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ESTIMATING EULER EQUATIONS WITH INTEGRATED SERIES

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Abstract

We consider the estimation of parameters in Euler equations where regressand or regressors may be non-stationary, and propose a several-stage procedure requiring only knowledge of the Euler equation and the order of integration of the data. This procedure uses the information gained from pre-testing for the order of integration of data series to improve specification and estimation. We can also offer an explanation of the frequent empirical finding that discount rates and adjustment costs are poorly estimated. Both analytical and experimental (Monte Carlo) results are provided.

1. Introduction.

There has recently been considerable attention devoted to the explanation of different forms of dynamic economic behaviour, in particular as reflected in the relationship between adjustment lags and multi-period forecasts based upon the rational expectations hypothesis (REH). For example, there exists a substantial literature concerned with the formal intertemporal theory of employment and price determination by competitive or oligopolistic firms which face convex costs of changing employment or prices (see, for example, Sargent (1978), Rotemberg (1982), and Nickell (1987)). The simplest model which could be called representative of this literature is the *intertemporal quadratic adjustment model* (QAC) which stems from intertemporal optimising behaviour subject to quadratic adjustment costs (Sargent (1981)) yielding linear behavioural rules, either in the open form of the Euler equation or in the closed form of the partial adjustment model with a target based upon expectations about the future paths of the forcing variables.

Faced with the choice of estimating such models by asymptotically fully-efficient methods (see Hansen and Sargent (1982)) based on the closed-form solution, or by less efficient but consistent methods (see Kennan (1979) and Wickens (1982)) based upon direct estimation of the Euler equation, the latter are in practice often preferred.[1] There are probably three especially important reasons for this choice.

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[1] It may also be used as a first step before implementation of the asymptotically fully efficient method.

First, the fully-efficient methods often involve a high computational cost. Second, these methods do not seem to be very robust to the imposition of incorrect *a priori* restrictions on the processes governing the forcing variables. Third, since the stability characteristics of the solution (stable, unstable or saddlepath) are important in determining the method by which the model can be estimated with full efficiency, it is preferable to have a method which can be implemented without the necessity of using an assumption about the characteristics of the unknown solution.

The present paper therefore examines the stochastic properties of the variables and parameter estimates which appear in linear Euler equations derived from the simple QAC model. In particular, we concentrate on the consequences of assuming that the forcing variables contain unit roots (i.e., stochastic trends) rather than being stationary about deterministic trends, and on the implications of this hypothesis for the use of consistent, but not asymptotically efficient, methods of estimation.

The assumption of stationarity around a deterministic trend is of course implicit in the standard practice in this literature of "de-trending" series before carrying out estimation by either of the classes of procedures (*e.g.* Sargent (1978), Kennan (1979), Blanchard (1983) and Shapiro (1986)). Since the validity of this assumption has recently been widely questioned (beginning with Nelson and Plosser (1982)), it seems worthwhile to examine the consequences of the hypothesis of stochastic trend, especially in the context of methods which make use only of the Euler equation. We show that, where series of interest are integrated of order one or greater, we can use the information gained from pre-testing for the order of integration to improve the specification and estimation procedure. In particular, we are able to avoid assumptions such as those of knowledge of the discount factor, or of the forms of the processes generating the forcing variables, which are implicit in some existing techniques.

In section 2 we set out the relevant economic theory and its implications for observable processes. In section 3 we examine what is perhaps the most popular method of consistent estimation, Kennan's (1979) two-step procedure. In section 4 we examine the consequences of assuming that the forcing variables are integrated processes, stating a theorem relating to the orders of integration and offering various several-stage alternatives to Kennan's procedure. Section 5 presents a small Monte Carlo experiment in which we examine the finite-sample properties of one estimation method, while Section 6 concludes.

2. The Model.

We will use a stylised intertemporal QAC model, of the type suggested by Kennan (1979), in which an economic agent is faced with the task of taking a sequence of decisions at each time period t. The values chosen, denoted by y_t ($t = 1, 2, 3, \ldots$), chase a stochastic target variable y_t^*; y_t is observable, and y_t^* is linearly related to an observed strictly exogenous forcing variable x_t according to

$$y_t^* = \beta x_t + e_t, \tag{1}$$

where β is a parameter capturing the desired relationship between y_t and x_t, and e_t reflects the influence of omitted variables in y_t^*. It is assumed that e_t is realised before y_t is determined and is a white noise process. It is also assumed that[2]

$$\rho(L)x_t = \varepsilon_t \tag{2}$$

where $\rho(L)$ is a rational lag polynomial containing, in general, d unit roots so that $\rho(L) = (1-L)^d\rho^*(L)$, with $d \geq 1$ and $\rho^*(L)$ having all roots outside the unit circle; ε_t is uncorrelated at all leads and lags with e_t. This last property follows from the strict exogeneity of x_t.[3]

Models of this type have been extensively analysed in the literature. The $\{y_t\}$ sequence is chosen to minimise the expected value of a quadratic loss function given by

$$E_t \sum_{s=0}^{\infty}\phi^s[\,(y_{t+s} - y^*_{t+s})^2 + c(\Delta y_{t+s})^2] \tag{3}$$

where $E_t(.)$ denotes the mathematical expectation conditional on the information set Φ_t; ϕ is the discount factor. The first-order condition for this minimisation is

$$y_t - y^*_t + c\Delta y_t - \phi c(E_t\Delta y_{t+1}) = 0 \tag{4}$$

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2 In order to focus our attention on the existence of stochastic trends (unit roots) we have abstracted from the presence of drifts and/or trends in the equations. Hence the variables may be interpreted as deviations from any deterministic components, and for the empirical analysis, detrending of the original series could therefore be appropriate.

