DISAGREEMENT ABOUT INFLATION AND THE YIELD CURVE

2015

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Documentos de Trabajo
N.º 1532

BANCO DE ESPAÑA

Eurosistema

DISAGREEMENT ABOUT INFLATION AND THE YIELD CURVE [(*)]

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Abstract

We show theoretically that inflation disagreement drives a wedge between real and nominal yields and raises their levels and volatilities. We demonstrate empirically that an inflation disagreement increase of one standard deviation raises real and nominal yields and their volatilities, break-even inflation, and the inflation risk premium by at least 30% of their respective standard deviations. Inflation disagreement is positively related to consumers' cross-sectional consumption growth volatility and trading in bonds, interest rate futures, and inflation swaps. Calibrating the model to disagreement, inflation, and yield data reproduces the economically significant impact of inflation disagreement on real and nominal yield curves.

Keywords: inflation disagreement, relative entropy, real and nominal yields, yield volatilities, break-even inflation, inflation risk premium, cross-sectional consumption growth volatility, trading on inflation.

JEL classification: D51, E43, E52, G12.

Resumen

Este trabajo muestra cómo las discrepancias en torno a la inflación esperada abren una brecha entre las rentabilidades reales y nominales y elevan sus niveles y volatilidades. Se demuestra empíricamente que un incremento de estas discrepancias en una desviación estándar aumenta las rentabilidades reales y nominales y sus volatilidades, la inflación *break-even* y la prima de riesgo de inflación por lo menos en el 30 % de sus respectivas desviaciones estándar. Las discrepancias en torno a la inflación esperada están positivamente relacionadas con la volatilidad de sección cruzada del crecimiento del consumo y con la participación de los consumidores en los mercados de bonos, futuros sobre tipos de interés y *swaps* de inflación. La calibración del modelo a los datos de discrepancias en la inflación esperada, inflación y rentabilidades reproduce el impacto económicamente significativo de las discrepancias en torno a la inflación esperada sobre las curvas de rentabilidades reales y nominales.

Palabras clave: discrepancias en torno a la inflación esperada, entropía relativa, rentabilidades reales y nominales, volatilidades de las rentabilidades, inflación *break-even,* prima de riesgo de inflación, volatilidad de sección cruzada del crecimiento del consumo, *trading* en inflación.

Códigos JEL: D51, E43, E52, G12.

Empirical evidence from survey data, such as the early work of Mankiw, Reis, and Wolfers (2004), shows that households, as well as professional forecasters, have different opinions about inflation. For example in June 2014, the interquartile range of annual inflation expectations is 1.7% to 5.0%, according to the Michigan Surveys of Consumers, and 1.7% to 2.2%, according to the Survey of Professional Forecasters. Figure 1 shows that inflation disagreement for both consumers and professional forecasters also varies substantially over time. Here inflation disagreement is defined as the cross-sectional standard deviation of inflation forecasts across survey participants.

Sims (2009) discusses how time-varying inflation disagreement could arise:

"For example in the period 1975-2000, the wide swings in US fiscal policy could easily have led to differing views about the implications of those swings for future inflation. And in the late 90s in the US, when unemployment and interest rates stayed persistently low, there were differences of view even among specialist economists about the long term implications for the inflation rate."

Malmendier and Nagel (2014) show that differences in life-time experiences of consumers lead to disagreement about expected inflation. Stock and Watson (2010) discuss the difficulties faced by professional forecasters in building inflation forecasting models. All of this evidence points toward the difficulties faced by households and professionals in accounting for inflation, which in turn, generates heterogeneity in investment and consumption decisions. For example as Malmendier and Nagel (2014) show, households who think that inflation will be high are more likely to borrow using fixed-rate mortgages and less likely to invest in long term bonds. Professional investors struggle with their views toward inflation too. For example, PIMCO's Total Return Fund bet on increased inflation after the Great Recession, which never materialized.[1] Given this evidence, we study theoretically and empirically how inflation disagreement impacts real and nominal Treasury bonds.

We consider a frictionless exchange economy with complete markets. Investors disagree about inflation, so they make different consumption-savings decisions because they perceive different real returns on investments. Thus, inflation disagreement raises the cross-sectional consumption growth volatility. Moreover, each investor perceives high real returns on their investments and so they think they will become wealthier. As consumption today is fixed in a pure exchange economy, real rates adjust for markets to clear. Therefore, inflation disagreement raises real yields and their volatilities relative to the no-disagreement benchmark,

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[1]See, for example, thereformedbroker.com/2014/09/28/do-we-need-to-fire-pimco/.
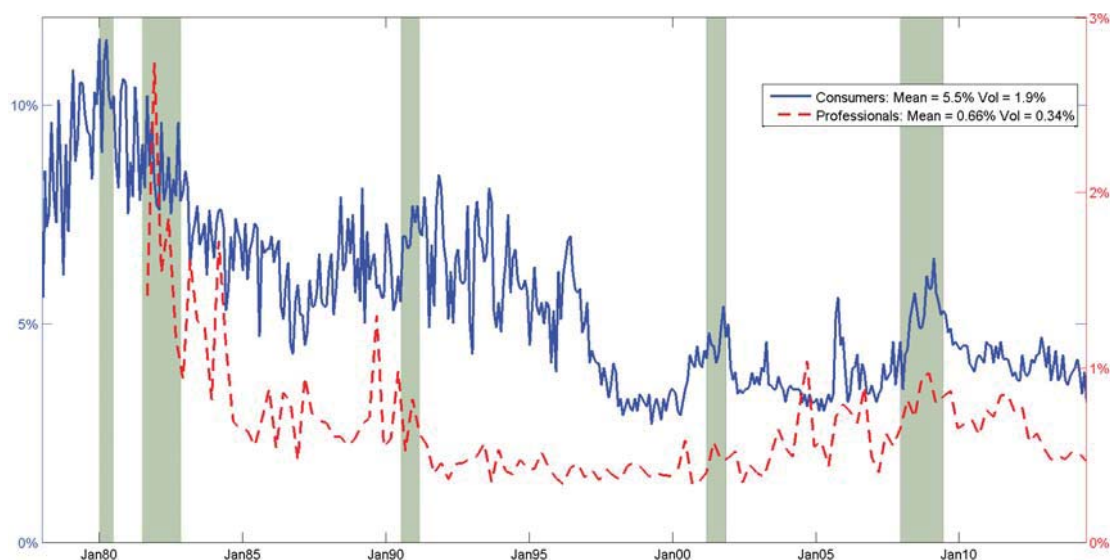
Figure 1: Inflation Disagreement of Consumers and Professionals
The blue solid line shows the cross-sectional standard deviation of one year ahead inflation expectations from the Michigan Surveys of Consumers at a monthly frequency, while the red dashed line shows the same measure from the Survey of Professional Forecasters at a quarterly frequency. The shaded regions denote NBER-dated recessions.

if the wealth effect dominates the substitution effect.[2] We show that these results are robust to the particular form of inflation disagreement. So, investors can disagree about the joint distribution of inflation and consumption, not just expected inflation, as long as they agree on the distribution of consumption.

To study the effect of changes in disagreement on real yields and the cross-sectional consumption growth volatility, we measure inflation disagreement more generally as the relative entropy of investors' inflation beliefs. This measure allows us to show that real yields and the cross-sectional consumption growth volatility increase with inflation disagreement when investors disagree about higher order moments of the inflation distribution, not just the mean, or have beliefs that do not even belong to the same class of distributions. In all our examples, real yield volatilities also increase in relative entropy.

The effects of inflation disagreement on nominal yields are ambiguous without making additional assumptions about investors inflation beliefs because investors may agree on a high expected inflation rate or disagree on a very low expected inflation rate, in which case it is impossible to compare nominal yields. We show that nominal yields increases

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[2]The real yield is decreasing, if the substitution effect dominates, which would require a risk aversion coefficient less than one with power utility and would be inconsistent with habit preferences.

with inflation disagreement when holding the consumption-weighted belief about the mean and risk premium of inflation in the respective homogeneous belief economies constant. This result does not immediately follow from an increase in real yields because inflation disagreement also drives a wedge between real and nominal yields as investors require an inflation risk premium for changes in the investment opportunity set due to speculative inflation trades. In all our examples, nominal yield volatilities are also higher with than without disagreement.

Empirically, we find broad support for our theoretical results. We use the Surveys of Consumers from the University of Michigan and the Survey of Professional Forecasters to compute an inflation disagreement measure for consumers and professionals, respectively. Statistically, real and nominal yields across all maturities increase with inflation disagreement. The effect of inflation disagreement on the real and nominal yield curve is also economically significant. An increase in disagreement of consumers/professionals by one standard deviation raises real yields on average by 58%/39% of their standard deviation and nominal yields on average by 54%/36% of their standard deviations. In addition, the volatilities of real and nominal yields increase with inflation disagreement and the coefficient estimates also have large economic significance. When inflation disagreement of consumers/professionals rises by one standard deviation, volatilities of real and nominal yields rise on average by 41%/63% and 48%/61% of their standard deviations.

Inflation disagreement not only shifts real and nominal yield curves up, but it also drives a wedge between real and nominal yields. Statistically, inflation disagreement of consumers/professionals raises the break-even inflation rate and the inflation risk premium at all maturities. When inflation disagreement of consumers/professionals rises by one standard deviation, break-even inflation rates rise on average by 60%/36% and inflation risk premiums by 72%/56% of their respective standard deviations.

To further strengthen our empirical results, we explore the channel through which inflation disagreement affects real and nominal Treasury bonds. Specifically, we use the Consumer Expenditure (CEX) Survey to verify that there is a positive relation between cross-sectional consumption growth volatility and inflation disagreement. The drawback of using cross-sectional consumption growth volatility is that the CEX data are of poorer quality than data based on trade in financial markets. Hence, we also show that inflation disagreement has a statistically positive effect on trading in nominal Treasury bonds, fixed income futures, and inflation swaps. These securities have very low basis risk and thus investors may use them to directly trade on their inflation beliefs.

By putting more structure on our model, we ask if it quantitatively matches our empirical results. To accomplish this, we consider a continuous-time model with two investors who disagree about the dynamics of expected inflation. We also allow for habits to help match asset pricing moments. The model admits closed-form solutions for bond prices,[3] is rich enough to capture average yields and yield volatilities, and generates upward sloping real and nominal yield curves. To compare the model to the data, we calibrate the inflation disagreement to the Survey of Professional Forecasters. In particular, we match the average and volatility of the disagreement and the mean and volatility of the consensus forecast. The calibrated model shows an economic significant relation between inflation disagreement and real and nominal yields and their volatilities with a reasonable risk premium and Sharpe ratio for inflation risk. The statistical and economic significance of inflation disagreement in model-based regressions are also very similar to the data.

Our paper is part of a growing literature that studies how disagreement impacts bond markets.[4] In earlier work, Doepke and Schneider (2006) quantitatively explore the impact of inflation on the U.S. wealth distribution under two different assumptions about inflation expectations. Xiong and Yan (2010), show that a moderate amount of heterogeneous expectations about inflation can quantitatively explain bond yield volatilities, the failure of the expectations hypothesis, and the Cochrane and Piazzesi (2005) forward factor predictability. Piazzesi and Schneider (2012), using an overlapping generations model with uninsurable nominal risk and disagreement about inflation, study the impact on wealth distributions due to structural shifts in the U.S. economy in the 1970s. Buraschi and Whelan (2013) and Whelan (2014) use survey data about various macroeconomic quantities to study the effects of disagreement on yield curve properties. Hong, Sraer, and Yu (2014) study how disagreement about expected inflation interacted with short-sale constraints can impact the pricing of long maturity bonds. Giacoletti, Laursen, and Singleton (2015) studies the impact of yield disagreement in a dynamic arbitrage free term structure model. Our paper differs from all of these works as we derive novel theoretical predictions that we empirically test on quantities including real and nominal yield levels and their volatilities, the cross-sectional consumption growth volatility, the break-even inflation rate, and the inflation risk premium. Another aspect of our work that differs from the literature is that we calibrate our model to disagreement data.

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[3]Our solution method relies on a binomial expansion similar to the approach in Yan (2008), Dumas, Kurshev, and Uppal (2009), and Bhamra and Uppal (2014). Alternatively, the model can be solved by the generalized transform analysis proposed in Chen and Joslin (2012).

[4]Many of these works grew out of the literature studying the equilibrium impact of heterogeneous beliefs. Earlier works include Harris and Raviv (1993), Detemple and Murthy (1994), Zapatero (1998), and Basak (2000). See Basak (2005) for a survey. Other papers that empirically explore the role of inflation beliefs on the term structure include Ang, Bekaert, and Wei (2007), Adrian and Wu (2010), Chun (2011), and Chernov and Mueller (2012).

I Theoretical Results

This section provides a general framework to study the impact of inflation disagreement on real and nominal yield curves, yield volatilities, cross-sectional consumption growth volatility, break-even inflation, and the inflation risk premium.

Our model is a pure exchange economy with a single perishable consumption good. The time horizon T' of the economy can be finite or infinite. Real prices are measured in units of the consumption good and nominal prices are quoted in dollars. Let C_t denote the exogenous real aggregate consumption process and Π_t the exogenous price process that converts real prices into nominal prices, that is, nominal consumption is $\Pi_t C_t$. The sample space Ω and the information set $\mathcal{F}_t$ on which we define all random variables and probability measures, in short beliefs, represent the uncertainty in the economy.

Two investors share a common subjective discount factor ρ, a Bernoulli utility function $u(C) = \frac{1}{1-\gamma} C^{1-\gamma}$ with $\gamma > 0$, and an exogenous habit process or, more generally, a preference shock H_t. Let $\mathbb{P}^i$ denote investor i's belief about inflation Π_t, consumption C_t, and the preference shock H_t. The investors have the same information set $\mathcal{F}_t$ and agree on the events of $\mathcal{F}_t$ that cannot occur. Hence, there is no asymmetric information and the likelihood ratio defined as $\lambda_t \equiv \frac{d\mathbb{P}^2}{d\mathbb{P}^1}$ is strictly positive and finite.

Both investors trade a complete set of Arrow-Debreu (AD) securities. Let ξ_t^i denote the state price density that represents the AD pricing functional under the probability measure $\mathbb{P}^i$. Each investor chooses a consumption process C_t^i to maximize

$$\mathbb{E}^i \left[\sum_{t=0}^{T'} e^{-\rho t} u \left(\frac{C_t^i}{H_t} \right) \right] \qquad \text{s.t.} \qquad \mathbb{E}^i \left[\sum_{t=0}^{T'} \xi_t^i C_t^i \, dt \right] \leq w_0^i, \qquad (1)$$

where $\mathbb{E}^i$ denotes the expectation under the probability measure $\mathbb{P}^i$ and w_0^i denotes initial wealth of investor i.[5] If time is continuous, then replace the sums in equation (1) with integrals.

We focus on inflation disagreement and thus make the following assumption.

Assumption 1 (Inflation Disagreement). *There is no disagreement about the distribution of consumption and the preference shock.*

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[5]Investors are either endowed with shares of a claim on aggregate consumption or with a fraction of the aggregate consumption process.

We determine the equilibrium consumption allocations C_t^1 and C_t^2 and state price densities ξ_t^1 and ξ_t^2 in the next proposition. The likelihood ratio λ_t summarizes the impact of disagreement on consumption allocations and state prices.

Proposition 1 (Consumption Allocations and State Price Densities)**.** *Optimal consumption allocations are $C_t^1 = f(\lambda_t)C_t$ and $C_t^2 = (1 - f(\lambda_t))C_t$ with*

$$f(\lambda_t) = \frac{1}{1 + (y\lambda_t)^{\frac{1}{\gamma}}}, \tag{2}$$

where $y = \frac{y^2}{y^1}$ and y^i is the constant Lagrange multiplier from the static budget constraint given in equation (1). State price densities are

$$\xi_t^1 = (y^1)^{-1}e^{-\rho t}C_t^{-\gamma}H_t^{\gamma-1}f(\lambda_t)^{-\gamma}, \qquad \xi_t^2 = (y^2)^{-1}e^{-\rho t}C_t^{-\gamma}H_t^{\gamma-1}(1 - f(\lambda_t))^{-\gamma}. \tag{3}$$

Edgeworth Box Example: The left plot of Figure 2 presents an Edgeworth box example with two dates 0 and 1. For simplicity, we set the subjective discount factor to zero and normalize aggregate consumption and the habit or preference shock to one. The price level today is normalized to one and the price level tomorrow is either Π_u or Π_d. There are two investors with different beliefs $\mathbb{P}^i = (p^i, 1 - p^i)$. The likelihood ratio λ equals $\frac{p^2}{p^1}$ with probability p^1 and $\frac{1-p^2}{1-p^1}$ with probability $1 - p^1$. The disagreement parameter is $\Delta = \frac{p^2-p^1}{p_1}$.[6]

Since there is no uncertainty about consumption, full insurance is Pareto efficient if there is no disagreement about inflation ($\lambda_u = \lambda_d = 1$). Hence, each investor consumes the same share of consumption in the high and low inflation state in equilibrium. Suppose investors are endowed with 0.5 units of the date zero consumption good in both states. Then, the initial endowment is an equilibrium if there is no disagreement (tangency point of blue indifference curves). This is no longer true when investors disagree about inflation. For instance, if the first investor thinks that the low inflation state is more likely, then she consumes a larger fraction of consumption in this state because $\lambda_u > \lambda_d$ and thus $f_u < f_d$. Therefore, full insurance is no longer an equilibrium and disagreement affects state prices. The tangency point of the red indifference curves in the left plot of Figure 2 denotes the equilibrium consumption allocation for this case.

Throughout this section, we consider two additional examples where consumption and the habit is normalized to one to illustrate our results. All three examples allow us to focus on how inflation disagreement impacts real and nominal bonds because $\xi_t^1 = \xi_t^2 = 1$ if there is no disagreement.

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[6]We divide by p_1 to make the disagreement parameter comparable across examples.
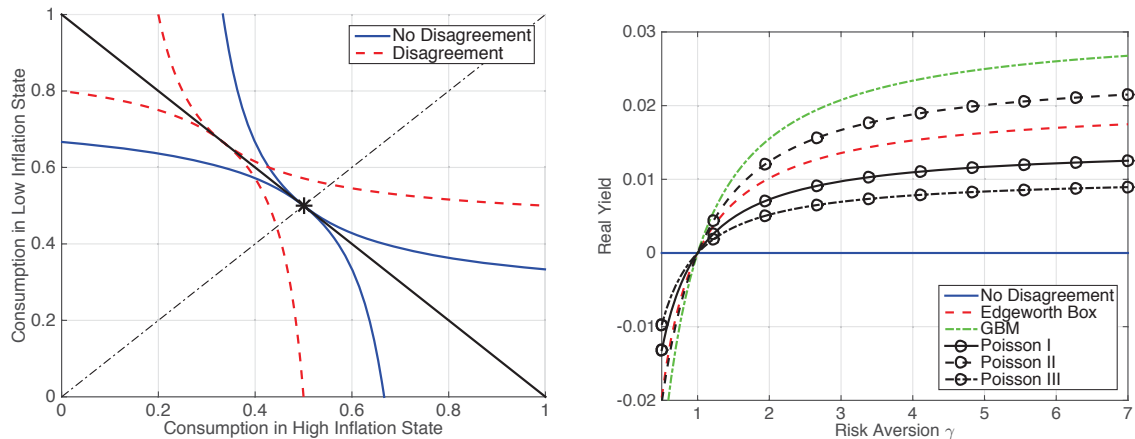
Figure 2: Edgeworth Box and Real Yields

The left plot shows an Edgeworth box example when both investors are endowed with 0.5 units of the date zero consumption good in both states and $\gamma = 2$. The red dashed indifference curves are tangent at the equilibrium allocation with disagreement and the blue solid indifference curves are tangent at the equilibrium allocation without disagreement. Full insurance is no longer an equilibrium when there is disagreement. The right plot shows real one-year yields as a function of γ. When there is no disagreement, then the real yield is zero (blue solid line). With disagreement, real yields are nonnegative if $\gamma \geq 1$ and negative otherwise.

