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(*) This research was conducted while the author visited the Banco de España. Errors and omissions are the author's own.

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Abstract

The nature of repeated interaction has been extensively studied in the repeated game literature. Abreu (1988), Abreu, Pearce and Stacchetti (1986, 1990), and Cronshaw and Luenberger (1994) develop a recursive approach to characterizing repeated games by focusing on the present values of subgame perfect strategies for each player, V. Judd and Conklin (1995), Cronshaw and Rutherford (1994) and Cronshaw (1996) have implemented these techniques computationally. Some of the most interesting examples of strategic interaction, however, arise in environments with state variables in which the recursive techniques cited above cannot be employed. In such environments the set of values of subgame perfect equilibrium becomes a function of the state variable–the object of interest becomes the value correspondence. This paper presents a general method for computing value correspondences under perfect monitoring and discounting.

1. Introduction

The nature of repeated interaction has been extensively studied in the repeated game literature. While the theory of repeated games has produced important qualitative results, one weakness of this theory is the lack of quantitative applications. A second weakness is that this theory has yet to be broadly extended to environments with state variables. In an earlier paper we address the first weakness, presenting a method for computing the set of subgame perfect equilibria in infinitely repeated games with perfect monitoring. Our approach is recursive, following the lead of Abreu (1988) and Abreu, Pearch and Stacchetti (1986, 1990), and Cronshaw and Luenberger (1994). Here, we extend that technique to games with state variables, restricting the state space to have finite elements.

The recursive tbeory of supergames introduces the concept of self-generating sets of payoffs to characterize the set of equilibria. The numerical method discussed below revolves around efficient ways to approximate and construct these self-generating sets. As in Cronshaw and Luenberger (1994) and Judd and Conklin (1995), we allow public randomization. This is advantageous since the resulting elements of the equilibrium value correspondence are convex, and a convex set can be represented by a simple function denoting the distance froman interior point to each point on its boundary. We choose a piecewise linear procedure to approximate these boundary functions. This method is of interest since it generates a "lower bound" to the true solution and is computationally cheap to evaluate and recompute. We illustrate the method with an example: computing subgame perfect equilibria for government fiscal policy in a primitive society.

This example needs moderate computer power. We give running times on Pentium-based personal computers. Furthermore, we used standard, freely available software. All the programs were written in FORTRAN, but could be implemented in any programming language, such as BASIC or Matlab.

Section 2 covers notation. Section 3 discusses the methods for approximating convex sets and describes the basic algorithm. Section 4 applies the algorithm to the two examples, and Section 5 concludes.

2. Supergames and Characterization of Equilibrium Payoffs

We examine an N-player infinitely repeated game with state variables. $A_i, i = 1, \cdots, N$, will denote the action space of player i in the stage game. $X = \bigcup_{k=i}^{K}\{x_k\}$ will denote the state space of K finite elements. We will call elements of $A \equiv A_1 \times A_2 \times \cdots \times A_N$ *action profiles*. Player i's payoff in the stage game will be $\Pi_i : A \times X \rightarrow R$. Abusing notation in the standard way, we define $a_{-i} = (a_1, \cdots, a_{i-1}, a_{i+1}, \cdots, a_N)$ and let

$$\Pi_i^*(a_{-i}, x) \equiv \max_{a_i \in A_i} \Pi_i(a_i, a_{-i}, x) \tag{1}$$

denote the best response payoff for player i given the actions a_{-i} by his opponents and the state x. Considering that for each value of x we can define a one-period game, we make the following assumptions:

Assumption 1: A_i, $i = 1, \cdots, N$, is a compact subset of R^m for some m.

Assumption 2: $\Pi_i(\cdot, x)$, $i = 1, ..., N$, is continuous.

Assumption 3: The stage game has a pure strategy Nash equilibrium for $x = x_k$, $k = 1, ..., K$.

Assumption 4: The value correspondence is non-empty for all values of $x \in X$.

Assumptions 1 and 2 are trivially satisfied by finite-play games. Assumptions 3 and 4 are satisfied by many interesting games, and are essential. Mixed strategy equilibria could be included if actions consisted of publicly observed randomization over pure strategy actions.

From the stage game, we can construct the corresponding supergame. The action space in the supergame is $A^\infty \equiv \times_{i=1}^{\infty} A$. We assume that player i aims to maximize his average discounted payoff, which equals

$$\frac{1-\delta}{\delta} \sum_{t=1}^{\infty} \delta^t \Pi_i(a_t, x_t) \tag{2}$$

where a_t is the action profile taken in period t. A *history*, y^t, is defined as the sequence $\{a_s, x_s\}_{s=0}^t$. By the continuity of Π_i and compactness of A_i,

$$\underline{\Pi_i}(x) \equiv \min_{a \in A} \Pi_i(a, x) \tag{3}$$

$$\overline{\Pi_i}(x) \equiv \max_{a \in A} \Pi_i(a, x) \tag{4}$$

are both finite. Therefore, for any given value of the state, the supergame payoffs are contained in the compact set

$$\mathcal{W}(x) = \times_{i=1}^{n} [\underline{\Pi_i}(x), \overline{\Pi_i}(x)]. \tag{5}$$

A *strategy* for player i is a mapping from the history of the game in time t into player i's action space for each time t: $\sigma_i : A_i^t \times X^t \rightarrow A_i, t = 0, 1, 2, ...$ A *strategy profile* is the set strategies of all players: $\sigma = \bigcup_{i \in N} \sigma_i$.

For each subgame perfect strategy profile σ, we define its value to player i to be

$$v_i(x_1, \sigma) = \frac{1-\delta}{\delta} \sum_{t=1}^{\infty} \delta^t \Pi_i(a_t, x_t | \sigma). \tag{6}$$

It follows that for each state there is a *value set*, representing the values that arise for all players across all strategies:

$$V(x) = \bigcup_{\sigma \quad SPE} v(x, \sigma), x \in X. \tag{7}$$

Finally, we define the value corrspondence to be

$$V(X) = \times_{x=1...K} V(x_k) \subseteq \times_{k=1}^{K} W(x_k) \subseteq R^{N \cdot K}. \tag{8}$$

Our objective is to compute the value correspondence $V(X)$.

The perspective implicit in the notation above does not provide us with a numerically tractable way of viewing the supergame since it focusses on infinite sequences of actions. While such descriptions are useful theoretically and descriptively, they are not useful when it comes to computation since infinite sequences are costly, if at all possible, to represent in a computer. Furthermore, examining the set of possible strategies and checking for subgame perfection is also difficult if one takes an infinite sequence approach. Naturally, we take a dynamic programming approach to the problem, compressing the information in the game's histories into continuation values and extend the approach to games with state variables.

This "dynamic progamming approach" to supergames with state variables can be expressed through a map, B. Let $W(x)$ be a candidate for $V(x)$ for each state x. Then the set of possible payoffs today are those consistent with Nash play today with continuation values in $W(x')$, and are defined by the map

$$B(W)(x) = \bigcup_{(a, w(x')) \in A \times co(W(x'))} \left\{(1-\delta)\Pi(a, x) + \delta w(x')\right\} \tag{9}$$

$$\text{subject to:} \qquad x' = h(a, x) \tag{10}$$

$$\tilde{x} = h(\tilde{a}_i, a_{-i}, x) \tag{11}$$

$$(1-\delta)\Pi_i(a, x) + \delta w_i(x') \geq \max_{\tilde{a}_i}\{(1-\delta)\Pi_i(\tilde{a}_i, a_{-i}, x) + \delta \underline{w}_i(\tilde{x}), \forall i \in N\}. \tag{12}$$

$h(a, x)$ is the law of motion of the state variable. Intuitively, a value $b(x)$ is in $B(W)(x)$ if there is some action profile, a, and continuation payoff, $w(x') \in W(x')$, such that $b(x)$ is the value of playing a today and receiving the continuation value $w(x')$ tomorrow, and, for each i, player i will choose to play a_i because he believes that to do otherwise will yield him the worst possible continuation payoff. The right hand side of equation 12 represents the value of defection from the prescribed equilibrium. For player i playing a_i implies that the state tomorrow will be $x' = h(a, x)$. However if player i defects and takes action a^*, the economy will not be in state x' tomorrow but state $x^* = h(a_i^*, a_{-i}, x)$. Hence when computing the value of defection, the punishment value must be drawn from the set $W(x^*)$.

Proposition: $V(X) = B^{\infty}W^0(X)$ if $W^0(X) \supseteq V(X)$.

proof: The numerical algorithm described in section 3 provides a constructive proof.

While this characterization of $V(X)$ is elegant and intuitive, there are important difficulties associated with computing it. The main problem is that $V(X)$ can be an ugly correspondence, difficult to represent in a computer. To deal with these problems, we alter the supergame by including public randomization. Intuitively, we assume that at the end of each stage game, a lottery is used to determine how the game continues. Formally, strategies will now map histories of players' actions, the history of the realizations of the lottery and the current lottery outcome, into current actions. Realized stage-game payoffs will differ from expected stage-game payoffs in strategies where non-trivial (non-degenerate) lotteries are employed. Operationally, we imagine the following scenario. All the players know that tomorrow's continuation value lies in $co(W(x'))$. However, they don't know what that continuation value will be until after a publicly observed random valuable is revealed, which will signal tomorrow's $w(x') \in W(x')$. Therefore, from today's perspective, we can *ex ante* construct a continuation value in $co(W)$.