3 For the sake of simplicity, we have assumed the existence of only *one* exogenous variable. The approach can however be generalised to allow the existence of several non-cointegrated forcing variables whose VAR representation requires differencing d times to achieve stationarity.

or equivalently

$$E_t\Delta y_{t+1} = \phi^{-1}\Delta y_t + (\delta/\phi)\{(y_t - \beta x_t) - e_t\}, \tag{5}$$

where $\delta \equiv c^{-1}$.

Since the model satisfies the Simon-Theil conditions for first-period certainty equivalence, it is well known (see Nickell (1987)) that upon imposing the terminal condition

$$\lim_{T\to\infty} \phi^T \cdot E_t\left[y_{t+T} - y^*_{t+T} + c\Delta y_{t+T}\right] = 0 ,$$

the closed-form solution of (5) has the partial adjustment representation

$$y_t(1-\mu L) = (1-\mu)(1-\phi\mu).\sum_{s=0}^{\infty} (\phi\mu)^s E_t y^*_{t+s}; \quad 0 < \mu < 1 \tag{6}$$

$$= \beta(1-\mu)(1-\phi\mu).\sum_{s=0}^{\infty} (\phi\mu)^s E_t x_{t+s} + (1-\mu)(1-\phi\mu)e_t \tag{7}$$

$$= \beta(1-\mu)(1-\phi\mu).\frac{[\rho(\phi\mu) - \phi\mu L^{-1}\rho(L)]}{\rho(\phi\mu)(1 - \phi\mu L^{-1})}.\ x_t + (1-\mu)(1-\phi\mu)e_t \tag{8}$$

where μ is the stable root of the saddle-path quadratic such that

$$f(\eta) = \eta^2 - (1 + \phi^{-1} + \delta/\phi)\eta + \phi^{-1} = 0,$$

where[4] $f(0) > 0$, $f(1) < 0$ and $f(\eta) \to \infty$ as $\eta \to \infty$.

There are several features of the simplification from (6) to (8) that are worth noting. First, in moving from (6) to (7), we have made use of the relation given by equation (1) and also of the standard

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[4] These conditions guarantee the existence of a stable root. The conditions may be verified by noting that δ and ϕ are both greater that 0.

assumption that $E_t e_t = e_t$ and $E_t e_{t+s} = 0$ for all $s > 0$.[5] Next, in moving from (7) to (8) we use the Wiener-Kolmogorov[6] prediction formula which, when applied to our example, implies that

$$\sum_{s=0}^{\infty} (\phi\mu)^s E_t x_{t+s} = \frac{[\rho(\phi\mu) - \phi\mu L^{-1}\rho(L)]}{\rho(\phi\mu)(1 - \phi\mu L^{-1})} x_t .$$

The joint estimation of (2) and (8), while exploiting the cross-equation restrictions imposed by the REH (Sargent (1978)), forms the basis of the fully asymptotically efficient method. In order to implement this procedure knowledge of the process generating the x_t series is required and it is thus subject to the objections discussed in the introduction. At the expense of efficiency, but retaining consistency, the Euler equation given by (5) can be estimated directly, either by a two-step method involving OLS regressions (Kennan's procedure) or by-errors-in-variables IV methods (see Wickens (1982)). In the latter case, use is made of the fact that the disturbance terms are serially correlated. Since the two-step method has become very popular in applied work (see Pesaran (1987) and the references cited therein), we examine it in the next section.

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5 Kennan (1979) introduced a further disturbance term in (8) to represent deviations of the actual values of y_t from their corresponding planned values. However under the assumptions that (i) e_t is realised before y_t is chosen, and (ii) the process generating the x_t process is known, the main points of the Kennan approach may be illustrated, without loss of generality, even in the absence of this additional disturbance term.

6 See, e.g., Hansen and Sargent (1982).

3. Kennan's Two-Step Procedure.

This procedure (see Muellbauer (1979), Muellbauer and Winter (1980) for examples of its use) uses knowledge of the closed-form solution, given by (8), and of the Euler equation, given by (5). Denoting the forward-looking target in the partial adjustment model (8) by d_t, we obtain

$$d_t = \beta(1-\phi\mu)\cdot\frac{[\rho(\phi\mu) - \phi\mu L^{-1}\rho(L)]}{\rho(\phi\mu)(1-\phi\mu L^{-1})}\cdot x_t = D(L)x_t \tag{9}$$

$$\Rightarrow y_t = \mu y_{t-1} + (1-\mu)D(L)x_t + (1-\mu)(1-\phi\mu)e_t. \tag{10}$$

Since, under previous assumptions, y_{t-k-1} $(k \geq 0)$ and x_{t-1} are uncorrelated with e_t, OLS applied to (10) yields a consistent estimator of μ. If the discount factor ϕ is known, a consistent estimator of δ is given by

$$\hat{\delta} = \frac{(1-\hat{\mu})(1-\phi\hat{\mu})}{\hat{\mu}}. \tag{11}$$

This is the first step of the Kennan procedure[7].

The second step uses knowledge of (5), the Euler equation, and constructs the variable s_t:

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[7] In Kennan's formulation the discount factor R is assumed known in the objective function $E\sum_{t=1}^{\infty} R^t \left\{a_1(X(t)-X^*(t))^2 + a_2(X(t)-X(t-1))^2\right\}$, which is minimised with respect to X(t). The solution will not be affected by dividing the terms in parentheses by $(1/a_1)$, which makes the problem analogous to that which we consider, expressed in (3). Hence assuming that R is known is equivalent to the assumption in our context that ϕ is known.

$$s_t = \Delta y_{t+1} - \phi^{-1}\Delta y_t - (\hat{\delta}/\phi)\cdot y_t.$$

From (5) it is clear that

$$s_t = -(\beta\delta/\phi)x_t + u_{t+1} \tag{12}$$

where $u_{t+1} = (y_{t+1} - E_t y_{t+1}) - (\delta/\phi).e_t - \phi^{-1}(\hat{\delta} - \delta)y_t$.

