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Abstract

Realized volatilities, when observed over time, share the following stylised facts: co-movements, clustering, long-memory, dynamic volatility, skewness and heavy-tails. We propose a dynamic factor model that captures these stylised facts and that can be applied to vast panels of volatilities as it does not suffer from the curse of dimensionality. It is an enhanced version of Bai and Ng (2004) in the following respects: i) we allow for long-memory in both the idiosyncratic and the common components, ii) the common shocks are conditionally heteroskedastic, and iii) the idiosyncratic and common shocks are skewed and heavy-tailed. Estimation of the factors, the idiosyncratic components and the parameters is simple: principal components and low dimension maximum likelihood estimations. A Monte Carlo study shows the usefulness of the approach and an application to 90 daily realized volatilities, pertaining to S&P100, from January 2001 to December 2008, evinces, among others, the following findings: i) All the volatilities have long-memory, more than half in the nonstationary range, that increases during financial turmoils. ii) Tests and criteria point towards one dynamic common factor driving the co-movements. iii) The factor has larger long-memory than the assets volatilities, suggesting that long–memory is a market characteristic. iv) The volatility of the realized volatility is not constant and common to all. v) A forecasting horse race against 8 competing models shows that our model outperforms, in particular in periods of stress.

Keywords: Realized volatilities, vast dimensions, factor models, long–memory, forecasting.

JEL classification: C32, C51, G01.

Resumen

Cuando se observan a través del tiempo, las volatilidades realizadas comparten una serie de características comunes: comovimiento, comportamiento de racimo *(clustering),* memoria larga, volatilidad de la volatilidad dinámica, asimetría y colas gruesas. En este artículo proponemos un modelo dinámico factorial que captura estas características y que puede ser aplicado a paneles de volatilidad de grandes dimensiones dado que no sufre la maldición de la dimensionalidad. El modelo es una adaptación del de Bai y Ng (2004) en los siguientes aspectos: i) permitimos memoria larga en los componentes comunes e idiosincráticos, ii) las sacudidas *(shocks)* comunes son condicionalmente heterocedásticas, y iii) las sacudidas comunes e idiosincráticas son asimétricas y con colas gruesas. La estimación de los factores, los componentes idiosincráticos y los parámetros es simple: componentes principales y estimaciones de máxima verosimilitud de baja dimensión. Un profundo estudio de Monte Carlo muestra la utilidad de la estrategia de estimación y una aplicación a un panel de 90 volatilidades realizadas correspondiente a compañías pertenecientes al índice S&P100, desde enero de 2001 a diciembre de 2008, muestra, entre otros resultados, que i) todas las volatilidades tienen memoria larga, más de la mitad en el rango no estacionario, que se incrementa durante períodos de estrés; ii) contrastes y criterios indican la presencia de un factor común dinámico; iii) el factor tiene memoria más larga que las volatilidades de las compañías, lo que sugiere que la memoria larga es una característica del mercado; iv) la volatilidad de la volatilidad realizada es dinámica y común para todas las compañías; v) una comparación entre 8 modelos en términos de predicción muestra que nuestro modelo es superior, sobre todo en períodos de estrés.

Palabras clave: volatilidad realizada, gran dimensión, modelo de factores, memoria larga, predicción.

Códigos JEL: C32, C51, G01.

1 Introduction

In the recent years markets for volatility products have developed rapidly. Volatility arbitrage is an example under the real world probability measure. To arbitrage, forecasts of the (realized) volatilities are needed. I.e. the objective is to take advantage of differences between the implied volatility of an option, and a forecast of future realized volatility of the option's underlying asset. As long as the trading is done delta–neutral, buying (selling) an option is a bet that the underlying's future realized volatility will be high (low). A trader may trade on several volatilities at the same time, or even a portfolio of them. To asses the total risk exposure, the time–varying dependencies across the realized volatilities have to be understood.

Volatility derivatives, i.e. securities whose payoff depends on the realized volatility of an underlying asset (e.g. volatility swaps) or an index return (e.g. VIX options), have also developed rapidly. VIX options have been tradable since February 2006, at which date the average volume was 7,896 contracts. Since then, it has grown steadily to 665,680 contracts at the end of August 2012. Options are exposed to a number of risks. One of them is gamma risk, or the risk that the realized volatility of the underlying stock over the option's lifetime will be larger or smaller than expected, which produces hedging errors (see Figlewski and Engle (2012) for more details). Under this risk neutral scenario, forecasts of the implied volatilities can be useful for pricing and developing hedging strategies.

We develop an econometric factor model for panels of volatilities. It captures the stylized facts and allows for forecasting. Indeed, when observed through time, these panels are characterized by the following stylized facts: co–movements, clustering, long–memory, dynamic volatility of the volatility, skewness and heavy–tails. Over the last ten years several articles have presented these facts (see, among others, Andersen et al. (2001) for a study of the stylized facts of realized volatilities for the 30 DJIA firms, and Andersen et al. (2001) for the analysis of the unconditional distribution of realized volatilities for exchange rates) and a handful of univariate models have been proposed to capture some of these facts. Andersen et al. (2003) proposes ARFIMA models for capturing long–memory.[1] Corsi et al. (2008) also models realized volatilities with ARFIMA models but specifying a heteroskedastic and fat–tailed distribution for the innovation term. Corsi (2010) takes a different avenue by considering the HAR (heterogenous autoregressive) model that captures the long–memory with sums of volatility components over different horizons.[2]

Recent literature has also focused on multivariate models. Gourieroux et al. (2009), Hautsch et al. (2010), Bauer and Vorkink (2010) and Halbleib and Voev (2011) propose models

[1] To be precise, Andersen et al. (2003) consider three realized volatilities and propose a tri–variate VARFIMA model. But since this model is a direct extension of the univariate ARFIMA and it is not feasible for vast dimensions, we classify it within the univariate set of models.

[2] Fractional integration has been considered in GARCH models since two decades ago. Ding et al. (1993), Ding and Granger (1996), Kirman and Teyssiere (2002), Bollerslev and Ole (1996) and Poon and Granger (2003) propose fractional integration models and methodologies for the dynamic variance of returns. On similar grounds, Granger and Starica (2005) propose a model for absolute value of log returns.

for realized volatilities and correlations. Alternative models, not proposed yet in this context, are fractionally cointegrated vector autoregressive models (see Johansen, 2008, Johansen and Orregaard, 2010, and references therein). However, it is not clear that they can be applied to vast dimensions and, at the same time, can account for the stylized facts observed in realized measures. Barigozzi et al. (2010) propose a parsimonious seminonparametric model for panels of realized volatilities that does not suffer from the curse of dimensionality, but the presence of a nonparametric curve makes forecasting challenging. To date, a model for panels of volatilities that is feasible for vast dimensions, captures the stylized facts, and is capable of forecasting reasonably well is missing.

In this article we introduce such a model. It has its roots in the macroeconometrics literature (Forni et al. (2000), Stock and Watson (2002), Bai (2003)) and it builds upon Bai and Ng (2004). We propose a dynamic factor model for a panel of N log–realized volatilities that depend on a small number of factors ($r \ll N$) and a vector of idiosyncratic components. The factors follow a r–dimensional VARFIMA model with conditional heteroskedasticity, and the idiosyncratics follow N independent ARFIMAs. The distributions of the common and idiosyncratic shocks are skewed and heavy–tailed. Estimation is based on principal components and maximum likelihood (ML). The dimensions of the ML problems are univariate for the idiosyncratics and r–variate for the factors, where r is typically a very small number (one in our case). As outlined above, this model has a number of advantages: i) it is able to mimic and explain the stylized facts of panels of realized volatilities, ii) inference is straightforward, iii) forecasting is simple and, when compared with the existing models, predictions are accurate, and iv) it is suitable for vast dimensional panels.

We apply the model to a panel of 90 daily realized kernels (Barndorff-Nielsen et al., 2008) spanning from Jan. 2001 to Dec. 2008. It consists of the constituents of the S&P100 index that have continuously been trading during the sample period. This data set has been used by Barigozzi et al. (2010) and the time span is convenient as it covers periods with different patterns in volatility: rough (2001-2003), calm (2004-2007), and very rough (2007-2008).

Four are the main estimation findings. First, two heuristic methods, one criteria and a test unanimously indicate that one factor drives the panel. This is robust with Barigozzi et al. (2010) and a consequence of the strong co–movements of the realized measures. Second, the long–memory found in the realized volatilities is a market feature, in the sense that the degree of fractional integration of the factor is significantly larger than that of the idiosyncratic components. Third, the volatility of the volatility is time varying (confirming Corsi et al. (2008)) and has a pattern that resembles that of realized volatilities. That is, peaks and trough somehow coincide. Fourth, the standardized common shocks present fatter tails than the idiosyncratic shocks, suggesting the market nature of the heavy–tails.

By combining long–memory with factor models, forecasts should be better than those of short–memory and/or univariate models. To verify this hypothesis we proceed with a thorough forecasting horse race. We compare 9 models, for 4 forecasting horizons (1 day, 1 week, 2 weeks and 1 month) and for the 90 firms. The results show that when markets are calm and volatilities are low, the factor structure does not play a significant role and the improvements in forecasting compared with simpler models are marginal. However, as volatilities increase, the co–movements are reinforced and the factor structure becomes, important in the sense that forecasting gains –relative to univariate models– are up to 10%.

The structure of the paper is as follows. Section 2 analyzes the panel of realized volatilities and unveils the six stylized facts. Section 3 shows the dynamic factor model, the assumptions and estimation. It also presents a Monte Carlo study carried out to asses the finite sample accuracy of the estimated parameters. Results are shown in Section 4, which are divided in two parts: estimation results and the forecasting horse race. The article concludes with Section 5 and an appendix with the assumptions and two lengthy tables. A web appendix, available in the author's websites, contains detailed results of the forecasting horse race.

2 The stylized facts of panels of realized volatilities

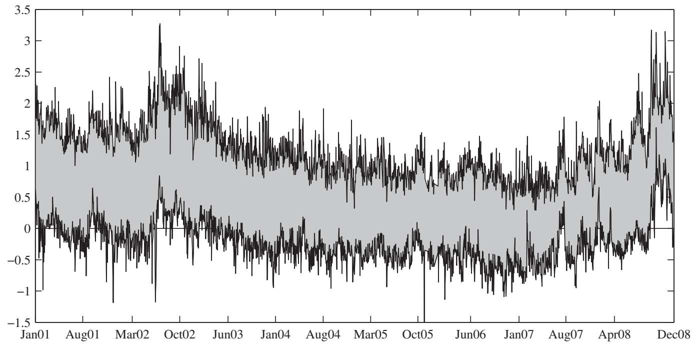
Our data is a comprehensive panel of log realized volatility measures from January 2, 2001 to December 31, 2008 as used in Barigozzi et al. (2010). It consists of U.S. equity companies that are part of the S&P100 index. It contains all the constituents of the index as of December 2008 that have been trading in the full sample period (90 in total). The assets are classified in the following sectors (with the abbreviation in parenthesis): consumer discretionary (CD), consumer staples (CS), financial (FIN), health care (HC), industrial (IND), information technology (IT), materials (MAT), energy (NRG), telecommunications service (TLC), and utilities (UT). The complete list of tickers, company names and sectors is reported in the Appendix.

Among the available estimators of the daily integrated volatility based on intraday returns, we adopt the realized kernels (Barndorff-Nielsen et al., 2008) and we follow the guidelines in Barndorff-Nielsen et al. (2009). Realized kernels are a family of heteroskedastic and autocorrelation consistent type estimators, robust to various forms of market microstructure noise present in high frequency data. Alternative estimators are the range (Parkinson, 1980; Alizadeh et al., 2002), the vanilla realized volatility (Andersen et al., 2003), and the two-scales estimator (Aït-Sahalia et al., 2005). Our primary source of data are tick–by–tick intra-daily quotes from the TAQ database. Since realized kernels are not robust to jumps, data are filtered using the methods described in Brownlees and Gallo (2006). In particular, we use a trimming method consisting on removing observations that are 3 sigma larger than the neighboring realizations (see page 2237 of Brownlees and Gallo (2006) for more details). For clarity in the exposition, from now on we denote the log of the realized kernel as simply realized volatility.

Figure 1 shows the time series envelop for the 90 realized volatilities.[3] Visual inspection

[3]An envelop plot enfolds, at any t, the realized volatilities with the maximum and minimum.