Geometric Brownian Motion Example: Consider a continuous-time economy in which the price level Π_t follows a geometric Brownian motion and two investors disagree on the expected inflation rate. The dynamics of the price level are

$$d\Pi_t = x^i \Pi_t \, dt + \sigma_\Pi \Pi_t \, dz_t^i, \tag{4}$$

where x^i denotes the expected inflation rate and z_t^i denotes the perceived nominal shock of investor i. The dynamics of the likelihood ratio λ_t are

$$d\lambda_t = \Delta \lambda_t \, dz_t^1, \qquad \Delta = \frac{x^2 - x^1}{\sigma_\Pi}. \tag{5}$$

Poisson Example: Consider a continuous-time economy in which the dynamics of the price level are

$$d\Pi_t = x\Pi_{t-} \, dt + \theta \Pi_{t-} dN_{t-}^i, \tag{6}$$

where x denotes a constant and θ denotes the constant jump size with $\theta \neq 0$ and $\theta > -1$. The two investors agree on the jump times of the Poisson process but disagree on the jump

intensity l^i. Hence, they disagree on the expected inflation rate $x + \theta l^i$. The dynamics of the likelihood ratio λ_t are

$$d\lambda_t = \Delta \lambda_{t-} \left(dN_{t-}^1 - l^1 dt \right), \qquad \Delta = \frac{l^2 - l^1}{l^1}. \tag{7}$$

We conclude this subsection by specifying the baseline parameters for all three examples. Edgeworth box example: $p^1 = 0.4, p^2 = 0.6, \Pi_u = 1.25$, and $\Pi_d = 0.9$. The GBM example, where (log) inflation rates are normally distributed with constant mean and volatility, focuses on the effects of disagreement about expected inflation on consumption allocations and asset prices. The baseline parameters for this example are $\sigma_\Pi = 2\%$, $x^1 = 1.5\%$, and $x^2 = 2.5\%$. The Poisson example illustrates how disagreement about expected inflation and higher order moments of inflation affect consumption allocations and asset prices. The baseline parameters are $x = 6\%, \theta = -10\%, l^1 = 12.5\%$, and $l^2 = 27.5\%$.

A   Real Yields and Cross-Sectional Consumption Volatility

A real bond is a default-free zero-coupon bond that pays one unit of the consumption good at its maturity. All real bonds are in zero-net supply and can be priced using a state price density from Proposition 1. Let $B_{t,T}$ denote the real price of a real bond maturing at T with continuously-compounded real yield $y_{t,T}^B = -\frac{1}{T-t} \log(B_{t,T})$, where $T \in [t, T']$. The real price of a real bond is

$$B_{t,T} = \mathbb{E}_t^i \left[\frac{\xi_T^i}{\xi_t^i} \right]. \tag{8}$$

The cross-sectional consumption growth variance from time t to T is

$$\sigma_{\text{CS}}^2(\lambda_t, \lambda_T) \equiv \frac{1}{4} \left(\log \left(\frac{C_T^1}{C_t^1} \right) - \log \left(\frac{C_T^2}{C_t^2} \right) \right)^2 = \frac{1}{4\gamma^2} \left(\log \left(\frac{\lambda_T}{\lambda_t} \right) \right)^2. \tag{9}$$

There are no fluctuations in the cross-sectional consumption distribution when there is no disagreement ($\lambda_T = \lambda_t = 1$). Moreover, there is less variation in cross-sectional consumption allocations if investors are more risk averse because they trade less aggressively on their beliefs. Trading on beliefs also leads to more volatile real yields. We formally show this in the next theorem, where we also discuss how disagreement affects the level of real yields.

Theorem 1. *If Assumption 1 is satisfied, then*

    (i) real yields and their volatilities do not depend on disagreement if $\gamma = 1$,

(ii) real yields are higher with than without disagreement if $\gamma > 1$ (the opposite is true if $\gamma < 1$),

(iii) the cross-sectional consumption growth volatility is higher with than without disagreement, and

(iv) the volatility of real yields is higher with than without disagreement if $\gamma \neq 1$ and λ_t is independent of C_t and H_t.

Why are real yields higher with disagreement if $\gamma > 1$ and lower if $\gamma < 1$? Intuitively, investors make different consumption and savings decisions based on their differing views about inflation. Both investors think they will capture consumption from the other investor in the future; hence, classical income and substitution effects then impact the demand for consumption today. If $\gamma > 1$, then the real interest rate rises to counterbalance increased demand for borrowing. If $\gamma < 1$, then the real interest rate falls to counterbalance lowered demand for borrowing.[7] There is no effect on real yields if the income and substitution effects exactly offset ($\gamma = 1$), as in Xiong and Yan (2010).

We determine bond prices in all three examples in closed form.[8] The right plot of Figure 2 shows real one-year yields as a function of γ. When there is no disagreement, then the real yield is zero (blue solid line). With disagreement, real yields are nonnegative if $\gamma \geq 1$ and negative otherwise. The red dashed, green dash-dotted, and solid black circle lines represent the baseline for the Edgeworth box, GBM, and Poisson examples. The black dashed circle line shows that real yields with lower jump intensities ($l^1 = 5\%$ and $l^2 = 20\%$) and the black dash-dotted circle line shows real yields with higher jump intensities ($l^1 = 20\%$ and $l^2 = 35\%$) than in the baseline case. The three Poisson examples show that real yields are increasing in $\Delta = \frac{l^2 - l^1}{l^1}$ if $\gamma > 1$ and decreasing if $\gamma < 1$.

In the remainder of this subsection, we generalize the results of Theorem 1 by defining a measure of disagreement to study the effects of changes in disagreement on real yield levels and the cross-sectional consumption growth volatility. Measuring disagreement is straightforward in all three examples because investors' beliefs belong to the same class of distributions and there is only disagreement about a single parameter. To measure disagreement among investors more generally, we define disagreement as relative entropy per year.[9] This measure

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[7]For details see Epstein (1988) or Gallmeyer and Hollifield (2008).

[8]We provide details in the Internet Appendix.

[9]The relative entropy or Kullback-Leibler divergence is widely used in statistics or information theory to measure the difference between two probability distributions (see Kullback (1959)). While this measure is not symmetric, the results do not change if we compute the relative entropy with respect to the second investor. Similarly, all our results still follow if we consider other divergence measures suggested in the literature (see Csiszár and Shields (2004)).
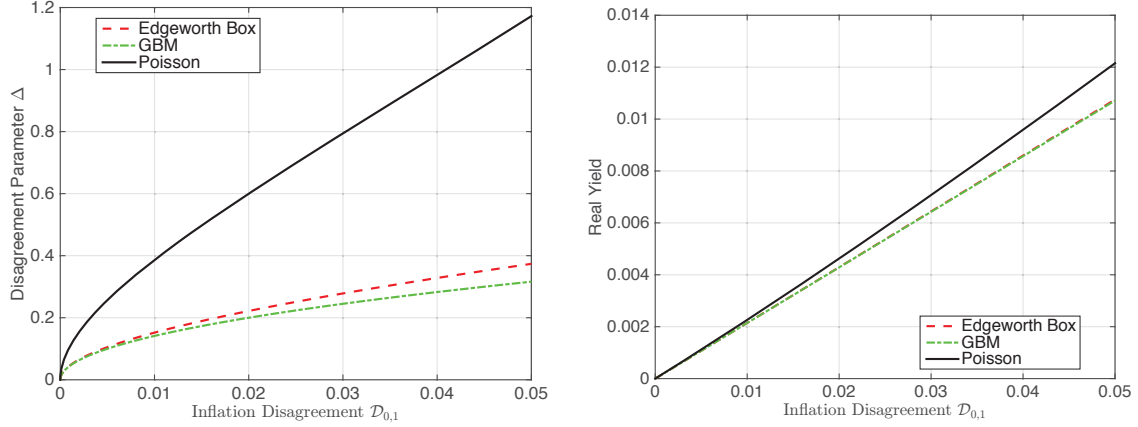
Figure 3: Disagreement Measure and Real Yields

The left plot shows the disagreement parameter Δ in all three examples as an increasing function of the inflation disagreement measure $\mathcal{D}_{0,1}$. The right plot shows that real yields are increasing in inflation disagreement $\mathcal{D}_{0,1}$ when $\gamma = 7$.

allows us to study the effects of disagreement on bond yields when investors have beliefs that differ by more than one parameter or do not even belong to the same class of distributions.

Definition 1 (Inflation Disagreement Measure). *Consider a belief structure $\mathcal{B}_{t,T} = (\mathbb{P}^1, \mathbb{P}^2)$ with the likelihood ratio $\lambda_u = \frac{d\mathbb{P}^2}{d\mathbb{P}^1} \mid_{\mathcal{F}_u}$ for all $t \leq u \leq T$. Define disagreement as*

$$\mathcal{D}_{t,T} = -\frac{1}{T-t} \mathbb{E}_t^1 \left[\log \left(\frac{\lambda_T}{\lambda_t} \right) \right]. \tag{10}$$

Disagreement $\mathcal{D}_{t,T}$ is nonnegative. It is zero if and only if the two investors have the same belief, in which case $\lambda_t = \lambda_T = 1$. The left plot of Figure 3 shows the disagreement parameter Δ for all three examples as a function of $\mathcal{D}_{0,1}$. The red dashed line represents the Edgeworth box example with $\Delta = \frac{p^2 - p^1}{p^1}$, the green dash-dotted line represents the Brownian example with $\Delta = \frac{x^2 - x^1}{\sigma_\Pi}$, and the black solid line represents the Poisson example with $\Delta = \frac{l^2 - l^1}{l^1}$. The plot shows that $\mathcal{D}_{0,1}$ strictly increases in the Δ's of all three examples and that it is zero if and only if $\Delta = 0$.

Consider the economies $\mathcal{E} = (\mathcal{B}_{t,T}, f(\lambda_t))$ and $\mathcal{E}_\eta = \left(\mathcal{B}_{t_\eta, T_\eta}^\eta, f(\eta_{t_\eta}) \right)$ populated by two investors with different beliefs, who share consumption according to the rule $f(\lambda_t)$ and $f(\eta_{t_\eta})$, respectively. We use different time subscripts to emphasize that the analysis below also allows for comparisons over time. Suppose there is more disagreement in economy $\mathcal{E}_\eta$ than in economy $\mathcal{E}$, that is, $\mathcal{D}_{t_\eta, T_\eta}^\eta \geq \mathcal{D}_{t,T}$. To compare consumption allocations and bond prices in the two economies, we make the following assumption about the likelihood ratios λ_t and η_{t_η}.

Assumption 2 (Likelihood Ratio Decomposition)**.** *Suppose the likelihood ratios η_u and λ_u are independent of C_u and H_u. Moreover, the distribution of $\frac{\eta_{T_\eta}}{\eta_{t_\eta}}$ conditional on $\mathcal{F}_{t_\eta}$ equals the distribution of $\frac{\lambda_T}{\lambda_t}\varepsilon$ conditional on $\mathcal{F}_t$, where ε denotes a strictly positive random variable with $\mathbb{E}^\varepsilon\left[\varepsilon \mid \lambda_T\right] = \mathbb{E}^\varepsilon\left[\varepsilon\right] = 1$ for all λ_T.*

While this assumption rules out disagreement about the correlation between consumption and inflation, it does not imply that inflation is independent of consumption and the habit. Moreover, it allows us to focus on one-dimensional decompositions of the conditional distribution of $\frac{\eta_{T_\eta}}{\eta_{t_\eta}}$. The multiplicative decomposition of the conditional distribution of $\frac{\eta_{T_\eta}}{\eta_{t_\eta}}$ nevertheless covers a large class of stochastic processes.[10] For instance, all three examples satisfy Assumption 2 if $\Delta_\eta \geq \Delta$. Intuitively, one can think of η_{T_η} as a noisy version of λ_T. We summarize the results in the next proposition.

Proposition 2 (Disagreement)**.** *Suppose the likelihood ratios η_t and λ_t satisfy Assumptions 1 and 2. Then, the belief structure $\mathcal{B}^\eta_{t_\eta,T_\eta}$ exhibits more disagreement than the belief structure $\mathcal{B}_{t,T}$, that is, $\mathcal{D}^\eta_{t_\eta,T_\eta} \geq \mathcal{D}_{t,T}$.*

We show in the next theorem that all results of Theorem 1, except for the yield volatility result, generalize when we compare economies with differing levels of disagreement (holding everything else fixed).

Theorem 2. *Consider two economies $\mathcal{E} = (\mathcal{B}_{t,T}, f(\lambda_t))$ and $\mathcal{E}_\eta = \left(\mathcal{B}^\eta_{t_\eta,T_\eta}, f(\eta_{t_\eta})\right)$ with*

- *the same time horizon, that is, $\tau = T_\eta - t_\eta = T - t$,*

- *the same current consumption allocations, that is, $f_t = f(\lambda_t) = f(\eta_{t_\eta})$, and*

- *the same distribution of real quantities, that is, the joint distribution of $\frac{C_{T_\eta}}{C_{t_\eta}}$ and $\frac{H_{T_\eta}}{H_{t_\eta}}$ conditional on $\mathcal{F}_{t_\eta}$ is equal to the joint distribution of $\frac{C_T}{C_t}$ and $\frac{H_T}{H_t}$ conditional on $\mathcal{F}_t$.*

Suppose there is more disagreement in economy $\mathcal{E}_\eta$ than in economy $\mathcal{E}$, that is, $\mathcal{D}^\eta_{t_\eta,t_\eta+\tau} \geq \mathcal{D}_{t,t+\tau}$, and adopt Assumptions 1 and 2. Then,

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[10]All results still follow if we consider additive mean independent and comonotone decompositions of the conditional distribution of $\frac{\eta_{T_\eta}}{\eta_{t_\eta}}$.

(i) real yields are the same in both economies if $\gamma = 1$,

(ii) real yields are higher in economy $\mathcal{E}_\eta$ than in economy $\mathcal{E}$ if $\gamma > 1$ (the opposite is true if $\gamma < 1$), and

(iii) the expected cross-sectional consumption growth volatility is higher in economy $\mathcal{E}_\eta$ than in economy $\mathcal{E}$ if $\frac{\lambda_T}{\lambda_t}$ and ε are independent.

The right plot of Figure 3 shows real yields and the left plot of Figure 4 shows the expected cross-sectional consumption growth volatility as an increasing function of disagreement $\mathcal{D}_{0,1}$ for all three examples when $\gamma = 7$. The right plot of Figure 4 shows the volatility of real one year yields as a function of disagreement. The black star and black diamond lines represent the Poisson example with $\gamma = 2$ and $\gamma = 0.5$, respectively. The green dash-dotted star and the green dash-dotted diamond lines represent the GBM example with $\gamma = 2$ and $\gamma = 0.5$, respectively. Real yield volatility in the GBM and Poisson example is higher for $\gamma = 0.5$ than for $\gamma = 2$ since the expected cross-sectional consumption growth volatility is decreasing with risk aversion. The plots show that real yield volatility is also increasing in disagreement.
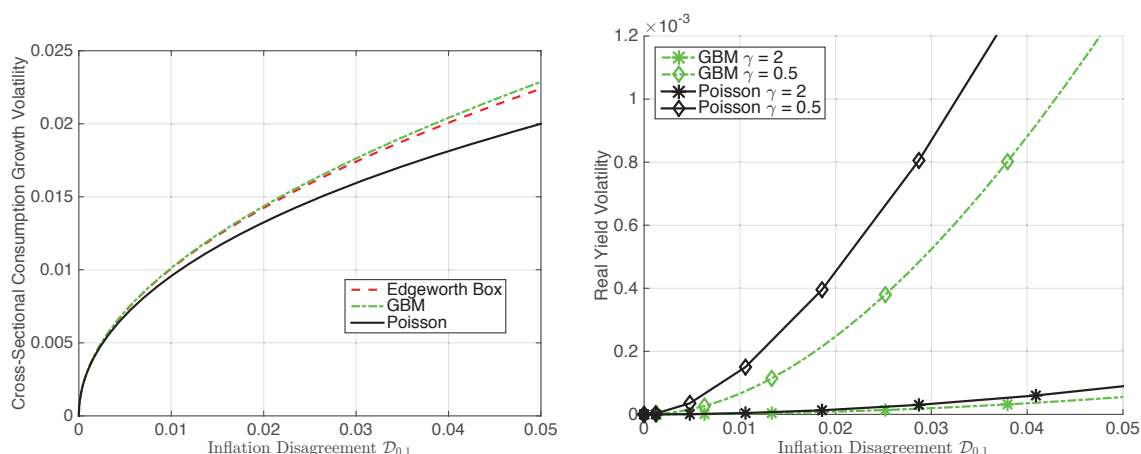
Figure 4: Volatility of Consumption and Real Yields

The expected cross-sectional consumption growth volatility and the volatility of real yields are strictly increasing in inflation disagreement $\mathcal{D}_{0,1}$. The expected cross-sectional consumption growth volatility is decreasing in risk aversion and thus real yield volatility is lower with $\gamma = 2$ than with $\gamma = 0.5$.

B Nominal Yields

A nominal bond is a default-free zero-coupon bond that pays one dollar at its maturity. All nominal bonds are in zero-net supply and are priced using the state price densities

from Proposition 1. Let $P_{t,T}$ denote the nominal price of a nominal bond maturing at T with continuously-compounded nominal yield $y_{t,T}^P = -\frac{1}{T-t} \log(P_{t,T})$, where $T \in [t, T']$. The nominal price of a nominal bond is

$$P_{t,T} = \mathbb{E}_t^i \left[\frac{\xi_T^i}{\xi_t^i} \frac{\Pi_t}{\Pi_T} \right]. \tag{11}$$

Let $\mathrm{RX}_{t,T}$ denote the real gross return on a T-year nominal bond in excess of the real gross return on a T-year real bond:

$$\mathrm{RX}_{t,T} = \left(\frac{P_{T,T}}{\Pi_T} \bigg/ \frac{P_{t,T}}{\Pi_t} \right) \bigg/ \frac{B_{T,T}}{B_{t,T}} = e^{(y_{t,T}^P - y_{t,T}^B)(T-t)} \frac{\Pi_t}{\Pi_T}. \tag{12}$$

The nominal return on a nominal bond is certain, while the real return on a nominal bond is a bet on inflation risk, that is, a bet on the real value of one dollar which is $\frac{1}{\Pi_T}$. The inflation risk premium perceived by investor i is defined as the expected real excess return on a T-year nominal bond:

$$\mathbb{E}_t^i [\mathrm{RX}_{t,T}] = e^{(y_{t,T}^P - y_{t,T}^B)(T-t)} \mathbb{E}_t^i \left[\frac{\Pi_t}{\Pi_T} \right], \qquad i = 0, 1, 2, \tag{13}$$

where $\mathbb{P}^0$ denotes the belief of a representative investor when there is no disagreement. Let $\mathrm{IRP}_{t,T}^i$ denote the annualized log inflation risk premium and $\mathrm{EINFL}_{t,T}^i$ the annualized expected log inflation rate perceived by investor i:[11]

$$\mathrm{EINFL}_{t,T}^i \equiv -\frac{1}{T-t} \log \left(\mathbb{E}_t^i \left[\frac{\Pi_t}{\Pi_T} \right] \right), \tag{14}$$

$$\mathrm{IRP}_{t,T}^i \equiv \frac{1}{T-t} \log \left(\mathbb{E}_t^i [\mathrm{RX}_{t,T}] \right) = y_{t,T}^P - y_{t,T}^B - \mathrm{EINFL}_{t,T}^i. \tag{15}$$

Hence, the nominal bond yield is

$$y_{t,T}^P = y_{t,T}^B + \mathrm{EINFL}_{t,T}^i + \mathrm{IRP}_{t,T}^i, \qquad i = 0, 1, 2. \tag{16}$$

In contrast to real yields, the effect of inflation disagreement on nominal yields is ambiguous without imposing additional assumptions on investors' inflation beliefs. For example,

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[11]Jensen inequality implies that

$$\mathrm{EINFL}_{t,T} = -\frac{1}{T-t} \log \left(\mathbb{E}_t \left[\frac{\Pi_t}{\Pi_T} \right] \right) \leq \frac{1}{T-t} \mathbb{E}_t \left[\log \left(\frac{\Pi_T}{\Pi_t} \right) \right] \leq \frac{1}{T-t} \log \left(\mathbb{E}_t \left[\frac{\Pi_T}{\Pi_t} \right] \right),$$

and thus $\mathrm{IRP}_{t,T}$ is higher than the inflation risk premium implied by other measures for expected inflation.

consider the case where investors agree on a very high expected inflation rate to the case
where investors have high disagreement about a very low expected inflation rate. Real yields
are elevated with high disagreement if $\gamma > 1$, but a high enough expected inflation rate
may cause nominal yields to be even higher although there is no disagreement. Similarly,
agreement on a very high inflation risk premium may lead to nominal yields that are higher
than when there is high disagreement about a low inflation risk premium.

