period will be given by:

$$L_\tau = \left[\frac{1}{\mu} A_\tau{}^\mu Z_\tau{}^{1-\mu} \frac{1}{w}\right]^{\frac{1}{1-\mu}}$$

For a given wage, the firm's labour demand will be therefore, an increasing function of productivity and demand. Substituting this optimal level of labour demand back into the firm's profit function (given by equation four) we obtain that the firm's profits in each period will be determined by:

$$\pi_\tau = Z_\tau\left(\frac{A_\tau}{w}\right)^{\frac{\mu}{\mu-1}}\left[\mu^{\frac{1}{1-\mu}}\left(\frac{1}{\mu}-1\right)\right]. \quad 7)$$

Recall that in the absence of adjustment costs the firm's value function is given by:

$$E\left\{\int_t^\infty e^{-(r+\delta)(\tau-t)}\left[\pi_\tau d\tau\right]\right\} =$$

which from equation seven is equal to:

$$= \mu^{\frac{1}{1-\mu}} (\frac{1}{\mu} - 1) \int_t^\infty e^{-(r+\delta)(\tau-t)} \mathrm{E}\{Z_\tau\} \left(\frac{A_\tau}{w}\right)^{\frac{\mu}{(\mu-1)}} d\tau$$

$$= \mu^{\frac{1}{1-\mu}} (\frac{1}{\mu} - 1) \left(\frac{A_t}{w}\right)^{\frac{\mu}{(\mu-1)}} \int_t^\infty e^{-(r+\delta)(\tau-t)} e^{\vartheta_a(\tau-t)\frac{\mu}{(\mu-1)}} \mathrm{E}\{Z_\tau\} d\tau, \qquad 8)$$

where $\mathrm{E}\{Z_\tau\}$ is the expected value of the mixed Brownian-Poisson process that demand strength is assumed to follow.

In order to evaluate the above function we need therefore to determine $\mathrm{E}\{Z_\tau\}$. To do so, we can make use of the definition of η_τ to rewrite the process followed by Z as:

$$\frac{dZ_\tau}{Z_\tau} = \vartheta_Z dt + \sigma_Z dW_\tau + \frac{1}{1-\mu}\left(\frac{\eta_0}{\eta_\tau} - 1\right) dq .$$

Thus demand will be at its equilibrium level, in the sense that jumps in demand do not occur, when $\eta_0 = \eta_\tau$. We can also make us of the fact that in the no-adjustment cost scenario the profit maximising condition implies that $\eta_\tau = w$ at all times, to set $\eta_0 = w$ and to then interpret η_o as the equilibrium value of η_τ. In other words the level it would obtain in the absence of adjustment costs. The process followed by Z can therefore he rewritten as:

$$\frac{dZ_\tau}{Z_\tau} = \vartheta_Z dt + \sigma_Z dW_\tau + \frac{1}{1-\mu}\left(\frac{\eta_0}{w} - 1\right) dq . \qquad 3a)$$

It follows then, that if $\eta_0 \neq w$ there exists the possibility that a jump in Z will occur, which will be negative or positive depending on whether $\eta_0 < w$ or $\eta_0 > w$.

Following Dixit and Pindyck (1994) chapter 3, p.86, $\mathrm{E}\{Z_\tau\}$ is the solution to the following stochastic differential equation:

$$\frac{E\{dZ_\tau\}}{Z_\tau}\frac{1}{d\tau} = \vartheta_Z + \frac{\rho}{1-\mu}\left(\frac{\eta_0}{w} - 1\right).$$

The solution to which is given by:

$$\mathrm{E}_t\{Z_\tau\} = Z_t e^{\left(\vartheta_Z + \frac{\rho}{1-\mu}\left(\frac{\eta_0}{w}-1\right)\right)(\tau-t)} .$$

Thus if $\eta_0 < w$ ($\eta_0 > w$) the expected level of Z at some point in time after time τ will be lower (higher) than that which prevails in the Bentolila and Bertola model. Substituting the above solution back into equation eight we have that in the absence of adjustment costs the firm's value function will be given by:

$$V = \mu^{\frac{1}{1-\mu}}\left(\frac{1}{\mu} - 1\right)\left(\frac{A_t}{w}\right)^{\frac{\mu}{\mu-1}} \int_t^\infty e^{-(r+\delta-\vartheta_a\frac{\mu}{\mu-1})(\tau-t)} Z_t e^{\left(\vartheta_Z + \frac{\rho}{1-\mu}\left(\frac{\eta_0}{w}-1\right)\right)(\tau-t)} d\tau,$$

$$= \mu^{\frac{1}{1-\mu}}\left(\frac{1}{\mu}-1\right)\left(\frac{A_t}{w}\right)^{\frac{\mu}{\mu-1}} Z_t \int_t^{\infty} e^{\left(\vartheta_z + \frac{\rho}{1-\mu}\left(\frac{\eta_0}{w}-1\right) + \vartheta_a \frac{\mu}{\mu-1} - r - \delta\right)(\tau - t)} d\tau,$$

$$= \mu^{\frac{1}{1-\mu}}\left(\frac{1}{\mu}-1\right)\left(\frac{A_t}{w}\right)^{\frac{\mu}{\mu-1}} Z_t \left(r+\delta+\frac{1}{1-\mu}\left(\mu\vartheta_a - \rho\left(\eta_0/w - 1\right)\right) - \vartheta_z\right)^{-1}, \qquad 8a)$$

which is bounded as long as $r+\delta+\frac{1}{1-\mu}\left(\mu\vartheta_a - \rho\left(\eta_0/w - 1\right)\right) - \vartheta_z > 0$.

The Firm's Optimisation Problem in the Presence of Linear Adjustment Costs:

We now turn to examine how the presence of linear asymmetric adjustment costs affects this optimal labour demand strategy. To do so, we assume that each addition to the work force by the firm incurs a hiring cost, H and that subsequent redundancies or other such contract terminations instigated by the firm incur a firing cost, F, with both H and F being payable by the firm[3]. The existence of adjustment costs requires that the firm's value function be modified in the following manner:

$$V_t = \underset{[X_\tau]}{Max}\, \mathrm{E}\left\{\int_t^{\infty} e^{-(r+\delta)(\tau-t)}\left[\pi_\tau d\tau - (1_{[dX>0]}H - 1_{[dX<0]}F)dX_\tau\right]\right\}, \qquad 9)$$

where the function $1_{[\cdot]}$ is equal to 1 when the firm is hiring, i.e. $dX_t > 0$, or firing in which case $dX_t < 0$. It is important to note however that, as the adjustment costs are a piece wise function, the cumulative labour process $\{X_t\}$ fails to be differentiable. Thus the adjustment costs component of the integral must be expressed in terms of Riemann-Stiltjes integrals.

Partially differentiating equation nine with respect to L in order to determine the optimal labour demand rule now yields the following three first order conditions, which must be valid at all times if the firm is in fact maximising its value function:

$$\mathrm{E}_t\int_t^{\infty}\{(\eta_\tau - w)e^{-(r+\delta)(\tau-t)}d\tau\} = -F \qquad \text{if } dX_t < 0, \qquad 10a)$$

$$-F < \mathrm{E}_t\int_t^{\infty}(\eta_\tau - w)e^{-(r+\delta)(\tau-t)}d\tau < H \qquad \text{if } dX_t = 0, \qquad 10b)$$

$$\mathrm{E}_t\int_t^{\infty}(\eta_\tau - w)e^{-(r+\delta)(\tau-t)}d\tau = H \qquad \text{if } dX_t > 0, \qquad 10c)$$

These conditions imply that if the firm is for example, considering whether to hire an additional worker it should calculate the discounted $E\{\eta\}$ he would provide in order to determine how this compares with

[3] F can be thought of as representing the severance costs imposed by firms legislation when dismissing a worker and H, as the screening and training costs associated with the recruitment of a new worker.

the wage it will have to pay the worker, discounted back to the current time period, plus any costs associated with the hiring. If the $E\{\eta\}$ is less than the discounted wage, plus the hiring costs then an optimising firm will not hire, since the costs associated with increasing the work force by one unit are greater than the additional revenue the extra worker brings to the firm.

Although intuitively obvious it is important to note the following. Firstly, in this analysis the two optimal trigger functions interact. In that increases in the cost of, for example, dismissing a worker affect the value of the hiring trigger point, since higher costs of abandonment makes firms more cautious about hiring. Similarly, reductions in the cost of hiring a worker will decrease the value of the firing trigger point. Secondly, the optimal trigger values and their Marshallian counterparts are not equivalent. In this framework, both the hiring and firing trigger points have higher and lower respective values than the Marshallian trigger conditions.

It should also be evident that the optimising firm will not, given that the total cost of employment adjustment is linear in the hiring and firing costs, continuously act on the margin with respect to its employment decisions. Instead the optimal hiring and firing policy is discontinuous. In some periods the optimal policy for the firm will be to fire or hire, thus the optimal labour demand will be given by the solution to either 10a or 10c depending on which condition is satisfied. Under other demand conditions the optimal strategy for the firm may be to take no action at all, but to instead allow the size of its work force to decline gradually through natural wastage. More specifically, inaction will be chosen when deviations of the expected marginal product of labour from its instantaneous maximum do not justify the costs of employment adjustment. Adjustments to its work force will only be observed when deviations in the expected marginal revenue product of labour from the optimal level are large enough to compensate for the hiring and firing costs.

Hiring and firing costs result therefore in the existence of a corridor within which the optimal policy for the firm is not to react to changes in demand. Employment within this region being determined by the previous period's employment level and the quit rate. As in the Bentolila and Bertola analysis this region is identified by a constant upper, u, and a lower, d, 'control barrier'. If the marginal revenue product of labour falls below the lower control barrier in response to say, an adverse shock the optimising firm will fire workers in order to raise $\{\eta\}$ back to the level of the lower control barrier. Similarly, if $\{\eta\}$ rises above the upper control barrier the firm will hire in order to restore $\{\eta\}$ back to the upper control barrier level.

To determine the optimal labour demand policy of the firm one needs therefore to identify this no-action region, this involves calculating the optimal upper and lower control barriers as functions of the parameters of the model. Thus we need to find an explicit expression for the integral in the first order conditions (FOCs) given by equations 10a to 10c. To find this, we can in the first instance rewrite these conditions exploiting the fact that wage rate is a constant. Doing so, condition 10a becomes:

$$E_t\int_t^{\infty}\eta_\tau e^{-(r+\delta)(\tau-t)}d\tau - E_t w\int_t^{\infty}e^{-(r+\delta)(\tau-t)}d\tau = -F,$$

$$E_t\int_t^{\infty}\eta_\tau e^{-(r+\delta)(\tau-t)}d\tau = w\left[-\frac{1}{r+\delta}e^{-(r+\delta)(\tau-t)}\right]_t^{\infty} - F,$$

$$E_t\int_t^{\infty}\eta_\tau e^{-(r+\delta)(\tau-t)}d\tau = \frac{w}{r+\delta} - F, \qquad \text{If } dX_t < 0. \qquad 10a')$$

Similarly, condition 10c can be re-written as:

$$E_t\int_t^{\infty}\eta_\tau e^{-(r+\delta)(\tau-t)}d\tau - E_t w\int_t^{\infty}e^{-(r+\delta)(\tau-t)}d\tau = H,$$

$$E_t\int_t^{\infty}\eta_\tau e^{-(r+\delta)(\tau-t)}d\tau = \frac{w}{r+\delta} + H, \qquad \text{If } dX_t > 0. \qquad 10c')$$

Now we simply have to evaluate the integrals on the left hand side of equation 10a' and 10c' taking into account the fact that η_τ is not free to move, but is instead "regulated" at points d and u so that the stochastic process, η, never goes beyond the control barriers.