The innovation $\eta_{t+1} = (y_{t+1} - E_t y_{t+1})$ can be obtained by using the Wiener-Kolmogorov formula, under the assumption that the forcing variable x_t is generated by (2). We therefore have

$$\eta_{t+1} = (1-\mu)(1-\phi\mu)[e_{t+1} + \beta\rho(\phi\mu)^{-1}\varepsilon_{t+1}]. \tag{13}$$

Hence u_{t+1} is given by

$$u_{t+1} = (1-\mu)(1-\phi\mu)[e_{t+1} + \beta\rho(\phi\mu)^{-1}\varepsilon_{t+1}] - (\delta/\phi)e_t - \phi^{-1}(\hat{\delta} - \delta)y_t; \tag{14}$$

$\hat{\delta} - \delta \to 0$ as $T \to \infty$ by the consistency of $\hat{\delta}$, and e_{t+1}, ε_{t+1} and e_t are all uncorrelated with x_t. It follows that plim $[T^{-1}\Sigma_t\ s_t u_{t+1}] = 0$ and that an OLS regression in (12) will yield a consistent estimator of β, the remaining unknown parameter of the model.

Note however that u_{t+1} has an MA(1) structure since $E(u_{t+1}u_t) = -(\phi)^{-1}\delta^2\mu \neq 0$ and the remaining cross-covariances are zero for any lag polynomial $\rho(L)$. Thus OLS will provide an inconsistent estimator of the standard error of $\hat{\beta}$. Kennan showed that the biases would generally be upwards. This of course might not be an important consideration if we wanted only a consistent estimator of β, say, to serve as an initial condition for the fully efficient approach.

To summarise, the Kennan procedure was developed to find a consistent estimator of the parameters of interest in a framework in which all the series of interest were stationary, so that interesting features of integrated processes were not considered. Moreover, there are several objections which could be made to the Kennan approach.

First, it exploits, fairly directly, knowledge of the process generating the x_t series. This information is crucial because in its absence the first step may be mis-specified and an inconsistent estimator of $\mu(\delta)$ may result. Second, the discount factor is assumed to be known. In some instances this may be regarded as an unreasonable assumption and we might therefore wish to estimate ϕ. Finally, and perhaps less importantly, the standard error of $\hat{\beta}$ is biased.

Given these objections, it is of interest to ask whether a simple estimation procedure could be found which makes use only of the Euler equation and disregards the specific characteristics of the process generating the forcing variable, apart from its order of integration. If such a method of estimation of β, δ, and ϕ existed, we could use the Generalised Method of Moments (GMM)[8] to obtain the correct variance-covariance matrix of the estimators. The next section is devoted to a discussion of this possibility.

4. Euler Equations with Integrated Variables.

We must now be more specific about the data generation process for the x_t and the y_t series. There are two special cases of the lag polynomial $\rho(L)$ in which we are particularly interested[9]. These are

$$\rho(L) = 1 - L \Rightarrow x_t = x_{t-1} + \varepsilon_t \tag{15a}$$

and

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8 See Hansen and Sargent (1982).

9 More generally, we consider series which are integrated of order d, denoted $x_t \sim I(d)$.

$$\rho(L) = 1 - 2L + L^2 \Rightarrow x_t = 2x_{t-1} - x_{t-2} + \varepsilon_t. \tag{15b}$$

We also specify, as initial conditions, $x_0 = 0$ for (15a), and $x_0 = x_{-1} = 0$ for (15b).

Equation (15a) corresponds to the case in which x_t is a random walk, while under the process given by (15b) the first difference of the x_t series is a random walk. In another terminology, x_t has one unit root, or is I(1), if it is generated by (15a) while it has two unit roots, or is I(2), if it is generated by (15b).[10]

Under (15a) it is easy to demonstrate that the process governing y_t, given by equation (8) above, simplifies to

$$y_t = \mu y_{t-1} + \beta(1-\mu)x_t + (1-\mu)(1-\phi\mu)e_t, \tag{16a}$$

while under (15b) we have

$$y_t = \mu y_{t-1} + \beta(1-\mu)(1-\phi\mu)^{-1}x_t - \beta(1-\mu)\phi\mu(1-\phi\mu)^{-1}x_{t-1}$$
$$+ (1-\mu)(1-\phi\mu)e_t. \tag{16b}$$

Since μ is within the unit circle, it may be seen from (16a) that y_t is I(1) when x_t is I(1). (16b) shows that y_t is I(2) when x_t is I(2)

Next, we examine the characteristics of the deviations from the long-run solution of (5): it has been suggested (Salmon (1982), for

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10 This is the terminology of Engle and Granger (1987). A series x_t with no deterministic component is said to be integrated of order d, denoted by $x_t \sim I(d)$, if $\Delta^d x_t$ has a Wold representation. Equivalently, x_t may be said to have d unit roots. As Nelson and Plosser (1982) showed, many economic time series seem to be adequately characterised by processes which have one (or two) unit roots which is our reason for concentrating below on I(1) and I(2) processes.

example) that a desirable property of any dynamic behavioural equation is that these deviations be zero in the steady state, which we interpret in this framework as implying I(0) processes. These deviations are calculated as $z_t = y_t - \beta x_t$. Consider first the case in which x_t and y_t are each I(1): that is, x_t and y_t are generated by (15a) and (16a) respectively. Then

$$z_t = y_t - \beta x_t = (1 - \mu L)^{-1}[-\beta\mu\varepsilon_t + (1-\mu)(1-\phi\mu)e_t]. \qquad (17a)$$

Clearly z_t is I(0); therefore in this case $y_t - \beta x_t$ is I(0) regardless of the value of ϕ. In the terminology of Engle and Granger (1987), y_t and x_t are CI(1, 1) $\forall\ \phi$. When y_t and x_t are generated by (15b) and (16b), similar operations yield

$$z_t = (1-\mu L^{-1})[-(1-\phi)\beta\mu(1-\phi\mu)^{-1}\Delta x_t + (1-\mu)(1-\phi\mu)e_t] \qquad (17b)$$

Note from (17b) that, in general, z_t is I(1); that is, y_t and x_t are CI(2, 1). The only exception would occur when $\phi = 1$; we omit this no-discounting case, as discussed below.