To avoid this problem, we need to hold expected inflation and the inflation risk premium
constant when studying the effects of inflation disagreement on nominal yields. However,
it is not clear which belief to hold constant when increasing inflation disagreement in a
heterogeneous beliefs economy. We could consider a mean preserving spread on inflation ex-
pectations and inflation risk premiums while keeping the average belief about them constant
to unambiguously increase disagreement. However, this does not take into account that the
belief of a wealthier investor has a stronger impact on nominal yields than the belief of a
poorer investor. Hence, we make the following assumption.

Assumption 3 (Belief Restriction). *The beliefs $\mathbb{P}^1$ and $\mathbb{P}^2$ satisfy*

$$\mathbb{E}_t^0 \left[\frac{\Pi_t}{\Pi_T} \right] = f(\lambda_t)\mathbb{E}_t^1 \left[\frac{\Pi_t}{\Pi_T} \right] + (1 - f(\lambda_t))\mathbb{E}_t^2 \left[\frac{\Pi_t}{\Pi_T} \right], \tag{17}$$

$$\mathbb{C}ov_t^0 \left[\frac{\Pi_t}{\Pi_T}, \frac{\xi_T^0}{\xi_t^0} \right] = f(\lambda_t)\mathbb{C}ov_t^1 \left[\frac{\Pi_t}{\Pi_T}, \frac{\xi_T^0}{\xi_t^0} \right] + (1 - f(\lambda_t))\mathbb{C}ov_t^2 \left[\frac{\Pi_t}{\Pi_T}, \frac{\xi_T^0}{\xi_t^0} \right], \tag{18}$$

*where $\mathbb{P}^0$ denotes the belief and $\xi_t^0 = e^{-\rho t}C_t^{-\gamma}H_t^{\gamma-1}$ is the state price density when there is
no disagreement.*

Equation (17) requires that the weighted average across each investor's expected real
value of a dollar is fixed when disagreement about inflation changes. The weights are given
by the fraction of output that each investors consumes $((f(\lambda_t), 1 - f(\lambda_t))$ as the wealthier
investor's belief impacts the nominal bond price the most. It is not only the belief about
expected inflation that impacts the nominal bond yield, but also the inflation risk premium
that investors require when betting on the real value of a dollar. So, we impose an additional
restriction given by equation (18). This restriction requires that the weighted average across
each investor's inflation risk premium belief is fixed when inflation disagreement changes. If
there is only disagreement about expected inflation, then equation (18) is trivially satisfied.
However, we also allow for disagreement about higher order moments of inflation and the
joint distribution of inflation and real quantities. Thus, the beliefs $\mathbb{P}^1$ and $\mathbb{P}^2$ about the
covariances in equation (18) do not have to be the same. We show in the next theorem
that, in this case, disagreement has qualitatively the same effect on nominal yields as on real
yields.

Theorem 3 (Nominal Yield). *If Assumptions 1 and 3 are satisfied, then*

(i) *nominal yields do not depend on disagreement if $\gamma = 1$ and*

(ii) *nominal yields are higher with than without disagreement if $\gamma > 1$ (the opposite is true if $\gamma < 1$).*

The left plot of Figure 5 shows nominal one-year yields as a function of risk aversion γ. The red dashed circle, green dash-dotted circle, and black circle lines represent the Edgeworth box, GBM, and Poisson examples, respectively, when there is no disagreement. The corresponding lines without circles represent the examples when there is disagreement and Assumption 3 is satisfied. The plot shows that in all three examples nominal yields are higher with disagreement than without disagreement if $\gamma > 1$ and lower if $\gamma < 1$.

We discuss the implications for nominal yields when equation (17) of Assumption 3 is not satisfied by means of the GBM example. Investors share aggregate consumption equally, that is, $f = 0.5$. The expected inflation rate is two percent $\bar{x} = 2\%$, if there is no disagreement in which case the nominal yield is 1.96% (green dash-dotted circle line). We consider three different cases with disagreement: (i) baseline with $x^1 = 1.5\%$ and $x^2 = 2.5\%$ (green dash-dotted line), (ii) $x^1 = 1\%$ and $x^2 = 2\%$ (green dash-dotted plus line), and (iii) $x^1 = 2\%$ and $x^2 = 3\%$ (green dash-dotted cross line). The consumption share weighted-average belief in the first case is approximately 2%, and thus Assumption 3 is satisfied.[12] If the consumption share weighted average belief is below 2%, then inflation disagreement lowers nominal yields if $\gamma < 1$, but does not always increase nominal yields if $\gamma > 1$. Intuitively, inflation disagreement pushes up real yields, but lowers the expected inflation rate. If the second effect dominates the first, then nominal yields are lower than in the no disagreement economy. The intuition is similar for the third case.

The right plot of Figure 5 shows that nominal yields in all three examples are strictly increasing in inflation disagreement $\mathcal{D}_{0,1}$ when $\gamma > 1$ and while keeping the consumption-share weighted expected value of one dollar fixed.[13]

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[12]In this example, we have that $0.5e^{-1.5\%} + 0.5e^{-2.5\%} \approx e^{-2\%}$.

[13]Equation (18) is trivially satisfied for all beliefs because $\xi_t^0 \equiv 1$.
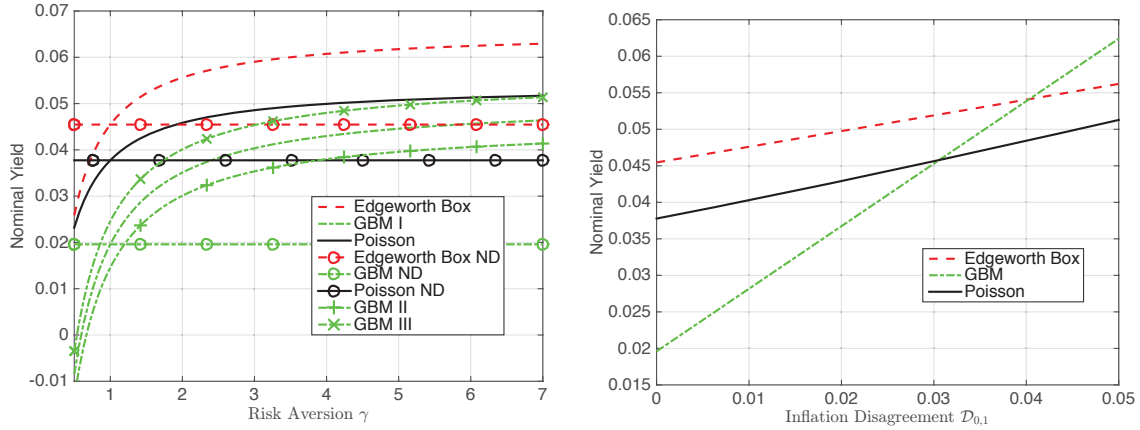
Figure 5: Nominal Yields and Disagreement

The left plot shows nominal one-year yields as a function of risk aversion. Nominal yields are higher (lower) with than without disagreement when $\gamma \geq (\leq)1$ except for the cases GBM II and III where Assumption 3 is not satisfied. The right plot shows the nominal one-year yield as an increasing function of inflation disagreement $\mathcal{D}_{0,1}$ when $\gamma = 7$ and the consumption-share weighted belief of both investors about the expected real value of one Dollar does not change with disagreement.

C Break-Even Inflation Rate

In this section, we study whether disagreement drives a wedge between real and nominal yields. Let $\mathrm{BEIR}_{t,T}$ denote the break-even inflation rate defined as the difference between the nominal and real yield of a $T-$year bond, that is, $\mathrm{BEIR}_{t,T} = y_{t,T}^P - y_{t,T}^B$.

Proposition 3 (Break-Even Inflation Rate). *If Assumptions 1 and 3 are satisfied, then*

    (i) the break-even inflation rate does not depend on disagreement if $\gamma = 1$ and

    (ii) the effects of disagreement on the break-even inflation rate are ambiguous if $\gamma \neq 1$.

The left plot of Figure 6 shows the difference between the break-even inflation rate in an economy with and without disagreement as a function of risk aversion. If $\gamma = 1$, the break-even inflation rate does not depend on disagreement. Thus, the red dashed lines (Edgeworth Box example), the green dash-dotted lines (GBM example), and the black lines (Poisson example) all intersect at zero. If $\gamma > 1$, then the break-even inflation rate is higher with disagreement than without disagreement in the GBM and Poisson example. The quantitative effect is smaller for the short end of the yield curve and it is larger in the Poisson example than the GBM example. In contrast to the real and nominal yield curve,
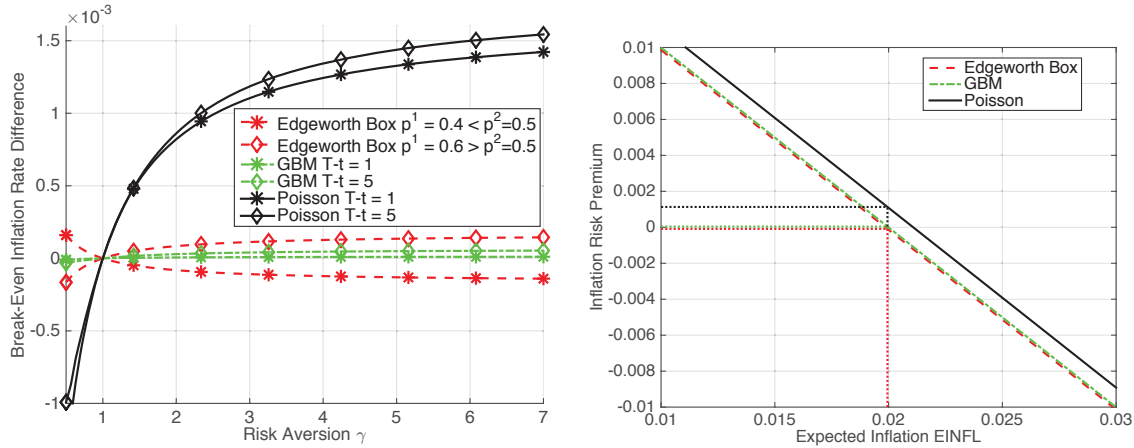
Figure 6: Break-Even Inflation and Inflation Risk Premium

The left plot shows the difference between the break-even inflation rate in an economy with and without disagreement as a function of risk aversion γ. If $\gamma \geq 1$, the break-even inflation rate is higher with than without disagreement in the GBM, Poisson, and second Edgeworth box example. The opposite is true in the first Edgeworth box example. The right plot shows the inflation risk premium when there is a disagreement as a function of perceived expected inflation of an econometrician. The inflation risk premium is sensitive to the belief of the econometrician.

the effects of inflation disagreement on the break-even inflation rate are ambiguous. For instance, consider an Edgeworth box example where risk aversion is greater than one and the second investor thinks that the high and low inflation state are equally likely. If the first investor thinks that the high inflation state is less likely (red dashed star line), than the break-even inflation rate is lower with than without disagreement. The opposite is true when the first investor thinks that the high inflation state is more likely (red dashed diamond line).

D Inflation Risk Premium

In contrast to the break-even inflation rate, which is a statement about prices, the inflation risk premium is sensitive to the belief chosen to determine inflation expectations. Specifically, let $\hat{\mathbb{P}}$ denote the belief of an econometrician. Then the nominal yield can be decomposed into:

$$y_{t,T}^P = y_{t,T}^B + \widehat{\text{EINFL}}_{t,T} + \widehat{\text{IRP}}_{t,T} = y_{t,T}^B + \text{EINFL}_{t,T}^i + \text{IRP}_{t,T}^i, \qquad \forall\, i = 0, 1, 2. \qquad (19)$$

Investors and econometricians agree on prices, so they agree on the break-even inflation rate $\text{BEIR}_{t,T} = y_{t,T}^P - y_{t,T}^B$. However, they may have different beliefs about inflation. If they disagree about expected inflation, then by equation (19) they have to disagree on the inflation risk premium. For example, consider the case when the first investor predicts lower inflation than the second investor, that is, $\text{EINFL}_{t,T}^1 < \text{EINFL}_{t,T}^2$. Subtracting the expected inflation rate from the agreed upon break-even inflation rate leads to a higher perceived compensation for inflation risk for the first investor, that is, $\text{IRP}_{t,T}^1 > \text{IRP}_{t,T}^2$.

The right plot of Figure 6 shows the inflation risk premium in an economy with disagreement perceived by an econometrician for different beliefs $\hat{\mathbb{P}}$. In all three examples, the first investor thinks expected inflation is 1% and the second investor thinks expected inflation is 3%, that is, $\text{EINFL}_{t,T}^1 = 1\% < \text{EINFL}_{t,T}^2 = 3\%$. Both investors consume the same fraction of consumption today, so the consumption-share weighted average belief about expected inflation is 2%. When the belief of the econometrician coincides with the consumption-share weighted average belief, then the inflation risk premium is slightly negative in the Edgeworth box example because the break-even inflation rate is smaller with than without disagreement. In the other two examples, the risk premium is positive. The plot of the inflation risk premium perceived by an econometrician in an economy without disagreement is very similar. In this case, the inflation risk premium is zero when we impose rational expectations, that is, if we impose that the belief of the econometrician coincides with the belief of the representative investor ($\mathbb{P}^0 = \hat{\mathbb{P}}$). If the econometrician underestimates expected inflation ($\widehat{\text{EINFL}}_{t,T} < \text{EINFL}_{t,T}^0$), then she perceives a positive inflation risk premium.

We characterize the inflation risk premium perceived by both investors in the following proposition.

Proposition 4. *The difference in investors' perceived inflation risk premiums is independent of preferences and consumption allocations. Specifically,*

$$IRP_{t,T}^2 - IRP_{t,T}^1 = EINFL_{t,T}^1 - EINFL_{t,T}^2 = \Delta EINFL_{t,T}. \tag{20}$$

Moreover, we have the following limits

$$\lim_{f_t \to 1} IRP_{t,T}^1 = IRP_{t,T}^{H,1}, \qquad \lim_{f_t \to 0} IRP_{t,T}^1 = IRP_{t,T}^{H,2} - \Delta EINFL_{t,T}, \tag{21}$$

$$\lim_{f_t \to 0} IRP_{t,T}^2 = IRP_{t,T}^{H,2}, \qquad \lim_{f_t \to 1} IRP_{t,T}^2 = IRP_{t,T}^{H,1} + \Delta EINFL_{t,T}, \tag{22}$$

where $IRP_{t,T}^{H,i}$ is the inflation risk premium in an economy populated by investor i only.

While the difference in inflation risk premiums is independent of preferences and consumption shares, the investor who actually perceives the largest (absolute) inflation risk

premium is not. Consider the case when investor one has a consumption share that is close to one. Then, bond prices reflect the view of investor one. Therefore, the speculative component, as captured by $\Delta \text{EINFL}_{t,T}$, is negligible from that investor's point of view. The entire speculative component is captured by the second investor. As the consumption shares become similar across investors, bond prices reflect both views and the perceived inflation risk premiums for both investors reflect the disagreement in the economy.
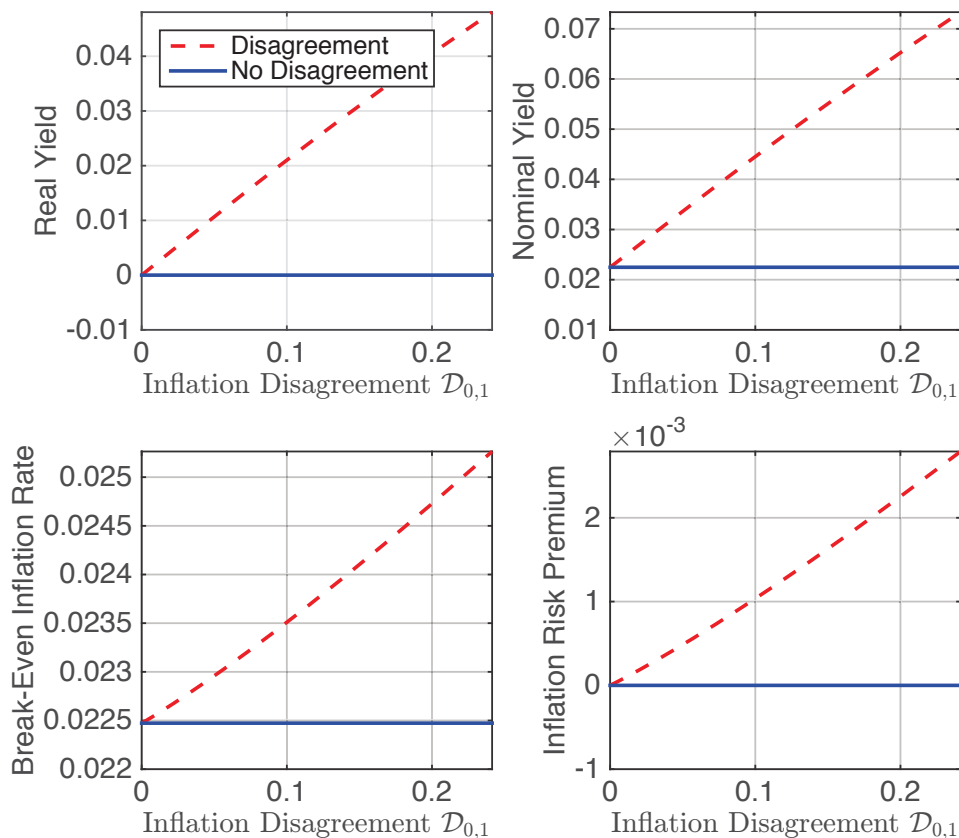
Figure 7: Inflation Risk Premium

The figure shows the real yield (top-left), nominal yield (top-right), break-even inflation (bottom-left), and inflation risk premium (bottom-right) as an increasing function of inflation disagreement $\mathcal{D}_{0,1}$ when $\gamma = 2$. There are three states with inflation given by $(0.9, 1, 1.125)$ in state one, two, and three, respectively. The probability as perceived by investor one over the three states are given by $(0.2, 0.4, 0.4)$. For the second investor, we vary the probability of the first state from 0.2 to 0.05 and then solve for the probability of the two other states such that $E^1 \left[\frac{1}{\Pi_1} \right] = E^2 \left[\frac{1}{\Pi_1} \right]$. There is a positive break even inflation rate and inflation risk premium even though investors agree on the expected real value of one dollar.