When the firm is neither hiring or firing the marginal revenue product of labour evolves according to a mixed Brownian-jump process. Applying Ito's lemma for a mixed Brownian-Jump Process (see Dixit and Pindyck, chapter three, page 87) we can show that in the absence of regulation the marginal revenue product of labour evolves, given that $\eta = F(A,Z,L)$, according to the following process:

$$d\eta = F_A dA + F_Z dZ + F_L dL + \frac{1}{2}F_{ZZ}(dZ)^2$$

$$= \mu^2 A^{\mu-1}Z^{1-\mu}L^{\mu-1}\vartheta_a A d\tau + \mu(1-\mu)A^{\mu}Z^{-\mu}L^{\mu-1}\cdot\left[\vartheta_Z Z d\tau + \sigma_Z Z dW_\tau + \frac{Z}{1-\mu}\left({}^{\eta_0}\!/_{\eta} - 1\right)dq\right]$$

$$-\mu(-1+\mu)A^{\mu}Z^{1-\mu}L^{\mu-2}\delta L d\tau - \frac{1}{2}\mu^2(1-\mu)A^{\mu}Z^{-\mu-1}L^{\mu-1}Z^2\sigma_z^2 d\tau$$

$$= \mu\eta\vartheta_a d\tau + (1-\mu)\eta\vartheta_Z d\tau + (1-\mu)\eta\sigma_Z dW_\tau + \eta\frac{\eta_0-\eta}{\eta}dq + (1-\mu)\eta\delta d\tau - \frac{1}{2}\mu(1-\mu)\eta\sigma_z^2 d\tau$$

$$= \eta\left[\mu\vartheta_a + (1-\mu)\left(\vartheta_z + \delta - \frac{\mu\sigma^2}{2}\right)\right]d\tau + \eta(1-\mu)\sigma_z dW_\tau + (\eta_0 - \eta)dq$$

$$\equiv \eta\vartheta_\eta dt + \eta\sigma_\eta dW + (\eta_0-\eta)dq,$$

where ϑ_η and σ_η are constant.

We need therefore to find the value of $\{\eta\}$, which solves the FOCs given by equations 10a' and 10c' under the assumption that $\{\eta\}$ follows the above process. To determine this value of η we are able to evaluate the integrals which appear in the FOCs using a similar argument to that developed in Bertola and Bentolila page p.386-7, modifying the procedure slightly to allow us to consider a mixed Brownian-jump process. More specifically we make use of the following mathematical result (the proof of which can be found in the appendix) that using the regulated mixed Brownian-jump process $\{\xi_t\}$, with starting point value $\bar{\xi}$ at time 0, whose dynamics in the absence of regulation are characterised by: $d\xi = \vartheta\xi_t dt + \sigma\xi_t dW_t + (\xi_0 - \xi_t)dq$ with an upper control barrier at u and a lower control barrier at d; then:

$$E_0\left\{\int_0^\infty \xi_t e^{-\lambda t}dt; \bar{\xi}, \xi_0, u, d\right\},$$

$$= \frac{\bar{\xi}}{\lambda+\rho-\vartheta} + \frac{B_1\bar{\xi}^{\alpha_1}}{(\lambda+\rho-\vartheta)} + \frac{B_2\bar{\xi}^{\alpha_2}}{(\lambda+\rho-\vartheta)},$$

$$+\frac{\rho}{(\lambda+\rho-\vartheta)\lambda}\left(\xi_0 + B_1\xi_0^{\alpha_2} + B_2\xi_0^{\alpha_2}\right),$$

$$\equiv f\left(\bar{\xi}, \xi_0; u, d, \vartheta, \sigma, \lambda\right), \qquad 11)$$

where α_1 and α_1 are the positive and negative roots of the quadratic equation:

$$\left(\frac{\sigma^2}{2}\right)\alpha^2 + \left(\vartheta - \left(\frac{\sigma^2}{2}\right)\right)\alpha = 0$$

and

$$B_1 = \frac{d^{\alpha_2}u - du^{\alpha_2}}{\alpha_1\left(d^{\alpha_1}u^{\alpha_2} - d^{\alpha_2}u^{\alpha_1}\right)},$$

$$B_2 = \frac{d^{\alpha_1}u - du^{\alpha_1}}{\alpha_2\left(d^{\alpha_2}u^{\alpha_1} - d^{\alpha_1}u^{\alpha_2}\right)}.$$

The Determination of the Optimal Hiring and Firing Trigger Points:

We can, using the mathematical result of the previous section, evaluate the marginal revenue product of labour $\{\eta\}$ in the same manner in order to determine the optimal hiring and firing trigger points. We know that at the lower control barrier d, $\eta = d$. Similarly at the upper control barrier u, $\eta = u$. We can therefore proceed by substituting the values of the constants of integration B_1 and B_2 into the general solution to our differential equation (equation A.7) for $\{\xi_t\}$. Setting $\{\xi_t\} = \{\eta_t\}$, $\lambda = r + \delta + \rho$, $\vartheta = \vartheta_\eta$ and $\sigma = \sigma_\eta$ and substituting our general solution into the first order conditions 10a and 10c we obtain the following two conditions, which have to be satisfied at points *d* and *u* in order for the firm to be willing to fire and hire respectively:

$$f(d, \xi_0; u, d, \vartheta_\eta, \sigma_\eta, r + \delta + \rho) = \frac{w}{r + \delta} - F, \qquad \text{10a'')}$$

$$f(u, \xi_0; u, d, \vartheta_\eta, \sigma_\eta, r + \delta + \rho) = \frac{w}{r + \delta} + H. \qquad \text{10c'')}$$

Once the function *f* is known, these equations can be used to solve for the control barriers *u* and *d* for given values of the hiring and firing costs *H* and *F*[5]. Having obtained numerical values for *u* and *d*, the optimal hiring and firing trigger points L_H and L_F can then be determined as functions of both demand and labour productivity. Inverting the MRPL and evaluating it at $\eta_\tau = d$ and $\eta_\tau = u$, we obtain the following two equations:

$$L_H = Z_t \left(\frac{\mu}{u}\right)^{\frac{1}{1-\mu}} A_t^{\frac{\mu}{1-\mu}}, \qquad \text{12a)}$$

$$L_F = Z_t \left(\frac{\mu}{d}\right)^{\frac{1}{1-\mu}} A_t^{\frac{\mu}{1-\mu}}, \qquad \text{12c)}$$

which describe the optimal labour demand policy of the firm.

In figure 3.3 we plot the firm's optimal hiring and firing points for a given level of demand and productivity under the assumption that demand strength fluctuates over time according to a mixed Brownian-jump process. For comparative purposes, we also illustrate the optimal hiring and firing trigger points under the Bentolila and Bertola assumption that demand strength evolves according to a geometric Brownian Motion (given by the long discontinuous lines) and for the no-adjustment cost case $H = F = 0$ (the short discontinuous line).

In contrast to the case in which there are no costs associated with employment adjustment, we see that the presence of adjustment costs results, as in the Bentolila and Bertola analysis, in a no-action region within which the optimal policy for the firm is not to adjust the size of its work force in response to changes in demand conditions. Employment within this region being instead determined by the employment level of the previous period and the number of quits. Adjustments in employment will only be observed, when changes in demand result in a movement of the marginal value product of labour to either the hiring or firing trigger points. It is interesting to note however, that the fact that an employer merely *'perceives'* there to be the possibility of a sudden change in demand strength at some point in

[5] As the solution to this function is highly non-linear, it has been obtained numerically using the computer programme Mathematica, version 2.2.

the future has a fundamental impact on the dynamic behaviour of employment, since in our model it results in a considerable widening of the no-action corridor[6].

Demand can therefore fluctuate considerably more in our analysis than in the Bentolila and Bertola framework without warranting changes in employment adjustment. Consequently, employment will be seen to exhibit a much higher degree of persistence in environments in which firms perceive prevailing demand conditions to be "transitory", in the sense that there exists the possibility that the present state of nature will be subject to frequent change. Moreover, as figures 3.3 and 3.4 illustrate the more "transitory" employers perceive present demand conditions to be in terms of the value given to ρ, the wider the no-action corridor becomes. Conversely, smaller values of ρ, in other words a lower average arrival rate of jumps result in a shrinking of the width of the corridor. Thus for very small values of ρ our model converges, as figure 3.4 illustrates, towards that of Bentolila and Bertola and a much lower degree of persistence is observed[7]. In the first instance then, our analysis illustrates that the assumptions made about the type of uncertainty the firm faces or more specifically, the process utilised to model demand uncertainty in this type of analysis has, in the presence of adjustment costs, important ramifications for the dynamic behaviour of employment.

The emergence of persistence within this framework in the aftermath of say, a negative demand shock, can be easily seen from the following example. Suppose that demand conditions were relatively buoyant prior to the shock, so that the MRPL of the marginal worker was such that the firm was located close to the hiring line. In contrast to the no-adjustment cost case where a negative shock would result in the instant dismissal of this worker, in our scenario the adverse shock may simply reduce the level of MRPL so that the firm shifts back towards the firing barrier. As the optimal labour demand policy for a firm located within the no-action barrier is to maintain its present employment level, one does not observe any employment adjustment in response to this negative shock. Moreover, as is evident from the width of the no-action corridor in figure 3.3 it can, particularly in more uncertain environments, take a considerable negative demand shock in order for firings to occur. The degree of persistence observed will therefore be dependent on the length of time it takes the firm to reach the firing barrier, which will itself will be a function of: a) the width of the corridor; and b) the location of the firm inside the corridor prior to the shock.

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[6] Note that this result is driven by the fact that employers *perceive* there to be the possibility of a discrete change in demand and is not therefore contingent on the change being realised.

[7] For comparative purposes figures 3.3 and 3.4 illustrate what could be considered, in terms of the value given to ρ, two polar cases. Since. $\rho = 0.1$ in our empirical work would imply that the firm perceived there to be a possibility of a change in economic conditions to be approximately 10 years. In contrast a value of 1.5 implies a change within eight months. Given the considerably shorter length of business cycles in the 1980's it seems reasonable to argue that the reality for this period lies somewhere in between these two extremes. For illustrative purposes however, this distinction is retained in the proceeding analysis.

An interesting implication of our analysis which emerges from figures 3.3 and 3.4 is that the possibility of a discrete change in future demand strength appears to have, in the presence of linear adjustment costs, an asymmetric impact on the optimal hiring and firing trigger points. More specifically, when the firm thinks that future demand is likely to be subject to more frequent change firing costs appear to have considerably more of an impact on the optimal demand policy of the firm, as reflected in a substantial upward shift of the firing line in more uncertain economic environments. In contrast changing the assumption as to the process which demand strength follows does not appear to have a significant impact on the firm's willingness to hire. In an increasingly transitory economic environment the optimal employment strategy would appear then, to imply a greater degree of labour hoarding over the business cycle. For, although greater uncertainty makes the firm more reluctant to hire, with the optimal policy stating that the firm should adjust less in the face of changing economic conditions given that it thinks demand strength is likely to change again suddenly in the near future, the positive employment effect (which arises from the upward shift in the firing function) would appear to offset any deterrent effect that this increase in uncertainty has on the hiring schedule. Naturally, the overall extent of these effects will be dependent on both the degree of risk aversion and the discount rate.

