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NEGATIVE HARMONY: THE SHADOW OF HARMONIC POLARITY ON CONTEMPORARY COMPOSITIONAL TECHNIQUES

By DEBORA HALLER

A PRODUCTION PAPER

Submitted in partial fulfillment of the requirements for the degree of Master of Music in Commercial Music, Composition and Arranging In the School of Music Of the College of Music and Performing Arts Belmont University

NASHVILLE, TENNESSEE

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Introduction

If we are to blame any composer for the puzzling question 'What is Negative Harmony?' that caused a worldwide itch on the creative minds of many young composers in this century, it would be Jacob Collier, who introduced the world with prodigious, colorful harmonies upon releasing his first album "In My Room" in 2016. Provided with brief answers during interviews with June Lee that explained the concept of tones and chords being mirrored on an axis center in the opposite direction, the surface of Negative Harmony was exposed but led to even more confusion. Prompted by even more questions, the internet exploded with articles and blogs about Collier's interpretations of Negative Harmony. Unfortunately, they all varied in their approach and results, which led curious fans such as myself back to the original question, 'What is Negative Harmony?'

Even though Collier shows innovation in his compositions, the musical principles applied are built on a solid foundation laid by composers and music theorists that was built centuries before him. For this reason, we can best learn about Negative Harmony if we evaluate it within its historical context.

Throughout history the art of composition has proven its progressive spirit to be anchored in more than just the mind of a musical genius. The deeper one observes each individual brushstroke within a musical masterpiece, the more one understands that music has always been part of a bigger picture; a cultural formation grounded within the spirit of its time. Born and evolved within a sociological context, the aesthetics of music have

1

been given a way to be measured objectively because they have been rooted in a reality formed by sciences such as philosophy, mathematics, and physics. It is only through an understanding of this context that the music theorist can provide a systematic thought, a "musical logic" that demonstrates a foundation of musical laws that give the untamed, creative mind structure and reason (Kraehenbuehl 1958, 22).

Ever since the emergence of harmony, which combines multiple voices simultaneously to create a phonic texture, Western music has brought forth countless efforts by musicians to establish a structure and framework consisting of rules on how tones are to be combined. The journey began with the introduction of counterpoint and polyphony in the medieval period and continued with chordal structures (homophony) and vocal textures up to six voices in the Renaissance. The shift towards chords built on top of a constant bass line (basso continuo) in the Baroque period began to show a shift towards tonal structure and chordal harmony as we know it today. Perfected in the classical period, harmony as well as the structure of a musical piece reflected symmetry, order, and balance. The rules of tonality were strictly followed and did not allow for much harmonic exploration until Romanticism interrupted the balance of harmony with tonal complexity and increased chromaticism. Gearing towards parallel tonalities went out of control when any chord was allowed to follow another, even if it led to something as radical as Schönberg's atonality. It was precisely in this context that music theorists looked for an unordinary musical logic behind what seemed to be a chaotic chordal sequence. The solution to the problem was found in the theory of dualism and polarity of tonal structures and harmonies, which reflected a symmetry, order, and balance in the midst of the chaos. Music theorists such as Moritz Hauptmann, Dr. Arthur von Öttingen,

and Hugo Riemann laid the groundwork for this particular modern thought of the nineteenth century. Despite the fact that their theories encountered difficulty due to the acoustic non-existence of the undertones, the idea of a harmonic polar opposite kept evolving, especially in the works of Ernst Levy during the twentieth century. Negative Harmony is a practical concept based on those theories, motivated by the efforts of explaining complex harmonies beyond any ordinary musical structure.

The first chapter of this paper dives into the rise of *harmonic dualism* within a scientific age by explaining the music theoretical concept from a scientific and historical perspective. It presents the theories of Hauptmann, Öttingen, and Riemann, who is renowned for the discussion on the existence of undertones, in contrast with the ordinary music theories of Western music as well as the criticism received in their time.

The second chapter elaborates on the evolution of a dualistic thought resulting in Ernst Levy's concept of *harmonic polarity*, which was established in the twentieth century. Approaching this theory from a mathematical and philosophical standpoint, Levy began to adapt his theory into his own compositions by posing harmonic paths towards the key center along with their polar opposite that reflects the same symmetry, function, and motion towards the key center despite their different character.

Ernst Levy's influence on commercial music began with the music of Steve Coleman as a melodic concept (1980s) and Jacob Collier as a harmonic concept (2010s). I will introduce both artists along with examples on their use of Negative Harmony in Chapter 3. More traits of polarity can also be traced in the music of Joni Mitchell, Herbie Hancock, Chick Corea and John Coltrane, who invite new colors into their compositions by presenting chord progressions or melodic improvisation that can be analyzed with the tools of Negative Harmony. Additionally, the last chapter will include examples of my own compositional use of harmonic polarity along with a reference to the artists and composers previously mentioned. Chapter 1: The Rise of Harmonic Dualism within a Scientific Age

The nineteenth century marks an era that was guided by a progressive and futuristic spirit. Romanticism provided a platform for the artist to break the rules and give way to his imagination. Following the sense of inspiration rather than structure musicians such as Liszt, Debussy, and Schönberg entertained a thought-provoking gamut of harmonic structures and progressions that find their climax in atonal space. Attempting to explain the logic behind these structures that allowed any chord to follow another, the mid nineteenth century gave birth to harmonic dualism; a music theory that approached harmonic structure from a symmetrical system of third and fifth relations. It redefined the relationship between chords and grounded the equivalence of major and minor chords in the thought that the minor tonality presents the symmetrical opposite of major. Germany was central to this movement, which faced its greatest difficulty in the particular way it had to be defended within the spirit of its time - the acoustical field of science.

The arts of the early nineteenth century was driven by the philosophy of idealism that holds to the belief that reality is born in the mind.¹ This worldview went hand in hand with romanticism, an art that "produced romantic poetry and painting and even romantic physics and chemistry" (Dahlhaus 1979, 99). Towards the middle of the century this standpoint was evaluated as too unsteady to classify an idea as real or truthful.

¹ Idealism was essentially a "belief in the necessary involvement of a perceiving mind in the constitution and comprehension of reality." Hegel, who later influenced music theorist and scholar Hermann von Helmholtz (1821–1894), argued that "all existence can only be understood as belonging to one mind," whether it refers to a human being or God (Davis 1995, 502).

Answers needed to be found in facts that could be scientifically proven as a natural phenomenon. By the end of the century, positivism became the spirit of a scientific age that foregrounded mathematics, reason, and logic. This thinking became essential for the music scholar who had to ground their theory in the laws of nature.² It represented Germany's *Weltanschauung* (worldview) which required the music theorist to discover the "underlying natural laws that are consciously or unconsciously obeyed by the artist" (Burnham 1992, 1).