The perceived inflation risk premiums are not bounded between the risk premiums in the homogeneous investor economies; that is, $min\left\{\text{IRP}^1_{t,T}, \text{IRP}^2_{t,T}\right\} \leq min\left\{\text{IRP}^{H,1}_{t,T}, \text{IRP}^{H,2}_{t,T}\right\}$ or $max\left\{\text{IRP}^1_{t,T}, \text{IRP}^2_{t,T}\right\} \geq max\left\{\text{IRP}^{H,1}_{t,T}, \text{IRP}^{H,2}_{t,T}\right\}$ can occur. The next example shows that investors can disagree about the distribution of inflation, but agree on the inflation risk premium. Consider a two date economy with two investors and three states, where the time discount factor is zero and aggregate consumption and habit are normalized to one. We choose probabilities perceived by the investors in such a way that they agree on expected inflation, $\text{EINFL}^1_{t,T} = \text{EINFL}^2_{t,T}$, but disagree about the distribution of inflation. In this case, the nominal yield in a homogeneous investor economy with beliefs given by investor one would be equal to that of a homogeneous investor economy with beliefs given by investor two and the inflation risk premium would be zero under both beliefs. However, once both investors are present in the same economy and $\gamma \neq 1$, then the inflation risk premium is non-zero due to changes in the investment opportunity set caused by speculative trade. Figure 7 shows the real and nominal yields, the break-even inflation, and the inflation risk premium as a function of disagreement. Both real and nominal yields are increasing in disagreement as predicted by Theorems 2 and 3. In addition, both investors agree on the inflation risk premium. Yet, the inflation risk premium differs from that of a homogeneous investor economy. Here, disagreement about the distribution of inflation creates a positive inflation risk premium that increases in disagreement.

II   Empirical Evidence

To validate the theory, we use the Survey of Professional Forecasters (abbreviated as SPF) and the Michigan Surveys of Consumers (abbreviated as MSC) to empirically test whether inflation disagreement affects (i) real yields (Table 3), (ii) nominal yields (Table 4), (iii) real and nominal yield volatilities (Table 5), (iv) break-even inflation and the inflation risk premium (Table 6), and (v) cross-sectional consumption growth volatility and trading on inflation disagreement (Table 7). The surveys differ with respect to the sophistication of the constituency, the size of the survey, and the data frequency. Thus, they provide independent evidence for inflation disagreement.

A   Data

Inflation Disagreement. Disagreement about inflation, our main explanatory variable, is the cross-sectional standard deviation of one year ahead inflation forecasts abbreviated

as DisInf. Disagreement of consumers is directly taken from the MSC database and disagreement of professionals is computed from the raw series of the SPF forecasts. The MSC inflation forecasts, conducted at a monthly frequency, are available since January 1978 while the SPF inflation forecasts, conducted at a quarterly frequency, are available since September 1981.[14]

Yields. The U.S. Treasury only began issuing TIPS in 1997, so we merge the implied real yields in Chernov and Mueller (2012), which are available at quarterly frequency from $Q3$-1971 to $Q4$-2002, with real yields on Treasury Inflation Protected Securities (TIPS) to build a longer time series of real bond yields. The available real yield maturities are 2, 3, 5, 7, and 10 years.[15] Monthly nominal Fama-Bliss discount bond yields are from CRSP.[16] The Fama-Bliss discount bond file contains yields with 1 to 5 year maturities with data going back to 1952. Lastly, from the real and the nominal yield series, we compute the time-series of real and nominal yield volatilities by estimating a $GARCH(1,1)$ model with an $AR(1)$ mean equation. We use all available data in the GARCH estimation.

Cross-Sectional Consumption. We calculate monthly cross-sectional consumption growth volatility, starting from April 1984, from consumption growth rates of consumers using data from the Consumer Expenditure Survey (CEX) of the Bureau of Labor Statistics.[17] For further information regarding the CEX data and how to construct consumption growth rates of households from the raw data see Malloy, Moskowitz, and Vissing-Jorgensen (2009) and the references therein.

Trading on Inflation Disagreement. We construct three measures for trading on inflation disagreement. First, we use the volatility of total treasury volume scaled by outstanding treasuries.[18] The trading volume data and the outstanding amount of treasuries are available from the Securities Industry and Financial Markets Association (SIFMA) at a monthly frequency since January 2001.[19] To measure the volatility of trading in Treasuries, we estimate a $GARCH(1,1)$ model with a constant mean term. Second, we use the open

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[14]See www.philadelphiafed.org/research-and-data for a detailed description of the Survey of Professional Forecasters, which is conducted by the Federal Reserve Bank of Philadelphia. The website www.sca.isr.umich.edu/ contains detailed information regarding the Michigan Surveys of Consumers.

[15]The real yield data are available at personal.lse.ac.uk/muellerp/RealYieldAOT5.xls. The TIPS data are available from www.federalreserve.gov/econresdata/researchdata/feds200628.xls. For the 5, 7, and 10 maturities, we use TIPS data from 2003 onwards. The 4 year yield is not available in Chernov and Mueller (2012). For 2 and 3 year maturities, we interpolate the rates for 2003 with cubic splines.

[16]The Fama-Bliss discount bond file is available from wrds-web.wharton.upenn.edu/wrds.

[17]We thank Jing Yu for advising us on the use of the CEX data including computing the cross-sectional consumption growth volatility.

[18]We follow Grossman and Zhou (1996) and Longstaff and Wang (2013) to capture the intensity of trading by using the volatility of turnover because turnover is not defined in a frictionless economy.

[19]The data are from SIFMA's website at this link: www.sifma.org.

interest in interest rate futures and scale it by the open interest in all financial futures to account for increased security trade over time. The open interest data for interest rate and financial futures are from the U.S. Commodity Futures Trading Commission (CFTC) at a monthly frequency since April 1986.[20] Third, we use de-trended log inflation swap notionals available at the monthly frequency since December 2005.[21] The monthly notional amounts correspond to averages of daily brokered inflation swap activity.

Inflation. We obtain quarterly and monthly CPI data from the FRED Economic Data base to compute inflation rates as logarithmic changes starting in January 1947. We estimate a GARCH$(1, 1)$ model with an ARMA$(1, 1)$ mean equation using the whole sample, to obtain a time series of monthly and quarterly expected inflation and inflation volatility forecasts over multiple horizons.

Summary Statistics. We conclude this subsection with summary statistics of all variables in Tables 1 and 2. All data series end in June of 2014 or $Q2$-2014 except the CEX data (consumption is available until December 2012 and income until March 2012), Treasury volume (available until August 2013), open interest data (available until December 2013), and the inflation swap notionals (available until February 2012).

B   Real Yield Levels

Empirically, we show that an increase in inflation disagreement raises real yields at all maturities. Table 3 shows the slope coefficients, t-statistics, the R^2's, and the number of observations (N) for a univariate and a multivariate regression model. For each maturity, we regress real yields on disagreement about inflation (DisInf) based on SPF (columns 2 to 6) and MSC (columns 7 to 11). To facilitate a comparison between SPF and MSC, we use the sample period $Q3$-1981 to $Q2$-2014 and standardize the regression coefficients in all tables.[22] Regression model 2 controls for expected inflation (ExpInf) and inflation volatility (SigInf) because both variables are correlated with disagreement and may effect real yields. The forecast horizons for ExpInf and SigInf correspond to yield maturities. We use Newey-West corrected t-statistics with 12 lags in all regressions to correct for serial correlation in error terms.

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[20]CFTC data are available from www.cftc.gov.

[21]See Fleming and Sporn (2013) for a description of the data. We thank Michael Fleming for sharing the aggregated inflation swap notional data with us.

[22]Using the same sample period on the quarterly data excludes the high inflation period $Q1$-1978 to $Q2$-1981 from the analysis.

Table 1: **Descriptive Statistics of Real and Nominal Yields and their Volatilities.** The table reports mean, median, standard deviation (Std), and number of observations (N) of percentage real and nominal yields and real and nominal yield volatilities. Quarterly real yields are from Chernov and Mueller (2012) merged with TIPS yields from Gürkaynak, Sack, and Wright (2010). Nominal yields at monthly and quarterly frequency are from Fama-Bliss. Yield volatilities are computed by estimating a GARCH$(1,1)$ model with an AR(1) mean equation. Real yield sample: Q3-1981 to Q2-2014. Nominal yield sample: January 1978 to June 2014.

Maturity	2y	3y	5y	7y	10y	1y	2y	3y	4y	5y	1y	2y	3y	4y	5y
	Quarterly Real Yields					Quarterly Nominal Yields					Monthly Nominal Yields				
Mean	1.927	2.027	2.255	2.415	2.586	4.851	5.160	5.421	5.647	5.809	5.412	5.672	5.884	6.080	6.220
Median	2.370	2.395	2.430	2.533	2.628	5.012	5.080	5.332	5.615	5.585	5.370	5.560	5.788	5.922	5.972
STD	1.976	1.836	1.594	1.433	1.247	3.390	3.424	3.380	3.311	3.219	3.690	3.631	3.528	3.432	3.328
N	132	132	132	132	132	132	132	132	132	132	438	438	438	438	438
	Quarterly Real Yield Volatilities					Quarterly Nominal Yield Volatilities					Monthly Nominal Yield Volatilities				
Mean	0.733	0.620	0.511	0.452	0.389	0.319	0.352	0.357	0.368	0.357	0.374	0.388	0.386	0.388	0.371
Median	0.639	0.555	0.468	0.423	0.368	0.258	0.313	0.335	0.339	0.329	0.278	0.322	0.340	0.342	0.326
STD	0.298	0.212	0.134	0.096	0.070	0.256	0.196	0.154	0.152	0.140	0.352	0.263	0.216	0.188	0.167

Table 2: **Descriptive Statistics of the Mean, Volatility, and Disagreement of Inflation, CEX and Trading Data.** The table reports mean, median, standard deviation (Std), and number of observations (N) of monthly and quarterly expected inflation, monthly and quarterly inflation volatility, MSC and SPF based measures of inflation disagreements (DisInf), CEX cross-sectional consumption growth volatility (Cons Vol) and income growth volatility (Income Vol), volatility of treasury volume (Vol Volume), open interest ratio in interest rate futures (Open Interest Ratio), and the notionals of inflation swaps (Inf Swaps). The reported statistics of one year forecasts of expected inflation (ExpInf) and inflation volatility (SigInf) are estimated using a GARCH$(1,1)$ model with an ARMA$(1,1)$ mean equation.

Forecast Horizon	1y	2y	3y	4y	5y	7y	10y	1y	2y	3y	4y	5y
	Quarterly Expected Inflation							Monthly Expected Inflation				
Mean	3.106	3.192	3.259	3.313	3.355	3.417	3.475	3.628	3.603	3.584	3.568	3.555
Median	3.100	3.186	3.254	3.308	3.351	3.414	3.473	3.197	3.239	3.272	3.299	3.320
STD	1.133	0.948	0.803	0.688	0.597	0.464	0.339	2.047	1.730	1.479	1.277	1.115
N	132	132	132	132	132	132	132	438	438	438	438	438
	Quarterly Inflation Volatility							Monthly Inflation Volatility				
Mean	1.746	1.483	1.359	1.286	1.237	1.175	1.124	0.973	0.722	0.608	0.538	0.489
Median	1.446	1.271	1.191	1.145	1.115	1.079	1.050	0.874	0.661	0.565	0.505	0.463
STD	0.942	0.668	0.533	0.450	0.392	0.316	0.249	0.363	0.217	0.153	0.115	0.091
N	132	132	132	132	132	132	132	438	438	438	438	438

	Quarterly			Monthly					
	DisInf SPF	DisInf MSC		DisInf MSC	CEX Cons Vol	CEX Income Vol	Vol Volume	Open Interest Ratio	Inf Swaps
Mean	0.660	5.192		5.537	0.367	0.898	0.036	0.688	0.000
Median	0.564	4.900		5.200	0.366	0.902	0.023	0.712	0.020
STD	0.339	1.584		1.947	0.022	0.178	0.023	0.103	0.496
N	132	132		438	345	330	152	333	70

Table 3: **Inflation Disagreement and Real Yields.** The table reports results from OLS regressions of real yields on disagreement about inflation (DisInf), expected inflation (ExpInf), and the volatility of inflation (SigInf). The t-statistics (t-stat) are Newey-West corrected with 12 lags. Regression coefficients are standardized. ExpInf and SigInf are predicted by a GARCH$(1,1)$ model with an ARMA$(1,1)$ mean equation over multiple horizons (T). Sample: Q3-1981 to Q2-2014.

Maturity	Survey of Professional Forecasters					Surveys of Consumers				
	2y	3y	5y	7y	10y	2y	3y	5y	7y	10y
DisInf	0.407	0.397	0.388	0.382	0.376	0.560	0.575	0.583	0.589	0.595
t-stat	3.48	3.33	3.23	3.18	3.12	3.04	3.18	3.29	3.39	3.50
adj. R^2	0.16	0.15	0.14	0.14	0.13	0.31	0.33	0.33	0.34	0.35
N	132	132	132	132	132	132	132	132	132	132
DisInf	0.290	0.285	0.281	0.280	0.280	0.452	0.472	0.487	0.501	0.515
t-stat	2.27	2.20	2.12	2.12	2.12	2.55	2.75	3.00	3.17	3.34
ExpInf	0.350	0.359	0.358	0.352	0.344	0.251	0.246	0.236	0.221	0.206
t-stat	2.19	2.17	2.03	1.95	1.88	1.98	1.98	1.87	1.77	1.64
SigInf	0.099	0.080	0.068	0.057	0.042	0.106	0.077	0.056	0.038	0.018
t-stat	0.71	0.57	0.48	0.39	0.29	1.04	0.80	0.62	0.43	0.20
adj. R^2	0.24	0.24	0.24	0.23	0.22	0.34	0.36	0.36	0.37	0.37
N	132	132	132	132	132	132	132	132	132	132

The top panel of Table 3 shows the univariate regression results. The coefficient estimates for disagreement are positive and statistically significant, at least at the 5% level, for SPF and MSC at all maturities. Inflation disagreement is economically significant for SPF and MSC. Specifically, an increase in disagreement by one standard deviation of SPF (0.339%) and MSC (1.584%) raises the two year real yield by 40.7% and 56.0% of its standard deviation (1.976%). The results are similar for other maturities.

The bottom panel of Table 3 shows that the coefficient estimates for disagreement remain positive and statistically significant when we control for the mean and volatility of inflation. Expected inflation is positive and statistically significant for short maturities. For longer maturities, the coefficients become insignificant for MSC. Inflation volatility produces statistically insignificant coefficient estimates in all regressions.

C   Nominal Yield Levels

Inflation disagreement also raises nominal yields at all maturities in the data. Table 4 shows regression results of nominal yields on inflation disagreement based on SPF (columns 2 to 6) and MSC (columns 7 to 11). Regression model 1 contains expected inflation as an additional explanatory variable because it affects nominal yields and is correlated with disagreement. To

alleviate concerns that inflation disagreement measures uncertainty about inflation instead of disagreement, regression model 2 also includes the volatility of inflation (SigInf). Nominal yields are available at the monthly frequency and, thus, we use monthly data starting in January 1978 for MSC. This increases the sample size from 132 to 438.

Table 4: **Inflation Disagreement and Nominal Yields.** The table reports results from OLS regressions of nominal yields on disagreement about inflation (DisInf), expected inflation (ExpInf), and the volatility of inflation (SigInf). The t-statistics (t-stat) are Newey-West corrected with 12 lags. Regression coefficients are standardized. ExpInf and SigInf are predicted by a GARCH$(1,1)$ model with an ARMA$(1,1)$ mean equation over multiple horizons (T). Samples: Q3-1981 to Q2-2014 and January 1978 to June 2014.

Maturity	Survey of Professional Forecasters					Surveys of Consumers				
	1y	2y	3y	4y	5y	1y	2y	3y	4y	5y
DisInf	0.354	0.356	0.363	0.364	0.377	0.470	0.513	0.548	0.571	0.594
t-stat	3.63	3.60	3.65	3.74	3.88	4.11	4.39	4.61	4.80	5.05
ExpInf	0.459	0.449	0.437	0.435	0.424	0.356	0.298	0.249	0.219	0.196
t-stat	4.36	4.37	4.26	4.26	4.19	3.49	2.73	2.16	1.81	1.62
adj. R^2	0.41	0.40	0.39	0.39	0.39	0.58	0.56	0.55	0.54	0.55
N	132	132	132	132	132	438	438	438	438	438
DisInf	0.364	0.374	0.381	0.384	0.397	0.488	0.542	0.582	0.613	0.636
t-stat	3.51	3.50	3.51	3.58	3.63	4.16	4.55	4.86	5.17	5.42
ExpInf	0.448	0.430	0.416	0.411	0.399	0.334	0.264	0.207	0.169	0.144
t-stat	3.21	3.07	2.94	2.89	2.83	3.15	2.41	1.84	1.46	1.24
SigInf	-0.024	-0.041	-0.044	-0.049	-0.050	-0.061	-0.091	-0.107	-0.126	-0.126
t-stat	-0.24	-0.40	-0.42	-0.46	-0.47	-0.86	-1.31	-1.55	-1.88	-1.97
adj. R^2	0.40	0.39	0.39	0.39	0.39	0.58	0.57	0.56	0.56	0.56
N	132	132	132	132	132	438	438	438	438	438

Like the real yield results, the coefficients for disagreement are positive as well as economically and statistically significant for SPF and MSC at all maturities, as shown in the top panel of Table 4. An increase in disagreement by one standard deviation of SPF (0.339%) and MSC (1.947%) raises the one year nominal yield by 35.4% and 47.0% of its standard deviation (3.390% and 3.690%, respectively). The economic significance of inflation disagreement is large and comparable to the one of expected inflation, which is 45.9% and 35.6%, respectively. The results are similar for other maturities. The bottom panel of Table 4 shows that the coefficient estimates for disagreement remain positive and statistically significant when we control for the mean and volatility of inflation. All coefficient estimates for inflation volatility are negative and insignificant, except for maturities 4 and 5 in the MSC regression which are negative and significant at the 5% level.

We conclude this section by mentioning an alternative interpretation for the nominal regression results. Subtracting a measure of expected inflation from nominal yields has been used in the literature to proxy for real yields. Therefore, the regression results in Table 4 provide additional evidence for a positive and statistically significant impact of inflation disagreement on real yields.

D   Real and Nominal Yield Volatilities

We now test whether real and nominal yield volatilities increase with disagreement about inflation. Table 5 presents standardized coefficients and Newey-West adjusted t-statistics with 12 lags for SPF in columns 2 to 6 and MSC in columns 7 to 11. In both regressions, we control for the mean and volatility of inflation. Like the real and nominal yield levels, the coefficients for disagreement are positive and economically significant for SPF and MSC for all maturities. Table 5 shows that an increase in disagreement by one standard deviation of SPF (0.339%) and MSC (1.584%) raises the two year real yield volatility by 52.3% and 33.2% of its standard deviation (0.298%) and the one year nominal yield volatility by 59.7% and 47.4% of its standard deviation (0.256%). The results are similar for other maturities.

E   Break-Even Inflation and Inflation Risk Premium

We find that inflation disagreement drives a wedge between real and nominal yields. The break-even inflation rate for maturity T is the difference between the nominal and real yield of a T-year bond. The corresponding inflation risk premium is the break-even inflation rate minus expected inflation (ExpInf) plus a convexity adjustment $\left(0.5\,\text{SigInf}^2\right)$.[23] For each maturity, we regress the break-even inflation rate and the inflation risk premium on disagreement about inflation (DisInf) based on SPF (columns 2 to 4) and MSC (columns 5 to 7). Regression model 1 contains expected inflation (ExpInf) as additional explanatory variable and regression model 2 controls for the volatility of inflation (SigInf).

Inflation disagreement raises the break-even inflation rate and the inflation risk premium at all maturities. In the two top panels in Table 6, the coefficient estimates for disagreement are positive and statistically significant, at least at the 5% level, for SPF and MSC. All coefficients are economically significant. Specifically, an increase in disagreement by one standard deviation of SPF (0.339%) and MSC (1.584%) raises the two year break-even inflation rate by 31.9% and 55.1% of its standard deviation (1.761%). The results are similar for other maturities and regression model 2.

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[23]See Subsection B in Section I for details.

Table 5: **Inflation Disagreement and Real and Nominal Yield Volatilities.** The table reports results from OLS regressions of real and nominal yield volatilities on disagreement about inflation (DisInf), expected inflation (ExpInf), and the volatility of inflation (SigInf). The t-statistics (t-stat) are Newey-West corrected with 12 lags. Regression coefficients are standardized. ExpInf and SigInf are predicted by a GARCH$(1,1)$ model with an ARMA$(1,1)$ mean equation over multiple horizons (T). Samples: Q3-1981 to Q2-2014 and January 1978 to June 2014.