4 Comparative Static Analysis:

Having illustrated that allowing for the possibility of transitory economic conditions has important ramifications for the dynamic behaviour of employment, we proceed in this section to use the theoretical model derived above to carry out a number of comparative static analyses in order to examine how changes in factors, argued to be more relevant to a firm's labour demand decision in the 1980's, affect the firm's willingness to hire and fire via their impact on the position of the optimal hiring and firing trigger schedules and thus ultimately the dynamic behaviour of employment.

4.1 Effects of Changes in the Level of Firing Costs on the Firm's Willingness to Hire and Fire:

Despite the fact that liberalisation of labour markets has ranked highly in the policy debate, few effective changes to the stringent nature of the employment constraints facing European firms appear to have been implemented over the last decade. The most recent European Commission report on the developments in and effects of employment protection legislation during the 1980's illustrates that despite the continual emphasis on the need for greater labour market flexibility only three countries can be considered to have weakened a number of employment protection regulations, with individual member states instead attempting to introduce greater labour market flexibility by loosening the regulations concerning the use of temporary contracts[8]. Moreover, in a number of the European countries the general trend toward greater employment protection would actually appear to have continued.

It would appear interesting given these developments and the predominance of the firing cost debate to examine the impact of changes in the level of firing costs under our assumption as to the way uncertainty enters the model in our model, to those attained when variations in demand strength are assumed to follow a geometric Brownian motion. For, it maybe the case that changes in the level of firing costs and increased uncertainty with respect to prevailing economic conditions has had an important impact on the evolution of employment during the 1980's. Moreover, given that the data illustrates marked differences in the evolution of employment between economies which are either tightly or loosely regulated in terms of employment protection legislation, our analysis is undertaken for both high and low adjustment cost regimes using the values calculated by Bentolila and Bertola for the level of firing costs in France and the UK, assuming the former to be an example of a higher cost regime and the latter a lower[9]. Throughout our empirical work we also make the more 'realistic' assumption for Europe that the level of firing costs is greater than that of hiring costs.

A priori the impact of higher firing costs on employment levels is somewhat ambiguous. For, on the one hand one would expect higher firing costs to have a positive impact on the employment level within the firm, since at the time of firing costs will be higher implying a lower marginal revenue product of labour and thus that less firing takes place. This positive impact may however, be offset if increases in firing costs have even more of an adverse impact on the firm's willingness to hire.

As is evident from the results of our comparative static analysis reported in the table 4.1 the effects of an increase in the level of firing costs on the optimal employment strategy appear to be dependent on: a) the prevailing level of uncertainty with respect to the nature of current economic conditions; and b) the original level of firing costs. More specifically, increases in the level of firing costs in an economy characterised by a relatively low level of both firing costs and uncertainty appear to have only a marginal impact on the firm's optimal employment strategy (see figure 4.1). Resulting in an upward shift of the firing function of approximately 2.6% and a downward shift in the hiring function of a mere

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[8] Note however, that whilst these contracts have undoubtedly increased the ability of employers to adjust the size of their work force in response to changing market conditions, it is only in the UK that the use of use of such contracts is not subject to tight regulatory controls.

[9] Bentolila and Bertola provide estimates of the expected cost of dismissing a worker based on the estimation of the following relationship: $F = N + (1 - P_a)SP + P_a\{(1 - P_u)(SP + LC) + P_u(UP + LC)\}$, where: N represents the legal minimum period of notice requirement; P_a, the probability that the worker will bring an unfair dismissal claim against his employer before the judicial system; P_u, the probability that an unfair dismissal ruling is given against the firm; *SP*, legally required severance pay; *LC*, the legal costs involved in any contested dismissal; and *UP*, payment made with respect to any unfair dismissal ruling. This estimation yields a value of 0.687 over the period 1975 to 1979 and 0.916 for France, over the period 1980 to 1986. The equivalent figures for the UK being 0.187 and 0.250 respectively. It should be noted however, as the authors themselves stress that these figures should be taken, given the nature of the data as *"ball park"* figures only.

0.5%. Whilst an increase in firing costs in an environment characterised by low costs, but a higher level of uncertainty results in a somewhat larger upward shift in the firing barrier, but has an even smaller impact on the hiring schedule. Resulting therefore, in a more noticeable increase in the width of the no-action corridor.

Table 4.1:

Percentage Change in the Location of the Optimal Hiring and Firing Schedules in Response to an Increase in the General Level of Firing Costs.

	Less Uncertain Regime		More Uncertain Regime	
	L_F	L_H	L_F	L_H
Low Cost Environment	2.6	-0.53	7.7	-0.19
High Cost Environment	9.7	-0.28	73.4	0

In common with other studies our results indicate that an increase in firing costs has an asymmetric impact on the hiring and firing schedules. This result should not however be that surprising. For, as earlier work has shown (see for example, Bentolila and Bertola (1990) and Bertola (1990)) increases in firing costs have a stronger impact on the firm's firing decision than on its hiring decision if the firm realistically discounts future revenues, since at the time the firing decision takes place firing costs are neither uncertain or discounted. Thus, although higher firing costs cause the firm to be more reluctant to hire and therefore imply an adverse employment effect, this adverse impact is actually offset due to the myopic nature of the firm's employment decision, by the fact that the increase in costs discourages firing by more than it does hiring. Moreover, the more the firm discounts into the future and the less certain it is about whether or not they will have to fire the worker, the stronger will be their impact on the optimal firing schedule and the larger will be the positive impact of firing costs on employment.

Somewhat more interesting results emerge from our analysis when we look at the interaction of changes in the level of firing costs in the so-called more uncertain environments. For, whilst an increase in the level of firing costs has a much more pronounced effect on the optimal employment strategy in both high and low cost regimes, such an increase appears to have a somewhat *"perverse"* impact on the optimal hiring and firing schedules in the high cost environment. In that, although as one might expect, it significantly increases the reluctance of firms to fire in response to fluctuating demand conditions, it appears to have no impact at all on the firm's willingness to hire as reflected in the position of the optimal hiring line and thus results in a substantial widening of the no-action region (see figure 4.2). This lack of response of the hiring function to a change in the level of firing costs arises simply because larger values of ρ result in a lower value of the final term on the right hand side of the hiring condition.

Turning our analysis on its head somewhat our results would also tend to imply that a reduction in the general level of firing costs in a more uncertain environment would, ceteris paribus, result in a considerable lowering of the firing barrier and thus in firms being prepared to fire much more quickly in response to adverse changes in demand conditions. Such a scenario would in fact appear to offer one explanation for why we do not observe large numbers of dismissals in the UK, particularly in the manufacturing sector in the aftermath of the second oil shock. It is only in the post-1982 period after the relaxation of firing constraints, which would have resulted in a downward shift of the firing schedule that we observe the fall out in employment that might otherwise have been expected given the prevailing economic conditions of the late 1970's and early 1980's.

Our somewhat simplistic empirical analysis would appear to indicate then, that changes in the level of firing costs *per se* may not be the most relevant issue on which to focus. For, what appears to be a more relevant factor to an explanation of the evolution of employment, but which has been somewhat overlooked by the literature, is the interaction between firing costs and the degree of uncertainty, and in particular, whether this interaction takes place in a high or low cost environment.

An additional appealing facet of this type of analysis is that it would also appear capable of explaining the behaviour of employment in the US over the last decade. For, despite some initial hesitancy in hiring in the US (reflected in a marked increase in the employment of temporary staff and the use of overtime) during the initial upswing of the early 1980's and 1990's recovery, the persistence effects associated with an adverse demand shock have clearly been much higher in Europe than in the US, where we have seen much more of a sustained increase in employment levels during the 1980's. Whilst it may not really be justifiable to argue that economic conditions have been significantly less uncertain in the US, the simple fact that the US labour market is much less regulated than those of the EC would appear to have had important consequences the dynamics of employment[10]. What may be particularly relevant to the lower levels of employment persistence and the more rapid pick up in employment in the 1980's is the fact that that employment decisions in some sectors, most notably the unskilled and lower skilled service sector positions where a large part of the rapid employment growth of the 1980's has been concentrated, are not, given that hiring and firing costs associated with such positions would appear to be negligible, subject to anywhere near the degree of irreversibility that European employment decisions are. Consequently, the hiring and firing barriers in these sectors will be located extremely close to the no-adjustment cost line. Other things being equal, one would therefore expect the persistence effects of

[10] Whilst one cannot argue that firms in the US operate in a no-adjustment cost environment, since they still have to pay costs of hiring and training, it is a widely accepted fact that dismissal costs are relatively insignificant compared to European levels. The industrial relations system in the US being dominated by the common law principle "hire at will" incorporated into the Labour Relations Act 1935, which stipulates that a firm in the private sector may dismiss a worker at anytime and for any reason without being legally obliged to give a minimum period of notice or pay a statutory predetermined minimum level of dismissal costs to its workers.

demand shocks to be significantly dampened in these sectors, to observe relatively more employment adjustment and a much lower degree of employment persistence[11].

The interaction between the level of firing costs and the degree of uncertainty cannot however, be the whole story behind the inferior employment experience of the member states during the 1980's, given that firing costs have actually been reduced in a number of European Community countries over recent years[12]. Moreover, in countries such as France and Spain considerable (if somewhat erratic) relaxation of regulations regarding the use of non-permanent contracts, which are not subject to the stringent employment regulations, have also been introduced. One would have expected, given the implications of our previous analysis, to have observed a sustained employment growth in those countries which in significant legislative changes have been made. Yet with the exception of the UK, these countries have been amongst the lowest performers in the Community in terms of employment creation, let alone sustained employment growth. This would appear to be particularly true of France, which despite a significant relaxation of firing restraints and a marked improvement in the macroeconomic climate from early 1986, has had one of the weakest job creation performances during the latter half of the 1980's.

More importantly however, the results of our firing cost/uncertainty scenario fail to account for the seemingly increased reluctance on the part of firm's to expand their permanent work force during most of the 1980's, since their impact on the firm's willingness to hire, as reflected in the position of the optimal hiring barrier, is found to be minimal. Such a result suggests that other factors may have become more relevant to the firm's labour demand decision than previously thought during the 1980's.

One explanation for the apparent reluctance of firms to hire during an upturn in economic activity is that: a) a number of legislative changes with regards to recruitment practices; b) developments in the system of wage negotiations; and c) the build up in unemployment and in particular long term unemployment, have resulted in a considerable increase in hiring costs and that these developments may have become a more important deterrent to hiring during this period of increased uncertainty, than changes in the already high level of firing costs. The most logical progression (and the one taken in the following section) would appear to be therefore, to extend our analysis further in order to examine the impact of such increases on the optimal employment strategy.