In order to understand the establishment and criticism of harmonic dualism we need to view it precisely in the ideological context of the late nineteenth century Germany. Only this perspective sheds light on why the dualist saw an urgent need to address physical acoustics in his methodology and why the idea of harmonic dualism became a problematic concept within a scientific age.

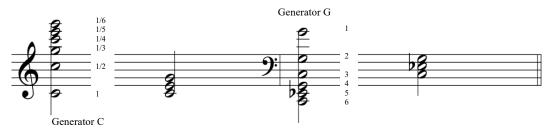
1.1 A Dual Theory: Term Definition

Harmonic Dualism is a theory of harmony that grounds the natural equivalence of major and minor modes in a two-fold system, presenting minor as the mathematical, acoustic, psychological, or philosophical opposition of major. Based on the overtone and undertone series that are calculated by harmonic division (above the fundamental) and arithmetical division (below the fundamental), the dual system defines minor as a mirrored image of major (see Example 1.1), (Snyder 1980, 45). They represent symmetrical opposites centered around the fundamental tone, which generates the

² "The German Romantic Weltanschauung is much in evidence here: Goethe's pronouncements proceed from the notion that music (and all art) is somehow an extension of nature, and that the origins of musical (and artistic) practices and phenomena are to be found in nature" (Snyder 1980, 47).

overtone series upwards and the undertone series downwards. We shall refer to the fundamental as the *generator*, which forms the major chord from the root upwards and the minor chord from the upper fifth downwards. Whereas the generator and fundamental of the major chord are the same tone, a conflict arises between the root and generating tone (fifth) of the minor chord; the root is not its fundamental.

Example 1.1 Overtones (left) and undertones (right) as the foundation for major and minor chords.



In addition, the opposition was justified in the emotional effects of both modes.

Moritz Hauptmann (1792 - 1868), who was the first to complete a dualist theory of

harmony, explains the opposition of major and minor in his book, Die Natur der

Harmonic und der Metrik (Nature of Harmony and Metre):

The minor triad, as an inverted major triad, must, in its meaning of being considered to originate from a negative unity, consist of a construction backwards...The minor triad thus being of passive nature, and having its starting-point above...and forming from it downwards, there is expressed in it, not upward driving force, but downward drawing weight, dependence in the literal, as well as in the figurative sense of the word. We therefore find in the minor chord the expression for mourning, the hanging boughs of the weeping willow as contrasted with the aspiring arbor vitae. (Hauptmann 1888, 16-17)

The basic principle is best summarized in the philosophic writings of Goethe, who

explained the concept in General Introduction to the Study of Music:

Thus, the fundamental C yields in an upward direction the C major harmony, downwards the F minor harmony, Major and minor are the polarity of harmony, the first principle of both. The major arises through ascending, through the upward tendency, through an extension of all intervals upwards; the minor through the extension of all intervals downwards...The carving out of such opposition is the basis for all music. (Moser 1949, 58)

According to those scientific establishments the dualist explains major and minor as "equal in importance, value, and conformity with facts of nature" (Jorgenson 1963, 31). It is the essential characteristic of harmonic dualism which lies "in conflict with the scientifically accepted concept of nature at that time" (Rehding 2003, 31). Though the ideas of a dual theory can be traced back to the sixteenth century, the nineteenth century theorists are most relevant to our study. For this reason, we limit our examination of harmonic dualism to the work of a group of German theorists – Moritz Hauptmann, Dr. Arthur von Öttingen, and Hugo Riemann – who devoted their work to explain the nature of the minor third and thus attempted to solve the much debated major-minor problem.

1.2 The Major-Minor Problem

Defining the minor triad has always presented a difficulty for the music theorist. Whereas the consonance of the major triad can be explained without trouble by referring to the first six ratios of the harmonic series, the "nature" of the minor triad remains a mystery (Hindemith 1937, 22).

The natural phenomenon of the overtone series states that a fundamental tone or pitch functions as a generator for other tones that vibrate simultaneously to their source. Though they are limitless, only the tones that are closest to the fundamental can actually be heard. As discovered by Pythagoras, the division of the string produces new tones that are multiples of the first fundamental frequency (see Figure 1.2.1).

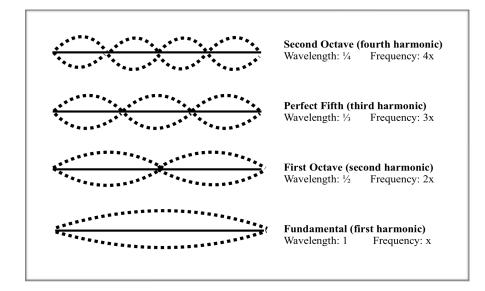


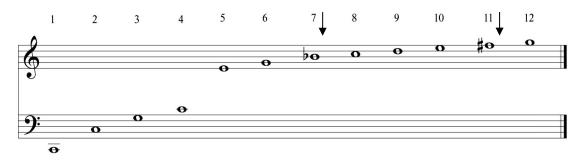
Figure 1.2.1 Formation of harmonics by string division according to Pythagoras.

The fundamental (first harmonic) generates all overtones, producing the primary intervals of an octave, fifth, and major third within the first six ratios. The octave is the most perfect consonance since it is produced by the first division of the string. Harmony and consonance are always dependent on the intervallic relation towards the fundamental tone. Rameau, the most influential music theorist on Western Music, explains the significance of relating consonant intervals to their origin:

From the different distances found between this fundamental sound and those it generates by its division, different intervals are formed. The fundamental sound is consequently the source of these intervals...Finally, from the union of these different intervals, different consonances are formed. The harmony of these consonances can be perfect only if the first sound is found below them, serving as their base and fundamental. (Rameau 1971, 8)

The reason we can classify a major triad as consonant is because it can be found in the first six ratios of the harmonic series (see Example 1.2), (Rameau 1971, 13). Referring to the overtone series as the only origin for consonance and harmony works for explaining the natural character of the major triad, however, it does not suffice as to define the character of the minor triad other than claiming it unnatural. By using only one system as a means of explaining the major mode as a natural phenomenon, the minor third becomes subordinate to major third and artificial in character (Levarie 1992, 29). This opens the door to two suggestions on how the problematic interval is derived: by chromatic alteration or harmonic inversion.

Example 1.2 Overtone series and generated intervals (first twelve ratios).³



French theorist and musician Guille-Louis Chrétien (1754 - 1811) proposed the theory of an altered third as an answer to the major-minor problem (Cotte 2001). In his book *La Musique etudiée come science naturelle* he suggests that the minor third is ultimately an interval that has been modified by being lowered a semitone; yet the minor chord remains perfect due to the perfect harmonic pillars of the tonic and the fifth.⁴ He responds to Rameau's writings in denying geometrical calculations based on the

³ The seventh and eleventh harmonics are slightly lower than the seventh interval that are not used in our tone system.