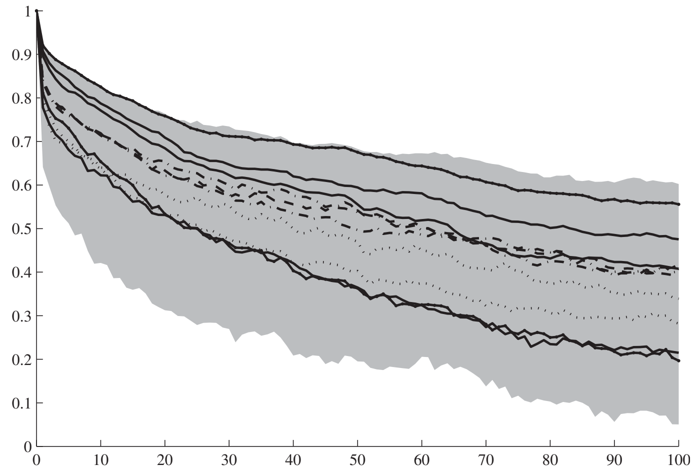
Figure 1: Realized Volatilities



Time series envelop plot of the 90 log realized volatilities of the U.S. equities that are part of the S&P100 index.

reveals the two first stylized facts: co–movements and clustering. The overall pattern suggests that they cluster around a common time–varying unobserved level that can be easily attached to well known economic events or common wide innovations. The downturn in volatility in 2001 corresponds to the aftermath of the burst of the dot com bubble while the rise around 2002 and 2003 corresponds to the US accounting scandals. Volatility then drops from 2004 to July 2007 when it starts to rise with the beginning of the financial crisis. It then skyrockets to the highest level of volatility in the last 20 years in the fall of 2008.

Figure 2: Autocorrelograms



Envelop plot of the autocorrelograms of the 90 realized volatilities of the U.S. equities that are part of the S&P100 index. Black lines are the autocorrelograms for 10 representative stocks pertaining to the 10 sectors.

Figure 2 displays the envelop of the autocorrelograms, up to lag 100, for the 90 assets. It also highlights the autocorrelations for 10 representative stocks pertaining to the above–mentioned sectors.[4] Although the decline of the autocorrelations is heterogeneous across assets

[4]Bank of America (FIN), Caterpillar (IND), DuPont (MAT), Exelon (UT), Home Depot (CD), Microsoft (IT), Pfizer (HC), Verizon (TLC), Wall-Mart (CS) and Exxon (NRG).

and sectors, they all show a decay that is slower than exponential. This is a distinctive feature of long–memory processes that can be caused by fractional integration. Structural breaks and jumps may produce long–memory as well (see e.g. Diebold and Inoue 2000, and Granger and Hyung 2004). However, understanding long–memory as fractional integration is appropriate as it explicitly models the persistent relation with the past, a convenient feature for forecasting. Fractional integration is characterized by a parameter, often denoted by d, that measures the degree of differentiation needed for rendering the realized volatilities short–memory.[5] Denote generically by X_{it} the realized volatility of the i–th asset at time t. Then X_{it} is said to be fractionally integrated of order d_i if $(1-L)^{d_i}X_{it}$ is $I(0)$. The values of d_i that we consider range between 0 and 1 and it is divided in two intervals: $0 < d_i < 0.5$ and $0.5 \leq d_i < 1$.[6] The former entails finite variance and mean reversion, while the latter means that X_{it} is not variance–stationary but mean reverting.

Estimates for the fractional integration parameters for each realized volatility are shown in the top panel of Table 1. We use three methods: an ARFIMA$(1, d, 0)$ where d is estimated following Beran (1995), the exact local Whittle of Shimotsu and Phillips (2005), and the Geweke & Porter–Hudak estimator of Geweke and Porter-Hudak (1983).[7] We display the minimum, the median and the maximum. Results for Beran (1995) and Shimotsu and Phillips (2005) are in some sense similar while the estimates using Geweke and Porter-Hudak (1983) are quite different, in particular its dispersion. It is known that the Geweke & Porter–Hudak estimators may have a severe bias. For all assets (column All), the distance between the minimum and maximum estimated d is of merely 0.19 for Beran (1995) and 0.22 for Shimotsu and Phillips (2005), indicating that the degree of long–memory is similar across assets and that it can be a market feature. Moreover, for Beran's estimators more than half of the realized volatilities have fractional integration beyond the threshold 0.5 (the median being 0.55) while all the exact local Whittle estimators are above 0.5, which implies lack of variance stationarity. Across sectors we observe the same degree of homogeneity.

We investigate further the long–memory aspects of volatilities by dividing the sample in subsamples of 2 years. Figure 3 show boxplots for the the fractional integration parameters (for the 90 firms) using Beran (1995). The degree of long–memory presents variations. Periods of turmoil are related not only with increases in the memory but also with an increase in the homogeneity across assets (i.e. narrower distance between the interquantile ranges). This is

[5] See Beran (1998), Robinson (2003) and Palma (2007) for survey textbooks on long–memory processes and time series models with long–memory.

[6] Values of d below 0 (anti–persistent) and beyond 1 (explosive) are theoretically admissible but not relevant in the context of realized volatilities.

[7] Other specifications for the conditional mean were estimated as well: ARFIMA$(1, d, 1)$, ARFIMA$(2, d, 0)$ and ARFIMA$(2, d, 1)$. The ARFIMA$(1, d, 0)$ turns out to be the most parsimonious and the residuals are white noises. Detailed results are available under request.

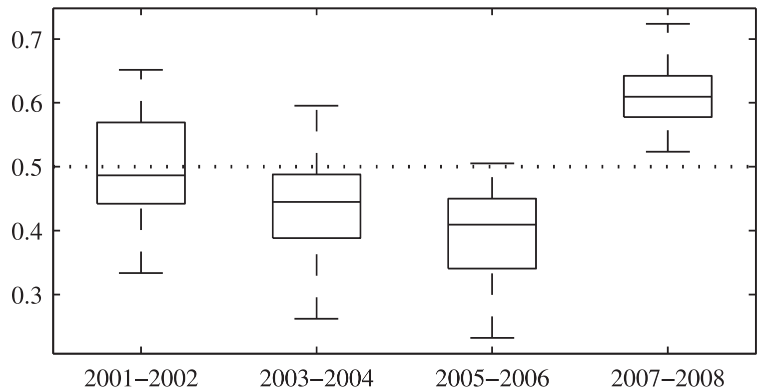
Table 1: Descriptive statistics

		All	CD	CS	NRG	FIN	HC	IND	IT	MAT	TLC	UT
		ARFIMA$(1, d, 0)$										
	min.	0.45	0.5	0.45	0.46	0.54	0.45	0.48	0.49	0.52	0.52	0.45
d	med.	0.55	0.54	0.5	0.53	0.58	0.51	0.56	0.56	0.55	0.55	0.54
	max.	0.64	0.57	0.55	0.57	0.62	0.57	0.64	0.62	0.57	0.56	0.56
		Exact Local Whittle										
	min.	0.56	0.64	0.56	0.62	0.67	0.63	0.60	0.60	0.61	0.67	0.67
d	med.	0.69	0.70	0.66	0.70	0.73	0.67	0.66	0.71	0.65	0.70	0.69
	max.	0.78	0.75	0.71	0.77	0.76	0.71	0.72	0.78	0.66	0.72	0.70
		Geweke & Porter–Hudak										
	min.	0.35	0.62	0.56	0.35	0.57	0.56	0.55	0.51	0.64	0.58	0.58
d	med.	0.67	0.71	0.69	0.67	0.64	0.64	0.71	0.65	0.67	0.66	0.65
	max.	0.85	0.78	0.77	0.75	0.76	0.71	0.85	0.69	0.69	0.77	0.73
		Autocorrelation demeaned squares										
	min.	0.44	0.51	0.44	0.60	0.70	0.45	0.51	0.53	0.59	0.57	0.59
Lag 1	med.	0.65	0.64	0.53	0.71	0.77	0.54	0.65	0.66	0.69	0.64	0.69
	max.	0.84	0.70	0.63	0.76	0.84	0.67	0.73	0.72	0.76	0.66	0.70
	min.	0.08	0.21	0.08	0.23	0.26	0.17	0.19	0.20	0.18	0.20	0.22
Lag 20	med.	0.28	0.28	0.22	0.37	0.37	0.24	0.29	0.29	0.32	0.32	0.26
	max.	0.56	0.42	0.28	0.52	0.56	0.32	0.42	0.36	0.43	0.36	0.33
		Autocorrelation demeaned absolute values										
	min.	0.44	0.54	0.44	0.58	0.68	0.48	0.50	0.54	0.57	0.58	0.60
Lag 1	med.	0.62	0.62	0.54	0.62	0.72	0.55	0.62	0.67	0.62	0.59	0.62
	max.	0.78	0.71	0.62	0.74	0.78	0.64	0.71	0.72	0.66	0.66	0.65
	min.	0.14	0.23	0.14	0.27	0.31	0.21	0.22	0.24	0.20	0.22	0.27
Lag 20	med.	0.30	0.30	0.24	0.33	0.38	0.27	0.28	0.34	0.30	0.29	0.29
	max.	0.53	0.44	0.32	0.51	0.53	0.32	0.43	0.41	0.33	0.37	0.30
		Skewness										
	min.	-0.15	0.04	-0.11	-0.07	-0.04	0.02	-0.15	-0.06	-0.14	-0.01	0.00
	med.	0.07	0.07	0.01	0.12	0.10	0.11	0.05	0.05	0.06	0.16	0.03
	max.	0.3	0.30	0.21	0.23	0.28	0.19	0.19	0.27	0.17	0.17	0.14
		Kurtosis										
	min.	3.27	3.42	3.68	3.41	3.45	3.7	3.27	3.59	3.37	3.54	3.61
	med.	4.03	4.12	4.49	4.45	4.00	4.22	3.9	3.97	3.75	3.84	3.92
	max.	5.91	4.83	5.91	4.70	4.91	5.12	4.92	4.89	4.02	4.16	4.55

Summary of descriptive statistics for the 90 realized volatilities for all the firms (All) and grouped in sectors: consumer discretionary (CD), consumer stable (CS), financial (FIN), health care (HC), industrial (IND), materials (MAT), energy (NRG), technology (TEC), and utilities (UT). For all panels the Table shows the minimum (min.), the median (med.) and the maximum (max.). Top panel shows the estimated fractional integration parameters using an ARFIMA$(1, d, 0)$ where d is estimated following Beran (1995), the exact local Whittle of Shimotsu and Phillips (2005), and the Geweke & Porter–Hudak estimator of Geweke and Porter-Hudak (1983). Second and third panels show the autocorrelation of lags 1 and 20 for the squares and absolute values of the demeaned realized volatilities. The last two panels show the skewness and kurtosis.

clear in the period 2001–2002 and, particularly, in the last financial crisis. Previous literature, see for instance Andersen et al. (2001), Andersen et al. (2001), Andersen et al. (2003), Granger et al. (2000) and Lieberman and Phillips (2008), has found the realized volatilities typically display an estimated d around 0.4. The sub–sample analysis confirms this value in calm periods but it goes beyond it in periods of turmoil. In all, the bottom line of this analysis is the third stylized fact of realized volatilities: there have long–memory and they are not necessarily variance–stationary but mean reverting.

Figure 3: d ACROSS 2–YEARS SUBSAMPLES



Boxplots of the estimates of the 90 fractional integration parameters (using Beran (1995)) for the sample divided in 2–years subsamples.

Second and third panels of Table 1 show the autocorrelations of order 1 and 20 (which correspond to one day and one month) for two measures of the volatility of realized volatility: the squares and absolute values of the demeaned realized volatilities. The dependence is remarkable and results for both measures are similar. The median of the autocorrelations of order 1 are 0.65 and 0.62 for the squares and absolute values respectively. After one month the dependence is still high: the medians are 0.28 and 0.30 for the squares and absolute values respectively. This is a clear indication of conditional dynamics in the realized volatilities. Moreover, the range of values across assets is tight suggesting that, as it happened with long–memory, the dynamic volatility is a market feature. Interestingly enough, there are not significant differences across sectors, which is seen as another corroboration of the market nature of the volatility of the volatility. This analysis allows us to conclude with the fourth stylized fact: realized volatilities have dynamic volatility and it is a market feature.

The last two panels of Table 1 display the skewness and the standardized kurtosis in a similar fashion to the previous descriptive statistics. Skewness is present in realized volatilities, in the sense that it is not concentrated around zero since it ranges from -0.15 to 0.30, and that there are substantial variations across sectors (and assets). On the other hand, the median of standardized kurtosis is 4.03 while the minimum and maximum are 3.27 and 5.91. Similar numbers are found for the different sectors. We conclude that the fifth and sixth stylized facts are that realized volatilities are skewed with an asymmetric pattern that is heterogeneous across firms, and that all the realized volatilities of all the assets show heavy–tails.