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11 What may also be an equally relevant factor to the high levels of employment growth experienced by the States in the early 1980's, but which is often overlooked in the literature is the fact that in the early 1980's firms were offered considerable fiscal incentives by the Reagan administration in order to encourage employment growth, which would have undoubtedly reduced the cost of hiring to firms.

12 In an attempt to gain greater labour market flexibility the British Government repealed a considerable amount of regulation concerning a firm's obligations to its workers in terms of redundancy payments from 1981 onwards. Similarly, in 1986 the French repealed legislation obliging firms to obtain permission from the Public Authorities for all economic dismissals. Prior to this, all dismissals for economic reasons required the direct authorisation of the regional labour office. The director of which had the power not only delay the dismissal procedure, but actually to prevent the dismissals from taking place. Although outright prevention was not frequently used, the major obstacle to mass dismissals was the procedure which firms had to go to in order to obtain authorisation, which was extremely costly and time consuming.

4.2 The Impact of Changes in Hiring Costs on the Optimal Trigger Points:

As already discussed the impact of firing costs on a firm's labour demand decision has tended to dominate the adjustment costs literature with only a passing reference, if any, being made to the role of hiring costs. We have two objections with regards to the appropriateness of this approach. Firstly, and perhaps more fundamentally, explanations for the differing employment and unemployment experiences of OECD countries based solely on higher firing costs are, as already mentioned, unable to explain why countries such as the Scandinavian ones have experienced superior employment performances over the 1970's and 1980's. Secondly, it would appear that the rise in hiring costs has, with increases in the stock of unemployment and the emergence of a body of long term unemployment undoubtedly raising the signal extraction problem facing European firms in the 1980's, been significantly larger than that of firing costs.

It should also be remembered that recruitment costs are only part of the costs that firms incur when recruiting additional workers. For, once a new employee has been hired, a firm then incurs costs of training and costs associated with loss of output during this training period. Moreover, in some countries, such as in Italy and to a lesser extent France where recruitment is channelled through national employment agencies, firms are not always free to hire the workers they want. In these countries the unemployed tend to be ranked according to a number of social criteria, such as marital status, number of dependants, length of unemployment duration. Criteria, which are used to calculate the individual's need for employment, as opposed to their suitability for available vacancies in terms of qualifications and willingness to work. Employers are then required to make at least some of their new hires from this ranking. Obviously the use of such a procedure does not always result in the best candidate being selected for the jobs. In fact, given the nature of the ranking system, it would appear more rational to assume that more often than not the best candidate is not selected. Costs in terms of training and lost output would therefore be expected to be considerably higher in: a) those economies in which such systems operate; and b) times of high unemployment and an increasing incidence of long term unemployment, given that it would appear likely that those individuals with longer unemployment duration will be positioned higher in the ranking order. Human capital and duration dependence arguments would imply that such individuals are likely to prove most costly to the firm in terms of training and output lost during training.

As one would expect, given that at the time of hiring these costs are neither uncertain or discounted, allowing for an increase of approximately 25% in the level of hiring costs has a direct impact on the hiring schedule[13]. Increases in their value resulting in the firm becoming less willing to hire in response

[13] This figure has, as in the Bertola and Bentolila analysis, been taken from Nollen (1987) and is based solely on training costs.

to changing economic conditions, with this increased hesitancy being reflected in a downward shift in the optimal hiring line. Once again however, it is evident from the results of table 4.2 that the interaction between changes in the level of hiring costs and the degree of uncertainty has important ramifications for employment dynamics. For, although an increase in hiring costs results, regardless of whether the firm is located in a high or low cost environment, in the firm becoming less willing to hire in response to changing economic conditions, this effect is greater in the more uncertain economic environment.

Table 4.2:

The Percentage Change in the Location of the Optimal Hiring and Firing Schedules in Response to an Increase in the General Level of Hiring Costs:

	Less Uncertain Regime		More Uncertain Regime	
	L_F	L_H	L_F	L_H
Low Cost Environment	1.4	-2.9	6.4	-5.6
High Cost Environment	0.9	-2.5	18.2	-5.5

Although the effects of an increase in the general level of hiring costs on the hiring decision are clearly evident from the FOC (given by equation 10c), their impact on the firing decision is not so readily apparent. To see this more clearly we can however, rewrite the FOC 10a using a similar argument to that developed in Bentolila and Bertola (p.391), so that hiring costs appear explicitly in the firing condition. To do so we assume: a) that the firm is at period t, a point in time at which the marginal worker has just be fired; and b) that T is the first moment in time after period t, in which demand strength is such that the firm considers hiring an additional worker. The firing condition can therefore be rewritten in the following manner:

$$E_t\left\{\int_t^T \eta_\tau e^{-(r+\delta+\rho)(\tau-t)}d\tau\right\}+E_t\left\{e^{-(r+\delta+\rho)(T-t)}\int_T^\infty \eta_\tau e^{-(r+\delta+\rho)(\tau-T)}d\tau\right\}=\frac{w}{r+\delta}-F,\ \text{if } dX_t<0. \quad 13)$$

We can then proceed, given that the strong Markov property of the regulated mixed Brownian-jump process implies that T and η_τ are independent, to separate the second term in parenthesis into the product of the two expectations. Rearranging and taking iterated expectations we obtain:

$$E_t\left\{\int_t^T \eta_\tau e^{-(r+\delta+\rho)(\tau-t)}d\tau\right\}$$
$$=\frac{w}{r+\delta}-F-E_t\left\{e^{-(r+\delta+\rho)(T-t)}\right\}E_t\left\{E_T\left\{\int_T^\infty \eta_\tau e^{-(r+\delta+\rho)(\tau-T)}d\tau\right\}\right\}. \quad 14)$$

As T is a hiring time we know from condition 10c that:

$$E_T\left\{\int_T^\infty \eta_\tau e^{-(r+\delta+\rho)(\tau-T)}d\tau\right\}=\frac{w}{r+\delta}+H.$$

Substituting the right hand side of which back into equation 14 the firing condition becomes:

$$E_t\left\{\int_t^T \eta_\tau e^{-(r+\delta+\rho)(\tau-t)}d\tau\right\}$$

$$=\frac{w}{r+\delta}\left(1-E_t\left\{e^{-(r+\delta+\rho)(T-t)}\right\}\right)-F+E_t\left\{e^{-(r+\delta+\rho)(T-t)}\right\}H, \qquad dX_t<0,\ dX_T>0,\ T>t.$$

Thus the firm fires the marginal worker, knowing that future demand conditions could be such that it becomes profitable for it to rehire him again at the random time T, if the loss the firm would incur between t and T (given by first term on the right hand side of the firing condition) if it did not fire the worker plus the future cost of rehiring the worker at T are, in expected discounted terms, equal to the current firing costs. Increases in the level of hiring costs will result therefore, in the firm becoming more reluctant to fire. Moreover, a higher degree of demand uncertainty, determined by a value of $\rho>0$, will lower both: a) the expected value of the loss incurred by the firm if the marginal worker is not fired; and b) the present value of future rehiring costs, thereby making, as is evident from the results in table 4.2, the firm more reluctant to fire.

It is interesting to note however, that from the results of table 4.2, the impact of such an increase results in a much larger reduction in hiring than firing in less uncertain regimes and thus will, other things being equal, lead to a reduction in the firm's employment level over the cycle

4.3 The Effect of a Regime Change on the Optimal Employment Strategy of the Firm:

European firms in the 1980's have undoubtedly faced a very different economic environment to that in which they had operated during the previous decade, not only in terms of the level of hiring and firing costs, but also with regards to the general level of uncertainty with respect to the nature of prevailing economic conditions. For, in contrast to the 1970's, the 1980's can, from the firm's perspective, be characterised as a period of: increasing hiring costs; a substantial reduction in employment to employment transitions (quits); and an increase in the level of uncertainty with respect to the nature of the prevailing economic conditions. From the analysis undertaken so far, it should be evident that the model derived above provides us with a particularly useful apparatus for examining the impact of such a regime change on the firm's willingness to hire and fire, as reflected in the behaviour of their optimal hiring and firing trigger points. In this section therefore, we allow for simultaneous changes in factors argued to have been relevant to the firm's labour demand decision during the 1970's and 1980's, in the hope that an analysis of the impact of such changes on the firm's optimal employment strategy can offer additional insights into the evolution of employment within the EC over the past fifteen years.

Perhaps the most significant change in the economic environment facing the firm over the last two and a half decades has been the increases in the level of uncertainty with respect to the prevailing economic conditions that have occurred during the 1980's. Changes which have been such that it would appear reasonable to argue that labour demand decisions have gone from being taken in a relatively low uncertain to a more uncertain economic environment.

Recall from section three that uncertainty enters into our model through changes in the evolution of demand strength, which is assumed to evolve according to the following mixed Brownian-jump stochastic process:

$$\frac{dZ_\tau}{Z_\tau} = \vartheta_Z dt + \sigma_Z dW_\tau + \frac{1}{1-\mu}\left(\frac{\eta_0}{\mu A_\tau{}^{\mu} L_\tau{}^{\mu-1} Z_\tau{}^{1-\mu}} - 1\right) dq.$$

Increases in uncertainty can therefore be allowed for via changes in the value of σ_z, as considered in the Bentolila and Bertola analysis, and by increases in the value of ρ. In our simulation we allow for changes in uncertainty with respect to the future economic conditions by: a) increases in the variance of demand σ_z; and b) by assigning a higher value to ρ. In making these assumptions then, we are not only allowing for the fact that economic conditions in the 1980's were more volatile (i.e. that demand has been subject to much larger fluctuations during the 1980's), but also that economic cycles were shorter.

The remaining parameter values are calculated as follows. In common with Bentolila and Bertola we assume hiring costs in the 1970's to be equal to one month's pay. We make the further assumption that hiring costs increased by 35% over the two periods. A caveat is in order here however, since this figure is based solely on the average increase in recruitment expenditure reported in the company accounts of 100 UK firms. It does not therefore take into account increases in the cost of training and loss of output that any deterioration in worker quality or prolonged recruitment procedures will have involved and thus should be treated as a lower bound [14]. The British quit rate for the two periods is taken from Burgess and Nickell (1987), whose empirical analysis suggests an average quit rate in Great Britain of 2.1 over the period 1973-1979 and 1.1% for the 1980's. The number of employment to employment transitions recorded in the French labour Force Survey is used to construct a French quit series for the two periods. We take the average of the annual rate of return, r, reported in the business sector as an indicator of the rate of return, which in the 1970's (1980's) is assumed to be equal to 9.85% (9.6%) in Great Britain and 12.2% (12%) in France. As in Bentolila and Bertola, the remaining parameters σ_Z and ϑ_Z were proxied using the average percentage change in the first differences of the logarithm of the industrial production index and three times the annual average standard deviation of the first differences of the logarithm of industrial production respectively.

The effects of such a regime change on the optimal hiring and firing trigger points for a firm operating in a low cost environment are clearly evident from figure 4.3, where the discontinuous lines represent the so-called lower-cost/low-uncertainty regime and the continuous lines a more volatile economic

[14] These firms were however, selected across industrial groupings in an attempt to obtain a more representative sample of recruitment costs in terms of occupational classification.

climate, with a higher demand uncertainty in terms of the values assigned to both σ_z and ρ and higher costs of adjustment. In order to facilitate the comparison of the effects of the regime change on the location of the hiring and firing barriers under the different cost environments we also report, in table 4.3, the percentage change in the hiring and firing trigger points as a result of the regime change.