⁴ "There is a difference that is presented in two chords, one major, the other minor; but it is the same constitutive interval that gives them the quality of a perfect harmony by preserving both the same tonic and the same dominant. Therefore, the added or changed sound, which can only be understood as a modification, cannot be given a more suitable name than that of a major mediant, or a minor mediant: Thus the constitutive interval, understood as a fifth or a fourth, remains the only immutable and determinative power." Translated directly from original text, in which Chrétien argues, "On distingue bien une difference qui présente deux accords, l'un majeur, l'autre mineur; mais c'est le meme intervalle constitutif qui leur donne la qualité d'accord parfait, en leur conservant à tous deux la même tonique et la même dominante. Le son ajouté ou changé, ne pouvant être entendu que comme modification, ne peut done pas recevoir une dénomination plus convenable que celle de médiante majeure, ou de médiante mineure: alors l'intervalle constitutif, entendu comme quarte, reste la seule puissance immuable et déterminative" (Chrétien 1811, 5).

monochord⁵ as a foundation for music theory and suggests that only the major harmony can be derived from the resonance of a sonorous body (Shirlaw 1917, 329). The problem that arises with this view is as to how much alteration an interval can undergo before it loses its characteristic as well as harmonic meaning. Goethe responds to this theory with the following question: "If the third is an interval provided by nature, how can it be flattened without being destroyed? How much or how little may one flat or sharp it in order that it may no more be a major third, and yet still be a third? And when does it cease being a third altogether" (Levy 1985, 3)?

Composer Sigfrid Karg-Elert (1877 – 1933), who was the last German theorist of the dualistic school, responds with even more contempt towards the altered chord approach. In his book *Polaristische Klang und Polaritätslehre* published in 1931, he writes: "Since today the minor chord is still considered by many theorists to be a major chord with altered third, one had better cry than laugh! A handworker knows the distinction between his materials better than these people, who consider themselves to be musically learned" (Snyder 1980, 59).

The universally accepted understanding of harmonic generation is the theory of Harmonic Inversion, which was developed by Jean-Phillipe Rameau (1684 - 1763) in his *Traité de l'harmonie (Treatise on Harmony)* in 1722 (Shirlaw 1917, 77). His work was essential to the progress of music theory since it was the first to incorporate the theory of overtones into harmonic generation. Before the overtones were discovered in the seventeenth century and first scientifically explained by Sauveur in 1701, the theory of

⁵ A single-string instrument in which the string was divided by a moveable bridge in order to develop a theory of proportions for the purpose of tuning, the mathematical relationship between intervals, and the division of the chromatic scale (Cecil 2001).

harmony was based solely on proportions and ratios. Thus, we find Rameau's treatise in a historic timeframe that shifted towards modern tonality (Westerby 1902, 25).

Based on the division of a monochord string, Rameau claims at the beginning chapters that "with regard to intervals...only the octave, the fifth, and the major third are the directly generated by the fundamental sound" (Rameau 1971, 15). The fourth on the other hand arises from the difference between the octave and the fifth rather than being directly generated by the fundamental and is therefore "merely a result of the octave" (Rameau 1971, 13). Considering the fundamental of the minor third he makes an exception. Being critical of Descartes' theory that "the minor third is generated by the major [third] as the fourth is by the fifth," Rameau suggests that the minor third should be related directly to the fundamental sound rather than being indirectly derived from the division of the fifth (Rameau 1971, 17). He states that the fifth is the source of all chord formation, but it cannot be used to determine the third. This role can only be given to the octave which serves as an aid to determine intervals that are indirectly derived such as the sixth is derived by the difference between the octave and the third. He grounds his belief in arithmetic by asserting that the minor third between the fifth and sixth harmonics (E and G) is directly generated by the octave 5:10 (E and E'), (see Example 1.3). Rameau concludes that there are only three primary consonances the perfect fifth, the major and minor third; and three secondary consonances arise from the primary consonances, the fourth, the major and minor sixth (Rameau 1971, 16). The arrangement of the thirds within the fifth can be reversed without changing the foundation of the harmony:

The minor perfect chord can be discussed in the same way as the major, since it is similarly constructed and gives, by its inversion, the same chords as the major. The only difference is in the arrangement of the thirds from which the fifth is formed. The third which had been major on the one hand becomes minor on the other; the sixths which arise from them behave similarly. (Rameau 1971, 42)

Feeling at liberty to juggle the major and minor third within the fifth without changing its fundamental origin, the problem arises why this exception can be made for the minor third but not for the perfect fourth. Why does Rameau insist on the minor third to be originated by the fundamental whereas the perfect fourth is produced through inversion? Furthermore, his argument to allocate the minor third interval 5:6 (E-G) to the octave 5:10 contradicts his argument that the source of harmony and consonance is found in the generating tone, which in this case is the fundamental sound C (Christensen 1993, 96). He continues to get himself lost in arguments by driving attention to the fifth as the most important determinant of harmony, and concludes in his book *Nouveau Systéme*: "Since the fifth is the most perfect of all consonances, along with the octave, and since it may be composed of a major third and a minor third, the order of those thirds must be immaterial. At least this is what the ear decides, and no further proof is necessary" (Christensen 1993, 96).

During the Enlightenment Rameau formed a new theory according to the phenomenon of resonance, accepting both arithmetic and harmonic division to establish a theory for the major and minor harmony. In his book *Génération harmonique* (1737) he proposes a dualist theory that suggests the major triad being formed by the upper twelfth and seventeenth partial, whereas the minor triad is formed by the lower twelfth and seventeenth partial. According to this principle the fundamental sound C will generate both C major and F minor triads (Snyder 1980, 46).

Rameau's research led him back to the dualist principle. Since the overtone series brings the equality of major and minor modes into question and lacks in sufficiency as to

explain the minor tonality, the dualist sees the need for a new theory which traces the fundamental origin of the minor harmony and finds it in the symmetrically mirrored reflection of the major harmony.

1.3 From Hauptmann to Öttingen: A Dualist Response

Moritz Hauptmann (1792 – 1868) counts as an essential representative of the dual theory who influenced latter theorists such as Öttingen and Riemann. His approach differs from those theorists as it is defended philosophically rather than based on natural sciences. Due to the difference in methodology it is debated whether Hauptmann should be considered a harmonic dualist. Being the first to establish a dual theory of harmony in his book *Die Natur der Harmonic und der Metrik* (1853), he briefly introduces his book with the acoustics of overtones and undertones. Yet he does not count it as a determining factor for his epistemology:

...we certainly cannot expect a theoretical establishment of harmony in the wider sense, an establishment of the laws governing the connection and succession of chords, from such data only as the acoustic ratios...Neither the truth not the falsehood of the acoustical presuppositions has any further influence upon the doctrine itself; although in view of the untruth and half-truth of these presuppositions this can only redound to the advantage of the doctrine. (Hauptmann 1888, xxxviii)

Though Hauptmann did not use physical acoustics to defend harmonic dualism, he found a way to approach it from a different angle: Rather than grounding his theory in natural sciences he rendered them as insufficient for the establishment of a valid music theory. Due to the fact that he was still forced into this scientific debate, we shall include him in the list of German representatives of harmonic dualism.