In sum, realized volatilities show co–movements, clustering, long–memory, conditional heteroskedasticity, skewness and heavy–tails. In the next section we propose a model that accounts for these facts. What if we ignore them? Ignoring co–movements (and therefore its factor structure) implies the use of traditional models (e.g. VARFIMA type) that may become hard to handle in large dimensions due to the curse of dimensionality. Not paying attention to clustering means that dynamics are not adequately captured and all subsequent analysis is doubtful. Neglecting long–memory entails a deterioration in the forecasts after few steps ahead, specially in periods of turmoil. Last, ignoring conditional heteroskedasticity, skewness and heavy tails, may lead to incorrect risk management and mispricing of volatility products.

3 A model for large panels of volatilities

3.1 The model

Factor models are based on the idea that each asset's volatility X_{it} can be decomposed into the sum of two mutually orthogonal components: the common component capturing the co–movement among volatilities, and a vector of idiosyncratic components $\boldsymbol{\xi}_t = (\xi_{1t}, \ldots, \xi_{Nt})$ capturing the asset's specific dynamics. Co–movements are summarized by $r \ll N$ common factors $\mathbf{F}_t$ that are loaded differently to each volatility through the matrix $\boldsymbol{\Lambda}$. Formally, let $\mathbf{X}_t$ $t = 1, \ldots, T$ be a $N \times 1$ vector of realized volatilities, the Dynamic Factor Model (DFM) is defined as:

$$\begin{aligned} \mathbf{X}_t =& \boldsymbol{\Lambda}\mathbf{F}_t + \boldsymbol{\xi}_t & (1) \\ \mathbf{D}(L)\mathbf{F}_t =& \mathbf{C}(L)\mathbf{H}_t^{1/2}\mathbf{u}_t \quad \mathbf{u}_t \sim \mathcal{D}(\mathbf{0}, \mathbf{1}, \gamma_u, \nu_u) & (2) \\ (1-L)^{\delta_i}\,\xi_{it} =& G_i(L)\epsilon_{it} \quad \epsilon_{it} \sim \mathcal{D}(0, \sigma_{\epsilon_i}, \gamma_{\epsilon_i}, \nu_{\epsilon_i}). & (3) \end{aligned}$$

The common factors evolve over time according to the VARFIMA model (2) with conditional heteroskedasticicty. The matrix $\mathbf{D}(L) = diag((1-L)^{d_1}, \ldots, (1-L)^{d_r})$ contains the polynomials of fractional integration, $\mathbf{C}(L) = \sum_{j=0}^{\infty} \mathbf{C}_j L^j$ is the pure MA representation of the VARMA model for the fractionally integrated factors $\mathbf{D}(L)\mathbf{F}_t$, $\mathbf{H}_t^{1/2}$ captures the conditional variance–covariance of the realized volatilities, and $\mathbf{u}_t$ are the orthogonal common shocks that follow a standardized skewed and heavy–tailed distribution, characterized by γ_u, a parameter that controls the asymmetry, and a tail index ν_u that generates the fatness of the tails.

The idiosyncratic components are modeled as N independent ARFIMA processes, as in (3). For the i–th volatility, δ_i is the degree of fractional integration of the idiosyncratic component, $G_i(L) = \sum_{j=0}^{\infty} G_{ij} L^j$ is an infinite MA polynomial, and ϵ_{it} is the i–th idiosyncratic shock distributed according to a zero–location distribution with dispersion, skewness and tail parameters σ_{ϵ_i}, γ_{ϵ_i} and ν_{ϵ_i} respectively. The common shocks and the idiosyncratic shocks are assumed to be uncorrelated at all leads and lags, while the idiosyncratic shocks are allowed to be both serially and cross-sectionally correlated albeit by a limited amount (precisely defined in assumption IC –see Appendix).

Model (1)–(3) is a generalization to fractional integration, conditional heteroskedasticity and heavy tails of Bai and Ng (2004) (in which $\mathbf{X}_{it}$ are Gaussian, homocedastic, and either stationary or have a unit root).[8] As the model of Bai and Ng (2004) relates to cointegration, our model is related to fractional cointegration. In Bai and Ng (2004) the r factors and the idiosyncratic components can be stationary or have stochastic trends. If the idiosyncratic components are all stationary, then the observations and the factors are cointegrated. In our model, the intuition is similar: if the volatilities and the factors are fractionally cointegrated, the idiosyncratic components have a lower degree of integration. Due to the empirical nature of this article, the theoretical underpinnings on fractional cointegration related with (1)–(3) constitute a future area of research.

3.2 Estimation

Estimation is divided in two steps. The first consists in estimating the factors, along with the loadings and the idiosyncratic components, while the second in estimating the dynamic models and the distributions in (2) and (3).

The factor, its loadings and the idiosyncratic components Suppose r, the number of factors, is known. Let $\mathbf{x}_t = \Delta\mathbf{X}_t$, $\mathbf{f}_t = \Delta\mathbf{F}_t$, and $\mathbf{z}_t = \Delta\boldsymbol{\xi}_t$. Taking first difference of (1), the differenced realized volatilities follow the model $\mathbf{x}_t = \boldsymbol{\Lambda}\mathbf{f}_t + \mathbf{z}_t$. Bai and Ng (2002) prove that the space spanned by the differenced factor $\mathbf{f}_t$ can be consistently estimated by principal components, and Bai (2003) shows that the maximum distance between the estimated differenced factor and loadings from the true ones is bounded, up to a scale.[9] Let $\hat{\mathbf{f}}_t$, $\hat{\boldsymbol{\Lambda}}$ and $\hat{\mathbf{z}}_t$ be the estimates obtained by principal components. Based on the set of assumptions listed on the Appendix, consistent estimators of $\mathbf{F}_t$ and $\boldsymbol{\xi}_t$ can be obtained (up to a rotation and an initial condition $\mathbf{F}_0$) by undoing the differentiation, i.e. cumulating: $\hat{\mathbf{F}}_t = \sum_{t=1}^{T}\hat{\mathbf{f}}_t$, and $\hat{\boldsymbol{\xi}}_t = \sum_{t=1}^{T}\hat{\mathbf{z}}_t$. A proof is not needed since none of the assumptions in the Appendix violate those in Bai and Ng (2004).

In practice, however, the number of factors is not known and it needs to be estimated. The literature has suggested different heuristic methods, criteria and tests to determine the number of common factors. We adopt four. The first is the percentage of variance explained by the i–th eigenvalue (in decreasing order) of the spectral density matrix of $\mathbf{x}_t$. We denote this method by μ_i^1. The second is the percentage of variance explained by the i–th eigenvalue (in decreasing order) of the variance–covariance matrix of $\mathbf{x}_t$. We denote this method by μ_i^2. The third is Bai and Ng (2002) information criteria, which we denote by IC. The last is

[8] Model (1)–(3) when $d_j = 1$, $j = 1 \dots, r$, $\delta_i = 1$, $i = 1 \dots, N$, and both the common and idiosyncratic shocks are normally distributed is studied in Alessi et al. (2009).

[9] Differentiating fractionally integrated random variables is a conservative approach, in the sense that $\mathbf{x}_t$ is over–differentiated. This is carried over in the idiosyncratic components $\mathbf{z}_t$. However, since they are stationary and weakly correlated, principal components can be used.

Onatski (2009) test, denoted by *Onat*, where the null hypothesis of $r-1$ common factors is tested against the alternative of r common factors. Though this test is developed for a more general model compared to (1)–(3), it is useful to consider it as a robustness check.[10] As shown in the empirical application bellow, the four methods unanimously indicate that there is one common factor: $r = 1$. Having one factor has a number of useful consequences. In general, for $r > 1$ factors, the common and idiosyncratic components are identified, but the factors and the loadings are not.[11] Hence we can only estimate consistently the space spanned by the factors, but not the factors themselves. By contrast, when $r = 1$ the lack of identification is alleviated since $\mathbf{R}$ becomes a scalar, denoted by R, and therefore factors are identified up to the sign. For the ease of exposition, from now on we consider $r = 1$.

The models The last step is the estimation of models (2) and (3) that were written in terms of the MA representation but in practice are ARFIMA–GARCH and ARFIMA for F_t and ξ_{it} respectively. Model (2) can be re–written as

$$\begin{aligned} \phi(L)(1-L)^d F_t &= \theta(L)\tilde{u}_t \\ \tilde{u}_t &= h_t^{1/2} u_t \\ h_t &= \omega + \alpha \tilde{u}_{t-1}^2 + \beta h_{t-1} \end{aligned}$$

where $\tilde{u}_t$ is the common residual, and u_t is the common shock that follows the standardized skewed–t distribution of Hansen (1994).[12] Since the dimensions of the ML problems are univariate, this methodology does not suffer from the curse of dimensionality. However, estimation of the parameters –in particular the fractional integration parameters– turns out to be computationally cumbersome. We adopt a pragmatic and parsimonious approach by proceeding in two steps. Denote by $\boldsymbol{\rho}_1$ the set of parameters in $\phi(L)$ and $\theta(L)$, and $\boldsymbol{\rho}_2 = (\omega, \alpha, \beta, \gamma_u, \nu_u)$. In the first step, and under quasi ML (QML) arguments, we maximize a Gaussian log–likelihood with respect to $\boldsymbol{\rho}_1$ and d. To estimate the parameter of fractional integration we rely on Beran (1995): estimation of d is based on a grid search algorithm while $\boldsymbol{\rho}_1$ are estimated by maximizing the (profiled) Gaussian log–likelihood. Second, the skewed–t log–likelihood of the residuals is maximized with respect to $\boldsymbol{\rho}_2$. The same procedure is followed for the estimation of model (3) but with $\boldsymbol{\rho}_{1i}$ the set of parameters in $\phi_i(L)$, $\theta_i(L)$ and δ_i, and $\boldsymbol{\rho}_{2i} = (\sigma_{\epsilon_i}, \gamma_{\epsilon_i}, \nu_{\epsilon_i})$.

The asymptotic theory of ARFIMA-GARCH models has been developed for Gaussian innovations by Ling and Li (1997). The extension to QML for non–Gaussian processes can,

[10] Other methods for determining the number of common factors are Bai and Ng (2007), Hallin and Liska (2007), Amengual and Watson (2007), Alessi et al. (2010), Kapetanios (2010), and Onatski (2010).

[11] This can be very easily seen by considering an orthonormal matrix $\mathbf{R}$ that rotates $\mathbf{F}_t$ and prevents its identification.

[12] There are alternative skewed and heavy–tailed laws. The tempered α–stable is appealing due to its theoretical properties. The normal mean–variance mixture class of distributions is also suitable for skewed and heavy–tailed random variables. Fernandez and Steel (1998) propose a general skewing mechanism that can be used for any unimodal symmetric distribution.

in principle, be generalized since the assumptions in Ling and Li (1997) also hold in our case. This is beyond the scope of this article and is left for future research. However the simulation results, shown below, confirm the good finite sample properties of the estimators. Moreover, regressors and regressands in (2) and (3) are estimates rather than the true values. This entails an estimation error that is carried over in the estimation of the parameters. Though Bai and Ng (2004) show that the estimation error vanishes as $N, T \to \infty$, and hence factors can be treated as if they are directly observed, a natural question is whether this error is meaningful in finite samples and if it affects significantly the accuracy of the estimated parameters. We address this problem in the following Monte Carlo study.

3.3 The finite sample performance of the estimated parameters

We proceed with a comprehensive Monte Carlo study with 36 different Data Generating Processes (DGP). For each DGP N equals 100 (roughly the same sample size as the panel of volatilities) and we simulate 1000 draws of sample sizes 500, 1000, and 5000. The model we simulate is a one factor model with the factor following an ARFIMA$(1, d, 0)$–GARCH$(1, 1)$, and the idiosyncratic components an ARFIMA $(1, d, 0)$:

$$\begin{aligned} \mathbf{X}_t &= \mathbf{\Lambda} F_t + \boldsymbol{\xi}_t \\ (1-\phi)(1-L)^d F_t &= h_t^{1/2} u_t \\ (1-\rho_i)(1-L)^{\delta_i} \xi_{it} &= \epsilon_{it} \end{aligned}$$

The 100 factor loadings are independent copies of a $\mathcal{N}(1, 1)$. The common shocks u_t are independent copies of a standardized skewed–t distribution with asymmetry parameter 0.1 and tail index 8. The law is denoted by $SkSt(0, 1, 0.1, 8)$. The GARCH parameters are fixed to $\omega = 0.002$, $\alpha = 0.05$ and $\beta = 0.92$. The autoregressive and fractional integration parameters of the factor are set to $[0, 0.5, 0.9]$ and $[0.2, 0.4, 0.6]$ respectively. These values produce processes ranging from almost no persistence ($\phi = 0$ and $d = 0.2$) to a great deal of persistence ($\phi = 0.9$ and $d = 0.6$).