Table 4.3:

Percentage Change in the Location of the Optimal Hiring and Firing Schedules as a Result of the Regime Change:

	Bentolila and Bertola Model		Jump Model	
	L_F	L_H	L_F	L_H
Low Cost Environment	3.6	-4.6	20.2	-8.9
High Cost Environment	3.7	-8.2	44.6	-11.5

These results illustrate that a switch to a combination of greater uncertainty, lower quits and higher hiring costs has a significant impact on the optimal employment strategy of the firm as reflected in the behaviour of the firm's optimal hiring and firing trigger points, with the firm becoming more reluctant to both hire and fire in response to fluctuating demand conditions, regardless of the type of environment in which these changes take place. Once again however, the results obtained illustrate that: a) this regime change would appear to have had more of an impact on the firm's firing decision, with the upward shift in the firing barrier clearly dominating any hiring effect and thus resulting in a considerable widening of the no-action corridor; and b) the adverse effects of such a regime change are more acutely felt by the firm operating in a high cost environment.

The interaction between the change in economic conditions experienced by European firms between the 1970's and 1980's and the changes that occurred in the institutional characteristics of the markets within which these firms operated would appear, therefore, to have had an important impact on the dynamic behaviour of employment. In particular, the considerable widening of the no-action corridor which occurs in the aftermath of such a regime change would imply that demand strength can fluctuate much more in the high cost/more uncertain environment without leading to changes in employment. Thus one would expect to observe ceteris paribus a much higher degree of employment persistence in a 1980's style environment than in the 1970's, with employment in those economies characterised by a relatively high level of adjustment costs exhibiting relatively more persistence.

It is interesting at this point of the analysis to compare the impact of such a regime change under our assumption that demand strength evolves according to a mixed Brownian-jump process, with that obtained under the Bentolila and Bertola assumption that demand evolves according to a geometric Brownian motion process. From the results of such a comparison (see table 4.3 and figure 4.4, where the functions labelled BB_F and BB_H represent the Bentolila and Bertola hiring and firing schedules), it is evident that allowing for this additional source of uncertainty, together with changes in the institutional characteristics of the environment in which firms operate has considerably more of an impact on the optimal employment strategy of the firm and thus on the dynamic behaviour of employment. More specifically, when we allow for the possibility of "transitory" economic conditions, such a regime change results in: a) a more pronounced increase in the width of the no-action corridor and thus in a much higher degree of employment persistence; and b) a more significant downward shift in the hiring function.

The implications of our model would appear therefore, to be more consistent with statistical evidence which indicates that firms have during the upswings in economic activity during the 1980's become more reluctant to make adjustments to their employment levels. Firms instead being prepared to increase overtime rates or to pay considerable amounts of commission to hire temporary staff in order to cope with fluctuations in economic conditions rather than make permanent expansions to their work force, when they are unsure whether economic conditions can be considered transitory or permanent. Furthermore, this approach would not only appear to offer an explanation for the higher degree of employment persistence observed in Europe during the 1980's, but also to go some way to providing an explanation for the differences in the behaviour of employment within individual Community member countries.

What, if anything, can our analysis tell us about long run labour demand? As already discussed, the employment outcome of such a regime switch will depend on the magnitude of the change in the hiring and firing trigger points. In other words which trigger point reacts more strongly to the change in regime. If, as in our case, the regime change deters firing more than hiring then overall one would expect to observe an increase in the firm's employment level over the course of the business cycle in both high and low cost environments. It is difficult however, given the model as it stands, to discuss the implications of such a regime change on long run labour demand, since in the long run labour demand will, amongst other things, be dependent on the level of wages, which for simplicity are assumed here to be constant. Moreover, if labour obtains an increasing share of output over time (as our assumption of constant wages and increasing employment would tend to imply) one would expect to observe a reduction in the firm's level of investment, which would itself have significant ramifications for the level of employment. An obvious extension to this analysis would be therefore, the relaxation of the constant wage assumption. Modifying the model in this manner would not only enable us to examine the robustness of our results of allowing for non-constant wages, but more importantly would allow us to analyse the impact of changes in both the level of adjustment costs and uncertainty on long run labour demand and thus on the level of employment itself.

5 Effects of Adjustment Costs and Uncertainty under Different Market Structures: a note.

Finally, an interesting feature of this model is that it allows us in a sense to look at the optimal labour demand policy in the presence of adjustment costs and uncertainty under different market conditions and to see therefore, how employment evolves over time under different market structures. Recall from our initial description of the model in section three that μ represents the inverse of the firm's mark-up. The assignment of a value of $\mu = 0.7$ in the above empirical work implies that we have up until now been looking at the impact of adjustment costs and changes in uncertainty on the optimal employment strategy of a firm operating in a relatively competitive market. It would appear worthwhile however, given the structure of a number of European Community economies, to focus briefly on the impact of adjustment costs and uncertainty on the optimal employment strategy of a firm operating under more monopolistic conditions. In order to examine how changes in the value of μ, in other words changes in the underlying market conditions in which the firm operates, affects the characteristics of the firm's optimal employment strategy and thus the behaviour of employment.

It is evident from figure 5.1, where we illustrate the optimal hiring and firing trigger points for a firm operating under relatively competitive market conditions (represented by the discontinuous lines) and a firm operating within a more monopolistic framework, that other things being equal the presence of adjustment costs has a much larger impact on the optimal employment strategy for a firm operating under more competitive market conditions. In contrast, the move towards a more monopolistic market structure (characterised in this analysis by setting $\mu = 0.3$) results in a considerable narrowing of the no-action corridor, with the corridor also appearing to pivot and shift down towards the horizontal axis. Indicating then, that an optimising firm operating in a more monopolistic environment will tend to adjust its work force considerably more than its competitive counterpart in the face of changing economic conditions. It would appear therefore, that the more competitive is the market in which the firm operates the wider the no-action corridor becomes and thus the more persistence employment exhibits. The width of the corridor being ceteris paribus an increasing function of μ, until it becomes infinitely wide under perfectly competitive market conditions, in other words when $\mu = 1$ (see figure 5.2).

In a monopolistic market employment protection legislation per se would not appear to have such important implications for the willingness of firm's to hire and fire in the face of fluctuating demand conditions. For, the optimal employment strategy, as reflected in the location of the hiring and firing barriers, implies that the monopolistic firm adjusts its work force considerably more in response to changing economic conditions than its competitive counterpart. This result would tend to suggest then, that the effectiveness oflegislation introduced in order to protect the individual worker would appear to be dependent on the characteristics of the market structure in which the worker is employed. More specifically, such legislation appears to offer, ceteris paribus, a considerably higher degree of protection to workers employed in a more competitive market. Consequently, the advantages of labour market liberalisation in terms of increased flexibility would appear to be greater in those economies with more competitive market structures.

6 Concluding Remarks:

Recently a number of authors (see for example, Alogoskoufis et al. (1995)) have claimed that deregulation is not the answer to the Community's relatively weak labour market performance, arguing instead that the characterisation of European labour markets as being sclerotic in nature would appear to be a misconception, given that the more recent studies of job creation and job destruction indicate that these rates are in Europe at levels comparable to those of the US. As further support for the inappropriateness of the "anti-deregulation" campaign, the authors emphasise that despite being a continual feature of the policy debate for much of the 1980's, the deregulation that has taken place would appear to have simply resulted in unemployment exhibiting a much greater degree of volatility and not as originally advocated in a sustained decrease in unemployment rates[15].

Such an argument is not inconsistent with existing theoretical work (see for example Bentolila and Bertola 1990) on the impact of adjustment costs on employment (and thus ultimately unemployment). In our analysis we have shown, using simple simulation techniques, that with a relatively low level of uncertainty with respect to general macroeconomic conditions (i.e. a small value of ρ) the no-action corridor remains relatively narrow. It is evident however from our analysis that hiring and firing costs become increasingly more relevant, in that their presence has more important ramifications for the dynamic behaviour of employment, as the level of uncertainty increases and particularly, when economic fluctuations occur more frequently. We find that allowing for the possibility of "transitory" demand conditions results in a considerable widening of the no-action region and thus a significantly higher degree of employment persistence is observed. Offering one explanation then, for Europe's apparent

[15] The increases in the volatility of unemployment that have been experienced in a number of European countries can however, be explained by the type of deregulation that has taken place. For, it is not that surprising, given the nature of these employment contracts together with the form of regulatory conditions imposed on their use, that the introduction of new legislation to allow firm to bypass the existing stringent employment protection by extending the use of part-time and short term contracts has led to a significant increase in labour market turnover and thus in the volatility of unemployment.

inability to generate sufficient employment growth, despite a general improvement in economic conditions from 1985 onwards.

Although one has to be careful when drawing conclusions from this type of analysis, given its comparative static nature, a number of interesting issues emerge from this study. Firstly, it would appear that focusing on the level of adjustment costs *per se* may be inappropriate, given that the results obtained indicate that it is the interaction between adjustment costs and the level of uncertainty, together with the type of environment in which this interaction takes place, which appear to have more important ramifications for employment dynamics. Of particular interest to the current policy debate is the fact that our analysis indicates that changes in the level of adjustment costs appear to have more of an impact on the firm's willingness to hire and fire (and thus on its overall employment level) in response to changing economic conditions, in environments characterised by both a high level of adjustment costs and a higher degree of uncertainty. The results of section 4.1 for example, imply that changes in the level of firing costs in countries such as France, Italy and Spain characterised as being high adjustment cost regimes will have considerably more of an impact, the higher the prevailing degree of uncertainty. It is not that surprising then, that the reductions in the level of firing costs which have taken place have not had such a significant impact in terms of their impact on sustained employment growth, since the most substantive steps towards labour market liberalisation have taken place in economies, such as the UK, already acknowledged to be low cost regimes.

Secondly, the results of our analysis of sections 4.2 and 4.3 would tend to suggest that allowing for changes in both: a) the economic environment within which firms have operated; and b) the considerable increases in hiring costs which have occurred over the past 15 or so years, in models of the firm's labour demand decision can go help to explain the differences in the times series behaviour of employment over the past two decades both within the European Community and across different economic areas, such as Europe and North America. Furthermore, such an analysis could prove to be useful in the identification of factors for policy targeting. For, our results would tend to suggest that a more effective area for labour market policy, in terms of encouraging sustained employment growth, may be to focus its attention on ways to lower the escalating hiring costs (and thereby reducing the signal extraction problem) that firms have had to bear since the early 1980's, rather than to continue to focus on the detrimental aspects of firing costs, which have not been subject to such significant changes during this period.

Appendix

Proof:

The methodology of Harrison (1985, Chapter Five) can also be adopted to the case of a regulated Brownian motion with jumps. Thus we redefine the regulated process, $\{\xi_t\}$ as follows:

$$\xi_t = \zeta_t D_t U_t$$

where:

i) $\{\zeta_t\}$ is a mixed Brownian jump process, which evolves according to the following stochastic differential equation: $d\zeta_t = \zeta_t \vartheta dt + \zeta_t \sigma dW_t + (\zeta_0 - \zeta_t)\sigma dq$ and has an initial value $\bar{\xi}$, $d \le \bar{\xi} \le u$;

ii) $\{U_t\}$ and $\{D_t\}$ are increasing and continuous processes, with $\bar{D} = \bar{U} = 1$;

iii) $\{D_t\}$ only increases when $\xi_t = d$, and $\{U_t\}$ only increases when $\xi_t = u$, where u and d are positive real numbers;

iv) $d \le \xi_t \le u$ for all $t \ge 0$.