While the main motivation for adding lotteries is to make the computation easier, it does generalize the notion of subgame perfect Nash equilibrium in an appropriate and interesting fashion. Since one aim of this research is to see what is possible in the absence of contracting, it is natural to add this public randomizing device. The addition of lotteries will possibly be inessential. In particular, if $V(X)$ without lotteries is convex, then adding lotteries will not expand $V(X)$ and our algorithm will compute $V(X)$. One could also further generalize the analysis to correlated equilibria. However, that would admit private information and assume some coordinating agent, unappealing assumptions in many contexts.

3. Computing the Equilibrium Value Correspondence

Choosing an initial set $W^0(X) \supseteq V(X)$, the sequence of sets $\{W^k(X)\}_{k=0}^{\infty}$, where $W^{k+1}(x) = B(W^k)(x)$, converges to $V(x)$ for each $x \in X$. Direct application of this technique, however, poses two computational obstacles. The first is that for many supergames, equilibrium value correspondence $V(X)$ is infinite and therefore impossible to represent on a computer. We treat this problem by exploiting the convexity of $W(X)$ and $B(W)(X)$: any ray originating from any interior point of a convex set intersects the boundary of that set only once. A convex set can be represented by its boundary, a fact we will exploit. By virtue of its "star-shapeness", a convex sets comprising the value correpondence can be represented in a particularly simple fashion; we discuss the details of this in section 2.1.

The second computational obstacle we confront is that even if we can represent $W(X)$ finitely, generating $B(W)(X)$ can involve exhaustive pairing of actions with all possible value pairs, $(w_i(x'), \underline{w}_i(x'))$ to derive incentive compatible actions and rewards. This technique is too crude for anything but the smallest problems. As an illustration, consider constructing $B(W)(X)$ from $W(X)$ in direct fashion: for a given state today, x, and the state that will arise tomorrow, $x' = h(a, x)$, applying the definition of $B(W)(x)$ for each player, you pair an action a_i with two continuation values $(w_i(x'), \underline{w}_i(\tilde{x}(a_i^*, x))) \in W_i(x') \times W_i(\tilde{x}(a_i^*, a_{-i}, x))$. If $(w_i(x'), \underline{w}_i(\tilde{x}(a_i^*, x)))$ satisfies *incentive compatibility*, $(1 - \delta)\Pi_i(a, x) + \delta w_i(x') \geq (1 - \delta)\Pi_i^*(a_{-i}, x) + \delta \underline{w}_i(\tilde{x}(a_i^*, x))$, then we know the new point $b_i(a, x, w_i(x')) = (1 - \delta)\Pi_i(a, x) + \delta w_i(x')$ will lie in $B(W)(x)$. Doing this exhaustively, for every possible combination of action profile and promise-threat pair, will generate the correspondence $B(W)(X)$. Exhaustive pairing is clearly impractical for most games. We avoid it by directly approximating the boundary of $B(W)$, ignoring interior points. We describe this method in section 2.1. Section 2.2 covers more specific details of the algorithm's implementation.

3.1 Piecewise Linear Approximations of Convex Sets

Since the sets the comprise the elements of the value correpondence are convex, we need efficient ways to approximate convex sets in the computer. Also, for theoretical reasons apparent below, we will want to use approximations which allow us to have useful and relatively precise information about the approximation error. For these reasons we will use piecewise linear approximations. Since we will only develop a two-player implementation of our methods, our discussion will focus on approximating two-dimensional convex sets, such as those in Figure 1.

3.2 Value Set Iteration for Convex Sets

To do this, we exploit the star-shapeness of each of the set $B(W)(x_k), k = 1, 2, ...K$ and the assumption that there exists some point $v^{Na}(x_k)$ for every state which is the equilibrium value of at least one subgame perfect equilibrium whose path passes through that state. Refer to Figure 1, where $v^{Na}(x_k)$ is in $W(x_k)$ and $B(W)(x_k)$. Any ray originating from $v^{Na}(x_k)$ intersects the boundary of the set $\partial B(W)(x_k)$ only once. Hence, we can determine the boundary at a given direction $\theta \in [0, 2\pi]$ by extending a ray from the set center $v^{Na}(x_k)$. Let α be the length of the ray and let $\vec{w}(\alpha, \theta, x_k)$ be the point at the "tip" of the ray. We want to increase α until $\vec{w}(\alpha, \theta, x_k)$ is no longer in $B(W)(x_k)$. To determine if $\vec{w}(\alpha, \theta, x_k) \in B(W)(x_k)$, we check two conditions. First, there must be both an action profile and a continuation value promise and threat, $(a, w(x_k), \underline{w}(a_i^*, a_{-i}, \tilde{x}_l(a_i^*, a_{-i},, x_k))$, that support $\vec{w}(\alpha, \theta)$. Second, the supporting triple $(a, w(x_k), \underline{w}(a_i^*, a_{-i}, \tilde{x}_l(a_i^*, a_{-i},, x_k))$ must satisfy incentive compatibility. Maximizing α at each $\theta \in [0, 2\pi]$, subject to these two constraints,

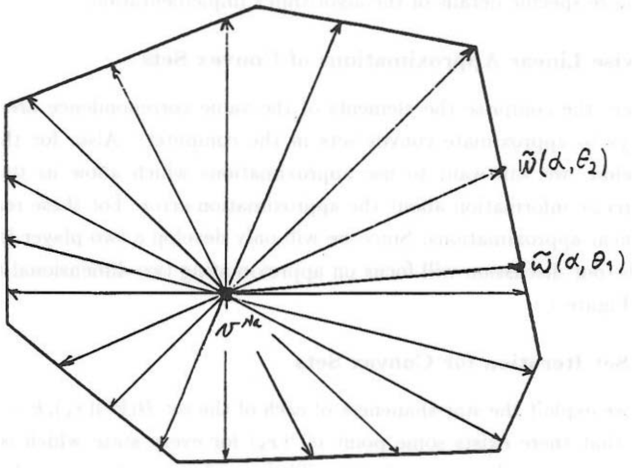

Figure 1: Star-shapeness of B(W)

defines a periodic function representing $\partial B(W)(x_k)$:

$$R(\theta)(x_k) = \max_{\alpha \geq 0, a \in A} \left\{ \alpha \;\middle|\; \begin{array}{l} i.)\ (v^{Na}(x_k) + \alpha v(\vec{\theta}) - (1-\delta)\Pi(a, x_k))/\delta \in W(x') \\ ii.)\ (1-\delta)\Pi(a, x_k) + \delta w(x_k) \geq (1-\delta)\Pi^*(a_{-i}, x_k) \\ \quad +\delta \underline{w}(a_i^*, a_{-i}, \tilde{x}_l(a_i^*, a_{-i},, x_k)) \end{array} \right\} \tag{13}$$

In Equation (9), the point $\vec{w}(\alpha, \theta, x_k)$ is written as $v^{Na}(x_k) + \alpha\vec{v}(\theta)$, where $\vec{v}(\theta) = (\cos\theta, \sin\theta)$. In order for $v^{Na}(x_k) + \alpha\vec{v}(\theta)$ to be in $B(W)(x_k)$, this point must be generated by a triple $(a, w(x_k), \underline{w}(a_i^*, a_{-i}, \tilde{x}_l(a_i^*, a_{-i},, x_k))$ where $v^{Na}(x_k) + \alpha\vec{v}(\theta) = (1-\delta)\Pi(a, x_k) + \delta w(x')$, for some $w(x') \in W(x')$. This implies that $[(v^{Na}(x_k) + \alpha\vec{v}(\theta) - (1-\delta)\Pi(a, x_k)]/\delta = w(x')$. Imposing $w(x') \in W(x'), x' = h(a, x_k)$ yields condition $i.$ in the equation. If we define a grid of M angles, $\Theta \subseteq [0, 2\pi]$, this representation reduces the problem of finding the boundary $\partial B(W)(x_k)$ in every direction θ for each state $k \in \{1, ..., K\}$ to $M \times K$ constrained maximization problems, a task that can be handled by conventional software and hardware. We compute the total approximation of the boundary function $R(\theta)$ by first convexifying the vertices defined by the $R(\theta_m)$ and computing the appropriate coefficients for the linear approximation between vertices.

Figure 2 illustrates the method applied with an angular grid. Starting at the "center" $v^{Na}(x_k)$, we select an angle $\theta_m \in \Theta$. At that angle, we extend a ray to the boundary of $B(W)(x_k)$ by maximizing α. At that maximized α^*, $\vec{w}(\alpha^*, \theta_m, x_k)$ is the interpolant vertex for the piecewise linear approximation of $B(W)(x_k)$. Note that our constraints appear in the set $W(x')$. For this example, the binding constraint on maximizing α is that we hit $\partial W(x')$ - $W(x')$ will afford us no more extravagant promises than $w(x')$.