The i–th idiosyncratic shock follows the distribution $SkSt(0, 1, \gamma_{\epsilon_i}, \nu_{\epsilon_i})$ where γ_{ϵ_i} and ν_{ϵ_i} are drawn from the uniform distributions $U(-0.15, 0.15)$ and $U(8, 50)$ respectively. We therefore allow for idiosyncratic shocks to be left or right skewed, and the tail indexes range from 8 to 50, producing an ample array of shocks, from heavy–tailed to Gaussian. The 100 autoregressive parameters are either set to zero or also drawn from the uniform distribution $U(0.5, 0.9)$. The parameters of fractional integration are set to two values corresponding to low ($\delta \sim U(0, 0.4)$) and high persistence ($\delta \sim U(0.4, 0.8)$). Table 2 gives an overview of the DGPs and left panel of Table 10 in the Appendix indexes them. It is noteworthy that the factor and the idiosyncratic components are generated so that the percentage of variance of each variable explained by the common component is between 25% and 95%.

Table 2: Monte Carlo design

Draws	1000
N	100
T	$500, 1000, 5000$
$\mathbf{\Lambda}$	$\sim \mathcal{N}(1, 1)$
u_t	$\sim SkSt(0, 1, 0.1, 8)$
ϕ	$0, 0.5, 0.9$
d	$0.2, 0.4, 0.6$
ω	0.002
α	0.05
β	0.92
ϵ_{it}	$\sim SkSt(0, 1, \gamma_{\epsilon_i}, \nu_{\epsilon_i})$
γ_{ϵ_i}	$\sim U(-0.15, 0.15)$
ν_{ϵ_i}	$\sim U(8, 50)$
ρ_i	$0, \sim U(0.5, 0.9)$
δ_i	$\sim U(0, 0.4), \sim U(0.4, 0.8)$

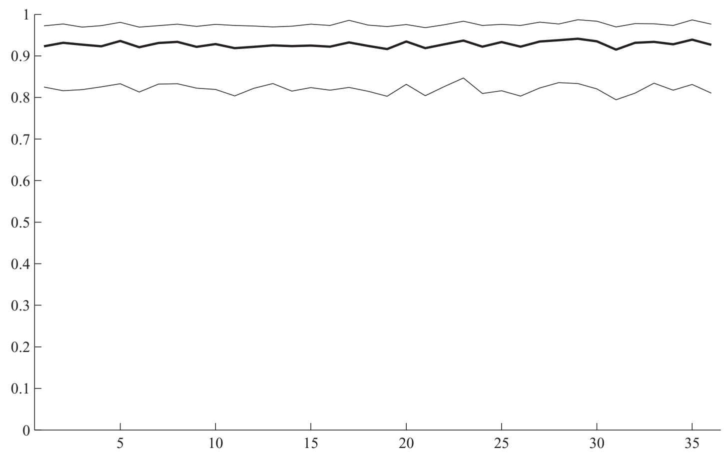
Specification of the Monte Carlo design. The top panel are the specifications for the number of draws, observations, dimension of the panel, and the factor loadings. Middle panel specify the distribution of the common shock and the parameters of the factor model. Bottom panel specify the distribution of the idiosyncratic shocks and the parameters of the model for the idiosyncratic component.

To estimate from the simulated data we differentiate the model, extract one factor and the idiosyncratic components with principal components, and estimate the parameters. We repeat this procedure 1000 times for each parameter configuration. To study the precision of the estimated factor we use the R^2 from the regression $\hat{F}_t = a + bF_t + \varepsilon_t$, while for the precision of the estimated parameters we compute the bias and the mean square error (MSE).

Figure 4 shows, for $T = 5000$, in the y–axis the median (thick line), the 25th and 75th quantiles (thin lines) over the 1000 draws of the R^2 from regressing the estimated factor on the true one. The x–axis shows the 36 DGP's. The estimation of the factor is very precise all the DGP's. Results for $T = 1000$ and $T = 500$ (available on request) are similar since for principal components the cross–sectional dimension is more important than the sample size. We therefore conclude that the estimation procedure of Bai and Ng (2004) is applicable when the variables are fractionally integrated.

Figure 5 shows the median bias and mean squared error (MSE) for the estimated parameters of the ARFIMA-GARCH model of the factor. In all graphs the solid line is the median bias or MSE over the 1000 replications for each DGP and for $T = 5000$. The dashed and dotted line is for $T = 1000$, while the dotted line is for $T = 500$. Overall the bias is small and it decreases as the number of observations increases. Detailed results on the median biases can be found in the right panel of Table 10 in the Appendix. Last, we verify the finite sample densities of the estimators. Figure 6 displays the kernel densities of the 1000 estimated parameters for the DGP 25 (see table 10 in the Appendix) which is a typical case found in

Figure 4: Accuracy of the estimated factor



In the x–axis the 36 DGPs, indexed in the left panel of Table 10 in the Appendix. On the y–axis the median (thick line), the 25th and 75th quantiles (thin lines) over the 1000 draws of the R^2 from regressing the estimated factor on the true one.

the empirical application, and for the three sample sizes. The densities approach symmetric bell shapes with smaller variance as the sample size increases. For $T = 500$ and $T = 1000$ the densities of $\hat{\omega}$ and $\hat{\beta}$ are highly skewed due to the presence of a few pathological estimates that disappear for $T = 5000$.

4 An application to S&P100 constituents

4.1 Estimation results

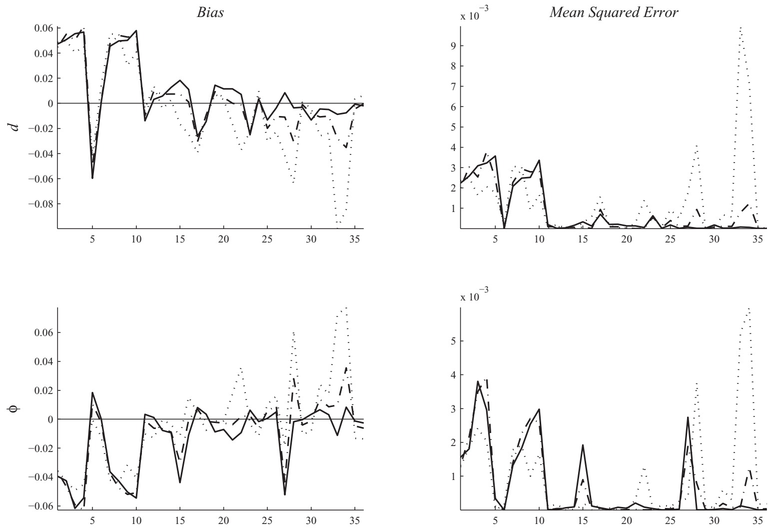
We apply the model to the panel of daily realized volatilities of the 90 S&P100 constituents. The precise characteristics of this database have been explained in Section 2. The presentation of the results proceed as follows: i) determining the number of factors, ii) estimating the factors, loadings and idiosyncratic components, and iii) estimating the dynamic models (2) and (3).[13]

Table 3 shows the results of the two heuristic methods, the test and the criteria for determining the number of factors. The first column contains the number of factors. The second and third columns show the percentage of variance explained by the i–th eigenvalue (in decreasing order) of the spectral density and variance–covariance matrices of $\mathbf{x}_t$ respectively. The fourth column shows the p–values of the Onatski (2009) statistic for the null of $r - 1$ common factors against the alternative of r common factors. The last column displays the Bai and Ng (2002) criteria. The evidence for one factor is strong: the percentage of variance explained by the first eigenvalue of the spectral density and the variance–covariance matrices overwhelming dominates, and the p–values of the test and the criteria clearly indicate towards one factor.

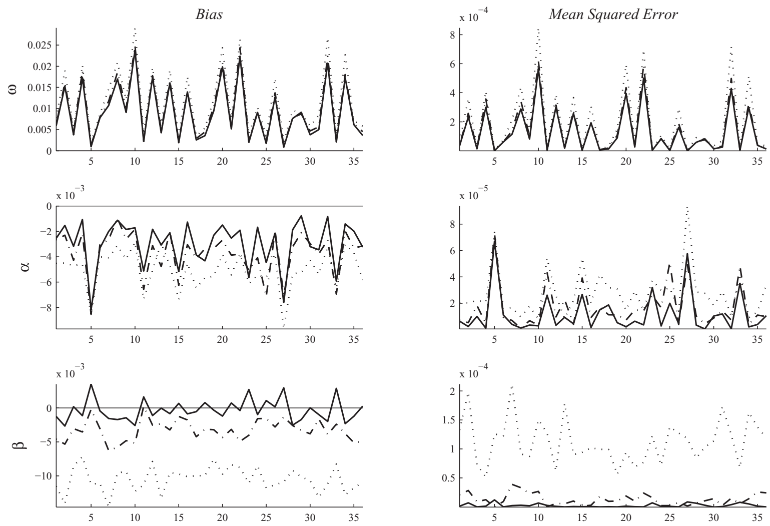
[13]Matlab codes are available on the website www.ecares.org/veredas.html.

Figure 5: Median bias and MSE

(a) ARFIMA parameters



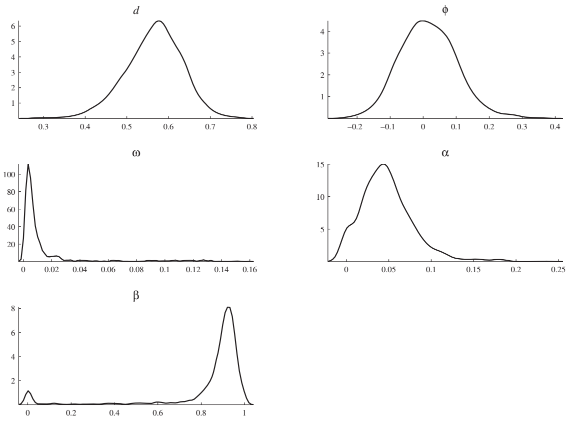
(b) GARCH parameters



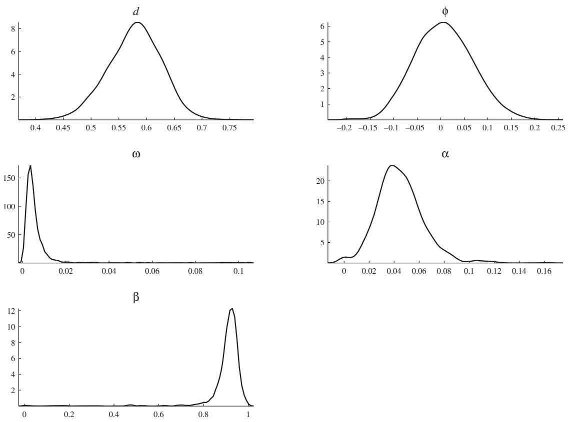
y–axis of the top panel shows the bias (left column) and MSE (right column) for the fractional integration (d) and autoregressive (ϕ) estimated parameters of the factor equation for the 36 DGPs (in x–axis). Bottom panel has the same structure and show the results for the GARCH estimated parameters. Dotted lines are for $T = 500$, dashed and dotted for $T = 1000$, and solid for $T = 5000$.

Figure 6: FINITE SAMPLE DENSITIES

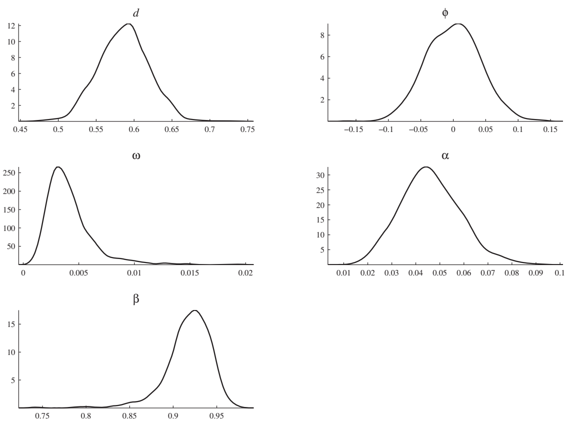
(a) $T = 500$



(b) $T = 1000$



(c) $T = 5000$



Kernel densities of the 1000 estimated ARFIMA-GARCH parameters of the factor equation, for the DGP 25 (see table 10 in the Appendix), and for the three sample sizes.