The arguments of Harrison (1985, p.22) illustrate that properties i to iv uniquely identify the processes $\{U_t\}$ and $\{D_t\}$, which ensure that the process $\{\xi_t\}$ remains within the control barriers, with control only occurring when $\{\xi_t\}$ would exit $[d,u]$with probability one in the absence of regulation. Applying Ito's lemma to $\xi_t = \zeta_t D_t U_t$ we obtain:

$$d\xi_t = \frac{D_t}{U_t} d\zeta_t + \frac{\zeta_t}{U_t} dD_t - \frac{D_t}{U_t^2} dU_t,$$

given that a non-decreasing process has a finite variance, $(dU_t)^2 = (dD_t)^2 = 0$.

If we now consider now a fuuction of $f(\xi)$:

$$f(x) = E_0\left\{\int_0^\infty e^{-\lambda t}\xi_t dt \middle| \bar{\xi} = x\right\} \qquad \text{A.1)}$$

From Harrison (1985) page 81, we know that $\{\xi_t\}$ is a Markov process in levels, the conditional expectation is therefore only a function of the starting value. Thus we have, by time homogeneity and Ito's lemma for a mixed Brownian-jump process, that:

$$\begin{aligned} df(\xi) &= f'(\xi)d\xi + \frac{1}{2} f''(\xi)(d\xi)^2 \\ &= f'\left[\vartheta\xi dt + \sigma\xi dW + (\xi_0 - \xi)dq\right] + \frac{1}{2} f''(\xi)\sigma^2\xi^2 dt \\ &= \left[f'(\xi)\vartheta\xi + \frac{\sigma^2}{2} f''(\xi)\xi^2\right]dt + \sigma f'(\xi)\xi dW \\ &\quad + (\xi_0 - \xi)f'(\xi)dq + f'\frac{\xi}{U}dD - f'\frac{D}{U^2}dU \end{aligned}$$

From property (iii), $dD_t = 0$ unless $\xi = d$ and $dU_t = 0$ unless $\xi = u$, we obtain that:

$$df(\xi) = \left(\vartheta f'(\xi)\xi + \frac{\sigma^2}{2} f''(\xi)\xi^2 \right) dt + \sigma f'(\xi)\xi dW$$

$$+(\xi_0 - \xi) f' dq + df'(d)\frac{dD}{D} - uf'(u)\frac{dU}{U}$$

Applying the integration by parts formula given in Harrison (1983, p.132) to the process $\{f(\xi)e^{-\lambda t}\}$ we obtain[23]:

$$e^{-\lambda t} f(\xi_t) = f(\xi_0) + \int_0^t e^{-\lambda v}\left[\vartheta f'(\xi_v)\xi_v + \frac{\sigma^2}{2} f''(\xi_v)\xi_v^2 - \lambda f(\xi_v) \right] dv$$

$$+\int_0^t e^{-\lambda v} \sigma f'(\xi_v)\xi_v dW + \int_0^t e^{-\lambda v}(\xi_0 - \xi_v) f'(\xi_v) dq$$

$$+ df'(d)\int_0^t e^{-\lambda v}\frac{dD}{D} - uf'(u)\int_0^t e^{-\lambda v}\frac{dU}{U} \qquad \text{A.2)}$$

Taking expectations of the above at time zero and letting $t \to \infty$, the left hand side vanishes for $\lambda > 0$, provided that $f(\xi_t)$ is bounded. The stochastic integral in dW, on the right hand side vanishes as long as $f'(\xi_t)\xi_t$ is bounded and the last two terms on the right vanish provided that:

$$df'(d) = 0 \qquad \text{A.3)}$$

and

$$uf'(u) = 0, \qquad \text{A.4)}$$

which we are going to impose.

Taking expectations of the integral: $E\left[\int_0^\infty e^{-\lambda t}\right]$

$$E\{f'(\xi_t)(\xi_0 - \xi_t)dq\} = \rho\{f(\xi_t + \xi_0 - \xi_t) - f(\xi_t)\}dt$$

$$= \rho[f(\xi_0) - f(\xi_t)]dt$$

Equation A.1 can therefore be rewritten as:

$$0 = f(\xi_0) + E_0\left\{\int_0^\infty e^{-\lambda t}\left[\vartheta f'(\xi_t)\xi_t + \frac{\sigma^2}{2} f''(\xi_t)\xi_t^2 - \lambda f(\xi_t) \right] dt \right\}$$

$$+\int_0^\infty e^{-\lambda t} \rho\left(f(\xi_0) - f(\xi_t)\right) dt.$$

[23] Which is also valid for a mixed Brownian-jump process, since we know from Royden (1968) that it is valid for any VF (Variation Finite) function.

As the term $\int_0^\infty e^{-\lambda t}\rho[f(\xi_0)-f(\xi_t)]dt$ is non-stochastic it can be taken inside the expectations operator. Thus:

$$f(\xi_0)=-E\int_0^\infty e^{-\lambda t}\left[f'(\xi_t)\xi_t+\frac{\sigma^2}{2}f''(\xi_t)\xi_t^2-(\lambda-\rho)f(\xi_t)+\rho f(\xi_0)\right]dt.$$

For this to be equal to equation A.1 the following differential equation must be satisfied:

$$-\xi_v=\vartheta f'(\xi_v)\xi_v+\frac{\sigma^2}{2}f''(\xi_v)\xi_v^2-(\lambda+\rho)f(\xi_v)+\rho f(\xi_0). \tag{A.5)}$$

The general solution to which is:

$$f(\xi)=\frac{1}{\lambda+\rho-\vartheta}\left[\xi+B_1\xi^{\alpha_1}+B_2\xi^{\alpha_2}\right]+\frac{\rho}{\lambda+\rho}f(\xi_0), \tag{A.6)}$$

where: B_1 and B_2 are constants of integration to be determined by the boundary conditions and:

$$\alpha_1\equiv\left(\frac{1}{\sigma^2}\right)\left[\left(\frac{\sigma^2}{2}-\vartheta\right)+\sqrt{\left(-\frac{\sigma^2}{2}+\vartheta\right)^2+2\sigma^2(\lambda+\rho)}\right]$$

and

$$\alpha_2\equiv\left(\frac{1}{\sigma^2}\right)\left[\left(\frac{\sigma^2}{2}-\vartheta\right)-\sqrt{\left(-\frac{\sigma^2}{2}+\vartheta\right)^2+2\sigma^2(\lambda+\rho)}\right].$$

From equation A.6, we can evaluate the value of $f(\xi_0)$ by solving the following:

$$f(\xi_0)=\frac{1}{\lambda+\rho-\vartheta}\left[\xi_0+B_1\xi_0^{\alpha_1}\quad B_2\xi_0^{\alpha_2}\right]+\frac{\rho}{\lambda+\rho}f(\xi_0)$$

Thus we obtain that:

$$f(\xi_0)\frac{\lambda}{\lambda+\rho}=\frac{1}{\lambda+\rho-\vartheta}\left[\xi_0+B_1\xi_0^{\alpha_1}+B_2\xi_0^{\alpha_2}\right],$$

or

$$f(\xi_0)=\frac{\lambda+\rho}{(\lambda+\rho-\vartheta)\lambda}\left[\xi_0+B_1\xi_0^{\alpha_1}+B_2\xi_0^{\alpha_2}\right].$$

The final general solution can be obtained by substituting $f(\xi_0)$ back into A.6 as follows:

$$f(\xi)=\frac{1}{\lambda+\rho-\vartheta}\left[\xi+B_1\xi^{\alpha_1}+B_2\xi^{\alpha_2}\right]+\frac{\rho}{(\lambda+\rho-\vartheta)\lambda}\left[\xi_0+B_1\xi_0^{\alpha_1}+B_2\xi_0^{\alpha_2}\right]. \tag{A.7)}$$

Note that ξ_0 is the point where the process ξ jumps to and that it is in principle different from the initial point $\bar{\xi}$.

To check that the fınction (A.7) is indeed a solution to our differential equation we can calculate its first two derivatives and substitute them back into the equation A.5 in order to show that it is in fact satisfied.

$$f'(\xi)=\frac{1}{\lambda+\rho-\vartheta}\left[1+\alpha_1 B_1\xi^{\alpha_1-1}+B_2\alpha_2\xi^{\alpha_2-1}\right]$$

$$f''(\xi)=\frac{1}{\lambda+\rho-\vartheta}\left[1+\alpha_1(\alpha_1-1)B_1\xi^{\alpha_1-2}+\alpha_2(\alpha_2-1)B_2\xi^{\alpha_2-2}\right].$$

Substituting into the differential equation we obtain:

$$-\xi=\frac{\vartheta}{\lambda+\rho-\vartheta}\left[\xi+B_1\xi^{\alpha_1}+B_2\xi^{\alpha_2}\right]+\frac{\sigma^2/2}{\lambda+\rho-\vartheta}\left[\alpha_1(\alpha_1-1)B_1\xi^{\alpha_1}+\alpha_2(\alpha_2-1)B_2\xi^{\alpha_2}\right]$$
$$-\frac{\lambda+\rho}{\lambda+\rho-\vartheta}\left[\xi+B_1\xi^{\alpha_1}+B_2\xi^{\alpha_2}\right].$$

This is true if:

$$\left[\frac{\vartheta}{\lambda+\rho-\vartheta}+1-\frac{\lambda+\rho}{\lambda+\rho-\vartheta}\right]\xi+\frac{B_1\xi^{\alpha_1}}{\lambda+\rho-\vartheta}\left[\vartheta\alpha_1+\frac{\sigma^2}{2}\alpha_1(\alpha_1-1)-(\lambda+\rho)\right]$$
$$+\frac{B_2\xi^{\alpha_2}}{\lambda+\rho-\vartheta}\left[\vartheta\alpha_2+\frac{\sigma^2}{2}\alpha_2(\alpha_2-1)-(\lambda+\rho)\right]=0,$$

which holds as long as α_1,α_2 are the solutions of:

$$\frac{\sigma^2}{2}\alpha^2+\left(\vartheta-\frac{\sigma^2}{2}\right)\alpha-(\lambda+\rho)=0.$$

Finally, to determine the constants of integration B_1 and B_2, we impose the boundary conditions:

$$df'(d)=0$$

and

$$uf'(u)=0,$$

to obtain:

$$\frac{d}{\lambda+\rho-\vartheta}\left[1+\alpha_1 B_1 d^{\alpha_1-1}+B_2\alpha_2 d^{\alpha_2-1}\right]=0$$

and

$$\frac{u}{\lambda+\rho-\vartheta}\left[1+\alpha_1 B_1 u^{\alpha_1-1}+B_2\alpha_2 u^{\alpha_2-1}\right]=0.$$

B_1 is therefore given by:

$$B_1 = \frac{d^{\alpha_2}u - du^{\alpha_2}}{\alpha_1\left(d^{\alpha_1}u^{\alpha_2} - d^{\alpha_2}u^{\alpha_1}\right)}$$

and B_2 by:

$$B_2 = \frac{d^{\alpha_1}u - du^{\alpha_1}}{\alpha_2\left(d^{\alpha_2}u^{\alpha_1} - d^{\alpha_1}u^{\alpha_2}\right)}.$$

Substituting these values into equation A.7 the proof is complete.