Direct application of this technique, however, poses two computational obstacles. The first is that for many supergames, equilibrium value sets $V(x)$ are infinite and therefore impossible to represent on a computer. We treat this problem by invoking lotteries to render $W(x)$ and $B(W)(x)$ convex. Any ray originating from any interior point of a convex set intersects the boundary of that set only once. By virtue of this quality, "star-shapeness", a convex set can be represented by its boundary, hence, in finite fashion. The second computational obstacle we confront is that even representing $W(x)$ finitely, generating $B(W)(x)$ can involve exhaustive pairing of actions with value pairs, $(w_i(x), \underline{w}_i(\tilde{x}))$. This technique is too crude for anything but the smallest problems. As an illustration, consider constructing $B(W)(x)$ from $W(x)$ in direct fashion: applying the definition of $B(W)$, you pair an action profile with a continuation value: $(a, w(x') \in A \times W(x')$. If $(a, w(x'))$ satisfies equations 20-23, then we know the new point $\hat{w}(a, w(x'), x) = (1-\delta)\Pi(a, x) + \delta w(x')$ will lie in $B(W)(x')$, $x' = h(a, x)$. Doing this exhaustively, for every possible action profile and promise pair, will generate the correspondence $B(W)(x)$. Exhaustive pairing is clearly impractical for most games. We avoid it by using a piecewise linear approximation of the boundary of $B(W)(x)$, ignoring interior points, and directly approximating the boundary of $B(W)(x)$ from the boundary of $W(x)$.

0.1 Specifics of Value Set Iteration for the 2-player Case.

At this point, we shall discuss the steps in detail for the special case where $N = 2$, A_1, the action-space of player 1, is finite with J elements, and A_2 is finite with L elements, and the state space has K elements. Starting with an initial $W(X)$ that we are sure contains the equilibrium value correspondence $V(X)$, we apply the operator $B(\cdot)$ to W until $\| B(W) - W \|_{v^{Na}} \leq \epsilon$, where ϵ is the convergence criterion and $\| \quad \|_{v^{Na}}$ is the root sum of squared differences of α^* at each θ_m between iterations. We summarize the principal steps for the operator $B(\cdot)$ with the following scheme:

0: Initialize

0.1: Select $v^{Na} \in V(x)$ for $x \in X$.

0.2: Set angle grid $\Theta = \{theta_1, ..., \theta_M\} \subseteq [0, 2\pi]$

0.3: Select initial $R(\theta)(x)$ lengths for each $\theta \in \Theta$, thereby constructing a $W(x) \supseteq V(x)$ for $x \in X$.

1: For each $x_k \in X$ do

2: For each $\theta_m \in \Theta$ do

2.1: For each action profile $(a_j, a_l) \in A$ do

$$\begin{aligned} \alpha(a_j, a_l) \;=\; & \max \quad \alpha \\ & \text{s.t.} \quad \text{(i)} \;\; \delta^{-1}[(v^N(x_k) + \alpha\vec{v}(\theta_m)) - (1-\delta)\Pi(a_j, a_l, x_k))] \\ & \qquad\quad \in W(x'(a_j, a_l, x_k)) \\ & \qquad\quad \text{(ii)} \;\; (1-\delta)\Pi(a_j, a_l, x_k) + \delta w(x') \geq (1-\delta)\Pi^*(a_{-i}, x_k) + \\ & \qquad\quad \underline{\delta w(\tilde{x}(a_j, a_l, x_k))} \\ & \qquad\quad \text{(iii)} \;\; \alpha \geq 0 \end{aligned}$$

where $\alpha(a_j, a_l, x_k) = 0$ is understood if the constraint set is empty.

2.2: $\alpha^*(\theta_m, x_k) = \max(\alpha(a_1, a_1, x_k), \alpha(a_1, a_2, x_k), \ldots, \alpha(a_J, a_L))$;

2.3: $\tilde{w}(\alpha^*(\theta_m, x_k), \theta_m, x') = v^{Na}(x_k) + \alpha^*(\theta_m, x_k)\vec{v}(\theta_m)$;

3: Determine those elements of the set $\bigcup_{m \in M} \tilde{w}(\alpha^*(\theta_m, x_k), \theta_m, x_k)$ which lie in $\mathcal{W}^*$, the boundary of the set's convex hull;

4: Compute piecewise linear approximation of the boundary of $B(W)(x_k)$; specifically, compute the convex hull of the points from (2).

5: Set $W(X) = B(W)(X)$. Stop if W has changed little; else go to 1.

Step 1 in the scheme specifies that the algorithm "loop" through each angle θ_m, $m = 1 \ldots M$. We maximize α in at each angle, subject to the constraints that appear in Equation (10). These constraints are functions of action profiles, and some profiles will be more constraining than others. We loop through all action profiles in step 1.1[1] to find the action profile that yields the maximal α. In step 1.1, θ_m and (a_j, a_l) are fixed, meaning that for each iteration on W, we maximize α $M \times J \times L \times K$ times. Note that in 1.1, there is the numerical subproblem of deciding if a point is in $W(x_k)$. These points show that the heart of the algorithm is a constrained maximization problem.

To understand this constrained maximization problem, we study how large α can be before hitting one of the two constraints. With reference to Figure 3, consider constraint *i.*). This constraint dictates that tomorrow's value promise, $w(x')$, used to support today's value, $\tilde{w}(x_k, \alpha, \theta_m) = v^{Na}(x_k) + \alpha v(\vec{\theta}_m) = (1-\delta)\Pi(a_j, a_l, x_k) + \delta w(x')$, must be in $W(x')$. Moreover, we can break $w(x')$ down into two components, one that varies with α, and one that is fixed:

$$w(x') = \frac{v^{Na}(x_k) - (1-\delta)\Pi(a_j, a_l, x_k)}{\delta} + \frac{\alpha}{\delta} v(\vec{\theta}_m) \equiv v_a(x_k) + \frac{\alpha}{\delta} v(\vec{\theta}_m) \tag{14}$$

This decomposition appears in Figure A3. We increase α so that $w(\alpha, x') = v_a(x_k) + \frac{\alpha}{\delta} v(\vec{\theta}_m)$ approaches the boundary of $W(x')$ along the angle θ_m. When $w(\alpha, x')$ strikes the boundary, this quantity as $\hat{\alpha}$.

Now consider constraint *ii.*), the incentive compatibility (IC) condition. We can rearrange this constraint as the inequality

$$w_i(x') \geq \delta^{-1}[(1-\delta)(\Pi_i(a_j, a_l, x_k) - \Pi_i^*(a_{-i}, x_k)) - \delta \underline{w}_i(x')], \quad i = 1,2. \tag{15}$$

Examine Figure A3. Since these constraints are not a function of α, they are vertical for player 1, horizontal for player 2, and shift as a function of the action profile a. They appear as ℓ_1 and ℓ_2 in Figure A3. The IC constraints are binding if a given player's value promise, $w_i(\alpha, x') = v_{i,a}(x_k) + \frac{\alpha}{\delta}\vec{v}_i$, is equal to the IC constraint. In Figure A3, this is shown happening for player 1 at $\alpha = \check{\alpha}$. Therefore, at the angle θ_m and the action profile (a_j, a_l), $\check{\alpha}$ is the constrained maximum for step 1.1; the IC constraint for player 2 and the constraint $w \in W$ are not binding. The maximization problem of step 1.1 boils down to solving a few linear equations that identify the intersections of the ray at angle θ_m originating at v_a, with the two IC constraints and the boundary of W.

We do the maximization in step 1.1 for all possible action profiles. Different action profiles imply different constrained maxima for α. This is shown in Figure A4. In that figure, we show how both v_a and the IC constraints shift as a function of a. Consider the action profile

[1] One might exploit game structure to economize on this search, as one would have to if A were continuous.