Hauptmann's theory of harmonic dualism is characterized by the fusion of Gioseffo Zarlino's and Guiseppe Tartini's mathematical principles of harmony and Hegel's dialectical principles, which hold the unity and reconciliation of absolute opposites such as nature and spirit, mind and matter, at their core (Westerby 1902, 29-30). Therefore, the laws that music depends on go beyond physics and transcend into a universal law that governs the human mind. This universal law is represented in three stages: unity, division or separation by its opposite, and reconciliation by a mediating element. According to Hauptmann, these stages are represented musically by the three intelligible components of the major triad: octave (I), the fifth (II) and major third (III). In his book *The Nature of Harmony and Metre* Hauptmann writes, "The Octave is the expression for unity; the Fifth expresses duality or separation; the Third, unity of duality or union. The Third is the union of Octave and Fifth…The Third fills out the emptiness of the Fifth, for it contains the separated duality of that interval bound up into unity" (Hauptmann 1888, 6).

These beliefs are combined with the theory of harmonic and arithmetic division established by Zarlino (1517-1590) and Tartini (1692-1770). Their writings recognize the symmetrical relation between major and minor and explain both triads according to the ratios of a vibrating string. According to their theory, the major chord is found within the ratios 1, $\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{4}$, etc. (*Divisione Armonica*) that result from dividing the string, while the minor chord functions in reverse according to the arithmetical progression 1, 2, 3, 4, etc. (*Divisione Arithmetica*), (Zarlino 1558, 46-50). Two hundred years after Zarlino's established mathematical principles, Tartini organized the results of the harmonic and arithmetical progression in a diagram of undertones in order to confirm the symmetrical relation between the major and minor triad (Figure 1.3.1). Both Zarlino and Tartini conclude that the minor triad is a major triad inverted. Just like the major triad is related to the overtones, the minor triad is related to their symmetrical opposite; the undertones.

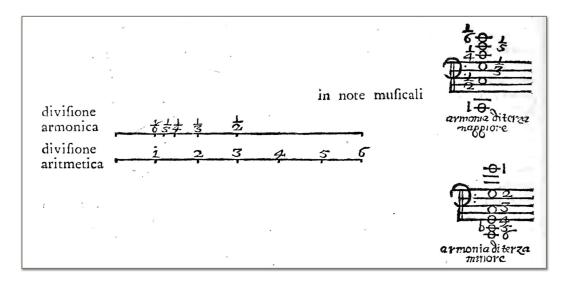
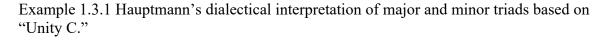
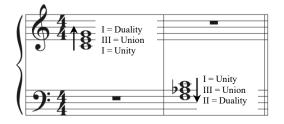


Figure 1.3.1 Tartini's harmonic division according to string division (overtones, upper right corner) and arithmetic division according to multiplication (undertones, lower right corner), (Tartini 1754, 66).

Following Tartini's theory, Hauptmann explains that the major chord is generated by the tonic - the positive element of unity which actively determines the fifth and the major third in an upward direction - whereas the minor chord is generated downwards from its fifth. Hauptmann concludes that the major chord is a positive generation determined by the tonic, whereas the minor chord functions in reverse as a negative generation originating in its fifth.





For example, the tone C generates not only a major chord upwards but also functions as a generator for the F minor triad which is constructed downwards (see Example 1.3.1). The generator, which is the tonic in the major triad and the fifth in the minor triad, represents the element of unity which determines the other components (duality and union) of the triads.

A tonality is formed when the central triad is surrounded by two triads within a distance of a fifth above, the dominant triad, and a fifth below, the subdominant. Thus, the major and minor tonalities are represented in the following illustrations:

I	III	II		Ι	III	II	
F	а	С	e	G	b	D	
		Ι	III	Π			
				→			

Figure 1.3.2 Hauptmann's dialectical interpretation of the major tonality. The roman numerals represent the tonal center establishing unity (I), the element of duality (II), and the reconciling element (III), (Hauptmann 1853, 27).

II d	III F	a	e	I b	
		•	 		

Figure 1.3.3 Hauptmann's dialectical interpretation of the minor tonality. The element of unity (I) is the fifth, the element of duality the root of the chord (II), and the reconciling element is the third (III), (Hauptmann 1853, 36).

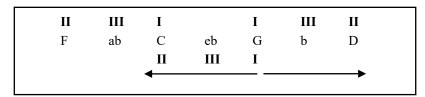


Figure 1.3.4 Hauptmann's dialectical interpretation of the harmonic minor tonality with two triads of "unity G" (Hauptmann 1853, 37).

The tension in Hauptmann's dialectical approach lies in his latter argument which

insists on the lowest tone (root) to be considered the starting point of the chord. Music

theorist Matthew Shirlaw refers to Hauptmann's analysis of the melodic minor system referring two triads to the unity of the fifth scale degree (see Figure 1.3.4), which leaves the tonality without a tonic function. From this standpoint it remains unclear how to interpret Hauptmann's harmonic progression (Snyder 1980, 51). Additionally, Hauptmann's ideas were "rendered inaccessible to any large circle of readers" due to their complexity (Westerby 1902, 29). Nevertheless, his concept is considered a big achievement as it presents a new approach to tonal harmony which finds its fullest expression in Hugo Riemann's theory of harmonic function (*Funktionstheorie*), (Caplin 1984, 257).

Only thirteen years after Hauptmann's work was published, Arthur von Öttingen (1836 - 1920) responded with a different approach on harmonic dualism. Given his career as a professor of physics, he based his theory entirely on natural sciences (Shirlaw 1917, 369). Based on Hauptmann's theory of unity, he establishes the components of the major chord as partial tones belonging to the fundamental whereas the components of the minor chord are in consonance with their first common harmonic. For example, the fundamental C generates a C major chord within its second, third, and fifth harmonic. The fundamental tone itself is in consonance with this chord since it also functions as its tonic. Applying this principle to the parallel minor chord, C minor, Öttingen evaluates its consonance according to its first shared overtone. The components C, E-flat, and G are all consonant with their phonic overtone G (see Example 1.3.2).