Table 3: Number of Factors

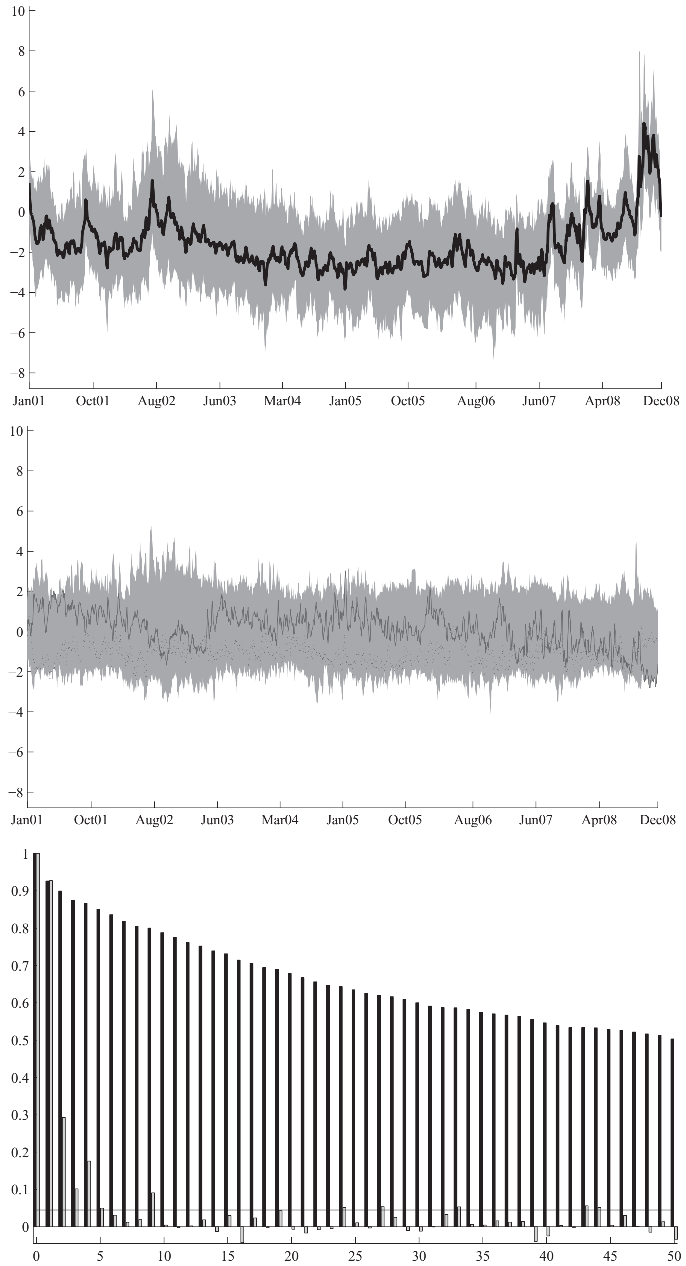
r	μ_i^1	μ_i^2	$Onat.$	IC
1	0.254	0.246	0.000	-0.232
2	0.040	0.023	0.664	-0.210
3	0.036	0.019	0.331	-0.184
4	0.033	0.017	0.119	-0.157
5	0.030	0.015	0.693	-0.127
6	0.028	0.015	0.767	-0.097
7	0.027	0.014	0.555	-0.067
8	0.025	0.013	0.465	-0.036

First column are the number of factors. μ_i^1 is the percentage of variance explained by i–th eigenvalue (in decreasing order) of the spectral density matrix of $\mathbf{x}_t$. μ_i^2 is the percentage of variance explained by i–th eigenvalue (in decreasing order) of the variance covariance matrix of $\mathbf{x}_t$. $Onat.$ is the p–value of the Onatski (2009) statistic for the null of $r-1$ common factors against the alternative of r common factors.IC is the criteria of Bai and Ng (2002).

Figure 7 shows the estimated factor and the idiosyncratic components. The top plot shows the factor along with the envelope of the realized volatilities. The factor tracks with a great deal of precision the co–movements of the volatilities. The middle plot shows the envelop of the idiosyncratic components, which seems to be rather stable in the sense that it does not track the trend in the realized volatilities. This does not mean that the idiosyncratic components are short–memory (as shown by the solid and dotted lines, representing two randomly chosen idiosyncratic components) but it is an indication that most of the long–memory is captured by the factor, as was already suggested in Section 2 and is further corroborated below. Indeed, the bottom plot shows the autocorrelogram (black bars) and the partial autocorrelogram (grey bars) of the factor. The autocorrelations decrease very slowly, possibly at hyperbolic rate, a typical pattern of long–memory processes.

Next we analyze the factor loadings, shown in Table 4. The top panel displays information for the factor loadings for the full sample (column All) and for two–years subsamples. In all cases the loadings are approximately equally distributed around one. More interestingly, the loadings do not seem to vary significantly across subsamples, which has the interpretation that the relation of the firm's volatility with the commonness is roughly the same regardless the state of the market. In order to further check this assessment, we test the equality of distributions across subsamples. We first test if they are cross–sectionally Gaussian. Jarque–Bera and empirical distributions tests (Kolmogorov, Cramer–von–Mises and Anderson–Darling) confirm the null hypothesis of Gaussianity, which implies that they are equally distributed across subsamples if they have equal variances. Levene and Brown–Forsythe tests have p–values closed to 0.10, which confirm that the exposures of the realized volatilities to the commonness are not affected by the market conditions. The bottom panel shows the information about the

Figure 7: Factor and idiosyncratic components



Top plot shows the estimated factor and the envelop of the realized volatilities. Middle plot displays the envelop of the estimated idiosyncratic components along with two of them. Bottom plot shows the autocorrelogram (in black) and the partial autocorrelogram (in grey) of the estimated factor.

factor loadings across sectors (using all the sample). Two sectors behave somehow differently: Financials and Information technology. The median and maximum loadings for these sectors are among the largest, meaning that for some of the firms belonging to these sectors, the sensitivity of the realized volatilities to the common volatility are among the largest of S&P100. Moreover, the dispersion in the loadings for Financials and Information Technology are among the smallest and largest respectively. This is an indication that the volatility of financial firms is significantly concentrated while for IT is very heterogeneous (the same happens for energy). Last, some sectors are more stable than the common volatility, like Utilities and Health Care.

Table 4: FACTOR LOADINGS

	All	01–02	03–04	05–06	07–08
min.	0.54	0.51	0.39	0.36	0.54
med.	0.96	0.97	0.98	1.00	0.96
max.	1.38	1.46	1.49	1.45	1.42

	CD	CS	NRG	FIN	HC	IND	IT	MAT	TLC	UT
min.	0.81	0.67	0.62	0.94	0.62	0.62	0.66	0.9	0.82	0.54
med.	0.99	0.91	0.88	1.13	0.88	0.92	1.2	0.96	0.92	0.84
max.	1.15	1.20	1.28	1.32	1.08	1.34	1.38	1.16	0.98	0.92

Minimum (min.), median (med.) and maximum (max.) of the 90 factor loadings. Top panel for all the assets and the full sample size (column All) as well as the 2–years subsamples. Bottom panel for the full sample but across sectors.

The model for (2) is an ARFIMA$(1, d, 0)$–GARCH(1,1).[14] The estimated autoregressive parameter $\hat{\phi}$ is -0.10, while the estimated fractional integration $\hat{d}$ equals 0.69, higher than any value in Table 1. As it is shown below, when comparing it with idiosyncratic fractional integrations, this high value reflects the fact that the factor is more persistent than individual assets. As a check, we estimated the ARFIMA model on the VIX and the realized volatilities of the S&P500 index. The fractional integration parameters are 0.62 and 0.94 respectively. The question however remains on why volatilities of aggregates have a higher long–memory than the assets'. Our factor, the S&P500 volatility, and VIX are in fact linear combinations that smooth out temporary firm–specific volatility shocks. This smoothness effect creates aggregates that move more slowly that the constituents, and hence longer memory.

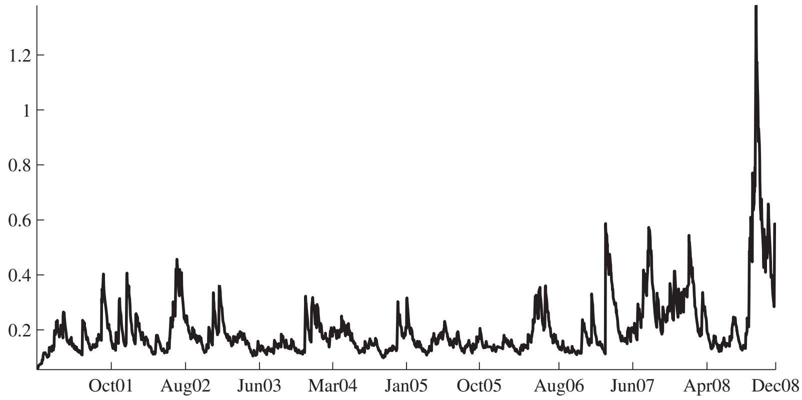
Figure 8 shows the estimated GARCH volatility of the factor. This is the dynamic component in the calculation of the conditional volatilities and covariances ($Var(X_{it}|\mathcal{I}_{t-1}) = \Lambda_i^2 h_t + \sigma_{\epsilon_i}^2$ and $Cov(X_{it}, X_{jt}|\mathcal{I}_{t-1}) = \Lambda_i \Lambda_j h_t + \sigma_{\epsilon_i}\sigma_{\epsilon_j}$) needed for pricing volatility options and risk management of swap volatilities. The similarity with the realized volatilities is striking as it increases and decreases with the events mentioned in Section 2. Moreover, the estimates of the GARCH parameters are of the same order of those estimated for returns: $\hat{\omega} = 0.01$, $\hat{\alpha} = 0.06$ and $\hat{\beta} = 0.90$. The volatility of the volatility is related to tail measurement of returns and Figure 8 clearly shows that it is time–varying.[15] The sharp increase during the 2008 crisis

[14] As in Section 2, other ARFIMA were estimated as well. The ARFIMA$(1, d, 0)$ is the most parsimonious and the residuals are white noises. Detailed results are available under request.

[15] The way in which the tails of the returns are related with the volatility of the realized volatility is an area that deserves further research and beyond the scope of this article.

is evident but a close inspection reveals that the risk of volatilities started to increase in mid 2007 (around June–July, or when Bear Stearns announced major losses). Prior to this date there were also episodes of sudden increases in risk.

Figure 8: Volatility of the common realized volatility

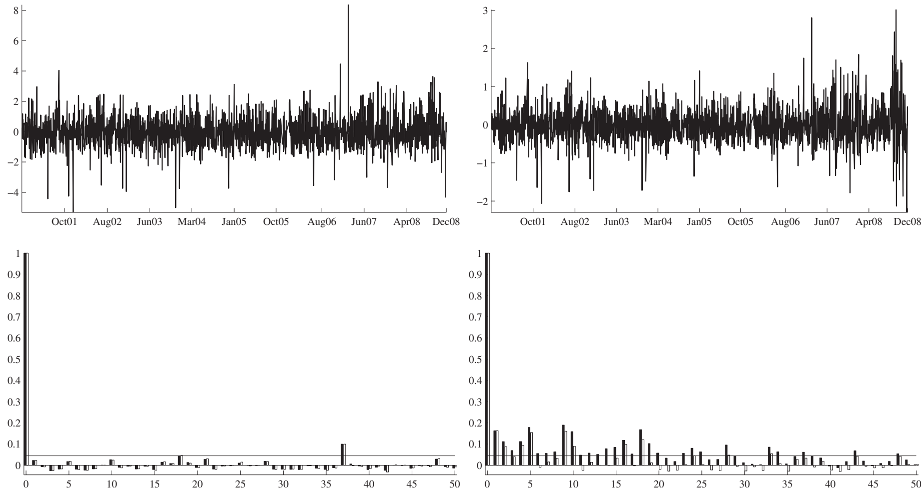


Estimated GARCH volatility of the common factor.

Figure 9 displays four plots with the estimated common shocks $\hat{u}_t$ and the estimated common residuals $\hat{v}_t = \hat{h}_t^{1/2}\hat{u}_t$, along with the autocorrelations (black bars) and partial autocorrelations (grey bars) of the squares. The common residuals show conditional heteroskedasticity that is captured by the GARCH model since the squares of the common shocks are uncorrelated. The plot of $\hat{u}_t$ also reveals events that are on the tails of the distribution. The estimated tail index is $\hat{\nu}_u = 5.72$, which confirms that tail thickness of the realized volatilities is a market feature. Note that the tail thickness of the factor (and of the common residual) is larger than 5.72 due to the presence of the GARCH effects, as it is well known that the unconditional fourth moment of $\hat{v}_t$ is larger than the fourth moment of $\hat{u}_t$. Likewise, the unconditional fourth moment of $\hat{F}_t$ is larger than the equivalent of $\hat{v}_t$. Last, the estimated asymmetry parameter $\hat{\gamma}_u$ equals 0.14 reflecting the fact that skewness across asset's volatilities, though heterogeneous, is more right–sided than left–sided.