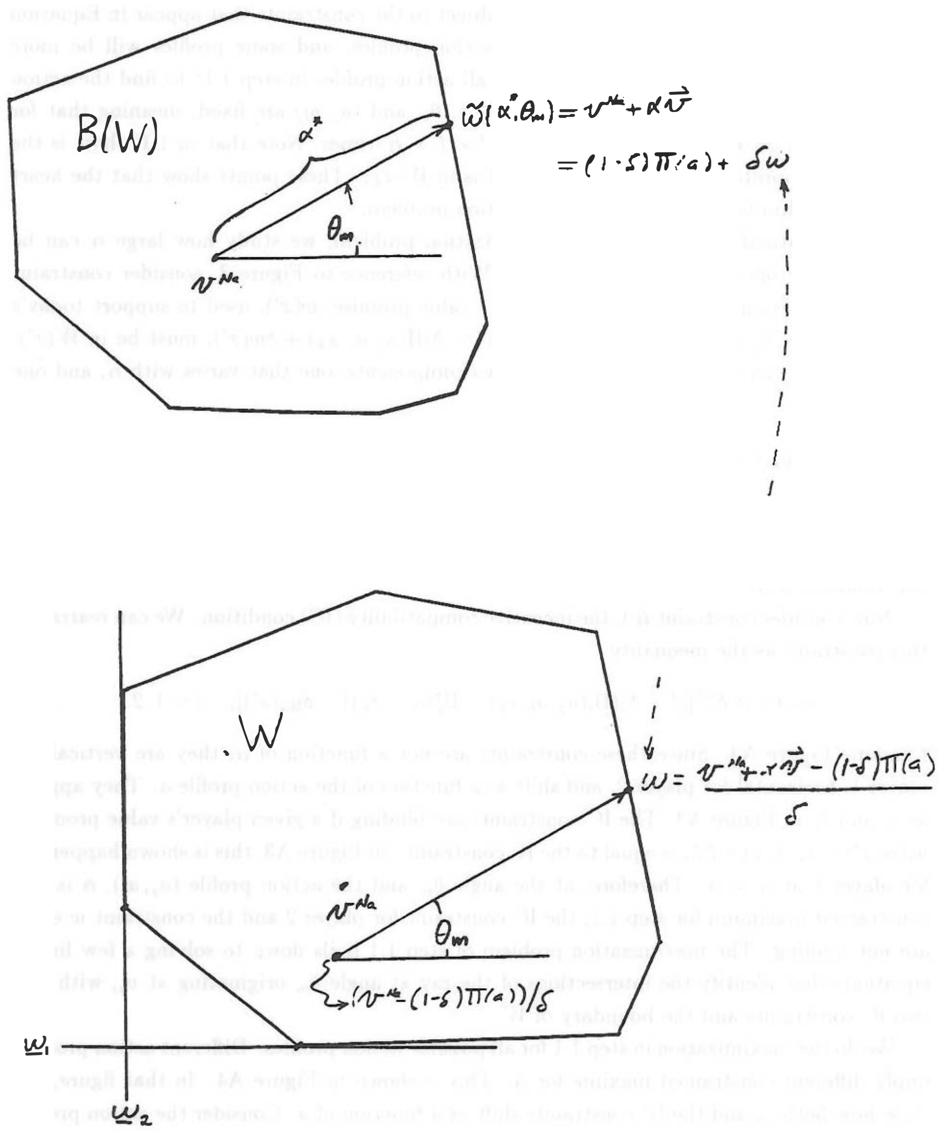

Figure 2: Extending the ray $v + \alpha\vec{v}$ at the boundary of $B(W)$

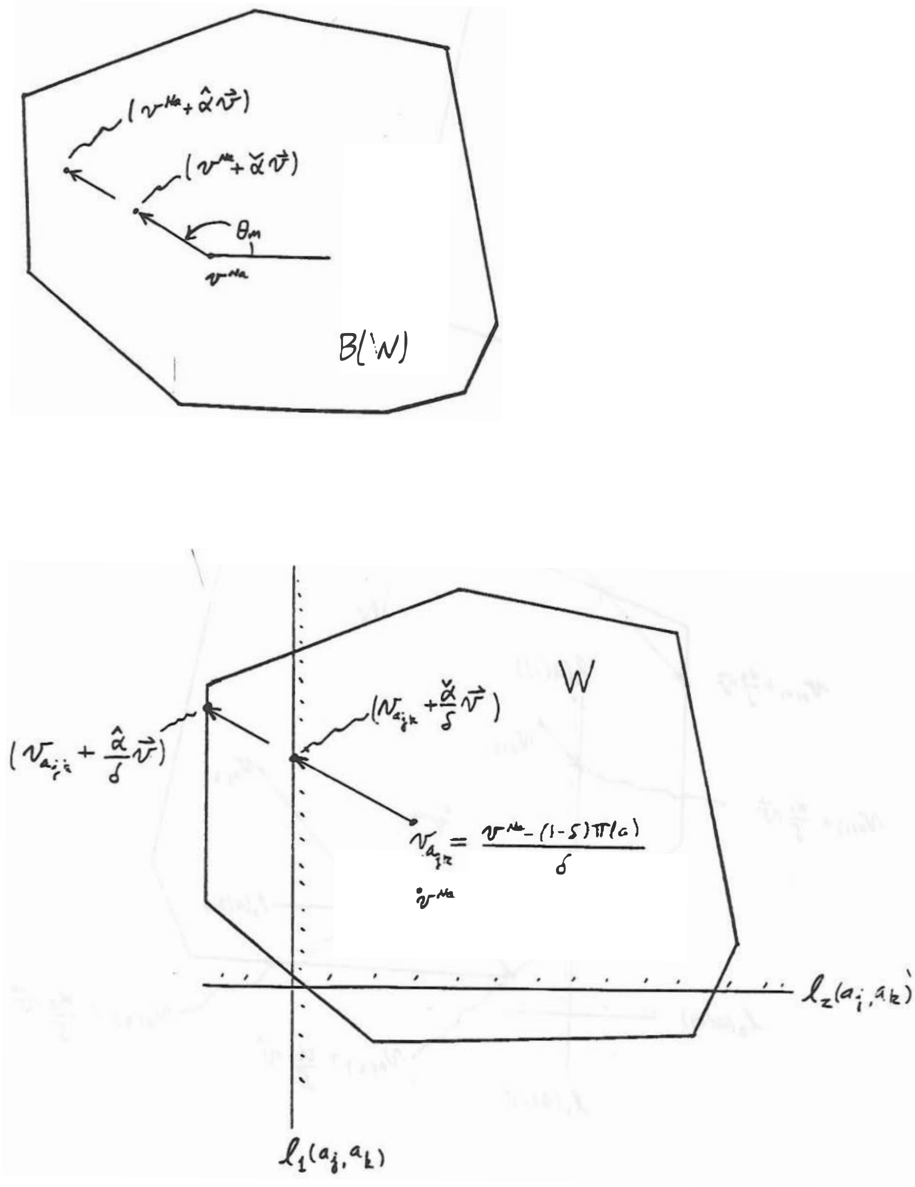

Figure 3: Extending the ray at angle θ with the action profile fixed

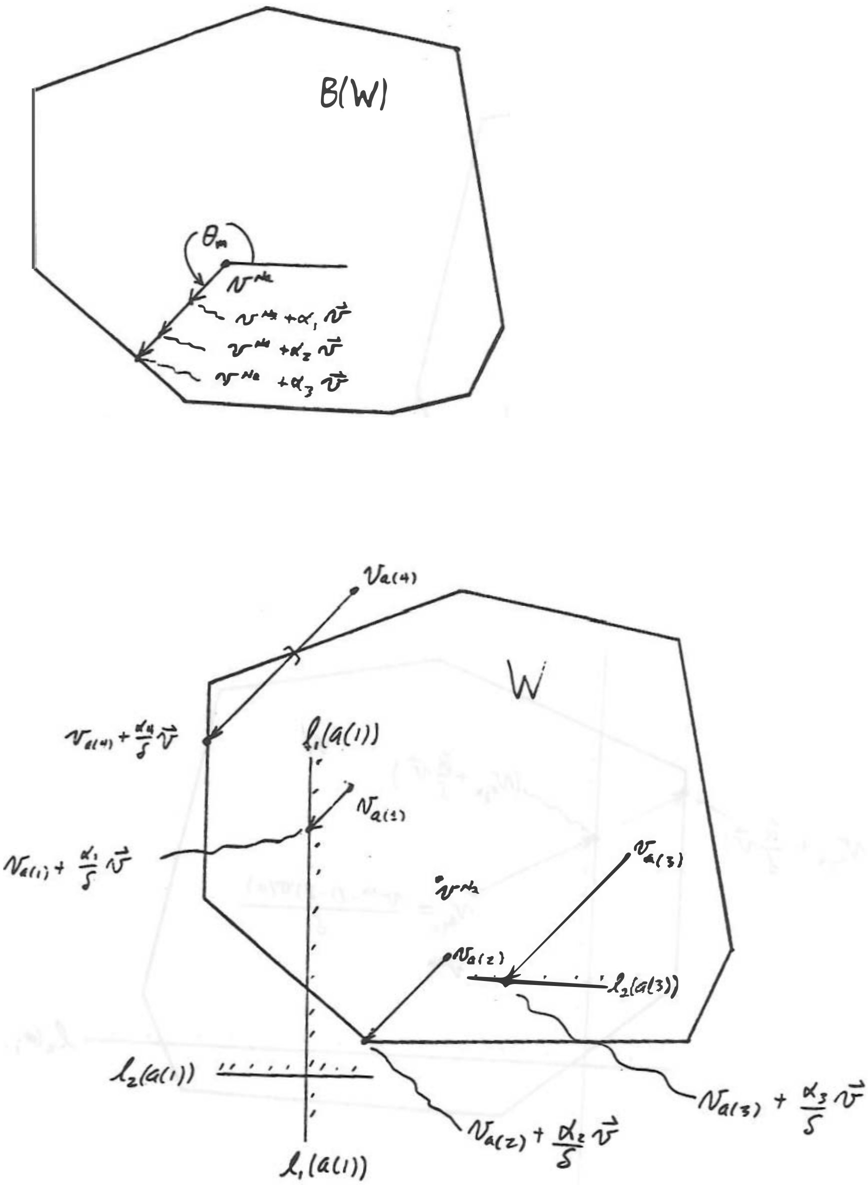

Figure 4: Ray extension from various action profiles

$a(1)$: it shifts $v_{a(1)}$ to the upper left of v^{Na}. As we increase α at angle θ_m, we find that we encounter player 1's IC constraint first, $\ell_1(a(1))$. Hence, action profile $a(1)$ produces a maximal α of length α_1. Moving on to action profile $a(2)$, $v_{a(2)}$ shifts directly below v^{Na}. The IC constraints for this action profile are non-binding, therefore we hit the constraint $v_{a(2)} + \frac{\alpha}{\delta}v(\vec{\theta}_m) = w(\alpha) \in W$ first. This determines that the maximal α for $a(2)$ is α_2. For action profile $a(3)$ we hit player 2's IC constraint first (the IC constraint for player 1 is not depicted) giving us a maximal α which we denote α_3. Action profiles do not always generate a point v_a which lies in W. We show this with action profile $a(4)$. As we extend α, the point $w(\alpha)$ enters the set W. Hence, it is possible that although $\alpha = 0$ is not in the constraint set, there are larger values of α that do fall in the constraint set. With θ_m remaining fixed, after having looped through each action profile we determine the largest value $\alpha(a_j, a_k)$ and set it equal to α^* (step 1.2). We determine the extremal elements of $B(W)$ at each angle θ_m in cartesian coordinates, $\tilde{w}(\alpha^*(\theta_m), \theta_m) = v_a^+ + \alpha^* v(\vec{\theta}_m)$ (step 1.3). We then convexify this set of extremal points and then fit an approximation of $R(\theta)$ (steps 2 and 3).