While the DGP's (15a) and (16a) and (15b) and (16b) have been used for the sake of illustration in analysing the integration and co-integration propoerties of y_t and x_t, it is possible to prove (see Appendix) the following theorem for general specifications of the lag polynomial $\rho(L)$ and any order of integration greater than or equal to one.

Theorem. If x_t is I(d), $d \geq 1$, e_t is I(0), x_t and y_t are generated by (2) and (8) respectively, and F(L) is the rational lag polynomial

$$F(L) = (1-\mu)(1-\phi\mu)\left[\frac{[\rho(\phi\mu) - \phi\mu L^{-1}\rho(L)]}{\rho(\phi\mu)(1 - \phi\mu L^{-1})(1 - \mu L)}\right]$$

with $\mu > 0$, $\phi < 1$, then the following statements hold:

(i) y_t is I(d)

(ii) Let $z_t \equiv y_t - \beta x_t$, $s_{xt} \equiv (\beta\Delta x_t, \beta\Delta^2 x_t, \ldots, \beta\Delta^{d-1} x_t)$, $\ell_j = F^{(j)}(1)(-1)^j/(j!)$ $(j = 1,2,\ldots d-1)$ and $\ell' \equiv (\ell_1, \ell_2, \ldots, \ell_{d-1})$, where $F^{(j)}$ denotes the j^{th} derivative of F(L) evaluated at L = 1. Then

$[z_t - \ell' s_{xt}]$ is I(0).

(iii) z_t is I(d-1).

(iv) Let $s_{yt} \equiv (\Delta y_t, \Delta^2 y_t, \ldots, \Delta^{d-1} y_t)$, with ℓ' defined as in (ii). Then $[z_t - \ell' s_{yt}]$ is I(d-2) $\forall\ d \geq 2$.

Corollary If x_t is I(1) and e_t is I(0), then y_t is also I(1) and z_t is I(0); if x_t is I(2) and e_t is I(0), then y_t is also I(2), z_t is I(1) and $z_t - \ell_1 \Delta y_t$ is I(0) for all values of ϕ not equal to 1.

This theorem suggests that if x_t is I(d), $d \geq 2$, then there is no linear combination of y_t and x_t which is stationary. This implies that if an agent discounts the future, his optimal strategy never involves choices of y_t such that the gap between y_t and the growing target, y_t^*, is asymptotically eliminated. In other words, given discounting (as stated in part (iii)), when $d \geq 2$, it is not worth incurring the additional adjustment costs necessary to catch up completely with a target the variance of which is exploding at a certain rate. This feature has also been discussed by Nickell (1985) and Pagan (1985), in the context of variables with deterministic growth rates. We extend their results to a framework in which growth is stochastic. Moreover part (ii) of the theorem characterises those deviations which vanish in the steady state, and part (iv) presents an alternative characterisation which is I(0) when d = 2 and in general reduces by one the order of integration of z_t.

The theorem also enables us to suggest consistent strategies for estimating directly the parameters of interest (β, γ, ϕ) in the Euler equation. The essential idea is to take advantage of the linear combinations given in parts (ii) to (iv) of the theorem to obtain a consistent estimate of β regardless of the order of integration of the underlying variables. This estimate can then be taken as given in estimating ϕ and δ.

There are two cases that we consider, in which both y_t and x_t are respectively I(1) and I(2); an even higher order of integration for the two series would not generally be regarded as likely to be a good characterisation of observed economic variables.

4.1: Estimation when x_t is I(1)[11].

First, check using the usual testing procedures (*e.g.* Dickey and Fuller (1979), (1981), Said and Dickey (1984)) that the orders of integration of y_t and x_t are indeed consistent. If x_t is I(1) but y_t is not, the DGP of (y_t, x_t) cannot have the characteristics of the QAC model. If y_t is also I(1), the next step is to test the null

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11 We do not consider the estimation of (16a) directly as this equation was derived using the specific assumption that x_t is a random walk; in this section we concentrate on general I(1) (or I(2)) processes.

hypothesis of no co-integration between x_t and y_t using procedures such as those suggested by Engle and Granger (1987) or Johansen (1988).