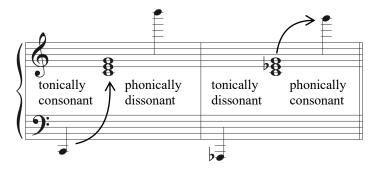
Example 1.3.2 Fundamental C ("tonischer Grundton") establishing C major in the overtone series (left), and its symmetrical opposite correspondent, C minor.



Therefore, the equality of major and minor triads is established by viewing them in light of their relationship towards the "tonic ground tone" (fundamental or chief combination tone) and "phonic overtone" (first common partial), (Hauptmann 1853, 36-37). The major chord is consonant with its tonic ground tone whereas the minor triad is consonant with its phonic overtone. Additionally, this experiment highlights the symmetry between both tones towards their tonal center, which can be measured in octaves. The symmetry becomes even more striking by considering that both triads are in consonance with their generator.

Öttingen takes his theory one step further by viewing the C major chord in relation to its first common partial, B. Since this tone is not a component of the major triad, he labels it phonically dissonant (Example 1.3.3).⁶

Example 1.3.3 C major as overtones of C (two octaves below) and their first common partial B, and its symmetrical opposite, C minor with the equivalent fundamental A-flat and its first common partial G.



He applies the same principle to the minor chord by mirroring the intervallic relationship between the generator and the phonic overtone of the major triad (two octaves and a major seventh) in the opposite direction. The point of mirroring in the

 $^{^{6}}$ The overtone B belongs to the overtone series of C as the fifteenth partial, the overtone series of E as the third partial, and the overtone series of G as the fifth partial. It is the first common overtone shared between C, E, and G.

minor chord is its generator, G. Creating a mirrored opposite, Öttingen establishes the tone A-flat below the C minor chord, which corresponds to the dissonant B in the previous major chord, as the tonic ground tone. Since A-flat is not consonant with the C minor chord, the triad becomes tonically dissonant.

In the same way the phonic overtone G which is located two octaves above the C minor chord, corresponds inversely to fundamental C two octaves below C major. This reserved process shows the symmetrical relationship between major and minor and labels minor as phonically consonant with the phonic overtone G, and tonically dissonant with A-flat (Example 1.3.3).

Öttingen concludes that major and minor are both equally consonant and dissonant. According to his theory of phonicity and tonicity, he continues to categorize the C major chord as a tonic sonority (C+) and the F minor chord built downwards from C as a phonic sonority (°C).⁷ With this theory being well assembled, however, it raised the question if the first common overtone can be assigned to the same important role as the fundamental:

It is not clear why in one case a lower tone...and in the other a higher tone...should be adduced. The basic error of this explanation is, however, that it reckons with actual tonal relations which are no sooner sited in the case of one triad than they are disregarded in the case of the other. If the overtones of the minor triad are significant, then so are those of the major triad. (Hindemith 1937, 77)

Even though Öttingen's theory of harmonic dualism evoked serious criticism in its time, it paved the way for Hugo Riemann (1849 - 1919), the most important representative of the dualistic movement.

⁷ Major and minor chords are both labeled according to their generator. For example, $G = G^+$, $Gm = {}^{\circ}D$.

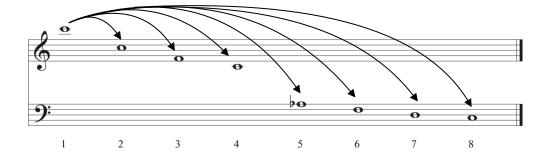
1.4 Concerning Undertones: Hugo Riemann's Theory of Harmonic Function

Inspired by Hauptmann and Öttingen's theory of presenting minor in a symmetrically opposite system, Riemann based his defense on the elements of nature, logic, and history (Rehding 2003, 99). During his career he was determined to prove the existence of the undertones in order to validate the natural consonance of the minor triad. Grounding the major and minor triad in two opposite systems, Riemann placed every chord in relation to its harmonic center, the tonic chord, and analyzed its function according to a harmonic "logic" found in a chord progression I-IV-V-I based on third and fifth relations.

Approaching harmonic dualism from a historic standpoint, Riemann sought to legitimize his theory by tracing dualistic traits in the historical evolution of harmony. In his compilation *Geschichte der Musiktheorie im IX. - XIX. Jahrhundert*, Riemann credits Zarlino with the discovery of dualism and reads his own theories into the works of Tartini and Rameau (Riemann 1920, 389). Even though they mention the harmonic and arithmetic divisions of the fifth, they have neither argued for the minor triad to be constructed downwards nor insisted upon the equality between major and minor. Rather, those divisions were used to derive intervallic relations through inversion (Dahlhaus 1957, 290). Nevertheless, Riemann's interpretations of Zarlino, Tartini, and Rameau's arguments show how much influence their concept of opposition through harmonic and arithmetic division had on the dual theory.

Riemann was the first to officially name the reversed order of all harmonics the *undertones*. His entire theory of harmonic dualism is based on the reality of undertones which prompted him to prove their existence throughout his career. He based the major

triad on the natural phenomenon of the overtone series and the minor chord entirely on the undertone series, which he perceived as equally natural (Riemann 1900, 571)⁸. He used the term "prime" for the tonal center which generates the major triad upwards and the minor triad downwards. Implementing Öttingen's ideas, he establishes the "phonic overtone" as the prime for the minor triad constructed downwards" (Riemann 1896, 10). Example 1.4.1 Undertone series based on "prime" C (first eight ratios).



Riemann seizes upon Hauptmann in posing only three intelligible intervals: the octave, perfect fifth, and major third. Not only the minor chord which functions in reverse, but "all other intervals are to be explained musically and mathematically as the results of multiplication and involution of these three" (Riemann 1896, 6). From these intelligible intervals evolves a harmonic logic represented in the chord progression I (tonic) — IV (lower-fifth) — V (upper-fifth) — I (tonic) and its dual opposite i - iv - v - i (Figure 1.4.1). They represent a circle of motion with the subdominant (antithesis) leading away from the tonic (thesis), striving up towards the dominant (synthesis) which forces the harmony through the leading tone back to the tonic. This harmonic logic served as an essential argument against modern music from the nineteenth century as it

⁸ Riemann writes regarding the consonance of major and minor: "Die Moll-Konsonanz wird in ganz derselben Weiße auf eine Untertonreihe bezogen wie die Dur-Konsonanz auf die Obertonreihe" (Riemann 1900, 571). Translated this means, "The minor consonance relates to the undertones in the same way as the major consonance to the overtones."

stood against the harmonic freedom that allowed any chord to follow another chord. According to Riemann, harmony had its boundaries in a logic that does not allow for any random chord progressions but is restricted to the significance of the scale degrees (Riemann 1872, 280). Each progression was only valid if it could be explained according to a harmonic logic (see Figure 1.4.1).