The models for the idiosyncratic components are also ARFIMA$(1, d, 0)$. Figure 10 shows the estimates of fractional integration. The thick and grey straight line is the estimated $\hat{d}$ of the factor equation, and the thin line represents the estimates of fractional integration $\hat{\delta}_i$ for the idiosyncratic components. For comparison we include two more lines: the thick black line are the estimates shown in the top panel of Table 1, and the dotted line is 0.5. On the x–axis are the 90 firms grouped in sectors, with the vertical lines acting as dividers. In Section 2 we concluded that realized volatilities are not necessarily stationary but mean reverting. Figure 10 reveals that the source of non–stationarity is common to all. Indeed, not only all the idiosyncratic estimates of fractional integration are in the stationary range, but

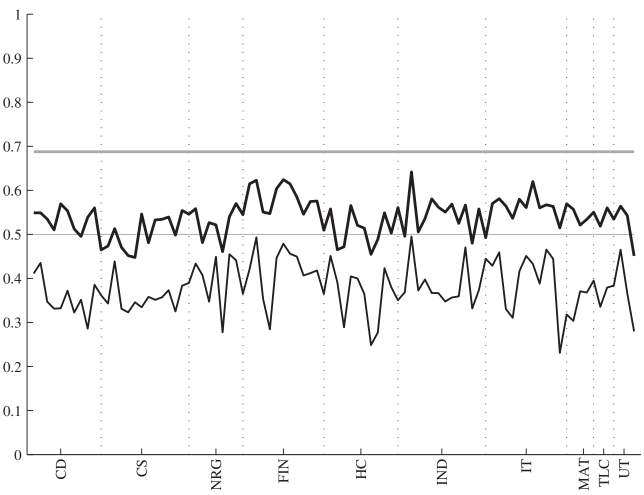
Figure 9: Common shock and common residual



Top plot shows the estimated common shock $\hat{u}_t$ (left) and estimated the common residual $\hat{v}_t = \hat{h}_t^{1/2}\hat{u}_t$ (right). Bottom plots show the autocorrelogram (in black) and the partial autocorrelogram (in grey) of $\hat{u}_t^2$ (left) and $\hat{v}_t^2$ (right).

also they are stable across assets with a maximum range of approximately 0.20. Last, across sectors, the largest idiosyncratic long–memory belongs to Financials, Information Technology and Industrials, while the two Consumer and Health Care have relatively low long–memory.

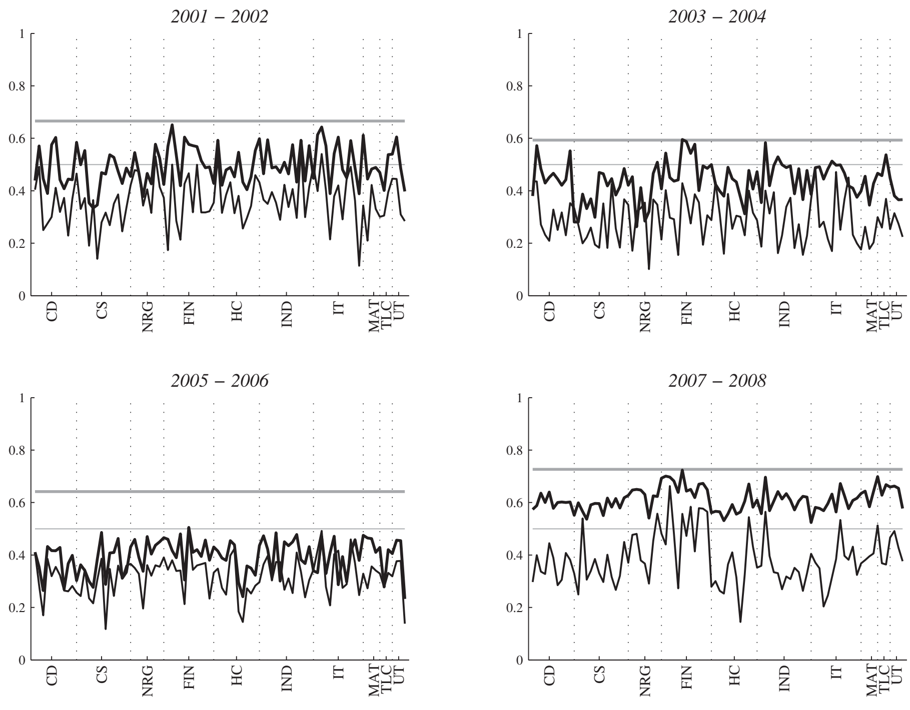
Figure 10: Fractional integration



x–axis are the 90 firms grouped in sectors, with the vertical lines acting are dividers. The solid thick straight line is the estimator $\hat{d}$ of the factor equation, and the solid thin line are the estimates of fractional integration $\hat{\delta}_i$ for the idiosyncratic components. The solid line with dots are the estimates shown in the top panel of Table 1, and the dotted straight line is 0.5.

Figure 11 shows the same analysis but with the 2 years subsamples. Results are in line with what we have found in Section 2. During periods of turmoil, the volatilities of all the firms become more dependent of the past as the degree of long–memory increases. From 2005-2006 to 2007-2008 the parameter of fractional integration of the factor increased sharply. The idiosyncratic long–memories also increased but the movement is less pronounced, though for some assets they crossed the non–stationary range. This is particularly clear for Financials, for which more than half of the firms had idiosyncratic long–memory beyond 0.5. By contrast the idiosyncratic long–memory of some sectors, like Health Care and Consumer Discretionary, were barely affected by the crisis. Another feature of the transition from 2005-2006 to 2007-2008 is that the dispersion of the idiosyncratic long–memory increased. This is in contrast with the finding in Section 2 that in periods of turmoil the degree of long–memory concentrates. A possible explanation is that during the crisis the common long–memory dominates in the sense that the factor explains a higher percentage of the volatility movements.

Figure 11: Fractional integration. Two–years subsamples



x–axis are the 90 firms grouped in sectors, with the vertical lines acting are dividers. The solid thick straight line is the estimator $\hat{d}$ of the factor equation, and the solid thin line are the estimates of fractional integration $\hat{\delta}_i$ for the idiosyncratic components. The solid line with dots are the estimates shown in the top panel of Table 1, and the dotted straight line is 0.5.

Finally, Table 5 shows a summary of the estimates of the parameters of the skewed–t distributions of the idiosyncratic shocks. The range of values for the dispersion parameter is very narrow, meaning that the volatilities of the idiosyncratic realized volatilities are very similar. This is related with the stylized fact that realized volatilities co–move. There are more differences however in the asymmetry parameter, mirroring the finding in Section 2. And as for the tail indexes, there is also a great deal of heterogeneity, ranging from very heavy (4.45) to nearly Gaussian tails (18.27). Across sectors, there are no apparent differences for the dispersion and the asymmetry. For the tail thickness we find the usual suspects: Financials, Energy and Information Technology have thicker idiosyncratic tails than the other sectors. It is somehow surprising that Industrials and Telecommunication Services firms are among the less idiosyncratically heavy–tailed.

Table 5: ESTIMATED PARAMETERS IDIOSYNCRATIC MODELS

		All	CD	CS	NRG	FIN	HC	IND	IT	MAT	TLC	UT
	min.	0.38	0.45	0.42	0.39	0.39	0.5	0.38	0.38	0.42	0.49	0.51
σ_{ϵ_i}	med.	0.5	0.51	0.49	0.53	0.45	0.52	0.5	0.47	0.49	0.54	0.55
	max.	0.61	0.56	0.61	0.6	0.54	0.6	0.61	0.54	0.53	0.55	0.61
	min.	-0.08	0.01	0	-0.08	-0.05	0.03	-0.02	0.01	0	0.06	-0.07
γ_{ϵ_i}	med.	0.07	0.06	0.08	0.01	0.04	0.09	0.07	0.07	0.04	0.06	0.01
	max.	0.15	0.11	0.15	0.11	0.08	0.13	0.15	0.15	0.07	0.06	0.07
	min.	4.45	4.48	4.45	5.17	5.49	4.63	4.66	6.23	5.52	7.32	5.44
ν_{ϵ_i}	med.	6.96	7.18	5.57	6.95	7.73	6.47	6.94	7.65	6.98	7.58	6.58
	max.	18.27	7.89	10.46	14.74	18.27	9.29	9.6	11.87	9.77	7.7	8.45

Minimum (min.), median (med.) and maximum (max.) of the estimates of the parameters of the idiosyncratic skewed–t distributions. Column All are for the 90 firm and the remaining columns for the sectors.

4.2 Forecasting: a horse race

One of the main advantages of long–memory models is forecasting, as the model remembers the recent and distant past more than short–memory models. On the other hand, factor models are a parsimonious way for capturing contemporaneous and spillover effects across large panels of volatilities. These two aspects, when combined in a single model, should provide better forecasts than short–memory and/or univariate models. In what follows we proceed with a thorough forecasting horse race, comparing 9 models, for 4 forecasting horizons, 1 loss function, and for the 90 firms.

The models are divided in three classes: univariate, short–memory dynamic factor models (denoted by SDFM) and long–memory dynamic factor models (denoted by LDFM). Five are the univariate models: a short–memory ARMA(1,1), an ARFIMA(0,d,0) that has long–memory but no dynamics, an ARFIMA(1,d,0), an ARFIMA(1,d,1), and the HAR model of Corsi (2010). Two are the SDFM, which are similar to (1)–(3) except that the ARFIMA models in (2) and (3) are replaced by short–memory models. The first one, SDFM1, forecasts both the factor and the idiosyncratic components with an ARMA(1,1). The second, SDFM2, forecasts both the factor and the idiosyncratic components with an AR(1). The two long–memory dynamic factor models follow the same lines. The first, LDFM1, forecasts the factor and the idiosyncratic components with an ARFIMA(1,d,0) while the second, LDFM2, uses a

HAR model. Table 6 summarizes the 9 models.

Table 6: Horse race

Acronym	Model
1,1	ARMA(1,1)
0,d,0	ARFIMA(0,d,0)
1,d,0	ARFIMA(1,d,0)
1,d,1	ARFIMA(1,d,1)
HAR	HAR
SDFM1	F_t and ξ_{it} with ARMA(1,1)
SDFM2	F_t and ξ_{it} with AR(1)
LDFM1	F_t and ξ_{it} with ARFIMA(1,d,0)
LDFM2	F_t and ξ_{it} with HAR

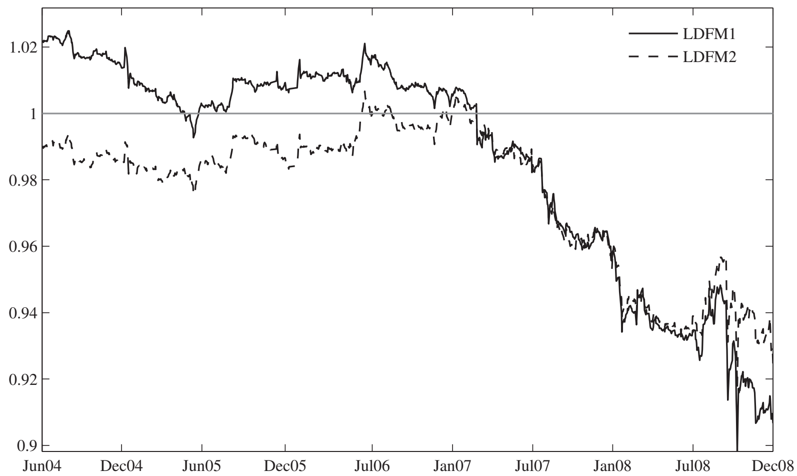
Summary of the 9 models used in the horse race. The left column shows the acronym used in the next tables.