Next, reparameterise the Euler equation (5) by substituting for $E_t y_{t+1}$ using the definition of u_{t+1} given in (12) ff; that is, we would estimate, if β were known,

$$\Delta^2 y_{t+1} = (\phi^{-1} - 1)\Delta y_t + (\delta/\phi)[y_t - \beta x_t] + \tilde{u}_{t+1}, \tag{18}$$

where $\tilde{u}_{t+1} = (1-\mu)(1-\phi\mu)[e_{t+1} + \beta\rho(\phi\mu)^{-1}\varepsilon_{t+1}] - (\delta/\phi)e_t$

The theorem states that y_t and x_t are CI(1, 1) for all values of ϕ. Thus, by the Stock (1987) super-consistency result[12], the estimate of β in a *static* regression of y_t on x_t converges to its true value at a rate proportional to the sample size; β may reasonably be taken as given when estimating (18), having been derived from a previous static regression; this constitutes the first stage in the estimation process. Altering equation (18) slightly by using $\hat{\beta}$, the super-consistent estimate of β, the final form of the equation we

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12 This establishes that static regressions integrated of order greater than or equal to one give T-consistent estimates of the long-run solution. Thus, possible sources of misleading inference such as simultaneity biases are not worrisome in such cases; these biases are of a lower order of integration than are the regressors and the regressand, and can be relegated to the residuals without affecting the estimate of the long-run solution. However, Hansen and Phillips (1988) propose an estimator with improved finite-sample properties which could be used in the first step of this and the following cases.

would estimate is given by

$$\Delta^2 y_{t+1} = (\phi^{-1} - 1)\Delta y_t + (\delta/\phi)[y_t - \hat{\beta}x_t] + \tilde{u}_{t+1} + (\delta/\phi)(z_t - \hat{z}_t) \quad (18')$$
$$= \theta_1 \Delta y_t + \theta_2 \hat{z}_t + [\tilde{u}_{t+1} + \theta_2(z_t - \hat{z}_t)].$$

Note that the bracketed error term follows an MA(1) process since $(z_t - \hat{z}_t)$ is $o(1)$ (see Engle and Granger (1987)).

Finally use an IV procedure to estimate (18'); since $\tilde{u}_{t+1}$ follows a first-order moving average process, estimation of ϕ and δ by IV is consistent. The instruments ought to be taken from the information set Φ_{t-1} (for example, $\hat{z}_{t-1}$, Δx_{t-1}, Δy_{t-1} and lags of these: see Hansen and Sargent (1982)); note that the regressand and the regressors in (18') are each I(0). The IV procedure will yield estimators of ϕ and δ consistent to $O_p(T^{-1/2})$. Note also that

$$\hat{\phi} = (\hat{\theta}_1 + 1)^{-1}; \; \hat{\delta} = (\hat{\theta}_1 + 1)^{-1}\hat{\theta}_2,$$

and thus both parameter estimates may be identified from the regression. If ϕ is restricted in (18') to a value not equal to its true value, the estimator of δ will be biased, the bias being $O(T^{-1/2})$.

4.2: Estimation when x_t is I(2).

Consider now the case in which the y_t and x_t processes are each I(2). The case $\phi = 1$ is now omitted, because the terminal condition will fail to hold under these circumstances; the closed-form solution (6) will no longer be valid. However since $\phi = 1$ corresponds to the case of no discounting, it is in any event of little economic interest.

If $0 < \phi < 1$, then by the theorem and corollary, y_t and x_t are CI(2, 1). This also implies that Δy_t and Δx_t are CI(1, 1) with the co-integrating vector given by $(1, -\beta)$. Hence in a regression of Δy_t

on Δx_t, we would reject the null hypothesis of a unit root in the residuals and would obtain a T-consistent estimator of β in the first step.

The estimator $\hat{\beta}$ of β may then be used in estimating the Euler equation (18'). This second step cannot be implemented directly because the regressand is I(0) while the regressors are each I(1); nevertheless, a modification will produce a consistent estimate of the co-integrating vector. The appropriate modification is given by part (iv) of the theorem, from which it may be seen that z_t and Δy_t are CI(1,1) with the co-integrating vector given by $(1, -\ell_1)$, where $\ell_1 = -(1-\phi)\delta^{-1}$. Normalising θ_1 to unity, differencing both sides of (16b) and using (17b), we can see that

$$\Delta y_t = -(1-\phi)^{-1}\delta z_t + \omega_t, \tag{19}$$

where ω_t is an I(0) series by (iv). Regressing Δy_t on $\hat{z}_t$ provides a consistent estimator (of $O_p(T^{-1})$) of $(1-\phi)^{-1}\delta$, because this co-integrating parameter is unique. Finally ϕ may be estimated by IV, using the following reparameterisation of (18):

$$\Delta^2 y_{t+1} = (\phi^{-1} - 1)[\Delta y_t + \hat{\pi}.\hat{z}_t] + [\tilde{u}_{t+1} + (\delta/\phi)(\hat{z}_t - z_t)] \tag{20}$$

where $\hat{\pi}$ is the estimate of $(1-\phi)^{-1}\delta$ derived from (19) and $\hat{z}_t$ is given, in the usual way, by $(y_t - \hat{\beta}x_t)$. Regressions (19) and (20) are the second and third steps of the procedure, respectively.

Note that in (20) both the regressand and the regressor are I(0). We will therefore obtain a consistent estimator of ϕ which converges to its true value at rate $T^{-1/2}$. Here, setting $\phi = 1$ in the original parameterisation of the Euler equation, (18), can have very unsatisfactory consequences, especially if ϕ is substantially below unity. This is easily seen in (18') where setting $\phi = 1$ eliminates the Δy_t regressor; the only remaining regressor is $\hat{z}_t$ which is I(1) while the regressand is I(0). This implies that $\hat{\theta}_2$ converges to zero at rate T, which in turn implies adjustment costs that are too high to be credible (consider Muellbauer's (1979) employment function, for example).

In summary, therefore, we have described a sequential (two-step for I(1) processes, three-step for I(2) processes) procedure for estimating β, δ and ϕ which makes use only of the Euler equation and knowledge of the number of unit roots in the DGP of the forcing variables. Simple co-integrating regressions yield estimates of some of the structural parameters; these estimates are super-consistent, converging to their true values at rates faster than $T^{-1/2}$. Pre-tests for the order of integration will in some cases rule out the QAC model as the DGP for the data at an early stage. By the theorem above, this will arise if x_t and y_t are found not to have the same order of integration.