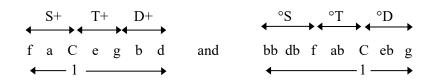
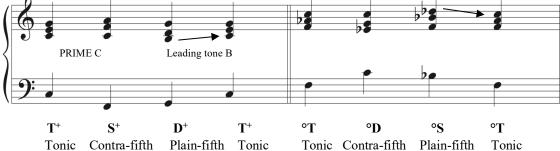


Figure 1.4.1 Complete cadence with its dual opposite. The number '1' marks the prime of the chord (based on Riemann's illustration in Harmony Simplified, 29).

The complete cadence can thus be summarized in the tonic surrounded by the "contra-fifth" (subdominant pushing downwards, leading the ear away from the tonic), and the "plain-fifth" (dominant pushing upwards, leading the ear towards the tonic). Whereas the "contra-fifth" requires motion, the "plain fifth" provides resolution towards the harmonic center. In major, this principle allows for the progression T^+ - S^+ - D^+ - T^+ . The leading tone is found in the dominant, a half step under the prime. In minor, the roles are reversed and result in °T - °D - °S - °T, which places the leading tone a half step above the prime (Example 1.4.2).



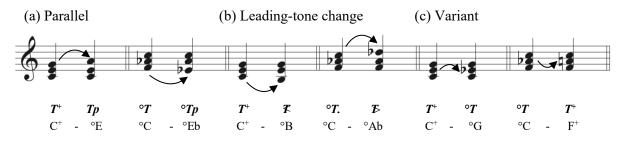
Example 1.4.2 Complete major $(C^+-F^+-G^+-C^+)$ and minor cadence $(^{\circ}C-^{\circ}G-^{\circ}F-^{\circ}C)$ as dual



The remaining chords within the diatonic scale function as representatives of this harmonic prototype. Though they are built on different scale degrees and are typically labeled accordingly with Roman Numerals, Riemann reminds the reader about the difference between the chord and its function. He focuses entirely on the harmonic pillars (T, S, D) represented within a cadential context. This idea is summarized in his theory of harmonic function, which allows for harmonic relation by sharing two common intervals while changing one tone to its neighbor tone.

For example, the chord built upon the sixth scale degree in the tonality of C major, which is Am (°E), functions in the same way as the tonic. Both chords share two common notes (C and E) while the fifth of the tonic (G) is modified to its upper neighbor-tone (A). This "parallel" modification constructs an *apparent consonance* (*Scheinkonsonanz*) since it carries the same function as the tonic chord (see Example 1.4.3.a).

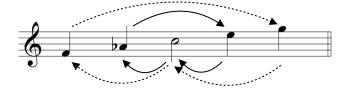
Example 1.4.3 Apparent Consonances for tonic function in major and minor (can be applied to all harmonic pillars).



Due to the fact that the tonic and submediant carry the same harmonic function, Riemann's analysis introduces the label Tp for the submediant chord (vi). The same concept applies to its dual opposite, Fm (°C). The fifth is considered from the starting point of the prime C, which leads to a motion towards the lower neighbor-tone E-flat (Example 1.4.3.a), (Rehding 2003, 55). Riemann's analysis also provides a new interpretation of the mediant chord built upon the third scale degree (iii) which functions as a tonic through a leading-tone change (Example 1.4.3.b), and the parallel tonic chord which is approached through a variation of the third (Example 1.4.3.c).

Riemann's theory of harmonic function becomes problematic in the process of explaining root-interval progressions with only one common tone, such as C+ and E+. To provide an effective solution, Riemann explains the concept of the dualistic *Klang* (sonority), which combines the major and minor triad around the tonal center and allows for fifth relations (IV - I - V) to be replaced with third-relations (bVI - I - III) in order to establish a tonality (see Example 1.4.4), (Riemann 1877, 18-19). Therefore, a major or minor triad located a major third below the tonic carries the same function as the subdominant (*°Sp*) while a triad located a major third above the tonic represents the upper-fifth.

Example 1.4.4 Riemann's Dualistic Klang (Prime "C") with third and fifth relations.



An example of the blending of the theory of harmonic function along with the theory of fifth and third relations is illustrated in Riemann's analysis of the chord progression C - Ab - D7 - G7 - C, provided in his book *Musik-Lexikon* (see Example 1.4.5). He begins by determining the tonal center as C major and follows with analyzing Ab major as a third relation functioning as a subdominant parallel (*Sp*) since it is carrying the same function towards the tonic center as the subdominant. He continues

with analyzing D major as the double dominant (V/V = DD), leading towards the dominant chord G7, which leads the listener back to the tonic chord C major.

Example 1.4.5 Riemann's harmonic analysis of third and fifth relations (Tonic, Subdominant-parallel, Secondary Dominant, Dominant, Tonic), (Riemann 1905, 668).



1.5 The Fallacy of Harmonic Dualism

The theory of harmonic dualism faced the greatest challenge in the justification of undertones through acoustics. Within the context of a scientific age, only the existence of undertones could legitimize Riemann's dualistic ideas and convince others of their truthfulness. Trying to validate his theory, he based his first argument on the premise that undertones were generated in the ear. His statement which he summarized in his doctoral dissertation *Über das Musikalische Hören* was rejected at the University of Leipzig and faced great opposition (Rehding 2003, 33).

Riemann's second attempt to prove his theory began with listening for undertones in the sound wave. In 1875, he claimed to hear undertones ringing simultaneously in the sound wave of the tone he played on the grand piano ("moonshine experiment"), (Rehding 2003, 15). With this experiment he tried to justify the undertones on the same "wave-length" as the harmonic series. Having already established that the fundamental vibrates simultaneously with higher frequencies only, Riemann's attempt appears impossible. This fact is validated on mathematical and acoustical grounds. On the other hand, the reversed principle would produce frequencies lower and larger than the fundamental (see Figure 1.5.1). They cannot be heard unless they are actually being played as separate tones (Constantinsen, 2019). For this reason, Riemann would have been right to argue that undertones exist as separate tones carrying the characteristics of new fundamentals rather than harmonics. Since they represent the opposite of the overtones, it would only make sense for their roles to be reversed as well.

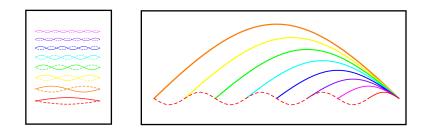


Figure 1.5.1 Visualization of wavelengths of overtones (left) and undertones (right) by Constantinsen.