The design of the forecasting exercise is the following. Starting with the first 500 observations, we estimate the models and forecast 1 day, 1 week (5 days), 2 weeks (10 days) and 1 month (20 days) ahead. Every day the model and the predictions are updated on the basis of a recursive scheme, i.e. we always use all the past information to estimate the model and update the predictions. The loss function is the Root Mean Square Error (RMSE) and this choice deserves some explanations. Since realized volatilities are proxies of the latent volatility, forecasting evaluation becomes complicated. The main tool for these evaluations are expected losses (or risk functions). Hansen and Lunde (2006) introduce the notion of robustness of loss functions: a loss function is robust if the ranking (using expected losses) of any two volatility forecasts is the same whether it is done using the true volatility or some estimator. Patton (2011) proposes a family of robust loss functions for forecasting volatility. Let Y_{t+k} be a generic realized volatility (not in logs) observed at time $t+k$ and $\hat{Y}_{t+k}$ its forecast. Let $L(\hat{Y}_{t+k}, Y_{t+k}, b)$ denote the loss function with $b+2$ degree of homogeneity. The family of robust loss functions is given by

$$L(\hat{Y}_{t+k}, Y_{t+k}, b) = \frac{\hat{Y}_{t+k}^{2b+4} - Y_{t+k}^{b+2}}{(b+1)(b+2)} - \frac{Y_{t+k}^{b+1}(\hat{Y}_{t+k}^{2} - Y_{t+k})}{b+1}$$

for $b \notin \{-1, -2\}$. The RMSE is a particular case for $b = 0$, while the Mean Absolute Error does not belong to this family. A priori this result does not hold for the log of realized volatilities. Standard theory shows that $\exp(\hat{X}_{t+k})$ is a biased forecast of Y_{t+k}, with bias depending on the accuracy of the forecast $\hat{X}_{t+k}$. However, this holds only if Y_{t+k} are log–normal and if the parameters are known (i.e. there is no estimation error). Realized volatilities are not log–normal since their logs are not Gaussian, as it was shown in Section 2. And Bardsen and Lutkepohl (2009) show that, in a VAR context, the optimal forecast (i.e. $\hat{Y}_{t+k}$) can be inferior to $\exp(\hat{X}_{t+k})$ if specification and estimation uncertainty are taken into account. They conclude that, in practice, using $\exp(\hat{X}_{t+k})$ may be preferable to using the optimal forecast. Based on this reasoning we perform the forecasting exercise for both $\hat{X}_{t+k}$ and $\exp(\hat{X}_{t+k})$.

Figure 12: Relative RMSE



Relative RMSE (relative to the ARFIMA(1,d,1), the univariate model that is expected to perform the best) of the two long–memory dynamic factor models. Each number is computed as the rolling mean of 250 root square errors of the ARIFMA divided by the corresponding long–memory dynamic factor model.

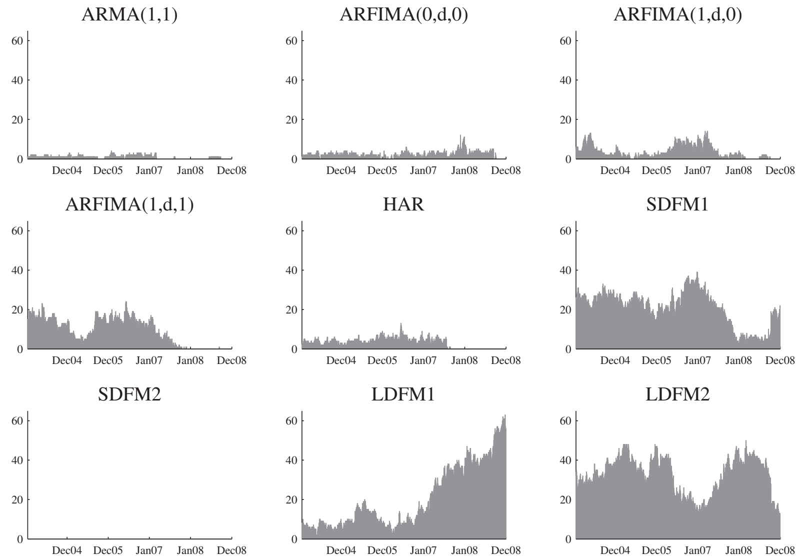
In the sequel we show aggregate results for one–step ahead forecasts and for the log of the volatility. Detailed results at asset and sectoral level, for other steps ahead, and for the volatilities are in an appendix available in the authors websites.

Figure 13: Percentage of smallest RMSE of dynamic factor models

Percentage of times that, for all assets and all steps aheads, the dynamic factor models have the smallest RMSE.

Figure 12 shows relative RMSE (relative to the ARFIMA(1,d,1), the univariate model that is expected to perform the best) of the two long–memory dynamic factor models. Each number is computed as the rolling mean of 250 root square errors of the ARIFMA divided by the corresponding long–memory dynamic factor model. During the period when volatilities were low and markets were calm, all models do similarly, with the relative RMSE ranging from 1.02 to 0.98. At the spring of the crisis, the volatilities started to exhibit a greater deal of co–movements, and improving the relative performances of LDFM1 and LDFM2, with gains up to 10%. The date at which the gains started is clearly identified with a jump around end February and early March 2007, a period of time identified by some researchers as the beginning of the crisis. For instance, Acharya and Richardson (2009) identify March 5 as the first date in the time line of the crisis. That day HSBC announced that one portfolio of subprime mortgages showed much higher delinquency that had been built into the pricing.

Figure 14: Models and assets



The panel shows, for each of the 9 models, the number of assets for which the models have smaller RMSE

Figure 13 complements the previous. It shows the percentage of times that, for all assets, the dynamic factor models have the smallest RMSE. All over the sample they outperform more than 50% of the times, except for one day, and the increase is steady since the beginning of the crisis, reaching 100% at the end of 2008. There are some jumps that can be identified with events. At the end of July and beginning of August 2007, Bear Stearns did not support one of its funds, carry trade experienced a six standard deviation move, several quant hedge funds collapse, and the European Central Bank and other Central Banks took non–standard monetary policy measures like injecting large amounts of liquidity in the overnight. Another increase happens in early 2008 when Citi and Merrill Lynch announced pronounced losses, AIG

Table 7: SECTORS DURING FINANCIAL CRISIS I

	CD	CS	NRG	FIN	HC	IND	IT	MAT	TLC	UT
1,1	1,008	1,009	1,006	1,007	1,013	1,016	1,012	1,007	1,014	1,001
	-	-	-	-	-	-	-	-	-	-
0,d,0	0,987	0,990	0,988	0,989	0,992	0,988	0,991	0,983	0,984	0,983
	1	-	-	-	-	-	3	-	-	-
1,d,0	0,992	0,994	0,994	0,996	0,994	0,998	0,994	0,993	0,991	0,992
	-	-	-	-	-	-	-	-	-	-
1,d,1	1,000	1,000	1,000	1,000	1,000	1,000	1,000	1,000	1,000	1,000
	-	-	-	-	-	-	-	-	-	-
HAR	1,011	1,012	1,008	1,009	1,008	1,010	1,010	1,014	1,007	1,016
	-	-	-	-	-	-	-	-	-	-
SDFM1	0,981	0,979	0,975	0,983	0,974	0,987	1,005	0,977	0,993	0,975
	-	-	1	-	1	-	-	-	-	1
SDFM2	1,062	1,062	1,041	1,088	1,073	1,059	1,122	1,075	1,105	1,072
	-	-	-	-	-	-	-	-	-	-
LDFM1	0,973	0,973	0,969	0,962	0,976	0,975	0,986	0,961	0,965	0,964
	4	3	2	9	1	5	5	3	3	2
LDFM2	0,970	0,965	0,967	0,969	0,965	0,970	0,992	0,969	0,978	0,974
	5	10	5	3	9	8	4	1	0	1

For each model, the first row is the Relative Root Mean Squared Errors, while the second line is the number of wins (i.e. the number of assets for which model m produces the smallest RMSE).

announces troubles in the valuation of CDS, and the UK Government nationalizes Northern Rock. The last jump is around September 15 2008, when Lehman Brothers files for bankruptcy. See Acharya and Richardson (2009), pp. 51-56, for more details.

A more detailed analysis is Figure 14. It shows, for each of the 9 models, the number of assets for which the models have smaller RMSE. The ARMA(1,1) model and SDFM2 perform very poorly while SDFM1 does reasonably well in calm periods, though LDFM2 does better. However, when the crisis starts the forecasting accuracy of LDFM1 increases and, by the end of the sample, is the clear winner of the horse race. A closer look to the figure reveals that a sharp difference between LDFM1 and LDFM2 during the middle of the crisis. Tables 7 and 8 show the performance of all the models during the crisis prior and posterior to February 17 2008, the day the UK Government nationalized Northern Rock, and in order to better understand the behavior, we do the analysis by sectors. For each model, the first row is the Relative RMSE for every sector, while the second line is the number of assets for which the model performs best. During the first part, long–memory dynamic factor models were the better forecasters for 83 out of 90 assets. For the second part, the number reduces to the still high: 66 out of 90. Moreover, during the first part of the sample LDFM2 seems to do best relative to LDFM1 (46 against 37) but during the second part the performance swaps and the long–memory dynamic factor model with ARFIMAs do significantly better than with the HAR model (51 against 15).

To sum up, when markets are calm and volatilities are low, the factor structure does not play a significant role and the improvements in forecasting compared with simpler models

Table 8: SECTORS DURING FINANCIAL CRISIS II

	CD	CS	NRG	FIN	HC	IND	IT	MAT	TLC	UT
1.1	0.996	0.993	1.000	1.001	0.997	0.992	0.994	0.996	0.990	0.984
	-	-	-	-	-	-	1	-	-	-
0,d,0	1.003	1.004	1.019	1.009	1.004	1.007	0.998	1.013	1.009	1.011
	-	-	-	-	-	-	-	-	-	-
1,d,0	0.999	0.998	1.010	1.001	1.000	1.000	0.997	1.003	0.999	1.002
	-	-	-	-	-	-	-	-	-	-
1,d,1	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
	-	-	-	-	-	-	-	-	-	-
HAR	1.004	1.002	0.998	1.012	1.003	1.003	1.003	1.004	1.006	1.003
	-	-	-	-	-	-	-	-	-	-
SDFM1	0.969	0.947	0.960	0.989	0.954	0.957	0.997	0.971	0.982	0.941
	2	6	1	-	6	3	-	1	1	3
SDFM2	1.069	1.051	1.041	1.080	1.075	1.050	1.112	1.111	1.200	1.050
	-	-	-	-	-	-	-	-	-	-
LDFM1	0.954	0.951	0.952	0.961	0.960	0.951	0.976	0.957	0.961	0.948
	7	3	4	12	2	8	9	3	2	1
LDFM2	0.961	0.945	0.951	0.994	0.955	0.957	0.989	0.964	0.986	0.948
	1	4	3	-	3	2	2	-	-	-

For each model, the first row is the Relative Root Mean Squared Errors, while the second line is the number of wins (i.e. the number of assets for which model m produces the smallest RMSE).

are marginal. However, as volatilities increase, the co–movements are reinforced and the factor structure becomes, important in the sense that forecasting gains –relative to univariate models– are up to 10%.

5 Conclusions

We propose a dynamic factor model for volatilities that is implementable for large dimensions and captures the stylized facts of the realized measures. This methodology has several advantages: i) it disentangles between commonness (or factors) and idiosyncrasies, ii) tests for the number of factors, iii) allows for long–memory, non stationarity and mean reversion, and iv) provides short–, medium– and long–run forecasts. We estimate the model on the panel of 90 daily realized volatilities, pertaining to S&P100, from January 2001 to December 2008, and we evince, among others, the following findings: i) All the volatilities have long–memory, more than half in the nonstationary range, that increases during financial turmoil. ii) Tests and criteria point towards one dynamic common factor driving the co–movements. iii) The factor has larger long–memory that the assets volatilities, suggesting that long–memory is a market characteristic. iv) The volatility of the realized volatility is not constant and common to all. v) A forecasting horse race against univariate short– and long–memory models and short–memory dynamic factor models shows that our model outperforms predictions, in particular in periods of stress.

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Appendix: Assumptions

These are the assumptions needed for the estimation procedure suggested by Bai and Ng (2004) to be used for model (1)–(3). The list of assumptions is followed by explanations. A word on notation: the sub–index i refers to the i–th firm, $\|A\| = trace(A'A)^{1/2}$, and $M < \infty$ is a generic positive.

FL) (i) $||\mathbf{\Lambda}_i|| \leq M$ for $i = 1, \ldots, r$, (ii) $||\mathbf{\Lambda}'\mathbf{\Lambda}/N - \mathbf{\Gamma}^{\mathbf{\Lambda}}|| \to 0$ as $N \to \infty$ and (iii) $\mathbf{\Gamma}^{\mathbf{\Lambda}}$ is positive definite.

H) Let $(\rho_{1t}, \ldots, \rho_{rt})$ be the eigenvalues of $\mathbf{H}_t^{1/2}$, then (i) $\rho_{jt} > 0$ $j = 1, \ldots, r$, and (ii) $\sup_t \max_r(\rho_{1t}, \ldots, \rho_{rt}) \leq M$.

CS) (i) $E||\mathbf{u}_t||^4 \leq M$, (ii) $\sum_{j=1}^{\infty} j||\mathbf{C}_j|| \leq M$, (iii) let $\mathbf{\Gamma}^{\mathbf{u}} = var(\mathbf{H}_t^{1/2}\mathbf{u}_t)$, then $var(\mathbf{D}(L)\mathbf{F}_t) = \sum_{j=1}^{\infty} \mathbf{C}_j \mathbf{\Gamma}^{\mathbf{u}} \mathbf{C}_j' \leq M$ is positive definite with rank r, and (v) $\mathbf{C}(1) = \sum_{j=1}^{\infty} \mathbf{C}_j$ has rank r.