In order to examine the finite sample properties of instrumental variables estimation of the model (18'), or alternative parameterisations of the same, we consider a set of Monte Carlo simulations in the next section.

5. Monte Carlo Results

This section describes the simulation study undertaken to supplement the analytical results reported above. We pay particular attention to the finite-sample behaviour of the IV estimators used in the last stage of the procedure. It is assumed in this exercise that the investigator has correctly deduced the orders of integration of the series from pre-testing; an obvious qualification lies in the frequently-observed low power of such tests.

The exercise is divided into two parts, corresponding to the I(1) and I(2) cases above. The first deals with variables y_t and x_t which are individually I(1), and which follow the DGP given by equations (15a) and (18), with $e_t \sim N(0, \sigma_e^2)$, $\varepsilon_t \sim N(0, \sigma_\varepsilon^2)$; $\sigma_\varepsilon^2 = \sigma_e^2 = 1$.[13] and $E(\varepsilon_t e_s) = 0$. We take $\beta = 1$, $\phi = (1.0, 0.95, 0.70)$ and $\mu = (0.95, 0.85)$. The sample size is T = 100, the number of replications is N = 1000 and the model is (18). To guarantee the existence of the first moments of the parameter estimates $\hat{\theta}_1$ and $\hat{\theta}_2$ we use an over-identified generalised instrumental variables estimator. In experimental practice, the consequence of not doing so is the occasional appearance of extremely large (in absolute value) values for the estimated parameters, such that there is no clear convergence to a reliable average as the number of replications is increased. This results from the fact that the coefficient estimates have no finite moments when exactly identified IV estimates are formed[14].

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[13] Finite-sample results are not invariant to these variances; this combination makes visible some of the important points to be made below, but of course represents only an example of the outcomes which can emerge.

[14] For the mean to exist, we must have at least one extra instrument; for the second moment to exist, we must have two extra. The non-existence can arise because of potential near-singularities in the moment matrix, leading to arbitrarily large parameter estimates (see Sargan (1981)).

The natural instruments to choose, since Δy_t and z_t are correlated with the error term in (18), are Δy_{t-1} and z_{t-1}. In order to obtain an over-identifying GIV estimator it is natural also to choose some lags of these quantities. This is the strategy followed here; we choose the values dated t-2 as well as those dated t-1. The instrumental variables $\hat{z}_{t-i}$ estimated in the first stage of the procedure were used in place of the z_{t-i}.

Table 1

Mean Values of Coefficient Estimates from Monte Carlo Experiment[15]

$T = 100$; $N = 1000$

DGP: (15a), (18); $x_t \sim I(1)$

ϕ	θ_1	θ_2	$\hat{\theta}_1$	$\hat{\theta}_2$	$\hat{\hat{\theta}}_1$	$\bar{\hat{\hat{\theta}}}_2$
			I: $\beta = 1$, $\mu = 0.95$			
1.00	0.0000	0.0026	-0.0212	-0.0102	0.	0.0021
			(0.0004)	(0.0011)		(0.0002)
			(0.0003)	(0.0009)		(0.0002)
0.95	0.0526	0.0054	-0.0231	0.0011	0.	0.0046
			(0.0005)	(0.0001)		(0 0004)
			(0 0005)	(0.0000)		(0.0003)
0.70	0.4286	0.0252	0.2362	-0.0025	0.	-0.0526
			(0.0021)	(0.0003)		(0.0081)
			(0.0016)	(0.0002)		(0.0067)
			II: $\beta = 1$, $\mu = 0.85$			
1.00	0.0000	0.0265	-0.0118	-0.0010	0.	0.0212
			(0.0001)	(0.0021)		(0.0054)
			(0.0001)	(0.0016)		(0.0046)
0.95	0.0526	0.0358	-0.0103	0.0062	0.	0.0236
			(0.0031)	(0.0021)		(0.0054)
			(0.0026)	(0.0025)		(0 0046)
0.70	0.4286	0.1021	0.2682	0.0941	0.	-0.1290
			(0.0093)	(0.0310)		(0.0160)
			(0.0086)	(0.0350)		(0.0133)

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15 θ_i: theoretical values; $\hat{\theta}_i$: unconstrained estimates; $\hat{\hat{\theta}}_i$: constrained estimates. Standard errors are in parentheses, the first being that from GIV estimation and the second from 2S2SLS estimation.

The second and third columns of each block in Table 1 represent the true values taken by the parameters θ_1 and θ_2 in equation (18) for each element of the range of values of the structural parameters given at the heading of each block[16]. Columns 4 and 5 show the parameter estimates when ϕ is not fixed but estimated along with the other parameters; finally columns 6 and 7 (yielding estimates denoted $\hat{\hat{\theta}}_i$) give the estimates of the parameters θ_i when ϕ is fixed at unity and Δy_t therefore vanishes from the DGP (and is removed from the model).

We report two sets of standard errors with these estimates. The first bracketed figure corresponds to the mean standard error obtained using the GIV estimation method described above; the second figure is the mean standard error obtained using the Cumby *et al.* (1983) two-step two-stage least squares method (2S2SLS). Denoting the regressors in (18) by X, the instrument set by W and the disturbance by u, we have that the asymptotic distribution of $\hat{\theta} \equiv (\hat{\theta}_1, \hat{\theta}_2)'$ is given by

$$T^{-1} \operatorname{plim} \left[\frac{X'W}{T} \Omega^{-1} \frac{W'X}{T} \right]^{-1}, \tag{21}$$

where $\Omega \equiv \operatorname{plim} T^{-1}W'uu'W$.