Facing the impossibility of finding undertones within the wavelength, he approached this concept from a new angle. After 1891, he argued that "undertones were necessarily inaudible due to acoustical interference between sound waves" (Rehding, 2003). After 1905, the paradigm of science shifted its standards to psychology which freed Riemann from the necessity to search for undertones in the acoustic realm.

Harmonic dualism was rejected on the basis of natural science. Given the discrepancy between Riemann's musical ideas and the musical standards of the nineteenth century, harmonic dualism suffered the most resistance over the debate on undertones. Nevertheless, Riemann's concept which evaluated the aesthetics of past composers, carried over into the 20th century and continued to impact the music world from an ethical standpoint.

Chapter 2: Harmonic Polarity: A New Perspective on Harmonic Norms

The establishment of harmonic dualism in the late-romantic period in Germany continued its influence on the musical arts and left a significant impression on Swiss composer, music theorist, pianist, and conductor Ernst Levy (1895 - 1985), who reconciled a dual system of harmony by spiritual and mathematical principles. After his death, Levy's writings on tonal harmony were edited by his friend Siegmund Levarie and published under the title *A Theory of Harmony*. Introducing a widened perspective on harmony as well as recognizing the generative force of the major third is central to Levy's innovations. Levarie ascribed great significance in Levy's musical efforts in harmonic polarity which continued dualistic ideas stretching from Zarlino to Riemann (Levy 1985, vii-viii). From today's musical standpoint, Levarie's instincts have proven to be very beneficial to the study on Negative Harmony. Not only has Levy reconciled the relationship between major and minor in a polar system of overtones and undertones, but he also left an impact on current compositional techniques that gave birth to Negative Harmony. For this reason, we shall evaluate Levy's hypothesis in depth.

2.1 Ernst Levy's Biographical Background

Levy's journey began in Basel, Switzerland, where he was born into a Jewish family. From early on he showed great interest in music. His ability to perform Haydn's D-Major Piano Concerto Hob. XVIII:11 at the age of six justified Levy as a prodigy child, whose talent and skill exempted Levy from public school in order to take piano classes at the Basel Conservatory under Hans Huber (Bräm 2010). After completing four years of studies in 1910, Levy left the Conservatory with a certificate of completion by the time he was only 15 years old (Levy by Falciola 1980, 23). He travelled to Paris and continued his studies under Raoul Pugno (1852 - 1921), a French composer and pianist. During his time in Paris he also became acquainted with composer Franz Liszt.

After receiving his high-school diploma in 1914, Levy was able to return to Basel to continue his studies in musicology, philosophy and art history. In 1916 he was offered to teach virtuoso classes alongside Hans Huber, which did not leave him with time to finish his studies (Baldassarre 2018, 194). One year later Levy took over Huber's position after being interrupted in the middle of his class to receive the news that Huber had fallen ill. About this incident, Levy says, "He never returned, so I became the successor of Hans Huber as professor —but not director— of the virtuoso class" (Levy by Falciola 1980, 23). In 1920, despite Levy's love for teaching, he took a one year's leave after feeling that "it was not anymore wine that came out but distillate" (Levy by Falciola 1980, 24). A letter from his childhood friend Paul Boepplé confirms that the leadership change at the Conservatory due to Huber's leave brought difficulties that affected Levy's treatment as a professor.⁹

Levy exhausted all possibilities in Basel and went to Paris in 1920. Despite the fact that he planned to reside there for only one year, he remained in France for the next twenty years. In order to make a living Levy continued to teach music and played recitals

⁹" Es ist einfach abscheulich und empörend wie Sie's Dir in Basel gemacht habe[n?]...Sie werden bald sehen, was sie durch ihre Eseleien verloren haben. Ich bin sicher dass Du bald Ersatz findest." (Paul Boepplé to Ernst Levy, 1921, 31)

in addition¹⁰ and devoted time writing musical articles for *Basler Nachrichten* (Bräm, 2010).

During Levy's years in Paris, he discovered that there was a great lack of knowledge about classical masterworks among the Parisians. Prompted by a great need for choirs, he founded the French choir *Chśur philharmonique*. Levy's investment earned Paris its first performance of *Ein Deutsches Requiem* by Johannes Brahms as well as Liszt's oratorio *Christus*. Levy's efforts also lead to the very first recording of Liszt's *Missa Choralis*, which was published by the label Polydor in 1935 (Ernst Levy Collection, 2017).

In the early morning of June 14, 1940 German troops invaded Paris (BBC January 23, 2019). The German invasion forced Levy to leave his teaching position in France. His first attempt to flee the war lead him back to Basel, Switzerland. Unable to find work due to the war and spirit of anti-semitism, he escaped to the US in 1941 (Ernst Levy by Falciola 1980, 35). This move proved to be advantageous for Levy, who received the opportunity to teach at the New England Conservatory of Music in Boston (1941 - 1945), at Bennington College in Vermont (1946 - 1951), the University of Chicago (1951 - 1954), the Massachusetts Institute of Technology (1954 - 1959), and Brooklyn College of the City University of New York (1959 - 1966), (Oron 2007). His lectures concerned composition techniques, music theory, musicology, and music theory, on which I will elaborate in this chapter.¹¹ Among his works that he composed were symphonies, orchestral suites, chamber music, sonatas, and pieces for piano. His lectures also reflected

¹⁰ Four days a month Levy travelled to La Chau-de-Fonds, Bienne, and Basel in order to give lessons. Due to the currency exchange his journeys to Switzerland allowed him to finance his living in Paris.

¹¹ "Catalogue des Ecrits de Ernst Levy" (1979) presented by the University of Chicago.

the correlation between physics, mathematics, and music. Levy states, "everything that has something to do with mathematics can somehow acquire a musical meaning and thereby become a kind of explication or initiation or revelation of things which materially or mathematically would not tell us anything" (Levy by Falcolia 1980, 42).

In 1963, Levy was forced to retire from playing piano due to Guillain-Barré syndrome, an excruciating sickness that begins with weakness in the limbs, eventually causing paralysis. In 1966, Levy returned to Switzerland in order to retire in Morges at Lake Geneva¹² where he died on April 19, 1981.

2.2 Levy's Musicalization of Mathematics

When Levy laid the foundation for the theory of harmonic polarity, he approached his ideas from a philosophical and spiritual standpoint that realized the "musicalization of mathematics."¹³ In order to establish harmonic rules formed by nature he looked into the natural character of music. He found a correspondence between an inner, spiritual force and an outer physical object, which is tone. Music carries both of those elements; the particular constellation of notes according to a music theory and the emotional effect it evokes. The formation of principles around tonal structure is considered the physical fact of music that informs us about potential tone relations and explain the science behind aesthetically pleasing tone combinations and mode mixtures. It is the tool we need in order to identify harmonic norms.

¹² Levy mentions the location of his retirement under its French name Lake Leman.