IS) For each i, (i) $E||\epsilon_{it}||^4 \leq M$, (ii) $\sum_{j=0}^{\infty} j||G_{ij}|| \leq M$, and (iii) $\omega_{\epsilon_i}^2 = G_i(1)^2\sigma_{\epsilon_i}^2 > 0$.

IC) (i) $E(\epsilon_{it}\epsilon_{jt}) = \tau_{ij}$, $\sum_{i=1}^{\infty} |\tau_{ij}| \leq M$ $\forall j$ and (ii) $E\left|N^{-\frac{1}{2}} \sum_{i=1}^{N} \left[\epsilon_{is}\epsilon_{jt} - E\left(\epsilon_{is}\epsilon_{jt}\right)\right]\right|^4 \leq M$, for every (t, s).

OR) $\{\mathbf{u}_t\}_{t=-\infty}^{+\infty}$ and $\{\boldsymbol{\epsilon}_t\}_{t=-\infty}^{+\infty}$ are mutually independent process, where $\boldsymbol{\epsilon}_t = (\epsilon_{it}, \ldots, \epsilon_{Nt})$.

C0) $E||\mathbf{F}_0|| < \infty$ and $E||\xi_{i0}|| < \infty, \forall i$.

Assumptions FL (Factor Loadings) guarantee that the factors are pervasive, i.e. they influence all variables. This is crucial for identification and is what distinguishes common from idiosyncratic shocks: the former affect all variables, the latter affect individual variables.[16] Assumptions H (Heroskedasticity) are needed for ensuring that the matrix $\mathbf{H}_t$ is positive definite and bounded above. These are high–order conditions and, for instance, the stationary multivariate GARCH models fulfill them. Assumptions CS (Common Shocks) ensures the existence of moments up to order four for the common shocks, positive definiteness of the the short–run variance of $\mathbf{D}(L)\mathbf{F}_t$, and that the long–run variance is full rank (i.e. the factors are not co–fractionally–integrated). Assumptions IS (Idiosyncratic Shocks) are similar to CS. Assumptions IC (Idiosyncratic Correlations) describe an approximate factor structure, meaning that the idiosyncratic components are allowed to be mildly cross-sectionally correlated (in the sense that the largest eigenvalue of the covariance matrix of the idiosyncratic components is bounded). Assumption OR (orthogonality) ensure that the common and the idiosyncratic shocks are independent sources of fluctuation for realized volatilities. Finally Assumptions C0 are standard initial conditions necessary when some variables in $\mathbf{X}_t$ are non–stationary.

[16] Bai and Ng (2004) is an extension to a non–stationary setting of Stock and Watson (2002), Bai and Ng (2002), Bai (2003), and Forni et al. (2005), which in turn are a restricted version of the generalized dynamic factor model of Forni et al. (2000) and Forni and Lippi (2001). In some of these papers assumption FL is stated in terms of the variance covariance matrix of $\mathbf{X}_t$ (denoted by $\mathbf{\Sigma_x}$). If the first r eigenvalues of $\mathbf{\Sigma_x}$ diverge as $N \to \infty$, the number of common factors is uniquely identified meaning that a common component with a different number of factors is not possible (see Chamberlain and Rothschild 1983, and Forni and Lippi 2001 for the general model). This result implies that if the variance covariance matrix of $\mathbf{X}_t$ has r diverging eigenvalues, a factor structure is admitted by our panel and hence we no longer assume FL, but it is satisfied.

Appendix: Tables

Table 9: S&P100 CONSTITUENTS

Ticker	Name	Sector
AA	Alcoa Inc	Materials
AAPL	Apple Inc.	Information Technology
ABT	Abbott Labs	Health Care
AEP	American Electric Power	Utilities
ALL	Allstate Corp.	Financials
AMGN	Amgen	Health Care
AMZN	Amazon Corp.	Consumer Discretionary
AVP	Avon Products	Consumer Staples
AXP	American Express	Financials
BA	Boeing Company	Industrials
BAC	Bank of America Corp.	Financials
BAX	Baxter International Inc.	Health Care
BHI	Baker Hughes	Energy
BK	Bank of New York Mellon Corp.	Financials
BMY	Bristol-Myers Squibb	Health Care
BNI	Burlington Northern Santa Fe C	Industrials
CAT	Caterpillar Inc.	Industrials
C	Citigroup Inc.	Financials
CL	Colgate-Palmolive	Consumer Staples
CMCSA	Comcast Corp.	Consumer Discretionary
COF	Capital One Financial	Financials
COST	Costco Co.	Consumer Staples
CPB	Campbell Soup	Consumer Staples
CSCO	Cisco Systems	Information Technology
CVS	CVS Caremark Corp.	Consumer Staples
CVX	Chevron Corp.	Energy
DD	Du Pont (E.I.)	Materials
DELL	Dell Inc.	Information Technology
DIS	Walt Disney Co.	Consumer Discretionary
DOW	Dow Chemical	Materials
DVN	Devon Energy Corp.	Energy
EMC	EMC Corp.	Information Technology
ETR	Entergy Corp.	Utilities
EXC	Exelon Corp.	Utilities
FDX	FedEx Corporation	Industrials
F	Ford Motor	Consumer Discretionary
GD	General Dynamics	Industrials
GE	General Electric	Industrials
GILD	Gilead Sciences	Health Care
GS	Goldman Sachs Group	Financials
HAL	Halliburton Co.	Energy
HD	Home Depot	Consumer Discretionary
HNZ	Heinz (H.J.)	Consumer Staples
HON	Honeywell Int'l Inc.	Industrials
HPQ	Hewlett-Packard	Information Technology
(cont.)		

(cont.)

IBM	International Bus. Machines	Information Technology
INTC	Intel Corp.	Information Technology
JNJ	Johnson & Johnson	Health Care
JPM	JPMorgan Chase & Co.	Financials
KO	Coca Cola Co.	Consumer Staples
LMT	Lockheed Martin Corp.	Industrials
LOW	Lowe's Cos.	Consumer Discretionary
MCD	McDonald's Corp.	Consumer Discretionary
MDT	Medtronic Inc.	Health Care
MMM	3M Company	Industrials
MO	Altria Group, Inc.	Consumer Staples
MRK	Merck & Co.	Health Care
MSFT	Microsoft Corp.	Information Technology
MS	Morgan Stanley	Financials
NKE	NIKE Inc.	Consumer Discretionary
NSC	Norfolk Southern Corp.	Industrials
ORCL	Oracle Corp.	Information Technology
OXY	Occidental Petroleum	Energy
PEP	PepsiCo Inc.	Consumer Staples
PFE	Pfizer, Inc.	Health Care
PG	Procter & Gamble	Consumer Staples
QCOM	QUALCOMM Inc.	Information Technology
RF	Regions Financial Corp.	Financials
SGP	Schering-Plough	Health Care
SLB	Schlumberger Ltd.	Energy
SLE	Sara Lee Corp.	Consumer Staples
SO	Southern Co.	Utilities
S	Sprint Nextel Corp.	Telecommunications Services
T	AT&T Inc.	Telecommunications Services
TGT	Target Corp.	Consumer Discretionary
TWX	Time Warner Inc.	Consumer Discretionary
TXN	Texas Instruments	Information Technology
TYC	Tyco International	Industrials
UNH	UnitedHealth Group Inc.	Health Care
UPS	United Parcel Service	Industrials
USB	U.S. Bancorp	Financials
UTX	United Technologies	Industrials
VZ	Verizon Communications	Telecommunications Services
WAG	Walgreen Co.	Consumer Staples
WFC	Wells Fargo	Financials
WMB	Williams Cos.	Energy
WMT	Wal-Mart Stores	Consumer Staples
WY	Weyerhaeuser Corp.	Materials
XOM	Exxon Mobil Corp.	Energy
XRX	Xerox Corp.	Information Technology

Table 10: The 36 Monte Carlo simulation designs

	d	ϕ	δ_i	ρ_i	$\hat{d}$	$\hat{\phi}$	$\hat{\omega}$	$\hat{\alpha}$	$\hat{\beta}$	$\hat{\nu}$	$\hat{\gamma}$
1	0.2	0	0	0	0.05	-0.04	0.01	0.00	0.00	0.86	-0.17
2	0.2	0	0	1	0.06	-0.04	0.02	0.00	-0.01	0.67	-0.17
3	0.2	0.5	0	0	0.06	-0.07	0.00	0.00	0.00	1.03	-0.17
4	0.2	0.5	0	1	0.06	-0.06	0.02	0.00	0.00	0.46	-0.17
5	0.2	0.9	0	0	0.00	-0.04	0.00	-0.01	0.00	2.29	-0.16
6	0.2	0.9	0	1	0.02	-0.02	0.01	0.00	0.00	0.83	-0.17
7	0.2	0	1	0	0.06	-0.04	0.01	0.00	-0.01	0.76	-0.17
8	0.2	0	1	1	0.06	-0.05	0.02	0.00	-0.01	0.77	-0.17
9	0.2	0.5	1	0	0.05	-0.05	0.01	0.00	0.00	0.69	-0.16
10	0.2	0.5	1	1	0.06	-0.06	0.03	0.00	0.00	0.63	-0.17
11	0.2	0.9	1	0	0.01	-0.03	0.00	0.00	0.00	1.59	-0.16
12	0.2	0.9	1	1	0.02	-0.01	0.02	0.00	0.00	0.59	-0.17
13	0.4	0	0	0	0.01	-0.01	0.01	0.00	0.00	0.86	-0.16
14	0.4	0	0	1	0.01	-0.01	0.02	0.00	0.00	0.53	-0.17
15	0.4	0.5	0	0	0.01	-0.04	0.00	0.00	0.00	1.40	-0.16
16	0.4	0.5	0	1	0.01	-0.01	0.02	0.00	0.00	0.46	-0.17
17	0.4	0.9	0	0	-0.02	0.00	0.00	0.00	0.00	1.16	-0.17
18	0.4	0.9	0	1	-0.01	0.00	0.00	0.00	0.00	1.13	-0.17
19	0.4	0	1	0	0.01	-0.01	0.01	0.00	0.00	0.63	-0.17
20	0.4	0	1	1	0.01	-0.01	0.02	0.00	0.00	0.53	-0.17
21	0.4	0.5	1	0	0.01	-0.01	0.01	0.00	0.00	0.95	-0.16
22	0.4	0.5	1	1	0.00	-0.01	0.03	0.00	0.00	0.64	-0.16
23	0.4	0.9	1	0	-0.02	0.00	0.00	-0.01	0.00	1.51	-0.16
24	0.4	0.9	1	1	0.01	-0.01	0.01	0.00	-0.01	0.76	-0.16
25	0.6	0	0	0	-0.01	0.00	0.00	0.00	0.00	1.30	-0.16
26	0.6	0	0	1	0.00	0.01	0.01	0.00	0.00	0.65	-0.17
27	0.6	0.5	0	0	0.00	-0.05	0.00	-0.01	0.00	2.22	-0.16
28	0.6	0.5	0	1	-0.01	0.01	0.01	0.00	-0.01	0.75	-0.17
29	0.6	0.9	0	0	0.00	0.00	0.01	0.00	-0.01	0.50	-0.17
30	0.6	0.9	0	1	-0.01	0.00	0.00	0.00	0.00	0.86	-0.16
31	0.6	0	1	0	0.00	0.01	0.01	0.00	0.00	0.98	-0.16
32	0.6	0	1	1	0.00	0.00	0.02	0.00	-0.01	0.45	-0.16
33	0.6	0.5	1	0	-0.02	0.00	0.00	-0.01	0.00	1.47	-0.17
34	0.6	0.5	1	1	-0.01	0.01	0.02	0.00	-0.01	0.52	-0.17
35	0.6	0.9	1	0	0.00	-0.01	0.01	0.00	0.00	0.56	-0.17
36	0.6	0.9	1	1	0.00	0.00	0.00	0.00	0.00	0.98	-0.17

Middle panel shows the specifications of the 36 Monte Carlo designs. Right panel displays the median biases for the estimates of the factor model. $\rho_i = 0$ means no autocorrelation in the idiosyncratic errors, while $\rho_i = 1$ means autocorrelated errors, $\rho_i \sim U(0.5, 0.9)$. $\delta_i = 0$ means low long memory in the idiosyncratic errors, $\delta_i \sim U(0, 0.4)$, while $\delta_i = 1$ means high long memory $\delta_i \sim U(0.4, 0.8)$.

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