Figure 3.1:

The Expected Value of the Demand Strength under the Two Alternative Demand Assumptions:

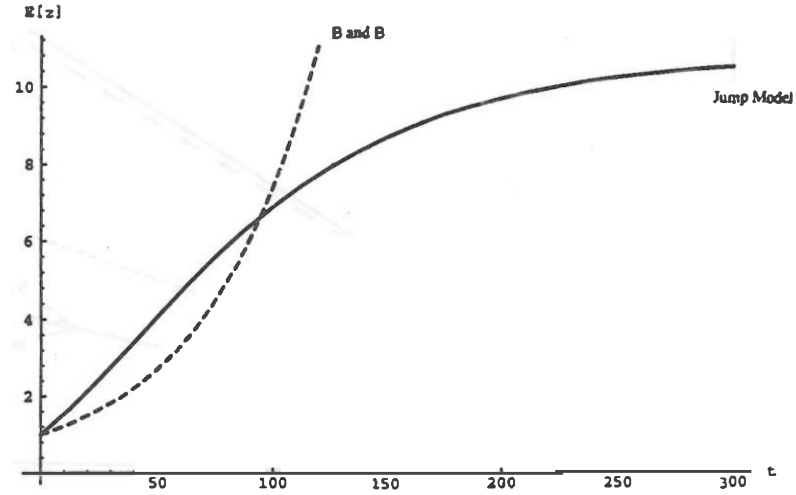


Figure 3.2:

The Expected Rate of Growth of Demand Strength:

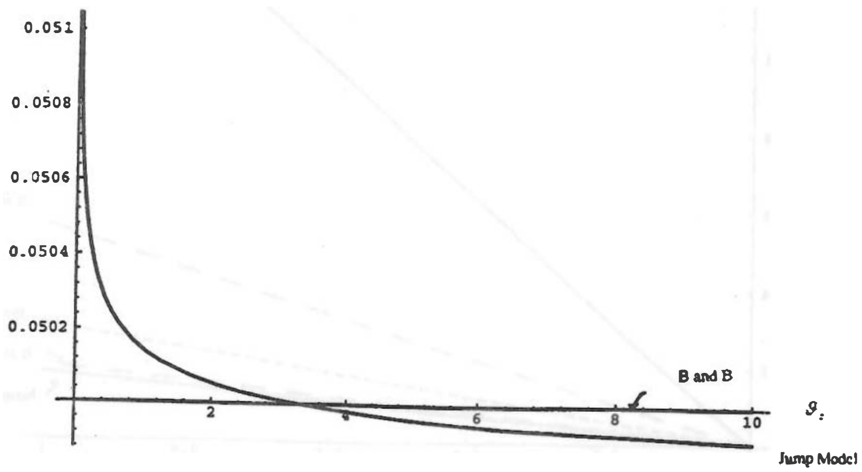


Figure 3.3:

The Optimal Hiring and Firing Trigger Points under
the Alternate Demand Assumptions:
$\rho = 0.1$

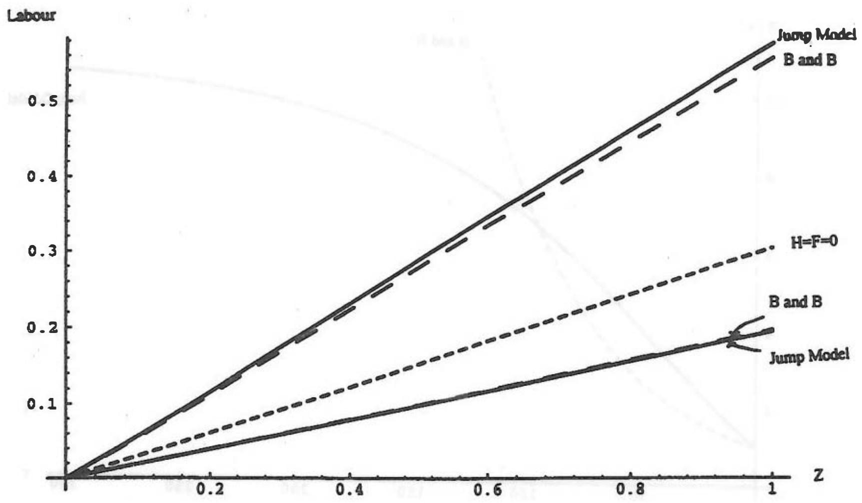


Figure 3.4:

The Optimal Hiring and Firing Trigger Points under
the Alternative Demand Assumptions:
$\rho = 1.5$

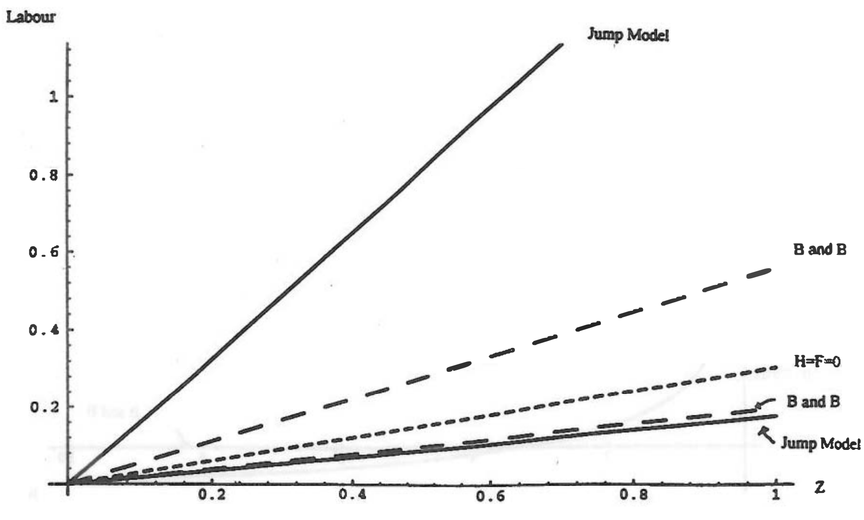


Figure 4.1:

The Impact of an Increase in the Level of Firing costs on the Optimal Trigger Points in a Low Cost Regime.

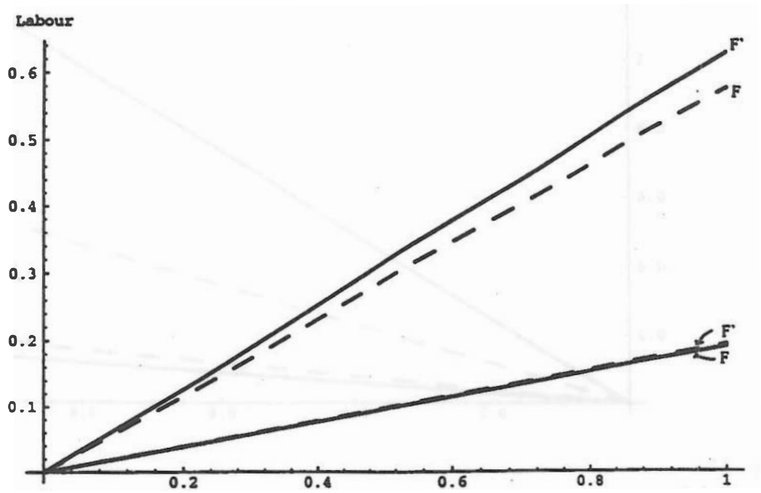


Figure 4.2:

The Impact of an Increase in the Level of Firing costs on the Optimal Trigger Points in a High Cost Regime.

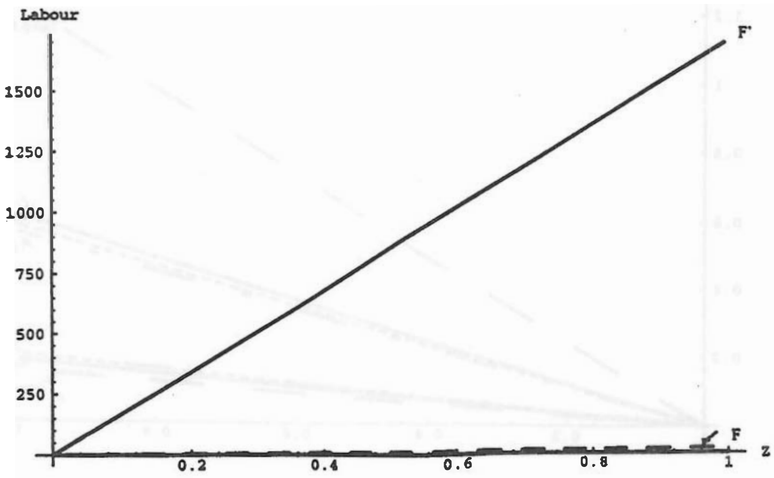


Figure 4.3:

The Effect of a Regime Switch on a Firm's Optimal Employment Strategy.

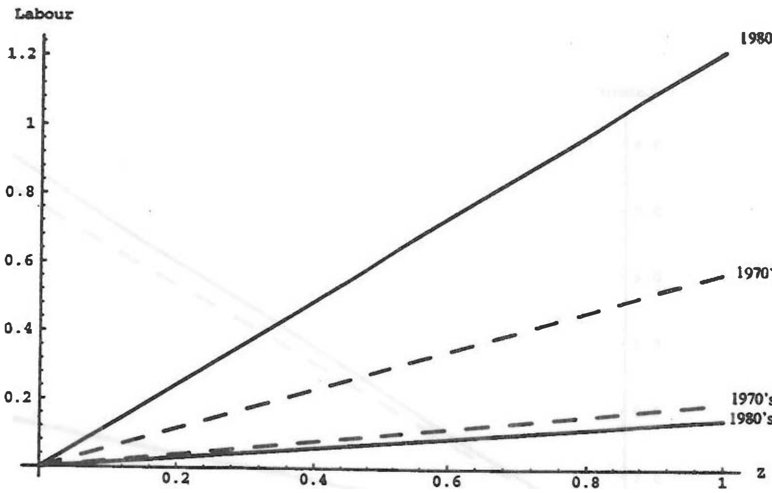


Figure 4.4:

The Effect of a Regime Switch on a Firm's Optimal Employment Strategy Under the Two Alternative Demand Assumptions:

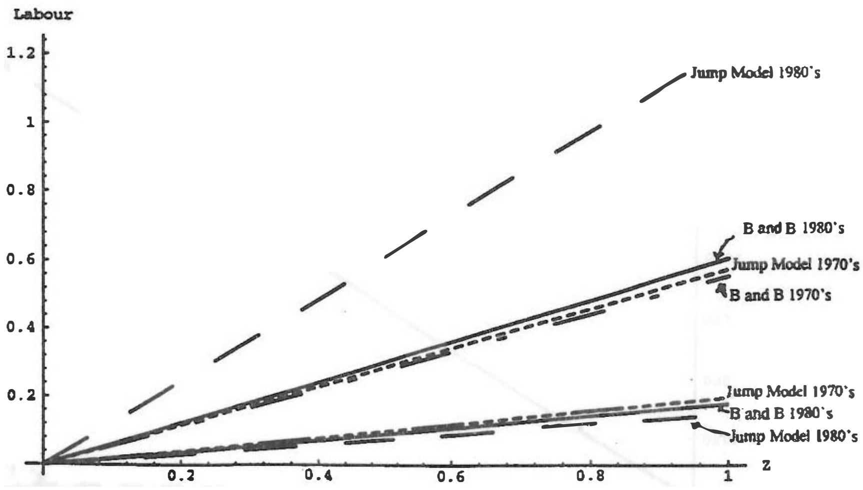


Figure 5.1:

The Optimal Employment Strategy Under Different Market Structures.