	Survey of Professional Forecasters					Surveys of Consumers				
	Real Yield Volatilities									
Maturity	2y	3y	5y	7y	10y	2y	3y	5y	7y	10y
DisInf	0.523	0.560	0.624	0.700	0.749	0.332	0.387	0.420	0.447	0.471
t-stat	8.13	8.76	8.61	9.32	9.52	1.97	2.16	2.15	2.04	1.97
ExpInf	0.018	0.065	0.081	0.055	0.025	0.074	0.110	0.137	0.129	0.108
t-stat	0.20	0.75	0.94	0.60	0.25	0.61	0.96	1.20	1.02	0.80
SigInf	0.238	0.228	0.183	0.114	0.016	0.391	0.380	0.351	0.305	0.219
t-stat	2.17	2.08	1.84	1.40	0.22	2.87	2.87	2.66	2.19	1.51
adj. R^2	0.40	0.44	0.50	0.56	0.57	0.28	0.32	0.34	0.33	0.31
N	132	132	132	132	132	132	132	132	132	132
	Nominal Yield Volatilities									
Maturity	1y	2y	3y	4y	5y	1y	2y	3y	4y	5y
DisInf	0.597	0.606	0.567	0.656	0.644	0.474	0.464	0.442	0.501	0.511
t-stat	5.24	5.20	5.94	6.92	8.13	4.40	4.03	4.01	3.67	3.65
ExpInf	0.287	0.265	0.260	0.204	0.205	0.287	0.261	0.283	0.170	0.126
t-stat	2.90	2.74	2.44	2.25	2.31	1.60	1.44	1.50	0.96	0.68
SigInf	0.129	0.116	0.113	0.088	0.063	0.174	0.174	0.150	0.153	0.114
t-stat	1.37	1.21	1.15	1.09	0.81	2.45	2.26	2.08	1.99	1.45
adj. R^2	0.55	0.54	0.48	0.56	0.53	0.52	0.47	0.46	0.42	0.38
N	132	132	132	132	132	438	438	438	438	438

In the two bottom panels in Table 6, the coefficient estimates for disagreement are again positive and statistically significant, at least at the 5% level, for SPF and MSC at all maturities. The effect of inflation disagreement on the inflation risk premium is economically significant. An increase in disagreement by one standard deviation of SPF (0.339%) and MSC (1.584%) raises the two year inflation risk premium by 52.6% and 62.6% of its standard deviation (2.475%). Like the break-even inflation rate regressions, the results for the inflation risk premium are similar for other maturities and regression model 2.

F   Economic Channel

We now provide evidence for the economic channel through which disagreement affects yields. First, inflation disagreement raises the cross-sectional consumption growth volatility. The top panel of Table 7 shows two regression specifications (columns 2 to 3). In the first specification, we regress CEX cross-sectional consumption growth volatility on MSC inflation disagreement and time-dummies that control for changes in the definition of food consumption and for missing data at the beginning of 1986 and 1996 due to changes in the household identification numbers. The second specification contains the CEX cross-sectional income growth volatility as control. The coefficient estimates on inflation disagreement in both regressions are positive with t-statistics of 2.22 and 2.89, respectively. Adding expected inflation and the volatility of inflation as additional explanatory variables into both regressions, shown in the bottom panel of Table 7 (columns 2 to 3), produces slightly lower coefficient estimates with t-statistics of 1.94 and 2.29. In the regressions shown in Table 7, we lag DisInf by two months. We motivate lagging DisInf given the quarterly frequency of CEX interviews for a household. Even if the survey participants adjust consumption contemporaneously with inflation beliefs, current innovations in consumption due to DisInf are reflected in CEX the earliest within the same month and the latest with a two month lag.

To provide further evidence for our economic channel, we consider three different classes of securities for which we expect increased trading when inflation disagreement is higher. First, inflation disagreement increases trading in nominal Treasury bonds. The top and bottom of column 4 in Table 7 shows a statistically positive relation between MSC inflation disagreement and trading in Treasuries measured by the volatility of total Treasury volume scaled by outstanding treasuries. The regressions differ in that in the bottom regression we add in ExpInf and SigInf as controls. The univariate regression produce a t-statistic of 2.33, while the multivariate regression produces a t-statistic of 3.78.

Table 6: **Break-Even Inflation and Inflation Risk Premium.** The table reports results from OLS regressions of break-even inflation and inflation risk premium on disagreement about inflation (DisInf), expected inflation (ExpInf), and the volatility of inflation (SigInf). Break-even inflation is the difference between nominal and real yields and the inflation risk premium is the break-even inflation rate minus expected inflation (ExpInf) plus a convexity adjustment $(0.5\,\text{SigInf}^2)$. The t-statistics (t-stat) are Newey-West corrected with 12 lags. Regression coefficients are standardized. ExpInf and SigInf are predicted by a GARCH$(1,1)$ model with an ARMA$(1,1)$ mean equation over multiple horizons (T). Samples: Q3-1981 to Q2-2014 and January 1978 to June 2014.

| | Survey of Professionals | | | Survey of Consumers | | |
| | Break-Even Inflation (BEIR) | | | | | |
Maturity	2y	3y	5y	2y	3y	5y
DisInf	0.319	0.358	0.397	0.551	0.597	0.647
t-stat	2.98	3.41	4.15	5.31	5.45	5.92
ExpInf	0.533	0.498	0.467	0.400	0.357	0.316
t-stat	6.77	6.59	6.72	5.47	5.69	6.06
adj. R^2	0.46	0.45	0.46	0.63	0.64	0.67
N	132	132	132	132	132	132
DisInf	0.407	0.436	0.469	0.604	0.644	0.693
t-stat	4.95	5.03	5.44	5.12	5.18	5.52
ExpInf	0.438	0.411	0.383	0.314	0.278	0.238
t-stat	4.23	3.76	3.54	4.53	3.69	3.12
SigInf	-0.201	-0.181	-0.172	-0.182	-0.163	-0.157
t-stat	-2.63	-2.44	-2.41	-1.89	-1.84	-2.15
adj. R^2	0.48	0.47	0.47	0.65	0.66	0.69
N	132	132	132	132	132	132
Inflation Risk Premium (IRP)						
DisInf	0.526	0.568	0.592	0.626	0.725	0.813
t-stat	4.89	5.77	6.54	3.64	4.26	5.24
ExpInf	-0.466	-0.348	-0.137	-0.577	-0.487	-0.304
t-stat	-1.70	-1.43	-0.70	-2.09	-2.09	-1.77
adj. R^2	0.36	0.34	0.32	0.44	0.48	0.56
N	132	132	132	132	132	132
DisInf	0.245	0.349	0.464	0.441	0.576	0.723
t-stat	3.72	4.24	4.92	6.44	5.94	5.91
ExpInf	-0.164	-0.105	0.012	-0.276	-0.241	-0.150
t-stat	-1.54	-0.87	0.09	-3.55	-2.82	-1.66
SigInf	0.641	0.509	0.305	0.630	0.506	0.309
t-stat	8.02	5.92	3.30	11.64	8.05	4.46
adj. R^2	0.65	0.51	0.38	0.76	0.69	0.63
N	132	132	132	132	132	132

Second, inflation disagreement increases trading in interest rate futures. We use open interest in interest rate futures scaled by open interest in financial futures and present the evidence for this trading channel in column 5 of Table 7. The t-statistics for the regression coefficients on MSC inflation disagreement are 2.60 (univariate) and 2.99 (multivariate using ExpInf and SigInf), respectively.

Third, inflation disagreement raises trading in inflation swaps. The univariate regression of inflation swap trading on MSC DisInf produces a t-statistics of 4.35. The multivariate regression, shown in the bottom panel of Table 7, does not yield a statistically significant coefficient estimate, which is likely caused by multicollinearity.[24] We measure inflation swap trading by detrending aggregated inflation notionals in both regressions.

G   Robustness

We check the robustness of our empirical results by analyzing different measures for dependent and independent variables for our empirical tests in the Internet Appendix. The different measures that we consider are: subtracting realized inflation from nominal yields to determine realized real yields, subtracting expected inflation estimated by an ARMA(1,1) from nominal yields to determine real yields, using an E-GARCH model instead of a GARCH model when estimating yield volatilities, using nominal yields extracted from U.S. Treasury security prices by the method of Gürkaynak, Sack, and Wright (2007), using the variance and interquartile range to measure disagreement for both SPF and MSC, and using realized or lagged realized inflation and their GARCH volatilities instead of predicted inflation and its volatility.

We also check the robustness of our empirical results by controlling for variables that effect our dependent variables and are correlated with inflation disagreement. Specifically, we consider the volatility of consumption growth, the volatility of GDP growth, the volatility of industrial production, the Jurado, Ludvigson, and Ng (2015) uncertainty measure, the Baker, Bloom, and Davis (2015) uncertainty measure, disagreement about GDP growth, and disagreement about earnings forecasts as additional control variables. Finally, we add lagged disagreement to address potential econometric issues with the persistence of disagreement. The robustness checks produce coefficient estimates, statistical significance, and economic significance for inflation disagreement which are similar to the ones we report in the paper.

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[24]The regression produces a high F-statistic with an insignificant t-statistic for each variable.

Table 7: **Cross-Sectional Consumption Growth Volatility and Trading.** The table reports OLS regression results. Dependent variables are cross-sectional consumption growth volatility, volatility of U.S. government bond trading volume, open interest of interest rate futures scaled by open interest in financial futures, and detrended inflation swap notional amounts. Explanatory variables are disagreement about inflation (DisInf), expected inflation (ExpInf), the volatility of inflation (SigInf), and CEX cross-sectional income growth volatility (SigInc). The CEX based regression contains a time-dummy and DisInf, ExpInf, and SigInf are lagged by two months. The t-statistics (t-stat) are Newey-West corrected with 12 lags. Regression coefficients are standardized. ExpInf and SigInf are predicted by a GARCH$(1, 1)$ model with an ARMA$(1, 1)$ mean equation. Samples: April 1984 - December 2012, January 2001 - August 2013, April 1986 - December 2013, May 2005 - February 2012.

	CEX Consumption Volatility I	CEX Consumption Volatility II	Volatility of Volume	Open Interest Ratio	Inflation Swaps
DisInf	0.162	0.146	0.332	0.314	0.265
t-stat	2.22	2.89	2.33	2.60	4.35
SigInc		0.303			
t-stat		4.31			
adj. R^2	0.37	0.49	0.10	0.10	0.06
N	345	330	151	333	70
DisInf	0.145	0.127	0.549	0.282	0.153
t-stat	1.94	2.29	3.78	2.99	1.34
ExpInf	0.036	0.068	-0.356	0.080	0.080
t-stat	0.43	1.06	-2.45	0.66	0.67
SigInf	-0.159	-0.069	-0.577	-0.402	0.228
t-stat	-2.24	-0.96	-3.15	-3.70	1.27
SigInc		0.281			
t-stat		3.92			
adj. R^2	0.40	0.50	0.31	0.28	0.05
N	345	330	151	333	70

III Model-Based Quantitative Evidence

Based on our theoretical and empirical evidence, we present a dynamic model that fits moments of inflation, inflation disagreement, and real and nominal yields and implies reasonable Sharpe ratios for inflation risk to quantitatively reproduce the impact of inflation disagreement on yield curves.

A Model

The exogenous real aggregate output process C_t follows a geometric Brownian motion with dynamics given by

$$dC_t = \mu_C C_t \, dt + \sigma_C C_t \, dz_{C,t}, \qquad C_0 > 0, \tag{23}$$

where z_C represents a real shock. The dynamics of the price level Π_t and the unobservable expected inflation rate x_t are

$$d\Pi_t = x_t \Pi_t \, dt + \sigma_\Pi \Pi_t \, dz_{\Pi,t}, \qquad dx_t = \kappa \left(\bar{x} - x_t \right) dt + \sigma_x \, dz_{x,t}, \qquad \Pi_0 = 1, \tag{24}$$

where $z_{\Pi,t}$ represents a nominal shock. The three Brownian motions $z_{C,t}$, $z_{\Pi,t}$, and $z_{x,t}$ are uncorrelated.

Investors agree on the long run mean $\bar{x}$ and the speed of mean reversion κ but have different beliefs about the volatility of expected inflation, σ_x.[25] The dynamics of the price level and the best estimator for expected inflation as perceived by investor i are[26]

$$d\Pi_t = x_t^i \Pi_t \, dt + \sigma_\Pi \Pi_t \, dz_{\Pi,t}^i, \qquad dx_t^i = \kappa \left(\bar{x} - x_t^i \right) dt + \hat{\sigma}_x^i \, dz_{\Pi,t}^i, \qquad x_0^i \sim N \left(\mu_{\bar{x},0}^i, \sigma_{x_0^i}^2 \right). \tag{25}$$

The volatility $\hat{\sigma}_x^i$ is a function of κ and σ_x^i. Investors observe the price level for a sufficiently long time so that the perceived volatility, $\hat{\sigma}_x^i$, has reached its steady state level.[27]

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[25]We model disagreement about the volatility of expected inflation because this leads to zero disagreement in the steady-state and to a tractable stochastic disagreement process.

[26]See Liptser and Shiryaev (1974a) and Liptser and Shiryaev (1974b).

[27]The steady state level is $\hat{\sigma}_x^i = \sigma_\Pi \left(\sqrt{\kappa^2 + \left(\frac{\sigma_x^i}{\sigma_\Pi} \right)^2} - \kappa \right)$. Note that the perceived volatility of expected inflation $\hat{\sigma}_x^i$ is lower than σ_x^i, due to learning.

Investors' nominal innovation processes are linked through the *disagreement process* Δ_t, which summarizes current disagreement about expected inflation. Specifically,

$$dz_{\Pi,t}^2 = dz_{\Pi,t}^1 - \Delta_t dt, \qquad \Delta_t = \frac{x_t^2 - x_t^1}{\sigma_\Pi}. \tag{26}$$

The disagreement process Δ_t follows an Ornstein-Uhlenbeck process

$$d\Delta_t = -\beta \Delta_t dt + \sigma_\Delta dz_{\Pi,t}^1, \qquad \beta = \frac{\kappa \sigma_\Pi + \hat{\sigma}_x^2}{\sigma_\Pi}, \qquad \sigma_\Delta = \frac{\hat{\sigma}_x^2 - \hat{\sigma}_x^1}{\sigma_\Pi}. \tag{27}$$

The dynamics of the likelihood ratio λ_t are

$$d\lambda_t = \Delta_t \lambda_t dz_{\Pi,t}^1. \tag{28}$$

We determine the disagreement measure over the horizon $T - t$ in the next Proposition.

Proposition 5. *The disagreement measure is*

$$\mathcal{D}_{t,T} \equiv \mathcal{D}\left(\Delta_t^2, T - t\right) = \frac{\sigma_\Delta^2}{4\beta} + \frac{1}{4\beta(T-t)}\left(\Delta_t^2 - \frac{\sigma_\Delta^2}{2\beta}\right)\left(1 - e^{-2\beta(T-t)}\right). \tag{29}$$

Disagreement is strictly increasing in Δ_t^2 and converges to $\frac{1}{2}\Delta_t^2$ and $\frac{\sigma_\Delta^2}{4\beta}$ if T goes to t and infinity, respectively. Hence, the instantaneous disagreement measure is given by $\frac{1}{2}\Delta_t^2$ and the long-run disagreement measure equals $\frac{\sigma_\Delta^2}{4\beta}$. In the empirical section, we measure disagreement as the standard deviation of expected inflation across investors, which in the model is $\frac{1}{2}\sigma_\Pi \frac{1}{\kappa}\left(1 - e^{-\kappa}\right) \mid \Delta_t \mid$. Therefore, the empirical disagreement measure is strictly increasing in $\mathcal{D}\left(\Delta(t)^2, T - t\right)$ for any maturity $T - t$.

Each investor solves the consumption-savings problem given in equation (1). We conclude the description of the model by specifying an external habit process which helps match asset pricing moments.[28] Specifically,

$$\log(H_t) = \log(H_0)e^{-\delta t} + \delta \int_0^t e^{-\delta(t-a)} \log(C_a) \, da, \qquad \delta > 0, \tag{30}$$

where δ describes the dependence of H_t on the history of aggregate output. Relative log output $\omega_t = \log(C_t/H_t)$, a state variable in the model, follows a mean reverting process

$$d\omega_t = \delta(\bar{\omega} - \omega_t) \, dt + \sigma_C \, dz_{C,t}, \qquad \bar{\omega} = (\mu_C - \sigma_C^2/2)/\delta. \tag{31}$$

Equilibrium consumption allocations and state price densities are given in Proposition 1.

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[28]See Abel (1990), Abel (1999), and Chan and Kogan (2002).

B   Real Yields

We provide closed-form solutions of real bond prices in the next proposition.

Proposition 6. *The real bond price, when γ is an integer is*

$$B_{t,T} \;=\; \sum_{k=0}^{\gamma} w_t^k B_{t,T}^k. \tag{32}$$

The stochastic weights w_t^k sum up to one and are given by

$$w_t^k = \binom{\gamma}{k} \frac{\lambda_t^{\frac{k}{\gamma}}}{\left(1 + \lambda_t^{\frac{1}{\gamma}}\right)^{\gamma}} = \binom{\gamma}{k} f(\lambda_t)^{\gamma-k}(1 - f(\lambda_t)^k. \tag{33}$$

$B_{t,T}^k$ *is an exponential quadratic function of the state vector $Y_{1,t} = (\Delta_t, \omega_t)$:*

$$B_{t,T}^k \;=\; exp\left(\mathcal{A}_B^k(T-t) + \mathcal{B}_B^k(T-t)'Y_{1,t} + Y_{1,t}'\mathcal{C}_B^k(T-t)Y_{1,t}\right), \tag{34}$$

where the coefficients $\mathcal{A}_B^k(\cdot), \mathcal{B}_B^k(\cdot), \mathcal{C}_B^k(\cdot)$ are solutions to ordinary differential equations summarized in the Internet Appendix.

The bond price in equation (32) is a weighted average of "artificial" bond prices that belong to the class of quadratic Gaussian term structure models. To gain intuition, we inspect the real short rate r_t which is the limit of the bond yield as maturity T approaches t:

$$r_t \;=\; \underbrace{\rho + \gamma\mu_C - \frac{1}{2}\gamma(\gamma+1)\sigma_C^2}_{CRRA} - \underbrace{\delta(\gamma-1)\omega_t}_{Habit} + \underbrace{\left(1 - \frac{1}{\gamma}\right)f(\lambda_t)(1 - f(\lambda_t))\frac{1}{2}\Delta_t^2}_{Disagreement}. \tag{35}$$

We see from equation (35) that the real short rate decomposes into the real short rate in an economy with a representative investor who has CRRA preferences and two additional terms. The additional terms account for habit preferences and inflation disagreement. The impact from inflation disagreement on the real yield curve depends on the consumption share $f(\lambda_t)$, risk aversion γ, and the instantaneous disagreement measure $\frac{1}{2}\Delta_t^2$. The real short rate does not depend on disagreement if $\gamma = 1$ and is increasing in disagreement when $\gamma > 1$ (the opposite is true when $\gamma < 1$).

C Nominal Yields

We provide closed-form solutions of the nominal price of a nominal bond in the next proposition.

Proposition 7. *The nominal bond price, when γ is an integer, is*

$$P_{t,T} = \sum_{k=0}^{\gamma} w_t^k P_{t,T}^k, \tag{36}$$

where w_t^k is given in equation (33). $P_{t,T}^k$ is an exponential quadratic function of the state vector $Y_t = (x_t^1, \Delta_t, \omega_t)$:

$$P_{t,T}^k = \exp \left(\mathcal{A}_P^k(T-t) + \mathcal{B}_P^k(T-t)'Y_t + Y_t'\mathcal{C}_P^k(T-t)Y_t \right), \tag{37}$$

where the coefficients $\mathcal{A}_P^k(\cdot), \mathcal{B}_P^k(\cdot), \mathcal{C}_P^k(\cdot)$ are solutions to ordinary differential equations summarized in the Internet Appendix.