4. An Example: Credible Fiscal Policy in a Primitive Society.

In this example a government must finance a varying but periodic series of (exogenous) expenditures, either through varying the taxes paid by its constituents, or through financing fluctuations in expenditures with the help of a lender. Government's objective is to maximize the welfare of its constituents. Included in the specification of the government's action space is a budget constraint. If an action violates the budget constraint, it is rendered unavailable. Though the government's source of revenue is a tax on the production of households, it cannot directly choose allocations of consumption and leisure on behalf of the households. Naturally, these allocations are chosen by the households themselves, in light of the tax rate. Taxes are distorting and therefore decrease households' welfare beyond losses that would arise through lump sum taxation. Due to its restriction to a distorting instrument of taxation, the government prefers to smooth taxes, since this maximizes household welfare subject to the government financing its expenditures.

The lender is risk neutral. He lends to the government out of an endowment he receives each period, ω. His payoff, d_t, is the sum of the endowment he receives each period less the net transfer made to the government. One can interpret the lender as a banking house or cartel of banks, assuming also that the coalition is robust to defection by the individual players that comprise it.

The fiscal problem faced by the government reduces to a two-player game between the government and its lender. The government is tempted to default on the lender once it has debt outstanding; the lender protects himself from such opportunism with the threat of withdrawing lending services indefinitely in the future should the government default.

Literature on sovereign lending[2] has shown that, perhaps surprisingly, this threat *is not sufficient* to support lending in all environments. That is, the Folk Theorem does not hold for all games of sovereign lending. The key to such "no lending results" is whether the government has access to an alternative channel for smoothing taxes: if the government does not have access to a storage technology, bankers effectively have a sanction technology. They can deny the government the means by which to smooth taxes in the face of fluctuating expenditures. If the government does have access to a storage technology (e.g., or a state-contingent form of savings deposit in an environment of uncertainty), it can self-smooth, and will always be better off not paying lenders, and using the proceeds of default to build a tax-smoothing fund.

As an illustration of the algorithm, we replicate these well-known results in the economy described above.

4.1 Program 1: the government cannot save

The bargaining session works as follows: all lending matures in one period. Lending of longer maturity must be arranged by successively "rolling over" principal. The government begins by stating how much interest and principal from last period's loans it will pay back, R_tB_{t-1}, where R_t is the gross factor of return (one plus the interest rate). Simultaneously, the government states how much fresh credit it wants, $\tilde{B}_t$. Next, the coalition of lenders state how much they are willing to lend, $\hat{B}_t$. New credit extended is $B_t = \min(\hat{B}_t, \tilde{B}_t)$, and the net transfer of funds in the period is $(B_t - R_tB_{t-1})$.[3] This specification reduces to an arrangement where the government and its lender bargain over net flows: let $\tilde{T}_t = (\tilde{B}_t - R_tB_{t-1})$ and $\hat{T}_t = (\hat{B}_t - R_tB_{t-1})$. Then $T_t = \text{sign}(\tilde{T}_t)\cdot\min(|\tilde{T}_t|, |\hat{T}_t|)$ if $\tilde{T}_t$ and $\hat{T}_t$ are the same sign, $T_t = 0$ if $\tilde{T}_t$ and $\hat{T}_t$ are of opposite sign.

The repeated game has a state variable, the level of government expenditure. For a given profile of actions, players' current and expected future payoffs differ according to the value of the state variable. The discounted present value of a strategy will also depend upon the current value of the state. Hence, the discounted value of an equilibrium is actually a set of values, indexed by the states that occur along the path arising in that equilibrium. The set of equilibrium values is comprised of many sets, one associated with each state of the world. Because the game is recursive, given the state of the world, the set of equilibrium values that can be achieved does not depend on time.

Formally, the lender's problem is

[2] Bulow and Rogoff (1989a, 1989b); Chari and Kehoe (1993a, 1993b).

[3] Technically, the timing scheme amounts to a device of exposition: the solution algorithm used to compute equilibruim utilizes the stage game in its normal form, which is equivalent to a version of the game where neither player observes the other's action at the time he chooses. We do not adopt a Rubenstein bargaining set-up for negotiations, common in other of the debt literature.

$$V(g_0) = \max_{\{\widehat{T}_t\}} \quad (1-\delta)\sum_{t=0}^{\infty} \delta^t d_t \tag{16}$$

$$\text{subject to:} \quad d_t = \omega - T_t \tag{17}$$

$$T_t = \begin{cases} \operatorname{sign}\left(\widetilde{T}_t\right) \min\left(\left|\widetilde{T}_t\right|, \left|\widehat{T}_t\right|\right) & \text{if } \quad \widetilde{T}_t \text{ and } \widehat{T}_t \text{ are of the same sign} \\ 0 & \text{if } \quad \widetilde{T}_t \text{ and } \bar{T}_t \text{ are of the opposite sign.} \end{cases} \tag{18}$$

$$(1-\delta)\sum_{t=s}^{\infty} \delta^t d_t \geq V(g_s) \tag{19}$$

In the problem δ is the discount factor, (1-δ) is a normalization factor that scales the discounted present value of utility to the range of single period utility values. Equation 3 is the bargaining rule on net transfers between government and banker explained above. Equation 4 imposes subgame perfect behavior on the lender.

The economy is inhabited by a number of identical households that produce the single consumption good by supplying labor. They have access to a production technology, $f(n_t)$, that allows them to convert labor, n_t, into the consumption good. This function is increasing in its first derivative and concave. Households optimize over an infinite horizon, δ is the discount factor, and preferences are defined over consumption and leisure, $U(c_t,\ l_t)$. The households' basic choice is how much to work versus how much to eat. The formal problem is

$$\max_{\{c_t, l_t\}} \quad (1-\delta)\sum_{t=0}^{\infty} \delta^t U(c_t, l_t) \tag{20}$$

$$\begin{aligned} \text{subject to:} \quad c_t &\leq (1-\tau_t) f(n_t) & (21) \\ l_t &= 1 - n_t. & (22) \end{aligned}$$

The family is a "tax-taker", that is, it accepts the tax levy τ_t as given each period. The family does not save because it is denied means to do so. Assume that production technology and preferences are such that the revenue maximizing tax rate, τ^*, always yields revenues sufficient to cover government expenditure: $g_t \leq \tau^* f(n_t(\tau^*))$. Households do not behave strategically. They go through life equating marginals in their repeated, static problem.

The government solves the problem

$$W(g_0) = \max_{\{\tau_t, \widetilde{T}_t\}} \quad (1-\delta)\sum_{t=0}^{\infty} \delta^t U(c_t, l_t) \tag{23}$$

$$\text{subject to:} \qquad g_t = T_t + \tau_t f(n_t), \tag{24}$$

$$U_2(c_t, l_t)/U_1(c_t, l_t) = (1 - \tau_t) f'(n_t) \tag{25}$$

$$c_t = (1 - \tau_t) f(n_t) \tag{26}$$

$$T_t = \begin{cases} \operatorname{sign}\left(\tilde{T}_t\right) \min\left(\left|\tilde{T}_t\right|, \left|\hat{T}_t\right|\right) & \text{if} \quad \tilde{T}_t \text{ and } \hat{T}_t \text{ are of the same sign} \\ 0 & \text{if} \quad \tilde{T}_t \text{ and } \hat{T}_t \text{ are of the opposite sign.} \end{cases} \tag{27}$$

$$(1 - \delta) \sum_{t=s}^{\infty} \delta^t U(c_t, l_t) \geq W(g_s) \tag{28}$$

Equation 8 is the government's objective function, equal to the discouted present value of the weighted utility of its subjects. $U_1(c_t,\ l_t) > 0$, $U_2(c_t,\ l_t) > 0$ and $U(c_t,\ l_t)$ is concave. Equation 9 is the government budget constraint. Domestic spending, g_t, must equal the net current flow of funds from the lenders plus total revenue taxes. Equations 10 and 11 are the households' incentive compatibility constraints. They determine how government taxes affect consumption and labor supply. Equations 2, 9 and 11 yield the aggregate resource equilibrium condition $c_t + d_t + g_t = f(n_t) + \omega$. Equation 12 is the outcome rule for bargaining over the net flow of funds between the lender and the borrower. Equation 13 imposes subgame perfection on the government.