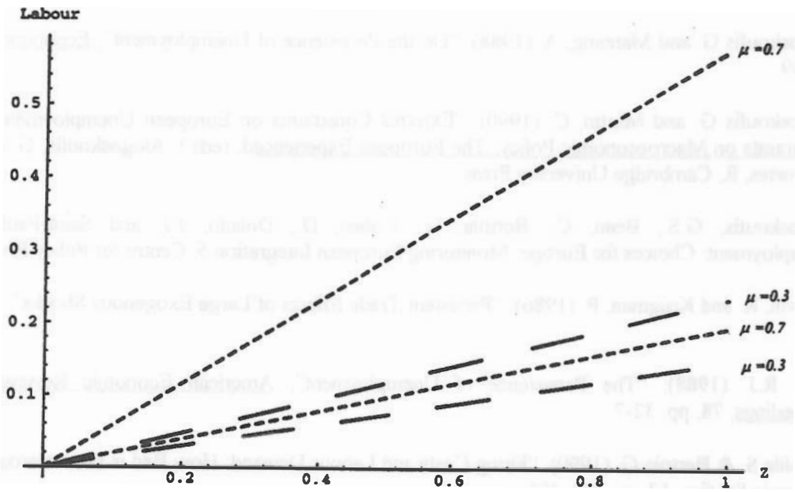
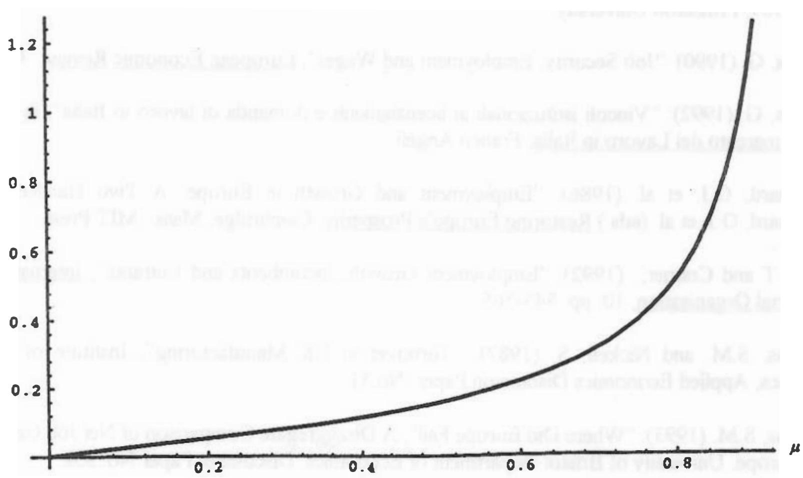


Figure 5.2:

The Impact of Moving to More Competitive Market Conditions on the Width of the No-Action Corridor:



References:

Abraham, K.G. and Houseman, S. (1993): "Does Employment Protection hinder Labour Market Flexibility: A lesson from France and Germany.", NBER Working Paper No. 4390.

Alogoskoufis G. and Manning, A. (1988): "On the Persistence of Unemployment", Economic Policy, 7, pp. 427-69.

Alogoskoufis G. and Martin, C. (1990): "External Constraints on European Unemployment", in External Constraints on Macroeconomic Policy: The European Experienced, (eds.): Alogoskoufis, G, Papademas, L, and Portes, R, Cambridge University Press.

Alogoskoufis, G.S., Bean, C., Bertola. G., Cohen, D., Dolado, J.J, and Saint-Paul, G. (1995): Unemployment: Choices for Europe: Monitoring European Integration 5, Centre for Policy Research.

Baldwin, R. and Krugman, P. (1986): "Persistent Trade Effects of Large Exogenous Shocks", MIT Working Paper.

Barro R.J. (1988): "The Persistence of Unemployment", American Economic Review, Papers and Proceedings, 78, pp. 32-7.

Bentolila S. & Bertola G. (1990): "Firing Costs and Labour Demand: How Bad is Eurosclerosis", Review of Economic Studies, 57, pp. 381-402.

Bentolila S. and Saint Paul, G. (1994): "A model of Labour Demand with Linear Adjustment Costs", Labour Economics, 1, pp. 303-326.

Bernanke, B.S. (1983): "Irreversibility, Uncertainty and Cyclical Investment," Quarterly Journal of Economics, 98, pp. 85-106.

Bertola, G. (1989): "Irreversible Investment", Mimeo, Massachusetts Insitute of Technology, 1987; revised June 1989, Princeton University.

Bertola, G. (1990): "Job Security, Employment and Wages", European Economic Review, 34, pp. 851-88.

Bertola, G. (1992): "Vincoli istituzionali ai licenziamenti e domanda di lavoro in Italia", in Padoa-Schioppa (ed) Il mercato del Lavoro in Italia, Franco Angeli.

Blanchard, O.J. et al. (1986): "Employment and Growth in Europe: A Two Handed Approach", in Blanchard, O.J. et al. (eds.) Restoring Europe's Prosperity, Cambridge, Mass.: MIT Press.

Boeri, T and Cramer, (1992): "Employment Growth, Incumbents and Entrants", International Journal of Industrial Organisation, 10, pp. 545-565.

Burgess, S.M. and Nickell, S. (1987): "Turnover in UK Manufacturing", Institute of Economics and Statistics, Applied Economics Discussion Paper, No.31.

Burgess, S.M. (1993): "Where Did Europe Fail", A Disaggregate Comparison of Net Job Generation in USA and Europe, University of Bristol Department of Economics, Discussion Paper No. 352.

Cabellero, R.J. (1991): "On the Sign of the Investment-Uncertainty Relationship", American Economic Review, 81, pp. 279-288.

Caplin, A. and Krishna, K. (1986): "A Simple Dynamic Model of Employment", Mimeo, Harvard University Cambridge, Mass.

Clark, K.B. and Summers, L.H. (1979): "Labor Market Dynamics and Unemployment: A Reconsideration", Brookings Papers on Economic Activity, 1, pp. 13-60.

Contini, B. and Revelli, R., Gavosto, A and Sestito, P. (1992): "Creazione e distruzione di posti di lavoro in Italia", Banca d'Italia, Temi di discussione del servizio studi, n.177.

Daniel, W.W. and Stilgoe, E. (1978): "The Impact of Employment Protection Laws", London: Policy Studies Institute.

Davis, S.J. and Haltiwanger, J. (1992): "Gross Job Creation and Gross Job Destruction and Employment Reallocation", Quarterly Journal of Economics, 107, pp. 819-63.

Dixit A. (1989): "Entry and Exit Decisions Under Uncertainty", Journal of Political Economy, 97, pp. 620-838.

Dixit A.K. (1992): "Investment and Hysteresis", Journal of Economic Perspectives, 6, pp.107-32.

Dixit A.K. and Pindyck, R. S. (1994): Investment under Uncertainty, Princeton University, Princeton, New Jersey.

Drèze, J.H. and Bean, C.R. (1990): Europe's Unemployment Problem, MIT Press, Cambridge, Mass.

Drèze, J.H. and Malinvaud, E. (1994): "Growth and Employment: The Scope for a European Initiative', European Economy - Reports and Studies, 1, pp.75-106.

Emerson, M. (1988): "Regulation or Deregulation of the Labour Market: Policy Regimes for the Recruitment and Dismissal of Employees in the Industrialized Countries", European Economic Review, 32: 775-817.

Episcopos, A. (1994): "Investment under uncertainty and the value of the firm", Economic Letters, 45, pp 319-322.

European Commission, (1993): "Employment Protection and Labour Force Adjustment in the Member States", Employment in Europe, pp.173-185.

Flanagan, R.J. (1987): "Labour Market Behaviour and European Economic Growth". In Lawrence, R.Z. and Schultze, E. (eds.) Barriers to European Growth, A Transatlantic View, Washington D.C., The Brookings Institution.

Gavosoto, A. and Sestito, P. (1992): "Job Turnover in Italy, Some Preliminary Evidence", Mimeo, International Centre for Economic Research, Turin.

Gennard, J. (1985): "Job Security: Redundancy Arrangements and Practices in Selected OECD Countries", Prepared for the Directorate fro Social Affairs, Manpower and Education of the OECD.

Hammermesh, D.S. (1986): "The Demand for Labor in the Long Run", in Ashenfelter A. and Layard, R. (eds.), Handbook of Labor Economics, I, Amsterdam: North-Holland.

Harrison, J.M. (1985): "Brownian Motions and Stochastic Flow Systems", New York: John Wiley and Sons.

Howitt, P. (1985): "Transaction Costs in the Theory of Unemployment", American Economic Review, 75 pp. 88-100.

Johnson, G.E. and Layard, R. (1986): "The Natural Rate of Unemployment Explanation and Policy", in Ashenfelter, O. and Layard, R. (eds.), Handbook of Labor Economics, II, Amsterdam: North-Holland.

Lazear, E.P. (1990): "Job Security Provisions and Employment", Quarterly Journal of Economics, 105(3), pp. 699-726.

Metcalf, D. (1986): "Labour Market Flexibility and Jobs: A Survey of Evidence from OECD Countries with Special Reference to Great Britain and Europe", London School of Economics Centre for Labour Economics, Working Paper No. 870.

Nickell S.J., (1978): "Fixed Costs, Employment and Labor Demand Over the Cycle", Economica, 45, pp. 329-345.

Nickell, S.J. (1986): "Dynamic Models of Labour Demand", in Ashenfelter, O. and Layard, R. (eds.), Handbook of Labor Economics, I, Amsterdam: North-Holland.

OECD (1986b): "Flexibility in the Labor Market. The Current Debate", Paris: OECD.

OECD, (1990b): "Labour Market Policies for the 1990s", Paris: OECD.

Pindyck, R.S. (1988): "Irreversible Investment, Capacity Choice, and the Value of the Firm", American Economic Review, 78, pp. 969-985.

Pindyck, R.S. (1988): "Irreversibility, Uncertainty and Investment", Journal of Economic Literature, 26 (3), pp.1110-1148.

Pissarides, C.A. (1990): "Unemployment and the Persistence of Employment Shocks", LSE, Centre for Labour Economics, Discussion Paper No. 377.

Saint Paul, G. (1991): "Dynamic labour demand with dual labour markets", Economic Letters, 36, pp. 219-222.

Saint Paul, G. (1994): "Searching for the Virtues of the European Model", CEPR Discussion Paper No. 950.

Snower, D. and Diaz-Vazquez, P. (1995): "How Unemployment Evolves", Presented at the CEPR Conference on Imperfections in the Labour Market, held at CEMFI, Madrid, March.

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