Similarly to the real bond price, the nominal bond price can be expressed as a weighed average of artificial bond prices that belong to the class of quadratic Gaussian term structure models. Taking the limit of the nominal bond yield as the maturity T approaches t, we obtain the nominal short rate

$$r_{P,t} = r_t + f_t x_t^1 + (1 - f_t)x_t^2 - \sigma_\Pi^2. \tag{38}$$

We see from equation (38) that the nominal short rate is the sum of the real short rate, the consumption share weighted average expected inflation belief, and a Jensen's inequality term. Thus, the consumption share weighted average belief replaces expected inflation in a standard economy with homogeneous beliefs. The intuition for this is straight forward; when an investor has a larger consumption share, her view is more important in determining the price of the nominal bond.

D Calibration

We set the preference parameters (ρ, γ, δ) to match the level of nominal yields. The consumption parameters (μ_C, σ_C) are from Chan and Kogan (2002). The inflation parameters $(\bar{x}, \kappa, \sigma_x)$ and disagreement parameters (σ_x^1, σ_x^2) match the mean, standard deviation, and autocorrelation of the consensus belief and disagreement in SPF. We set the belief of the econometrician such that $\hat{\sigma}_x$ equals $(\hat{\sigma}_x^1 + \hat{\sigma}_x^2)/2$. We use SPF instead of MSC because it

explicitly asks professionals about CPI growth and thus leads to lower disagreement. The last parameter σ_Π matches the volatility of inflation. Table 8 reports the parameters.

Table 8: **Parameters**

Parameter	Description	Value
Investors		
ρ	Time preference parameter	0.006
γ	Common risk aversion	7
δ	Habit parameter	0.050
f_0	Initial consumption allocation	0.5
Consumption		
μ_C	Expected consumption growth	0.0172
σ_C	Volatility of consumption growth	0.0332
Inflation		
σ_Π	Inflation volatility	0.02
$\bar{x}$	Long run mean of expected inflation	0.0317
κ	Mean reversion of expected inflation	0.19
$\hat{\sigma}_x$	Volatility of expected inflation	0.01
Disagreement		
$\hat{\sigma}_x^1$	Estimated volatility of expected inflation investor 1	0.0044
$\hat{\sigma}_x^2$	Estimated volatility of expected inflation investor 2	0.0156

To analyze the quantitative implications of the model, we generate 10,000 sample paths of 50 years of data by simulating from the model under the belief of the econometrician (σ_x) instead of the belief of one of the investors (σ_x^1 or σ_x^2). All statistics are based on averages across the 10,000 sample paths.[29] Table 9 shows the mean, volatility, and autocorrelation of the consensus forecast in the first panel and disagreement in the second panel. We compute the mean and volatility of expected inflation across investors to determine the consensus belief and disagreement. The model matches the mean, volatility, and to a lesser extent the autocorrelation of the consensus belief and disagreement. Table 10 reports the mean, standard deviation, and autocorrelation of real and nominal yields in the model and in the data. The model matches the level and volatility of real and nominal yields. The persistence of nominal yields in the model is lower than in the data, that is, the average autocorrelation across maturities is 0.65 in the model and 0.89 in the data.[30]

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[29]Our model, as most heterogeneous belief models, is not stationary and thus we cannot compute unconditional moments.

[30]The mean, volatility, and Sharpe ratio of the market portfolio defined as a claim to aggregate output are 3.8%, 16.4%, and 0.23, respectively.

Table 9: **Disagreement in the Model and the Data.** The table reports mean, volatility, and annual autocorrelation for the consensus belief and disagreement about inflation. We compute the mean and volatility of expected inflation across investors to determine the consensus belief and disagreement. SPF statistics are based on the Survey of Professional Forecasters available at the quarterly frequency from $Q3-1981$ to $Q2-2014$. Model statistics are based on averages across 10,000 sample paths of 50 years of simulated data under the belief of the econometrician.

	Model	SPF
Consensus Belief		
Average	0.032	0.031
Volatility	0.013	0.012
Autocorrelation	0.703	0.683
Disagreement		
Average	0.005	0.007
Volatility	0.004	0.003
Autocorrelation	0.168	0.190

E Quantitative Effects of Inflation Disagreement

Figure 8 shows real and nominal yields with maturities ranging from 1 to 5 years for two realizations of current disagreement Δ. In the two plots, the black solid line corresponds to the steady state level of Δ, which is 0, and the blue dashed line corresponds to a one standard deviation increase in Δ, which is 0.5143. The plots show that inflation disagreement has an economically significant impact on real and nominal yields. The economic magnitudes are comparable to the data. Specifically, an increase in disagreement by one standard deviation raises the two year real yield by 0.94% and the one year nominal yield by 1.43%. The effects in the data are $0.407 \times 1.976 = 0.80\%$ for the two year real yield and $0.354 \times 3.124 = 1.11\%$ for the one year nominal yield. The economic significance for longer maturities is lower in the model than in the data because disagreement is less persistent in the model.

Table 11 shows regression results of nominal and real yield levels and volatilities on disagreement. As in the empirical analysis, the t-statistics are Newey-West corrected with 12 lags and coefficients are standardized in all four univariate regressions. First, the coefficients and t-statistics for the real and nominal level regressions are similar to the data at the short end. For longer maturities, the coefficients are smaller in the model. This is driven by the lower persistence of disagreement and yields in the model than in the data. Second, the volatility regressions are also close to the data counter parts, especially for shorter maturities. Similarly to level regressions, we see a steeper decline in the coefficients in the model relative

Table 10: **Yields in the Model and the Data.** The table reports summary statistics for real and nominal yields. Quarterly real yields are from Chernov and Mueller (2012) merged with TIPS yields from Gürkaynak, Sack, and Wright (2010) for the period Q3-1981 to Q2-2014. Monthly nominal Fama-Bliss discount bond yields are from CRSP for the period January 1978 to June 2014. Model statistics are based on averages across 10,000 sample paths of 50 years of simulated real and nominal yields and their volatilities under the belief of the econometrician.

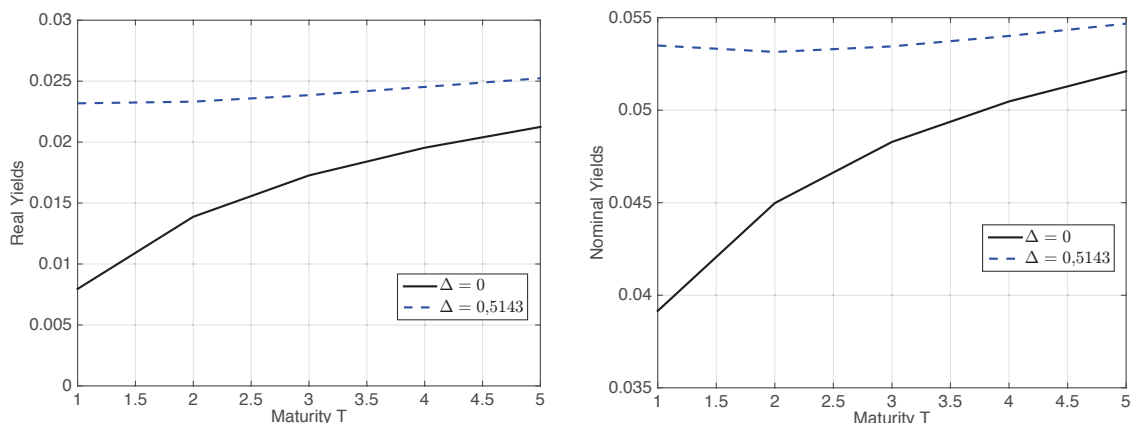| Maturity	Real Yields		Nominal Yields	
	Model	Data	Model	Data
Average				
1	0.021		0.052	0.049
2	0.021	0.019	0.053	0.052
3	0.022	0.020	0.053	0.054
4	0.023		0.054	0.056
5	0.024	0.023	0.055	0.058
Volatility				
1	0.028		0.029	0.034
2	0.023	0.020	0.025	0.034
3	0.021	0.018	0.022	0.034
4	0.019		0.021	0.033
5	0.019	0.016	0.020	0.032
Autocorrelation				
1	0.45		0.47	0.75
2	0.59	0.66	0.60	0.76
3	0.68	0.70	0.68	0.77
4	0.73		0.73	0.76
5	0.76	0.73	0.76	0.78

Figure 8: Real and Nominal Yields

The left plot shows real yields and the right plot shows nominal yields as function of time to maturity for two realizations of current disagreement Δ. The black solid line corresponds to the steady state level of Δ and the blue dashed line corresponds to a one standard deviation increase in Δ. Both plots show that an increase in inflation disagreement has an economically significant positive impact on real and nominal yields.

to the data as the maturity increases. The volatility of yields are decaying quicker in the model than in the data. Hence, we would except the effect of disagreement on the volatility of yields at longer maturities to be lower in the model.

F Inflation Risk Premium and Sharpe Ratio

We specify a simple asset structure that dynamically completes the market to inspect quantitatively the inflation risk premium and the corresponding Sharpe ratio.

Suppose investors can continuously trade an inflation-protected money market account with real price $B_{t,0}$, a nominal money market account with nominal price $P_{t,0}$, and a security, called a stock, with real price S_t and unit volatility that is locally perfectly correlated with real consumption growth. The dynamics of the inflation-protected money market account and stock in equilibrium are

$$dB_{t,0} = B_{t,0}\, r_t\, dt, \qquad B_{0,0} = 1, \qquad \text{and} \qquad dS_t = S_t \left((r_t + \theta_{C,t})\, dt +\, dz_{C,t} \right), \qquad S_0 = 1,$$

where $\theta_{C,t} = \gamma\, \sigma_C$ is the market price of risk for the real shock $z_{C,t}$. The dynamics of the real price of the nominal money market account, $p_{t,0} = P_{t,0}/\Pi_t$, in equilibrium are

Table 11: **Disagreement Regressions.** The table reports results from OLS regressions of the level and volatility of real and nominal yields on disagreement about inflation. We use the Survey of Professional Forecasters (SPF) to measure inflation disagreement (Q3-1981 to Q2-2014). Quarterly real yields are from Chernov and Mueller (2012) merged with TIPS yields from Gürkaynak, Sack, and Wright (2010). Monthly nominal Fama-Bliss discount bond yields are from CRSP. Real and nominal yield volatilities are computed by a GARCH$(1, 1)$ with an AR(1) mean equation. Model coefficients and standardized t-statistics are based on averages across 10,000 sample paths of 50 years of simulated real and nominal yields and their volatilities under the belief of the econometrician. The t-statistics (t-stat) are Newey-West corrected with 12 lags.

Maturity	Real Yields Model	Real Yields SPF	Nominal Yields Model	Nominal Yields SPF
Level				
1 year	0.644		0.598	0.465
t-stat	7.77		6.57	3.65
adj. R^2	0.43		0.38	0.21
2 year	0.497	0.407	0.453	0.466
t-stat	5.56	3.48	4.67	3.64
adj. R^2	0.27	0.16	0.24	0.21
3 year	0.390	0.397	0.354	0.469
t-stat	4.20	3.33	3.55	3.68
adj. R^2	0.18	0.15	0.16	0.21
4 year	0.318		0.289	0.469
t-stat	3.35		2.87	3.75
adj. R^2	0.13		0.12	0.21
5 year	0.268	0.388	0.245	0.480
t-stat	2.80	3.23	2.42	3.90
adj. R^2	0.10	0.14	0.10	0.21
Volatility				
1 year	0.609		0.566	0.709
t-stat	8.27		7.06	5.10
adj. R^2	0.38		0.33	0.50
2 year	0.569	0.604	0.499	0.707
t-stat	7.21	8.51	5.66	5.10
adj. R^2	0.33	0.36	0.26	0.50
3 year	0.519	0.648	0.429	0.665
t-stat	6.18	9.27	4.53	5.75
adj. R^2	0.28	0.41	0.20	0.44
4 year	0.464		0.366	0.732
t-stat	5.28		3.70	6.47
adj. R^2	0.23		0.15	0.53
5 year	0.407	0.700	0.313	0.712
t-stat	4.50	8.35	3.09	7.11
adj. R^2	0.18	0.49	0.12	0.50

$$dP_{t,0} = P_{t,0}\left(\left(r_{P,t} - x_t^i + \sigma_\Pi^2\right)dt - \sigma_\Pi\,dz_{\Pi,t}^i\right),$$
$$= P_{t,0}\left(\left(r_t - \sigma_\Pi\theta_{\Pi,t}^i\right)dt - \sigma_\Pi\,dz_{\Pi,t}^i\right), \qquad P_{0,0} = 1, \qquad i = 1,2, \tag{39}$$

where $\theta_{\Pi,t}^i$ denotes the market price of risk of the perceived inflation shock $z_{\Pi,t}^i$. Specifically,

$$\theta_{\Pi,t}^1 = (f_t - 1)\Delta_t \qquad \text{and} \qquad \theta_{\Pi,t}^2 = f_t\Delta_t. \tag{40}$$

An increase in inflation is bad news for real asset prices and thus the market price of risk for the inflation shock has a different sign then the Sharpe ratio of the asset. The inflation risk premium and the Sharpe ratio for the nominal money market account perceived by investor i are

$$\text{IRP}^i = -\sigma_\Pi\theta_{\Pi,t}^i \qquad \text{and} \qquad \text{SR}^i = \frac{\text{IRP}^i}{\sigma_\Pi} = -\theta_{\Pi,t}^i.$$

Hence, investors have opposing views on the real return of the nominal money market account due to their different views about expected inflation.

We focus on the case where the first investor takes a long position in the nominal money market account because she perceives a positive inflation risk premium due to a lower expected inflation rate than the second investor, that is, $\Delta \geq 0$. The left plot of Figure 9 shows that the inflation risk premium and the Sharpe ratio perceived by the first investor are strictly increasing in disagreement Δ. The maximal Sharpe ratio and inflation risk premium when both investors share output equally ($f = 0.5$) and $\Delta = 0.5143$, which corresponds to a one standard deviation increase from the steady state of zero, are 0.2571 and 0.0051, respectively. As shown more generally in Proposition 4, the right plot of Figure 9 confirms that the inflation risk premium and the Sharpe ratio perceived by the first investor declines when her consumption share in the economy increases. When her consumption share is close to one, then prices reflect only her view about inflation, and thus the inflation risk premium and the Sharpe ratio are close to zero. However, in this case the second investor perceives the highest Sharpe ratio and inflation risk premium in absolute terms because he is short the nominal money market account.
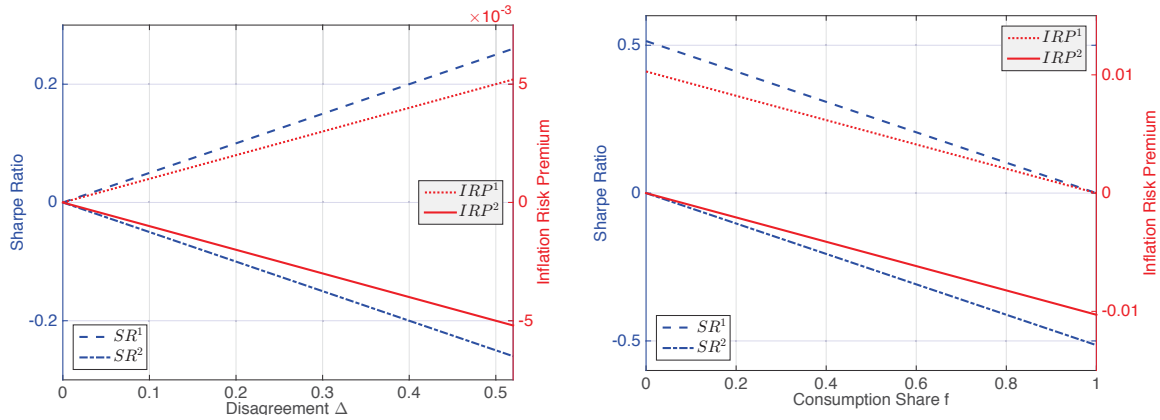
Figure 9: Sharpe Ratio and Inflation Risk Premium

The left plot shows the inflation risk premium and Sharpe ratio perceived by the first investor as strictly increasing functions of disagreement Δ. The second investor perceives a negative inflation risk premium and Sharpe ratio and thus is short the nominal money market account. The right plot shows that the inflation risk premium and Sharpe ratio perceived by the first investor goes down when her consumption share in the economy increases.

IV   Concluding Remarks

Surveys of consumers and professionals show that there is disagreement about inflation. But does this disagreement affect asset prices or individual consumption? We consider a pure exchange economy with a frictionless, complete securities market to answer this question theoretically. We show that disagreement about inflation has a strong impact on the cross-sectional consumption growth volatility as well as real and nominal yield curves. Intuitively, investors make different consumption savings decisions based on their different beliefs about real returns on investments which raises the volatility of individual consumption and yields. Each investor thinks the high real returns on their investments will make them wealthier and thus interest rates have to rise for consumption markets to clear. In addition, disagreement drives a wedge between real and nominal yields curves due to changes in the investment opportunity set caused by speculative trade in equilibrium.

We find empirical support for our theoretical predictions using a survey of consumers (MSC) and a survey of professional forecasters (SPF). Specifically, real and nominal yields are higher and more volatile when there is inflation disagreement which also increase the break-even inflation rate and inflation risk premiums. The effects are economically and statistically significant. Empirically, an inflation disagreement increase of one standard deviation raises real and nominal yield levels and volatilities, break-even inflation, and the inflation risk

premium by at least 30% of their respective standard deviations. We provide empirical support for the economic channel through which disagreement effect asset prices by showing that there is more trade in nominal Treasuries, interest rate derivatives, and inflation swaps and there is higher cross-sectional consumption growth volatility when disagreement about inflation is high. Calibrating a dynamic model where investors disagree about the dynamics of expected inflation to disagreement, inflation, and yield data reproduces the economically and statistically significant impact of inflation disagreement on real and nominal yield curves.

We document that inflation disagreement raises individual consumption volatilities, real interest rates and their volatilities which seems to be an undesirable outcome for policy-makers. Clearly, it is optimal for investors to trade on their inflation beliefs in our complete market economy. However, all investors cannot have correct beliefs and thus it is not clear whether trading on their beliefs is ex-post welfare improving. Recent studies such as Brunnermeier, Simsek, and Xiong (2014), Gilboa, Samuelson, and Schmeidler (2014), and Heyerdahl-Larsen and Walden (2015) show that policies that reduce disagreement, restrict trade on disagreement, or taxes, and hence avoid an increase in individual consumption volatilities, may be socially optimal in this case. Better understanding how central banks respond to inflation disagreement and potentially impact bond markets could be fruitful for future work.

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A   Proofs

Proof of Proposition 1. See Detemple and Murthy (1994) or Basak (2005) and the references therein.