The matrix given in (21) is the true variance-covariance matrix of the GIV estimators, so it is interesting to compare it with the reported matrix. In order to implement (21), we have used a simple estimator proposed by Newey and West (1987), which ensures that Ω is positive definite. The estimate is

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16 Estimates of the co-integrating parameter β were generally close to unity, although in some cases small sample biases could certainly emerge (see Banerjee *et.al.* (1986)).

$$\hat{\Omega} \equiv T^{-1}\sum_{t=1}^{T}\left[W_t'\hat{u}_t\hat{u}_t'W_t + \sum_{\substack{k=-m\\k\neq 0}}^{m}(1-(k/m+1))\cdot W_t'\hat{u}_t\hat{u}_{t-k}'W_{t-k}\right], \qquad (22)$$

where $\hat{u}_t$ are the GIV residuals and m=1 for an MA(1) disturbance.

With a sample of 100 observations, the estimated values of $\hat{\theta}_1$ are negative for high values of μ and ϕ, indicating estimated values of ϕ slightly greater than unity. While the values are not plausible as estimates of a discount factor, we may have in these results some explanation of the commonplace empirical result that discount factors are not estimated to fall in, say, the interval [0.9, 1.0], as one might have expected *a priori*. However when we take a relatively low value of ϕ (*e.g.* ϕ = 0.7), $\hat{\theta}_1$ becomes positive. A possible explanation lies in the small-sample biases which appear in the coefficient of the lagged dependent variable when estimating equations such as (18) (see, *e.g.*, Grubb and Symons (1987)), which may also affect the estimates of $\hat{\theta}_2$. It is important to note also, from columns 5 and 6, that imposing ϕ = 1 when the true value of ϕ is, or is close to, unity improves the estimation of θ_2; the estimate is positive and close to the true value. However when ϕ = 0.7, imposing ϕ = 1 leads to poor estimates $\hat{\hat{\theta}}_2$ of θ_2, as would be expected from the now-inconsistent estimator. Parameter estimates generally seem somewhat more accurate for lower values of μ, perhaps reflecting the fact that z_t then looks more like an I(0) variable (recall (17a)[17]).

With respect to the standard errors, we observe that in most cases the GIV-estimated standard errors are higher than those from

17 This might be a possible explanation of the poor performance of Kennan's non-durable employment equation, where $\hat{\mu}$ seems high.

2S2SLS, confirming Kennan's result concerning upward biases; this also appears in Table 2. However the differences for the DGP that we have examined here are small.

The second simulation exercise uses (15b) and (18), with other features of the DGP (the generating processes for the disturbances) unchanged. Results, again for N=1000 and T=100 are found in Table 2.

Table 2

Mean Values of Coefficient Estimates from Monte Carlo Experiment[18]

T = 100; N = 1000
DGP: (15b), (18); $x_t \sim I(2)$

ρ	θ_1	θ_2	$\hat{\theta}_1$	$\hat{\theta}_2$	$\hat{\hat{\theta}}_1$	$\hat{\hat{\theta}}_2$
			II: $\beta = 1,\ \mu = 0.85$			
0.99	0.0101	0.0283	-0.0009	0.0228	0.	0.0231
			(0.0001)	(0.0094)		(0.0092)
			(0.0001)	(0.0084)		(0.0072)
0.95	0.0526	0.0358	-0.0008	0.0308	0.	0.0234
			(0.0000)	(0.0097)		(0.0096)
			(0.0000)	(0.0082)		(0.0081)
0.70	0.4286	0.1021	0.1215	0.0352	0.	0.0056
			(0.0521)	(0.0172)		(0.0012)
			(0.0482)	(0.0181)		(0.0010)

18 Again, θ_i: theoretical values; $\hat{\theta}_i$: unconstrained estimates; $\hat{\hat{\theta}}_i$: constrained estimates. Standard errors are in parentheses, the first being that from GIV estimation and the second from 2S2SLS estimation.

As in the previous discussion, we distinguish three cases: ϕ almost equal to unity (0.99), ϕ close to unity (0.95) and ϕ relatively far from unity (0.70). Only μ = 0.85 is considered, as the power of the co-integration test is relatively low when μ = 0.95. The second and third columns of Table 2 correspond to the true values given in the the same columns of Table 1.

When ϕ = 0.99, we observe again a negative value for $\hat{\theta}_1$, although this is very small in absolute value consistent with the fact that $\hat{\theta}_1$ is a T-consistent estimator of θ_1= 0.0101; moreover when ϕ = 0.99 $\hat{\theta}_2$ is quite well estimated, and similar results obtain when ϕ = 0.95, reflecting the finite sample continuity in the neighbourhood of ϕ = 1. *Imposing* ϕ = 1 renders the estimate $\hat{\theta}_2$ only slightly more accurate on average.

Finally when ϕ = 0.7, the ratio $\hat{\theta}_2/\hat{\theta}_1$, at 0.29, is a good estimate of its theoretical counterpart $\delta/1-\phi$ (recall (19)), which takes a value of 0.24; the individual coefficients, however, are not especially good estimates of θ_1 and θ_2. In this last case, imposing ϕ = 1 yields an estimate $\hat{\hat{\theta}}_2$ which is quite small, reflecting again a T-consistent estimator of a parameter with a true value of zero. Similarly, the second-step regression of Δy_t on $\hat{z}_t$ gave a value of $\hat{\delta}/(1-\phi)$ of 0.26, while the third-step regression (20) yielded an estimate of θ_1 of 0.35, in each case, again, fairly close to the true values of .24 and .43 respectively.

To summarise the results of the experiments, it seems that implementation of the two-step and three-step procedures give reasonably accurate results for the processes used here, although sizeable finite-sample biases can appear in the estimation of ϕ when the stable root is close to unity. Imposition of the restriction ϕ = 1 or ϕ equal to some value close to one is generally a reasonable strategy if the restriction is close to being valid, not surprisingly, but can lead to noticeable biases in results if the restriction is invalid in the case where the forcing variable is I(1), and to very high estimated values of the adjustment costs when the forcing variable is I(2).