Solutions to Program 1 show that in punishment phases of paths arising in subgame perfect equilibria, the government must move taxes up and down with fluctuating expenditure. In high welfare equilibria, the government borrows and repays to allow smoothing of taxes. These results are graphed and discussed in greater detail in Section 4.

4.2 Program 2: the government can save

In this program, the government has access to a storage technology. This makes the lender's penalty of cutting off future credit much less severe. The government can smooth without creditors, but it must save in advance. The government now solves the problem

$$W(g_0, A_0) = \max_{\{\tau_t, \tilde{T}_t, A_{t+1}\}} \quad (1 - \delta) \sum_{t=0}^{\infty} \delta^t U(c_t, l_t) \tag{8}$$

$$\text{subject to:} \qquad g_t = \tau_t f(n_t) + T_t + R_t A_t - A_{t+1}, \tag{14}$$

$$U_2(c_t, l_t)/U_1(c_t, l_t) = (1 - \tau_t)f'(n_t) \tag{10}$$

$$c_t = (1 - \tau_t)f(n_t) \tag{11}$$

$$T_t = \begin{cases} \operatorname{sign}\left(\tilde{T}_t\right) \min\left(\left|\tilde{T}_t\right|, \left|\hat{T}_t\right|\right) & \text{if} \quad \tilde{T}_t \text{ and } \hat{T}_t \text{ are of the same sign} \\ 0 & \text{if} \quad \hat{T}_t \text{ and } \tilde{T}_t \text{ are of the opposite sign.} \end{cases} \tag{12}$$

$$(1 - \delta)\sum_{t=s}^{\infty} \delta^t U(c_t, l_t) \geq W(g_s, A_s) \tag{13}$$

Households' and the lender's problems are unchanged from Program 1.

Granting the government a storage technology causes lending to break down. This is because in the first period the government owes the lender a repayment it can instead save the resources that were to be used to repay. In the next period the government has need of resources in order to smooth taxes, it draws down its savings. The lender, in anticipation of this, never lends. These results are sensitive to the assumptions: $\tau^* f(c(\tau^*)_t) \geq g_t$ is critical. It is also important that the sequence $\{g_t\}_{t=0}^{\infty}$ is known with certainty. If $\{g_t\}_{t=0}^{\infty}$ was stochastic, a no-lending result would depend the on distribution and support of g_t. With a storage technology and no lending the government is worse off at the best equilibria of Program 2 than it would be at the best equilibria of Program 1: in those states of the world where the government has low savings and high expenditures, it must raise taxes and defer a plan of smooth taxes until the next period of low government expenditure. If it could borrow, it could begin smoothing right away. These findings are sensitive to the efficiency of the government's storage technology. If it can save and earn interest equal to or greater than the lender's discount rate, there will be no lending whatsoever. As interest earned by the government approaches zero, or becomes negative, small amounts of debt can be supported. As the storage technology becomes less efficient, lending nears levels for Program 1. This confirms results of Chari and Kehoe (1993b) and Bulow and Rogoff (1989a) in this environment.

4.3 Computational results

For the computation of all programs, we assume the following functional forms:

$$U(c_t, l_t) = c_t^{(1-\sigma_1)}/(1 - \sigma_1) + l_t^{(1-\sigma_2)}/(1 - \sigma_2), \tag{15}$$

$$f(n_t) = n_t^{\gamma}. \tag{16}$$

The period over which the experiments will be run will be about three or four years. We choose this period length because we are modeling expenditure fluctuations and this seems a reasonable duration to posit for a typical cause of expenditure fluctuation, war. Therefore $\delta = 0.9$ for most experiments.

4.3.1 Computations for Program 1

To run the first computation we set $\sigma_1 = 0.6$, $\sigma_2 = 0.2$. While these elasticities are not particularly "realistic", for the additive separable specification of utility and the households' environment it yields negative labor supply response to the tax rate. (For log preferences, i.e. $\sigma_1 = \sigma_2 = 1$, labor supply is inelastic; for $\sigma_1 > 1$ and $\sigma_2 > 1$, labor supply rises in response to higher taxes.) The production parameter γ is set at 0.65. The process for exogenous government expenditure is of period two: one period of high expenditure, one of low (four years of war, four years of peace). $\{g_t\}_{t=0}^{\infty} = \{0, \quad 0.24, \quad 0, \quad 0.24, \ldots\}$. The endowment for the lender, $\omega = 0.24$. The lender is always capable of funding the government's full need for finance. Utility functions were scaled up or down by affine transformation so that payoffs lie in the interval [0, 10]. Calling the government player 1, the lender player 2, and their respective payoffs $\Pi_1(a)$ and $\Pi_2(a)$, we have

$$\Pi_1(a) = \xi_1 + \xi_2 U(c_t, l_t)$$

and

$$\Pi_2(a) = \zeta_1 + \zeta_2 d_t.$$

The values $\xi = (-26, 15.5)$ and $\zeta = (0, 20.8)$ map government and lender payoffs into $[0, 10] \times [0, 10]$.

Finally, we discretize both action and state spaces of the government and bankers. Discretization is not necessary for variables c_t, and l_t, for they are determined as a result of the government's choice of τ and can be solved exactly without increasing computational complexity. For the government's actions,

$$\tau_t \in \{0.0, \quad 0.2, \quad 0.46, \quad 0.715\};$$

$$\bar{T}_t = \bar{B}_t - R_t B_{t-1} \in \{-0.24, \quad -0.18, \quad -0.12, \quad -0.06, \quad 0.0, \quad 0.06, \quad 0.12, \quad 0.18, \quad 0.24\}.$$

For the lender's action:

$$\hat{T}_t = \bar{B}_t - R_t B_{t-1} \in \{-0.24, \quad -0.18, \quad -0.12, \quad -0.06, \quad 0.0, \quad 0.06, \quad 0.12, \quad 0.18, \quad 0.24\}.$$

Negative transfers go from government to lender; positive transfers go from lender to government. We have chosen the grids on g_t, T_t and τ_t so that the accounting of the government's

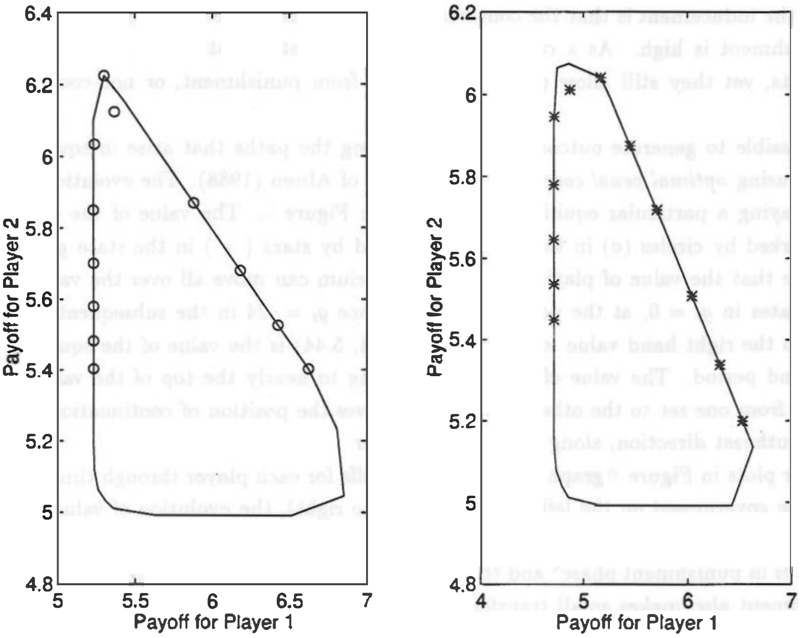

Figure 1: Value sets, Program 1

budget leaves only small amounts of taxed resources unallocated. However, the government still has the choice to give these inter-grid rounding errors back to consumers lump sum, or to give them to the lender, as a small adjustment to the bargained transfer in the lender's favor.

The value sets for Program 1 are shown in Figure 1 for the two values of the state variable g_t. The axes of the graphs represent the average of the present discounted values each player receives from playing a subgame perfect equilibrium. The value set corresponding to peace ($g_t = 0$) appears on the left, war ($g_t = .24$) on the right. There are many possible equilibria, and by extension, many equilibrium values in each state of the world. The punishment values on the southwest border of the value sets arise in two types of equilibria: the first are autarkic—the government never borrows, and the lenders never lend. The second are "carrot and stick" punishments—in these equilibria, it is not best to play non-cooperatively, but rather to go along with your own punishment. In such equilibria, the player that is known as the deviator deliberately chooses an action that results in a low one-shot payoff.

Of course the inducement is that the continuation promise associated with going along with one's punishment is high. As a consequence, carrot and stick paths can generate severe punishments, yet they still allow players to emerge from punishment, or non-cooperative play.