Proof of Theorem 1. We split this proof into three parts

(i) Real yields:

Let ξ_T^0 denote the state price density when there is no disagreement. Specifically,

$$\xi_t^0 = e^{-\rho t} C_t^{-\gamma} H_t^{\gamma-1}.$$

By Assumption 1, there is no disagreement about the distribution of output C_t and the habit H_t and, thus, the real price of a real bond when there is no disagreement and the representative investor has belief $\mathbb{P}^0$ is

$$B_{t,T}^0 = \mathbb{E}_t^0 \left[\frac{\xi_T^0}{\xi_t^0} \right] = \mathbb{E}_t^1 \left[\frac{\xi_T^0}{\xi_t^0} \right] = \mathbb{E}_t^2 \left[\frac{\xi_T^0}{\xi_t^0} \right].$$

The real price of a real bond with disagreement is

$$B_{t,T} = \mathbb{E}_t^1 \left[\frac{\xi_T^1}{\xi_t^1} \right] = \mathbb{E}_t^1 \left[\frac{\xi_T^0}{\xi_t^0} \left(\frac{f(\lambda_T)}{f(\lambda_t)} \right)^{-\gamma} \right].$$

We have that

$$\left(\frac{f(\lambda_T)}{f(\lambda_t)} \right)^{-\gamma} = \left(\frac{1 + (y\lambda_T)^{\frac{1}{\gamma}}}{1 + (y\lambda_t)^{\frac{1}{\gamma}}} \right)^{\gamma} = \left(f_t + (1 - f_t) \left(\frac{\lambda_T}{\lambda_t} \right)^{\frac{1}{\gamma}} \right)^{\gamma},$$

and, hence,

$$B_{t,T} = \mathbb{E}_t^1 \left[\frac{\xi_T^0}{\xi_t^0} \left(f_t + (1 - f_t) \left(\frac{\lambda_T}{\lambda_t} \right)^{\frac{1}{\gamma}} \right)^{\gamma} \right].$$

Suppose $\gamma = 1$. Then the bond price simplifies to

$$B_{t,T} = \mathbb{E}_t^1 \left[\frac{\xi_T^0}{\xi_t^0} \left(f_t + (1 - f_t) \left(\frac{\lambda_T}{\lambda_t} \right) \right) \right] = f_t \mathbb{E}_t^1 \left[\frac{\xi_T^0}{\xi_t^0} \right] + (1 - f_t) \mathbb{E}_t^1 \left[\frac{\lambda_T}{\lambda_t} \frac{\xi_T^0}{\xi_t^0} \right]$$

$$= f_t \mathbb{E}_t^1 \left[\frac{\xi_T^0}{\xi_t^0} \right] + (1 - f_t) \mathbb{E}_t^2 \left[\frac{\xi_T^0}{\xi_t^0} \right] = f_t B_{t,T}^0 + (1 - f_t) B_{t,T}^0 = B_{t,T}^0.$$

This concludes the proof of the case $\gamma = 1$.

Consider the function $h(x) = x^{\frac{1}{\gamma}}$, which is strictly increasing and convex if $\gamma < 1$ and strictly concave if $\gamma > 1$. Suppose $\gamma > 1$ and thus $h(x)$ is strictly concave. The case of $\gamma < 1$ is similar and thus omitted.

The real price of a real bond with disagreement is

$$B_{t,T} = \mathbb{E}_t^1 \left[\frac{\xi_T^0}{\xi_t^0} \left(f_t + (1 - f_t) \left(\frac{\lambda_T}{\lambda_t} \right)^{\frac{1}{\gamma}} \right)^{\gamma} \right]$$

$$= \mathbb{E}_t^1 \left[\frac{\xi_T^0}{\xi_t^0} \right] \mathbb{E}_t^1 \left[\frac{\frac{\xi_T^0}{\xi_t^0}}{\mathbb{E}_t^1 \left[\frac{\xi_T^0}{\xi_t^0} \right]} \left(f_t + (1 - f_t) h \left(\frac{\lambda_T}{\lambda_t} \right) \right)^{\gamma} \right]$$

$$= B_{t,T}^0 \, \hat{\mathbb{E}}_t^1 \left[\left(f_t + (1 - f_t) h \left(\frac{\lambda_T}{\lambda_t} \right) \right)^{\gamma} \right],$$

where $\hat{\mathbb{E}}_t^1$ denotes the conditional mean using the bond price $B_{t,T}^0$ as numeraire. Specifically,

$$\frac{\zeta_T^1}{\zeta_t^1} \equiv \frac{d\hat{\mathbb{P}}^1}{d\mathbb{P}^1} = \frac{\xi_T^0}{\xi_t^0} \frac{1}{B_{t,T}^0}.$$

We have that

$$\hat{\mathbb{E}}_t^1 \left[\frac{\lambda_T}{\lambda_t} \right] = \mathbb{E}_t^1 \left[\frac{\lambda_T}{\lambda_t} \frac{\zeta_T^1}{\zeta_t^1} \right] = \mathbb{E}_t^2 \left[\frac{\zeta_T^1}{\zeta_t^1} \right] = \frac{\mathbb{E}_t^2 \left[\frac{\xi_T^0}{\xi_t^0} \right]}{B_{t,T}^0} = \frac{B_{t,T}^0}{B_{t,T}^0} = 1.$$

Strict concavity of $h(\cdot)$ leads to

$$f_t h(1) + (1 - f_t) h \left(\frac{\lambda_T}{\lambda_t} \right) < h \left(f_t \cdot 1 + (1 - f_t) \cdot \frac{\lambda_T}{\lambda_t} \right).$$

Hence,

$$B_{t,T} = B_{t,T}^0 \, \hat{\mathbb{E}}_t^1 \left[\left(f_t + (1 - f_t) h \left(\frac{\lambda_T}{\lambda_t} \right) \right)^\gamma \right] < B_{t,T}^0 \, \hat{\mathbb{E}}_t^1 \left[h \left(f_t \cdot 1 + (1 - f_t) \cdot \frac{\lambda_T}{\lambda_t} \right)^\gamma \right]$$

$$= B_{t,T}^0 \left(f_t + (1 - f_t) \hat{\mathbb{E}}_t^1 \left[\frac{\lambda_T}{\lambda_t} \right] \right) = B_{t,T}^0 \left(f_t + (1 - f_t) \right) = B_{t,T}^0.$$

(ii) *Consumption Growth Volatility*

The consumption allocations are

$$C_t^1 = f(\lambda_t) \, C_t, \quad \text{and} \quad C_t^2 = (1 - f(\lambda_t)) \, C_t, \quad \text{where} \quad f(\lambda_t) = \frac{1}{1 + (y\lambda_t)^{\frac{1}{\gamma}}}.$$

The cross-sectional variance of consumption growth from time t to T is

$$\sigma_{\text{CS}}^2(\lambda_t, \lambda_T) = \frac{1}{4} \left(\log \left(\frac{C_T^1}{C_t^1} \right) - \log \left(\frac{C_T^2}{C_t^2} \right) \right)^2 = \frac{1}{4} \left(\log \left(\frac{f(\lambda_T)}{f(\lambda_t)} \frac{1 - f(\lambda_t)}{1 - f(\lambda_T)} \right) \right)^2$$

$$= \frac{1}{4\gamma^2} \left(\log \left(\frac{\lambda_T}{\lambda_t} \right) \right)^2.$$

If there is no disagreement, then $\lambda_t \equiv 1$ and thus $\sigma_{\text{CS}}^2(\lambda_t, \lambda_T) = 0$.

(iii) *Real Yield Volatility*

If $\gamma = 1$, then real yields with disagreement are equal to real yields when there is no disagreement and thus the volatility of yields does not depend on disagreement.

Suppose $\gamma \neq 1$. The real price of a real bond with disagreement is

$$B_{t,T} = B_{t,T}^0 \, \hat{\mathbb{E}}_t^1 \left[\left(f_t + (1 - f_t) h \left(\frac{\lambda_T}{\lambda_t} \right) \right)^\gamma \right],$$

where $\hat{\mathbb{E}}_t^1$ denotes the conditional mean using the real bond price without disagreement, $B_{t,T}^0$, as numeraire. Let $y_{t,T}^B$ denote the real yield when there is disagreement and $y_{t,T}^{B^0}$ the real yield when there is no disagreement. We have that

$$
\begin{aligned}
y_{t,T}^B &= -\frac{1}{T-t}\log\left(B_{t,T}\right) \\
&= -\frac{1}{T-t}\log\left(B_{t,T}^0\right) - \frac{1}{T-t}\log\left(\hat{\mathbb{E}}_t^1\left[\left(f_t + (1-f_t)h\left(\frac{\lambda_T}{\lambda_t}\right)\right)^{\gamma}\right]\right) \\
&= y_{t,T}^{B^0} - \frac{1}{T-t}\log\left(\hat{\mathbb{E}}_t^1\left[\left(f_t + (1-f_t)h\left(\frac{\lambda_T}{\lambda_t}\right)\right)^{\gamma}\right]\right).
\end{aligned} \tag{41}
$$

λ_t is independent of C_t and H_t and, hence,

$$
\mathbb{V}^i\left[y_{t,T}^B\right] \geq \mathbb{V}^i\left[y_{t,T}^{B^0}\right], \qquad \forall i = 0,1,2,
$$

with equality if the conditional expectation in equation (41) is constant.

Corollary 1. *[Independent Multiplicative Decomposition] Consider the probability space $(\Omega, \mathcal{F})$ and the three strictly positive random variables $\tilde{x}, \tilde{y}$, and $\tilde{\varepsilon}$ with corresponding probability measures $\mathbb{P}^x$, $\mathbb{P}^y$, and $\mathbb{P}^\varepsilon$. Suppose that (i) $\tilde{y}$ and $\tilde{x}$ have unit mean, that is, $\mathbb{E}^y\left[\tilde{y}\right] = \mathbb{E}^x[\tilde{x}] = 1$, (ii) $\tilde{y}$ and $\tilde{x}\tilde{\varepsilon}$ are equal in distribution, that is, $\tilde{y} \overset{d}{=} \tilde{x}\tilde{\varepsilon}$, and (iii) $\tilde{x}$ and $\tilde{\varepsilon}$ are mean independent, that is, $\mathrm{E}^\varepsilon[\tilde{\varepsilon} \mid \tilde{x} = x] = \mathrm{E}^\varepsilon[\tilde{\varepsilon}] = 1$, $\forall x$. Then the following three statements hold:*

(i)
$$
\mathbb{E}^y\left[g\left(\tilde{y}\right)\right] \leq \mathbb{E}^x[g(\tilde{x})],
$$

for all concave functions g,

(ii)
$$
\mathbb{V}^y\left[\tilde{y}\right] \geq \mathbb{V}^x\left[\tilde{x}\right],
$$

(iii) and
$$
\mathbb{E}^y\left[\left(\log\left(\tilde{y}\right)\right)^2\right] \geq \mathbb{E}^x\left[\left(\log\left(\tilde{x}\right)\right)^2\right],
$$

if $\tilde{x}$ and $\tilde{\varepsilon}$ are independent.

Proof. We split the proof into three parts:

(i) It follows from the definition of equality in distribution, mean independence, and Jensen's inequality that

$$
\begin{aligned}
\mathbb{E}^y\left[g\left(\tilde{y}\right)\right] = \mathbb{E}^y\left[g\left(\tilde{x}\tilde{\varepsilon}\right)\right] = \mathbb{E}^x\left[\mathbb{E}^\varepsilon\left[g\left(\tilde{x}\tilde{\varepsilon}\right) \mid \tilde{x}\right]\right] &\leq \mathbb{E}^x\left[g\left(\mathbb{E}^\varepsilon\left[\tilde{x}\tilde{\varepsilon} \mid \tilde{x}\right]\right)\right] = \mathbb{E}^x\left[g\left(\tilde{x}\mathbb{E}^{\mathbb{P}}\left[\tilde{\varepsilon} \mid \tilde{x}\right]\right)\right] \\
&= \mathbb{E}^x\left[g\left(\tilde{x}\right)\right].
\end{aligned}
$$

(ii) It follows from the definition of equality in distribution, mean independence, and Jensen's inequality that

$$
\begin{aligned}
\mathbb{V}^y\left[\tilde{y}\right] = \mathbb{V}^y\left[\tilde{x}\,\tilde{\varepsilon}\right] &= \mathbb{E}^y\left[\tilde{x}^2\tilde{\varepsilon}^2\right] - \left(\mathbb{E}^y\left[\tilde{x}\,\tilde{\varepsilon}\right]\right)^2 = \mathbb{E}^x\left[\mathbb{E}^\varepsilon\left[\tilde{x}^2\tilde{\varepsilon}^2 \mid \tilde{x}\right]\right] - \left(\mathbb{E}^x\left[\mathbb{E}^\varepsilon\left[\tilde{x}\,\tilde{\varepsilon} \mid \tilde{x}\right]\right]\right)^2 \\
&= \mathbb{E}^x\left[\tilde{x}^2\mathbb{E}^\varepsilon\left[\tilde{\varepsilon}^2 \mid \tilde{x}\right]\right] - \left(\mathbb{E}^x\left[\tilde{x}\,\mathbb{E}^\varepsilon\left[\tilde{\varepsilon} \mid \tilde{x}\right]\right]\right)^2 \geq \mathbb{E}^x\left[\tilde{x}^2\left(\mathbb{E}^\varepsilon\left[\tilde{\varepsilon} \mid \tilde{x}\right]\right)^2\right] - \left(\mathbb{E}^x\left[\tilde{x}\right]\right)^2 \\
&= \mathbb{E}^x\left[\tilde{x}^2\right] - \left(\mathbb{E}^x\left[\tilde{x}\right]\right)^2 = \mathbb{V}^x\left[\tilde{x}\right].
\end{aligned}
$$

(iii) Since $g(x) = \log(x)^2$ is convex for $0 < x < 1$ and concave for $x > 1$, we cannot apply the first result to show the third result. However, if $\tilde{x}$ and $\tilde{\varepsilon}$ are independent, then

$$
\begin{aligned}
\mathbb{E}^y\left[\left(\log\left(\tilde{y}\right)\right)^2\right] = \mathbb{E}^y\left[\left(\log\left(\tilde{x}\,\tilde{\varepsilon}\right)\right)^2\right] &= \mathbb{E}^y\left[\left(\log\left(\tilde{x}\right) + \log\left(\tilde{\varepsilon}\right)\right)^2\right] \\
&= \mathbb{E}^x\left[\left(\log\left(\tilde{x}\right)\right)^2\right] + 2\mathbb{E}^y\left[\log\left(\tilde{x}\right)\log\left(\tilde{\varepsilon}\right)\right] + \mathbb{E}^\varepsilon\left[\left(\log\left(\tilde{\varepsilon}\right)\right)^2\right] \\
&= \mathbb{E}^x\left[\left(\log\left(\tilde{x}\right)\right)^2\right] + 2\mathbb{E}^x\left[\log\left(\tilde{x}\right)\right]\mathbb{E}^\varepsilon\left[\log\left(\tilde{\varepsilon}\right)\right] + \mathbb{E}^\varepsilon\left[\left(\log\left(\tilde{\varepsilon}\right)\right)^2\right].
\end{aligned}
$$

The first and third terms are non-negative and thus it remains to show that the second term is nonnegative. We know that $\tilde{x}$ and $\tilde{\varepsilon}$ have unit mean and thus the average of the log of both variables is nonpositive because by Jensen's inequality

$$
\mathbb{E}^x\left[\log\left(\tilde{x}\right)\right] \leq \log\left(\mathbb{E}^x\left[\tilde{x}\right]\right) = 0.
$$

Hence,
$$
\mathbb{E}^x\left[\log\left(\tilde{x}\right)\right]\mathbb{E}^\varepsilon\left[\log\left(\tilde{\varepsilon}\right)\right] \geq 0,
$$

which concludes the proof of the third statement.

Proof of Proposition 2. We need to show that

$$
\mathcal{D}_{t_\eta,t_\eta+\tau} = -\frac{1}{\tau}\mathbb{E}_{t_\eta}^{\eta,1}\left[\log\left(\frac{\eta_{t_\eta+\tau}}{\eta_{t_\eta}}\right)\right] \geq -\frac{1}{\tau}\mathbb{E}_t^1\left[\log\left(\frac{\lambda_{t+\tau}}{\lambda_t}\right)\right] = \mathcal{D}_{t,t+\tau},
$$

which is equivalent to showing that

$$
\mathbb{E}_{t_\eta}^{\eta,1}\left[\log\left(\frac{\eta_{t_\eta+\tau}}{\eta_{t_\eta}}\right)\right] \leq \mathbb{E}_t^1\left[\log\left(\frac{\lambda_{t+\tau}}{\lambda_t}\right)\right]. \tag{42}
$$

The function $g(x) = \log(x)$ is concave and thus it follows from Corollary 1 that inequality (42) is satisfied if η_t and λ_t satisfy Assumption 2.

Proof of Theorem 2. We split this proof into two parts

(i) Real yields:

Let ξ_T^0 denote the state price density when there is no disagreement. Specifically,

$$
\xi_t^0 = e^{-\rho t}C_t^{-\gamma}H_t^{\gamma-1}.
$$

It follows from Assumption 2 that there is no disagreement about the distribution of output C_t and the habit H_t and thus the real price of a real bond when there is no disagreement and the representative investor has belief $\mathbb{P}^0$ is

$$B_{t,T}^0 = \mathbb{E}_t^0 \left[\frac{\xi_T^0}{\xi_t^0} \right] = \mathbb{E}_t^1 \left[\frac{\xi_T^0}{\xi_t^0} \right] = \mathbb{E}_t^2 \left[\frac{\xi_T^0}{\xi_t^0} \right].$$

It follows from Assumption 2 that λ_t is independent of ξ_t^0 and thus the real price of a real bond with disagreement is

$$B_{t,T} = \mathbb{E}_t^1 \left[\frac{\xi_T^0}{\xi_t^0} \left(f(\lambda_t) + (1 - f(\lambda_t)) \left(\frac{\lambda_T}{\lambda_t} \right)^{\frac{1}{\gamma}} \right)^{\gamma} \right]$$

$$= \mathbb{E}_t^1 \left[\frac{\xi_T^0}{\xi_t^0} \right] \mathbb{E}_t^1 \left[\left(f(\lambda_t) + (1 - f(\lambda_t)) \left(\frac{\lambda_T}{\lambda_t} \right)^{\frac{1}{\gamma}} \right)^{\gamma} \right]$$

$$= B_{t,T}^0 \mathbb{E}_t^1 \left[\left(f(\lambda_t) + (1 - f(\lambda_t)) \left(\frac{\lambda_T}{\lambda_t} \right)^{\frac{1}{\gamma}} \right)^{\gamma} \right].$$

Similarly,

$$B_{t_\eta,T_\eta}^\eta = B_{t_\eta,T_\eta}^0 \mathbb{E}_{t_\eta}^{\eta,1} \left[\left(f(\eta_{t_\eta}) + (1 - f(\eta_{t_\eta})) \left(\frac{\eta_{T_\eta}}{\eta_{t_\eta}} \right)^{\frac{1}{\gamma}} \right)^{\gamma} \right].$$

We have that $\tau = T_\eta - t_\eta = T - t$ and thus it follows from Assumption 2 that $B_{t,t+\tau}^0 = B_{t_\eta,t_\eta+\tau}^0$. Moreover, $f_t = f(\lambda_t) = f(\eta_{t_\eta})$ and, hence,

$$B_{t,t+\tau} = B_{t,t+\tau}^0 \mathbb{E}_t^1 \left[\left(f_t + (1 - f_t) \left(\frac{\lambda_{t+\tau}}{\lambda_t} \right)^{\frac{1}{\gamma}} \right)^{\gamma} \right],$$

$$B_{t_\eta,t_\eta+\tau}^\eta = B_{t,t+\tau}^0 \mathbb{E}_{t_\eta}^{\eta,1} \left[\left(f_t + (1 - f_t) \left(\frac{\eta_{t_\eta+\tau}}{\eta_{t_\eta}} \right)^{\frac{1}{\gamma}} \right)^{\gamma} \right].$$

Suppose $\gamma = 1$. Then the bond prices simplify to

$$B_{t,t+\tau} = B_{t,t+\tau}^0 \mathbb{E}_t^1 \left[f_t + (1 - f_t) \frac{\lambda_{t+\tau}}{\lambda_t} \right] = B_{t,t+\tau}^0,$$

$$B_{t_\eta,t_\eta+\tau}^\eta = B_{t,t+\tau}^0 \mathbb{E}_{t_\eta}^{\eta,1} \left[f_t + (1 - f_t) \frac{\eta_{t_\eta+\tau}}{\eta_{t_\eta}} \right] = B_{t,t+\tau}^0.$$

This concludes the proof of the case $\gamma = 1$.