6. Concluding remarks

There have been a number of empirical studies in which investigators have estimated Euler equations in attempts to understand dynamic adjustment processes in the context of the QAC model. The results of such estimation need not, however, yield accurate estimates of critical parameters, especially if the integration properties of the data are disregarded as where non-stationarity is implicitly dealt with through commonplace procedures such as de-trending. By assuming that the forcing variables are integrated, we characterise the order of integration of the control variable and of the deviations from the target stemming from the optimal control rule. Several co-integrating relationships are found to be implied.

In view of these findings, we propose the use of an alternative several-stage procedure to that of Kennan (1979), which requires only knowledge of the Euler equation and the order of integration of the data. Some of the stages in estimation require the use of IV estimators rather than OLS as in Kennan's approach. The results reported here suggest that, even when estimation is by IV, the fact that regressions may be "inconsistent" (in the sense of having a regressand of different order of integration than the regressors) can lead to parameter values which approach zero rather than the correct theoretical values. In particular, we find that procedures such as Kennan's may be biased toward the finding of overly low (even negative, in small samples) estimates of discount factors and overly high costs of adjustment. However, we also find that the standard procedure of fixing the discount factor to unity (or slightly less than unity) seems to perform reasonably well when the true discount factor is indeed in that range. Nonetheless, it is risky even here to apply Kennan's method, which assumes knowledge of the discount factor, because of the considerable uncertainty surrounding this estimate: when it is well below unity the consequences of fixing it to unity can be serious, especially if the forcing variable is I(2).

In general, these results suggest the importance of consideration of the orders of integration of underlying series in determining the outcome of estimation of Euler equations.

Appendix

Proof of Theorem.

(i) Since y_t is generated by (8) it can be written as

$$y_t = \beta F(L)x_t + (1-\mu L)^{-1}(1-\mu)(1-\phi\mu)e_t. \qquad \text{(A1)}$$

Since $0 < \mu < 1$ and e_t is I(0), the order of integration of y_t is given by the order of integration of $F(L)x_t$. This is in turn equal to the order of integration of x_t if $F(1) \neq 0$. Otherwise, F(L) contains at least one unit root, leading to a lower order of integration. F(L) can be written as

$$(1-\mu)(1-\phi\mu)\left[\frac{L - \phi\mu[\rho(L)/\rho(\phi\mu)]}{(L-\phi\mu)(1-\mu L)}\right]$$

$$\Longrightarrow F(1) = 1 - \phi\mu[\rho(1)/\rho(\phi\mu)]. \qquad \text{(A2)}$$

If x_t is I(d), $d > 0$, then given the representation (2), we must have $\rho(1) = 0$, and by A(2) $F(1) = 1$ ($\neq 0$). Thus y_t has the order of integration of x_t and part (i) of the theorem is proven.

(ii) Define $z_t = y_t - \beta x_t = \beta[\,F(L) - 1\,]x_t + I(0)$. (A3)

If x_t is generated by (2), then through a Taylor expansion around $L = 1$ it can be shown (see Stock (1987)) that

$$\rho(L)x_t = \left[\rho(1) + \sum_{j=1}^{d-1}\frac{\rho^{(j)}(1)}{j!}(-1)^j(1-L)^j + \tilde{\rho}(L)(\,1-L)^d\right]x_t, \qquad \text{(A4)}$$

where $\rho^{(j)}$ denotes the j^{th} derivative of $\rho(L)$ with respect to L^j and $\tilde{\rho}(L)$ has all roots outside the unit circle. If $x_t \sim I(d)$, then by (A4), $\rho(1) = \rho^{(1)}(1) = \ldots = \rho^{(d-1)}(1) = 0$.

By a similar expansion, we obtain from (A3),

$$F(L)x_t = \left[F(1) + \sum_{j=1}^{d-1}\frac{D^{(j)}(1)}{j!}(-1)^j(1-L)^j + \tilde{F}(L)(\,1-L)^d\right]x_t,$$

$$\text{and } z_t = \beta\left[(F(1)-1)x_t + \sum_{j=1}^{d-1}\ell_j\Delta^j x_t\right] + I(0), \qquad \text{(A5)}$$

where $\ell_j = F^{(j)}(1)(-1)^j/(j!)$ $(j = 1,2,...d-1)$ and we have used the fact that $\tilde{F}(L)(1 - L)^d$ must be I(0). Since $F(1) = 1$, x_t will not appear in (A5); this completes the proof of part (ii). Note that the coefficients ℓ_j will be functions of the underlying parameters and that $\ell_j = g(\rho^{(j)}(1), \phi, \mu)$. The formulae for the coefficients ℓ_j can be obtained by repeated differentiation of F(L).

(iii) Note that since $F(1) - 1 = 0$ and $\ell_1 \neq 0$ in (A5), the leading term in (A5) is I(d-1), as required to prove (iii).

(iv) Finally note from the definition of z_t that $\Delta^j z_t = \Delta^j y_t - \beta\Delta^j x_t$ and therefore $z_t - \ell' s_{yt}$ is equal to $z_t - \ell' s_{xt} + \ell' s_{zt}$, where $s_{zt} \equiv (\Delta z_t, \Delta^2 z_t, ..., \Delta^{d-1} z_t)$. From (ii), $z_t - \ell' s_{xt}$ is I(0); from (iii) $\Delta^j z_t$ is I(d-(j+1)). The leading term in s_{zt} is I(d-2), and this is therefore the order of integration of $z_t - \ell' s_{yt}$. ∎

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