It is possible to generate outcome allocations along the paths that arise in equilibrium from $V(x)$ using *optimal penal codes*, in the language of Abreu (1988). The evolution of the value of playing a particular equilibrium is shown in Figure 5. The value of the outcome path is marked by circles (o) in the state $g_t = 0$ and by stars ($*$) in the state $g_t = .24$. Firstly note that the value of playing a given equilibrium can move all over the value sets. Play originates in $g_t = 0$, at the point (5.2, 5.4). Since $g_t = .24$ in the subsequent period, skip over to the right hand value set: the star at (4.4, 5.44) is the value of the equilibrium in the second period. The value of play evolves rising to nearly the top of the value sets, alternating from one set to the other. Then, play moves the position of continuation values down, in southeast direction, along the Pareto frontier.

The four plots in Figure 6 graph the one-shot payoffs for each player through time on the top row (the government on the left, the lender on the right), the evolution of value on the bottom row. In periods 1-11, period payoffs for the government are volatile, reflecting the fact that it is in punishment phase[4] and that it is not being allowed to smooth by the lender. The government also makes small transfers to the lender out of revenues not allocated to g_t. This is part of "going along with its own punishment." In period 10 the government begins to make larger transfers to the lender during periods of peace (declining peaks in the top left plot). With that show of "good faith", the lender starts lending to the government. Government taxes, and period-payoffs are then smooth for the remainder of play. The lender now aborbs the volatility of the sequence g_t through financial flows. The four plots in Figure 3 graph the underlying economic allocations through time.

Note that tbis path is only one among literally infinite paths that arise in equilbrium. We were able to see allocations during a punishment because we chose to start the path at a punishment value. Paths moving along the northeast frontiers of the equilibrium value sets always have the government smoothing taxes.

4.3.2 Computations for Program 2

In Program 2, the government has the capacity to save. As with Program 1, parameters are set as $\sigma_1 = 0.6$, $\sigma_2 = 0.2$, and $\gamma = 0.65$. The process for exogenous government expenditure remains $\{g_t\}_{t=0}^{\infty} = \{0, \quad 0.24, \quad 0, \quad 0.24, \dots\}$, and the endowment for the lender $\omega = 0.24$. The values $\xi = (-26, 15.5)$ and $\zeta = (0, 20.8)$.

[4]Once on a cooperative path, punishments do not arise in equilibrium as can happen in a game with imperfect information. However by initiating the game at a punishment value, we are able to trace out punishment paths that would otherwise lie "off path."

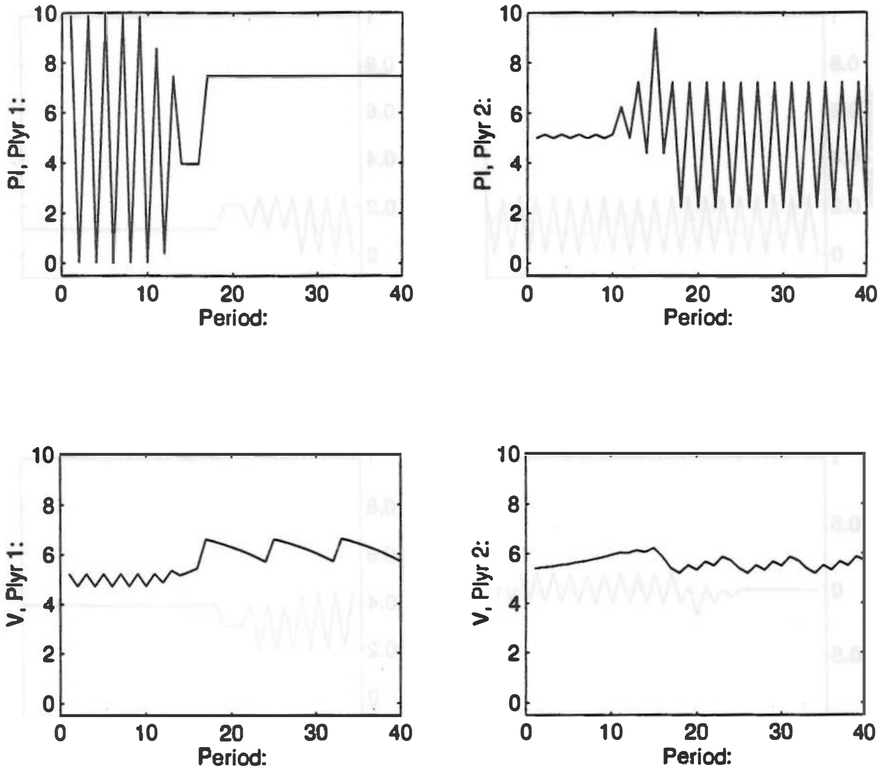

Figure 6: Evolution of Value, Payoffs over time, Program 1

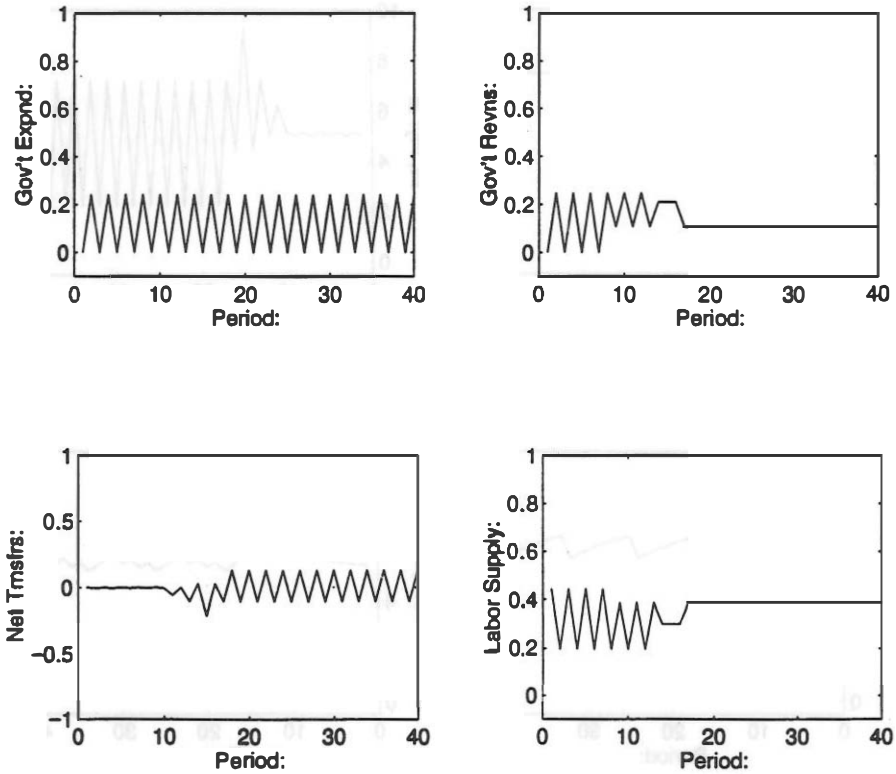

Figure 7: Evolution of allocations over time, Program 1

For the government's action space, τ is discretized as

$$\tau_t \in \{0.0, \quad 0.2, \quad 0.46, \quad 0.715\}.$$

The gross rate of interest for the government, R, is constant at $1/\delta$ or 1.111. The government's assets lie in the grid

$$A_t \in \{0.0, \quad 0.06, \quad 0.12\}.$$

The government's bargaining position for transfers lies in

$$\tilde{T}_t = \tilde{B}_t - R_t B_{t-1} \in \{-0.24, \quad -0.18, \quad -0.12, \quad -0.06, \quad 0.0, \quad 0.06, \quad 0.12, \quad 0.18, \quad 0.24\}.$$

The lender's transfer offers lie in

$$\hat{T}_t = \hat{B}_t - R_t B_{t-1} \in \{-0.24, \quad -0.18, \quad -0.12, \quad -0.06, \quad 0.0, \quad 0.06, \quad 0.12, \quad 0.18, \quad 0.24\}.$$

There are six possible states in the game, one for each combination of g_t and A_t.

The value sets for Program 2 are shown in Figure 8. The only four states are shown, $(g_t, A_t) = \{(0,0), \quad (0,0.12), \quad (.24,0), \quad (.24,.12)\}$. The value set in each state has collapsed to a single point. This is because lending breaks down, and there is a unique equilibrium action (and hence a unique subsequent path of play and a unique value) in each state. The lenders never lend, and the government smooths using its own assets. The path shown is marked by a $+$ and a $\times$: in state 1 ($g_t = 0, \quad A_t = 0$), the government saves 0.12. In state 6 ($g_t = .24, \quad A_t = .12$), it raises revenues of .12, draws down savings, and gives interest back to the households as a lump-sum payment. The period payoffs and evolution of values through time are shown in Figure 9, and Figures 10 and 11 show the economy's allocations through time.

5. Conclusion

In this paper we have described and implemented a computer algorthm to solve discounted sugergames with perfect monitoring and state variables. Key assumptions are restriction to pure strategies, the inclusion of public randomization and a finite state space. We represent the key object in the "dynamic programming approach to supergames", the value set, by its boundary. This parsimonious representation of the value set allows us to compute an inner approximation of the set rapidly, far more rapidly than would be possible using "brute force" methods such as exhaustive search.

We demonstrate the algorithm for a fiscal policy game played by a government and its lender. The level of government expenditures to be financed is the state variable. Computations verify theoretical results of Cronshaw and Luenberger and Abreu, Pearce and Stacchetti, Bulow and Rogoff and Chari and Kehoe. This computational approach is generalizable to games of imperfect information, stochastic games, and games of asymmetric information. Areas of research we are exploring are ways to produce an "outer-approximation" of the value set, and other applications.