Define the function $g(x) = \left(f + (1 - f) x^{\frac{1}{\gamma}} \right)^{\gamma}$ which is strictly concave if $\gamma > 1$ and strictly convex if $\gamma < 1$. Suppose $\gamma > 1$ and thus $h(x)$ is strictly concave. The case of $\gamma < 1$ is similar and thus omitted. We need to show that $B_{t_\eta,t_\eta+\tau}^\eta < B_{t,t+\tau}$, which is equivalent to showing that

$$\mathbb{E}_{t_\eta}^{\eta,1}\left[g\left(\frac{\eta_{t_\eta+\tau}}{\eta_{t_\eta}}\right)\right] < \mathbb{E}_t^1\left[h\left(\frac{\lambda_{t+\tau}}{\lambda_t}\right)\right],$$

which follows directly from Assumption 2 and Corrolary 1.

(ii) *Consumption Growth Volatility*

The cross-sectional variance of consumption growth from time t to T in both economies is

$$\sigma_{\text{CS}}^2(\lambda_t, \lambda_{t+\tau}) = \frac{1}{4\gamma^2}\left(\log\left(\frac{\lambda_{t+\tau}}{\lambda_t}\right)\right)^2, \quad \text{and} \quad \sigma_{\text{CS}}^2(\eta_{t_\eta}, \eta_{t_\eta+\tau}) = \frac{1}{4\gamma^2}\left(\log\left(\frac{\eta_{t_\eta+\tau}}{\eta_{t_\eta}}\right)\right)^2.$$

Hence, we need to show that

$$\mathbb{E}_{t_\eta}^{\eta,1}\left[\sigma_{\text{CS}}^2(\eta_{t_\eta}, \eta_{t_\eta+\tau})\right] \geq \mathbb{E}_t^1\left[\sigma_{\text{CS}}^2(\lambda_t, \lambda_{t+\tau})\right],$$

which is equivalent to showing that

$$\mathbb{E}_{t_\eta}^{\eta,1}\left[\left(\log\left(\frac{\eta_{t_\eta+\tau}}{\eta_{t_\eta}}\right)\right)^2\right] \geq \mathbb{E}_t^1\left[\left(\log\left(\frac{\lambda_{t+\tau}}{\lambda_t}\right)\right)^2\right]. \tag{43}$$

If η_t and λ_t satisfy Assumption 2 and if ε and $\lambda_{t+\tau}$ are independent, then inequality (43) follows from Corollary 1.

Proof of Theorem 3. Let ξ_T^0 denote the state price density when there is no disagreement and the representative investor has belief $\mathbb{P}^0$. Specifically,

$$\xi_t^0 = e^{-\rho t}C_t^{-\gamma}H_t^{\gamma-1}.$$

The nominal price of a nominal bond when there is no disagreement and the representative investor has belief $\mathbb{P}^i$ is

$$\bar{P}_{t,T}^i = \mathbb{E}_t^i\left[\frac{\xi_T^0}{\xi_t^0}\frac{\Pi_t}{\Pi_T}\right], \qquad i = 0, 1, 2.$$

The nominal price of a nominal bond with disagreement is

$$P_{t,T} = \mathbb{E}_t^1\left[\frac{\xi_T^0}{\xi_t^0}\frac{\Pi_t}{\Pi_T}\left(f_t + (1-f_t)\left(\frac{\lambda_T}{\lambda_t}\right)^{\frac{1}{\gamma}}\right)^\gamma\right].$$

Suppose $\gamma = 1$. Then the bond price simplifies to

$$P_{t,T} = \mathbb{E}_t^1\left[\frac{\xi_T^0}{\xi_t^0}\frac{\Pi_t}{\Pi_T}\left(f_t + (1-f_t)\left(\frac{\lambda_T}{\lambda_t}\right)\right)\right] = f_t\mathbb{E}_t^1\left[\frac{\xi_T^0}{\xi_t^0}\frac{\Pi_t}{\Pi_T}\right] + (1-f_t)\mathbb{E}_t^1\left[\frac{\lambda_T}{\lambda_t}\frac{\xi_T^0}{\xi_t^0}\frac{\Pi_t}{\Pi_T}\right]$$

$$= f_t\mathbb{E}_t^1\left[\frac{\xi_T^0}{\xi_t^0}\frac{\Pi_t}{\Pi_T}\right] + (1-f_t)\mathbb{E}_t^2\left[\frac{\xi_T^0}{\xi_t^0}\frac{\Pi_t}{\Pi_T}\right] = f_t\bar{P}_{t,T}^1 + (1-f_t)\bar{P}_{t,T}^2.$$

It remains to show that

$$f_t\bar{P}_{t,T}^1 + (1-f_t)\bar{P}_{t,T}^2 = \bar{P}_{t,T}^0.$$

We have for all beliefs indexed by $i = 0, 1, 2$ that

$$\bar{P}_{t,T}^i = \mathbb{E}_t^i\left[\frac{\xi_T^0}{\xi_t^0}\frac{\Pi_t}{\Pi_T}\right] = \mathbb{C}ov_t^i\left[\frac{\xi_T^0}{\xi_t^0}, \frac{\Pi_t}{\Pi_T}\right] + \mathbb{E}_t^i\left[\frac{\xi_T^0}{\xi_t^0}\right]\mathbb{E}_t^i\left[\frac{\Pi_t}{\Pi_T}\right].$$

By Assumption 1, there is no disagreement about the marginal distribution of output and the habit and, hence,

$$B_{t,T} \equiv \mathbb{E}_t^i\left[\frac{\xi_T^0}{\xi_t^0}\right], \qquad \forall i = 0, 1, 2.$$

Hence,

$$\bar{P}_{t,T}^1 = \mathbb{C}ov_t^1\left[\frac{\xi_T^0}{\xi_t^0}, \frac{\Pi_t}{\Pi_T}\right] + B_{t,T}\mathbb{E}_t^1\left[\frac{\Pi_t}{\Pi_T}\right], \quad \text{and} \quad \bar{P}_{t,T}^2 = \mathbb{C}ov_t^2\left[\frac{\xi_T^0}{\xi_t^0}, \frac{\Pi_t}{\Pi_T}\right] + B_{t,T}\mathbb{E}_t^2\left[\frac{\Pi_t}{\Pi_T}\right].$$

Multiplying the first equation with f_t and the second equation with $(1-f_t)$ and adding them up leads to

$$P_{t,T} = f_t\bar{P}_{t,T}^1 + (1-f_t)\bar{P}_{t,T}^2 = f_t\mathbb{C}ov_t^1\left[\frac{\xi_T^0}{\xi_t^0}, \frac{\Pi_t}{\Pi_T}\right] + (1-f_t)\mathbb{C}ov_t^2\left[\frac{\xi_T^0}{\xi_t^0}, \frac{\Pi_t}{\Pi_T}\right]$$

$$+ B_{t,T}\left(f_t\mathbb{E}_t^1\left[\frac{\Pi_t}{\Pi_T}\right] + (1-f_t)\mathbb{E}_t^2\left[\frac{\Pi_t}{\Pi_T}\right]\right) = \mathbb{C}ov_t^0\left[\frac{\xi_T^0}{\xi_t^0}, \frac{\Pi_t}{\Pi_T}\right] + B_{t,T}\mathbb{E}_t^0\left[\frac{\Pi_t}{\Pi_T}\right] = \bar{P}_{t,T}^0.$$

This concludes the proof of the case $\gamma = 1$.

Consider the function $h(x) = x^{\frac{1}{\gamma}}$, which is strictly convex if $\gamma < 1$ and strictly concave if $\gamma > 1$. Suppose $\gamma > 1$. The case of $\gamma < 1$ is similar and thus omitted.

The nominal price of a nominal bond with disagreement is

$$P_{t,T} = \mathbb{E}_t^1 \left[\frac{\xi_T^0}{\xi_t^0} \frac{\Pi_t}{\Pi_T} \left(f_t + (1 - f_t) \left(\frac{\lambda_T}{\lambda_t} \right)^{\frac{1}{\gamma}} \right)^{\gamma} \right]$$

$$= \mathbb{E}_t^1 \left[\frac{\xi_T^0}{\xi_t^0} \frac{\Pi_t}{\Pi_T} \right] \mathbb{E}_t^1 \left[\frac{\frac{\xi_T^0}{\xi_t^0} \frac{\Pi_t}{\Pi_T}}{\mathbb{E}_t^1 \left[\frac{\xi_T^0}{\xi_t^0} \frac{\Pi_t}{\Pi_T} \right]} \left(f_t + (1 - f_t) h \left(\frac{\lambda_T}{\lambda_t} \right) \right)^{\gamma} \right]$$

$$= \bar{P}_{t,T}^1 \, \hat{\mathbb{E}}_t^1 \left[\left(f_t + (1 - f_t) h \left(\frac{\lambda_T}{\lambda_t} \right) \right)^{\gamma} \right],$$

where $\hat{\mathbb{E}}_t^1$ denotes the conditional mean using the bond price $\bar{P}_{t,T}^1$ as numeraire. Specifically,

$$\frac{\zeta_T^1}{\zeta_t^1} \equiv \frac{d\hat{\mathbb{P}}^1}{d\mathbb{P}^1} = \frac{\xi_T^0 \Pi_T^{-1}}{\xi_t^0 \Pi_t^{-1}} \frac{1}{\bar{P}_{t,T}^1}.$$

We have that

$$\hat{\mathbb{E}}_t^1 \left[\frac{\lambda_T}{\lambda_t} \right] = \mathbb{E}_t^1 \left[\frac{\lambda_T}{\lambda_t} \frac{\zeta_T^1}{\zeta_t^1} \right] = \mathbb{E}_t^2 \left[\frac{\zeta_T^1}{\zeta_t^1} \right] = \frac{\mathbb{E}_t^2 \left[\frac{\xi_T^0}{\xi_t^0} \frac{\Pi_t}{\Pi_T} \right]}{\bar{P}_{t,T}^1} = \frac{\bar{P}_{t,T}^2}{\bar{P}_{t,T}^1}.$$

Strict concavity of $h(\cdot)$ implies that

$$f_t h(1) + (1 - f_t) h \left(\frac{\lambda_T}{\lambda_t} \right) < h \left(f_t \cdot 1 + (1 - f_t) \cdot \frac{\lambda_T}{\lambda_t} \right).$$

Hence,

$$P_{t,T} = \bar{P}_{t,T}^1 \, \hat{\mathbb{E}}_t^1 \left[\left(f_t + (1 - f_t) h \left(\frac{\lambda_T}{\lambda_t} \right) \right)^{\gamma} \right] < \bar{P}_{t,T}^1 \, \hat{\mathbb{E}}_t^1 \left[h \left(f_t \cdot 1 + (1 - f_t) \cdot \frac{\lambda_T}{\lambda_t} \right)^{\gamma} \right]$$

$$= \bar{P}_{t,T}^1 \left(f_t + (1 - f_t) \hat{\mathbb{E}}_t^1 \left[\frac{\lambda_T}{\lambda_t} \right] \right) = \bar{P}_{t,T}^1 \left(f_t + (1 - f_t) \frac{\bar{P}_{t,T}^2}{\bar{P}_{t,T}^1} \right)$$

$$= f_t \bar{P}_{t,T}^1 + (1 - f_t) \bar{P}_{t,T}^2 = \bar{P}_{t,T}^0.$$

Proof of Proposition 3. Suppose $\gamma = 1$. We know from Theorem 1 that $B_{t,T} = \bar{B}_{t,T}^0$ if $\gamma = 1$. Similarly, we know from Theorem 3 that $P_{t,T} = \bar{P}_{t,T}^0$ if $\gamma = 1$. Hence, $\frac{P_{t,T}}{B_{t,T}} = \frac{\bar{P}_{t,T}^0}{\bar{B}_{t,T}^0}$ and thus the break-even inflation rate does not depend on disagreement.

Counterexamples: Figure 10 shows the difference between the break-even inflation rate in an economy with and without inflation disagreement as a function of risk aversion. The price level today is normalized to one. In the high inflation state, it is 1.25. In the low inflation state, it is 0.9. The second investor thinks that both inflation states are equally

likely. Suppose the first investor thinks that the probability of a high inflation state is less likely than the second investors thinks. The red area shows that the break-even inflation rate is lower with disagreement if $\gamma > 1$ and higher if $\gamma < 1$. Suppose the first investor thinks that the probability of a high inflation state is more likely than the second investors thinks. The blue area shows that the break-even inflation rate is higher with disagreement if $\gamma > 1$ and higher if $\gamma < 1$.
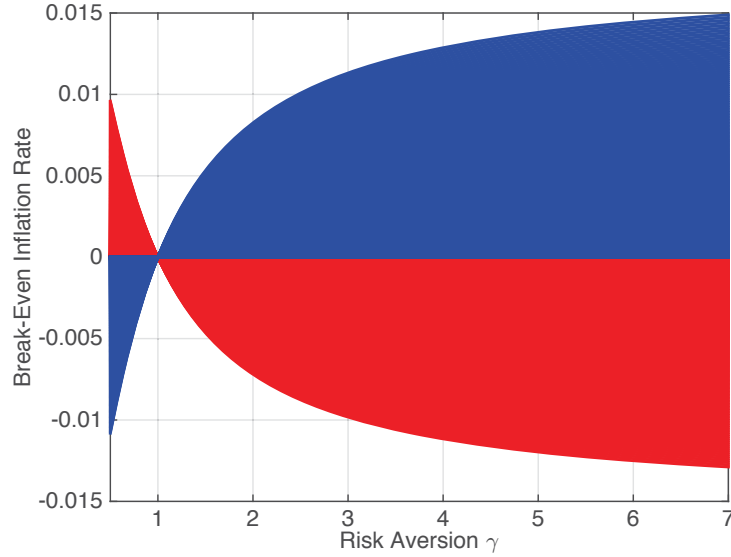

Figure 10: Break-Even Inflation Rate in Edgeworth box

This plot shows the difference between the break-even inflation rate in an economy with and without inflation disagreement as a function of risk aversion. The price level today is normalized to one and it is 1.25 in the high and 0.9 in the low inflation state tomorrow. The second investor thinks that both inflation states are equally likely.

Proof of Proposition 4. Straightforward.

Proof of Proposition 5. The disagreement measure is

$$\mathcal{D}_{t,T} = \frac{1}{2(T-t)} E^1 \left[\int_t^T \Delta_s^2 ds \right] = \frac{1}{2(T-t)} \int_t^T E^1 \left[\Delta_s^2 \right] ds$$

To evaluate the above we need $E^1 [\Delta_s^2]$. To this end, note that by Ito's lemma

$$d\Delta_t^2 = 2\beta \left(\frac{\sigma_\Delta^2}{2\beta} - \Delta_t^2 \right) dt - 2\beta \Delta_t dz_{\Pi,t}^1.$$

Using the dynamics of Δ_t^2, we have $E^1 [\Delta_s^2] = \frac{\sigma_\Delta^2}{2\beta} + e^{-2\beta} \left(\Delta_t^2 - \frac{\sigma_\Delta^2}{2\beta} \right)$. Inserting this back into

the expression for the disagreement measure and integrating yields the result.

Proof of Proposition 6. Assume γ is integer. The real bond price is $B_{t,T} = E_t^1 \left[\frac{\xi_T^1}{\xi_t^1} \right]$. From Proposition 1, we have that the SDF is

$$
\begin{aligned}
\xi_t^1 &= (y^1)^{-1} e^{-\rho t} C_t^{-\gamma} H_t^{\gamma-1} f(\lambda_t)^{-\gamma} = (y^1)^{-1} e^{-\rho t} C_t^{-\gamma} H_t^{\gamma-1} \left(1 + (y\lambda_t)^{\frac{1}{\gamma}} \right)^{\gamma} \\
&= \sum_{k=0}^{\gamma} \binom{\gamma}{k} (y^1)^{-1} e^{-\rho t} C_t^{-\gamma} H_t^{\gamma-1} (y\lambda_t)^{\frac{k}{\gamma}}.
\end{aligned}
$$

Inserting the above into the expression for the bond price we have

$$
\sum_{k=0}^{\gamma} w_t^k E_t^1 \left[\left(\frac{C_T}{C_t} \right)^{-\gamma} \left(\frac{H_T}{H_t} \right)^{\gamma-1} \left(\frac{\lambda_T}{\lambda_t} \right)^{\frac{k}{\gamma}} \right], \quad \text{where} \quad w_t^k = \binom{\gamma}{k} \frac{\lambda_t^{\frac{k}{\gamma}}}{\left(1 + \lambda_t^{\frac{1}{\gamma}} \right)^{\gamma}}.
$$

Define $\frac{\xi_T^k}{\xi_t^k} = \left(\frac{C_T}{C_t} \right)^{-\gamma} \left(\frac{H_T}{H_t} \right)^{\gamma-1} \left(\frac{\lambda_T}{\lambda_t} \right)^{\frac{k}{\gamma}}$. We can think of this as a stochastic discount factor in an artificial economy. Applying Ito's lemma we have

$$
\frac{d\xi_t^k}{\xi_t^k} = -r_t^k dt - \theta_t^k dz, \quad \text{where} \quad dz = \left(dz_{C,t}, dz_{\Pi,t}^1 \right)
$$

and

$$
\theta_t^k = \left(\gamma \sigma_C, \frac{k}{\gamma} \Delta_t \right), \quad \text{and} \quad r_t^k = \rho + \gamma \mu_C - \frac{1}{2} \gamma(\gamma+1)\sigma_C^2 - \delta(\gamma-1)\omega_t + \frac{1}{2} \frac{k}{\gamma} \left(1 - \frac{k}{\gamma} \right) \Delta_t^2.
$$

Define the state vector $Y_{1,t} = (\Delta_t, \omega)$. We have that $Y_{1,t}$ follows a multidimensional Ornstein-Uhlenbeck process. Moreover, the real short rate in the artificial economies are quadratic in the state vector and the market prices of risk are linear in the state vector. Hence, the artificial state price densities are in the class of Quadratic Gaussian Termstructure Models (QGTM) and the solution to $E_t^1 \left[\left(\frac{C_T}{C_t} \right)^{-\gamma} \left(\frac{H_T}{H_t} \right)^{\gamma-1} \left(\frac{\lambda_T}{\lambda_t} \right)^{\frac{k}{\gamma}} \right] = E^1 \left[\frac{\xi_T^k}{\xi_t^k} \right]$ is an exponential quadratic function of the state vector with time dependent coefficients that are solutions to ordinary differential equations.[31]

Proof of Proposition 7. The proof follows similar steps as in the proof of Proposition 6. In particular, the bond price can be written as

$$
\sum_{k=0}^{\gamma} w_t^k E_t^1 \left[\left(\frac{C_T}{C_t} \right)^{-\gamma} \left(\frac{H_T}{H_t} \right)^{\gamma-1} \left(\frac{\lambda_T}{\lambda_t} \right)^{\frac{k}{\gamma}} \frac{\Pi_t}{\Pi_T} \right],
$$

and we can define a set of artificial nominal stochastic discount factors

$$
\frac{\xi_{\Pi,T}^k}{\xi_{\Pi,t}^k} = \left(\frac{C_T}{C_t} \right)^{-\gamma} \left(\frac{H_T}{H_t} \right)^{\gamma-1} \left(\frac{\lambda_T}{\lambda_t} \right)^{\frac{k}{\gamma}} \frac{\Pi_t}{\Pi_T}.
$$

---

[31]We derive solutions to bond prices that belong to the class of Quadratic Gaussian Term Structure Models in the Internet Appendix.

Applying Ito's lemma we have

$$\frac{d\xi^k_{\Pi,t}}{\xi^k_{\Pi,t}} = -r^k_{\Pi,t}dt - \theta^k_{\Pi,t}dz, \quad \text{where} \quad \theta^k_{\Pi,t} = \theta^k_t + \sigma_\Pi, \quad r^k_{\Pi,t} = r^k_{\Pi,t} + x^1_t + \frac{k}{\gamma}\Delta_t - \sigma^2_\Pi.$$

Define the state vector $Y_t = (x^1_t, \Delta_t, \omega)$. We have that Y_t follows a multidimensional Ornstein-Uhlenbeck process. Moreover, the real short rate in the artificial economies are quadratic in the state vector and the market prices of risk are linear in the state vector. Hence, the artificial state price densities are in the class of Quadratic Gaussian Termstructure Models (QGTM) and thus we can solve for the bond price in closed form up to the solution of ordinary differential equations.

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