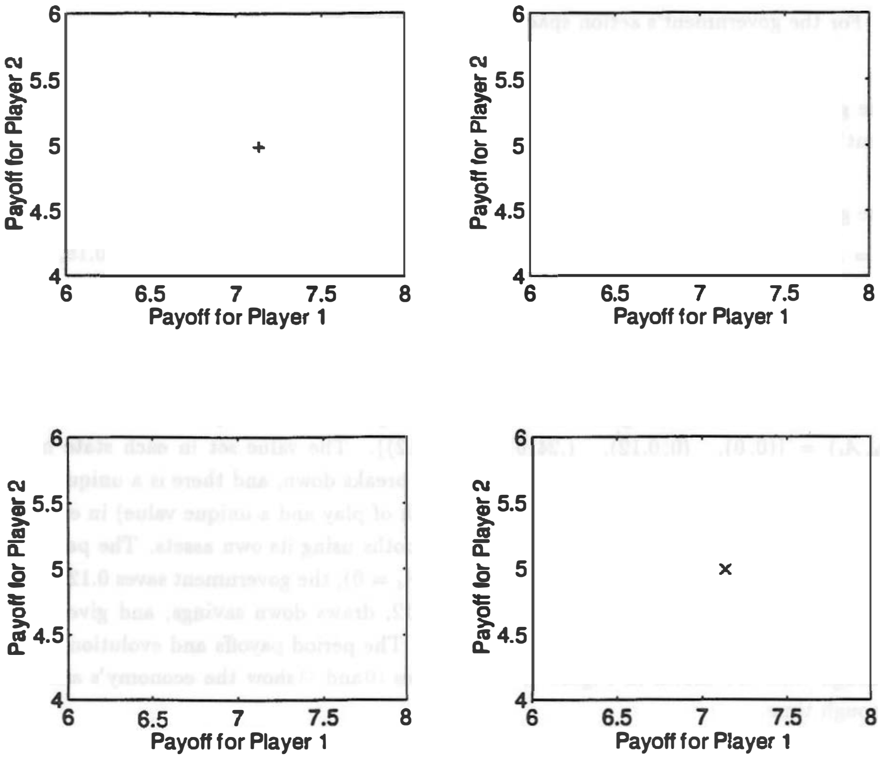

Figure 8: Value sets, Program 2

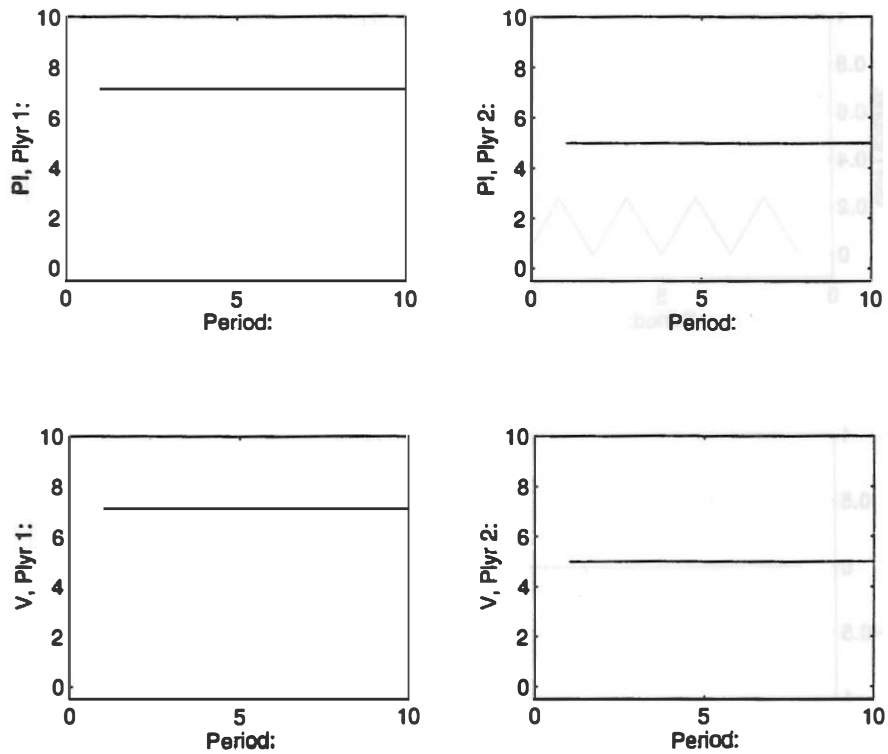

Figure 9: Evolution of Value, Payoffs over time, Program 2

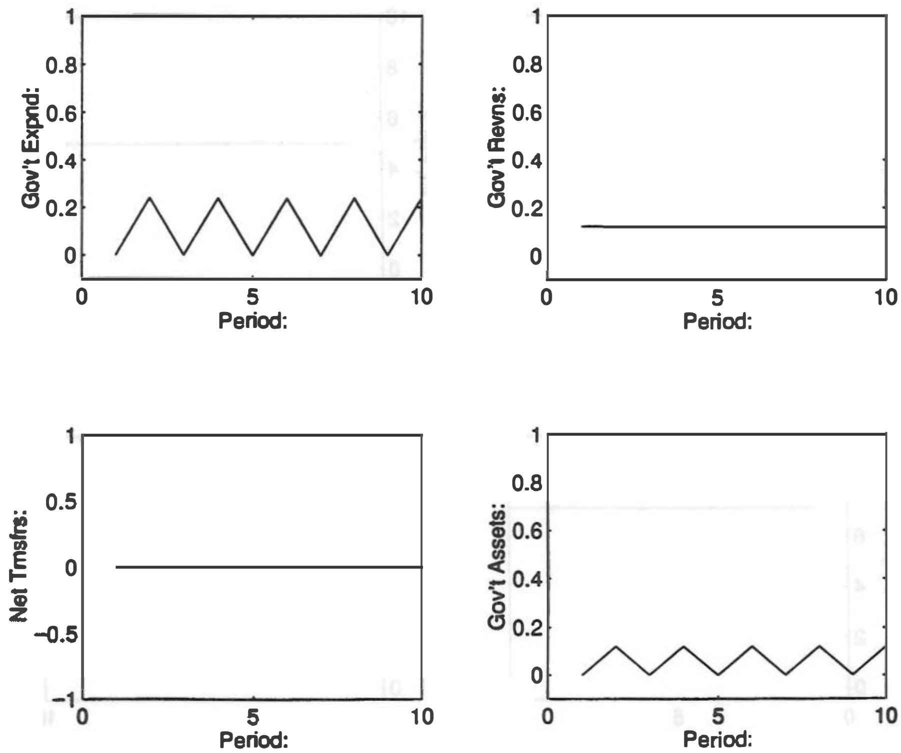

Figure 10: Evolution of allocations over time, Program 2

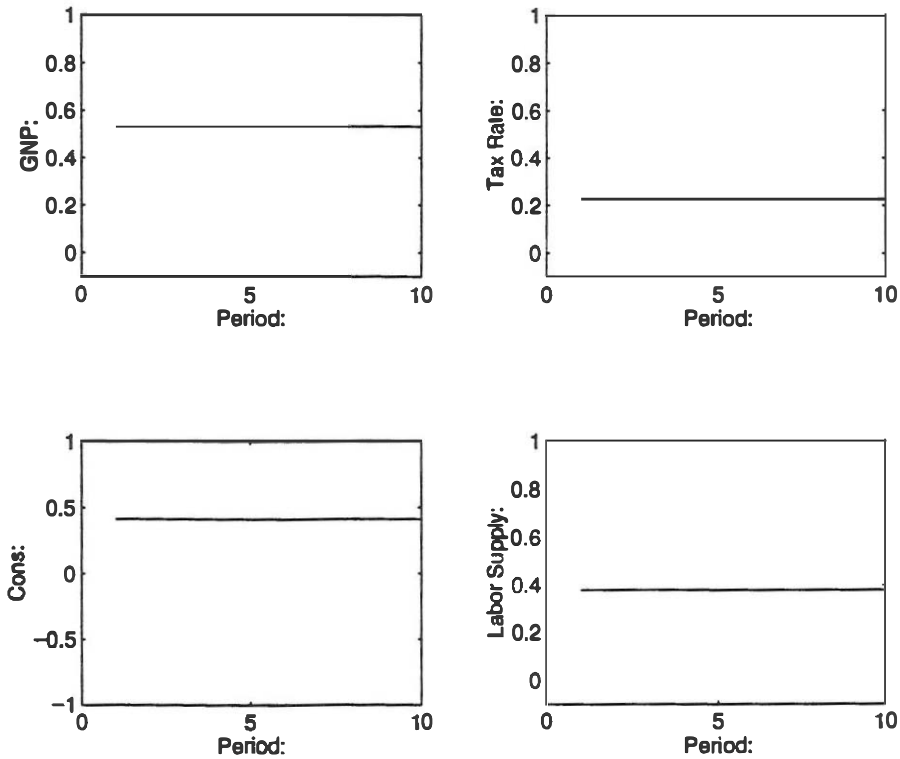

Figure 11: Evolution of allocations over time, Program 2

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