¹³ Levy explains in his lecture to the Musicological Society of America in New York, "Pythagoras, who is said to have discovered the reaction between the length of strings and the pitch, was delighted not by the fact that music could be transformed into numbers, but by the fact that numbers could be heard. Here we have that "musicalization" of mathematics." (Levy 1942, 3)

Continuing Riemann's line of thought, Levy based his framework of tonal structure on the principles of Pythagoras, who found a correlation between the length of strings and pitch (Wagner 1977, 136). It considers the division of the string as well as its multiplication as the foundation for a dual tone-system, which portrays notes being generated from a fundamental tone in an upward and downward direction - the overtones and undertones. By adopting this framework, Levy found himself in the same debate over undertones as Hugo Riemann, but his solution lay far away from acoustics.

Levy was greatly aware that a string can technically not be multiplied since it cannot increase its length (Levy 1985, 51). Though it is considered highly impractical to double strings lengths since the tones would simply become too low to bear significance, it can be done on a mathematical level. By multiplying the string according to whole numbers and transposing the results into proper octaves, the tones keep their core identity while presenting an acoustical roster of a mirrored version of our harmonic series - the undertones (Levy 1985, 5). Levy created a parallel natural phenomenon from a spiritual and mathematical standpoint that is independent of the physical existence of undertones. Holding this perspective, he avoided the need to justify them acoustically all together:

The basis for that [Pythagorean table] development is the string experiment, that is to say, the division and multiplication of a string. This statement is important, as it means absolute abstention from taking into consideration the natural overtones and hence abstention from entering the ill-famed discussion over the existence of under-tones. Our work is based upon a spiritual principle, not upon an acoustic fact. (Levy 1945, 5)

Despite denying the existence of undertones in nature, Levy argued that "undertones...do exist musically in our psyche" (Levarie and Levy 1980, 51). The experience itself becomes a theoretical reality validated in mathematics. Establishing this new standpoint allowed Levy to present major and minor as equivalent consonances without having to defend their natural character. Having arrived at this conclusion we can define harmonic polarity along with the following statement:

The polarity theorist will define the triad as the musical aspect of the senarius [first six ratios] in its two reciprocal forms. He will further say: major and minor are perfect and equivalent consonances. They are reciprocal phenomena, and a reciprocal mathematical operation presides over their physical production. Hence they are a manifestation of polarity, one of the great principles fashioning not only the outer world of nature but also the inner world of thought and imagination. (Levy 1985, 13)

2.3 The Pythagorean Table: An Approach to Tone Structure and Tone Relations

The theory of polarity according to the *reciprocation method* - the symmetric mirroring or reproduction of intervals¹⁴ - produces the *positive series* through the wavelength division that doubles frequencies (overtones), and the *negative series* through the wavelength multiplication which divides frequencies (undertones). Levy presents the mathematical results of this method in form of the Pythagorean Table (see Table 2.3.1).

The point of mirroring is a single tone called generator. Levy chose C (1/1) as the main generator and established a system of wavelength division ($\frac{1}{2}$, $\frac{1}{3}$., $\frac{1}{4}$, etc.) and multiplication (2/1, 3/1, 4/1). It is to be noted that any tone in our twelve-tone system can function as a generator, meaning this table can be applied to any note. Once a generator is chosen, the Pythagorean Table fully reveals the positive and negative series of this generator as well as the positive series of each "undertone" (horizontally), and the negative series of each "overtone" (vertically). It is limited to an *index*, a number of nine divisions and multiplications for our purposes.

¹⁴ Levy explains, "reciprocation means reproducing an interval in the opposite direction - an operation clearly distinguished from inversion, which is the reproduction of a tone in the opposite direction." (Levy 1985, 6)

•	POSITIVE SERIES								
	• 1/1 c	1/2 c1	1/3 _{G1}	1/4 C2	1/5 E2	1/6 _{G2}	1/7 < Bb2	1/8 c3	1/9 D3
	2/1	2/2	2/3	2/4	2/5	2/6	2/7	2/8	2/9
	c1	c	G	C1	E1	_{G1}	< Bb	^{C2}	D2
	3/1	3/2	3/3	3/4	3/5	3/6	3/7	3/8	3/9
	F2	F1	c	F	^	c1	< Eb	F1	_{G1}
SERIES	4/1	4/2	4/3	4/4	4/5	4/6	4/7	4/8	4/9
	c2	c1	_{G1}	c	E	G	< Bb	c1	D1
NEGATIVE SERIES	5/1	5/2	5/3	5/4	5/5	5/6	5/7	5/8	5/9
	Ab3	Ab2	Eb1	Ab1	c	^{Eb}	< Gb	Ab	_{Вb}
N	6/1	6/2	6/3	6/4	6/5	6/6	6/7	6/8	6/9
	_{F3}	F2	c1	F1	A1	c	< Eb	F	G
	7/1	7/2	7/3	7/4	7/5	7/6	7/7	7/8	7/9
	> D3	> D2	> A2	> D1	> F#1	> A1	c	> D	> E
	8/1	8/2	8/3	8/4	8/5	8/6	8/7	8/8	8/9
	c3	^{C2}	_{G2}	c1	E1	_{G1}	< Bb1	c	D
	9/1 вь4	9/2 Bb3	9/3 F2	9/4 Bb2	9/5 A1	9/6 F1	9/7 < Ab1	9/8 Bb1	• 9/9 c

Table 2.3.1: Pythagorean Table according to Levy – identity rays crossing through C1 (0/0, 1/2, 2/4, 3/6, 4/8).

Levy's illustration shows that "every tone is located at the intersection of a major and a minor series" (Levy 1985, 6). Each tone becomes part of a chain that reveals how the two systems of overtones and undertones are related and interlocked. Ultimately, each tone carries a magnetic force towards the main generating tone. Levy illustrated this magnetic force by drawing a straight line starting from the main generator C through the center of each square which represents the same note. Those *identity rays* connect each tone of the same character. Drawing lines diagonally starting from the upper left corner (0/0) within different angles shows the connection between notes that carry the same tone identity (Levy 1985, 9).

Just like suggested by Hugo Riemann, the first six ratios of the positive series referred to as *senarius* reveal the formation of a major-chord in index one (1/1), three ($\frac{1}{3}$), and five ($\frac{1}{5}$), whereas the negative series organizes the symmetrical distances in the opposite direction, resulting in a minor chord on the same index (1/1, 3/1, 5/1), (Riemann 1920, XXI). Table 2.3.2 illustrates the major and minor chord formation around the generator C.

Table 2.3.2 Senaric				

Ratio	5/1 (third)	3/1 (fifth)	2/1 (octave)	1/2 (octave)	1/3 (fifth)	1/5 (third)	
Senarius	5	3	2	2	3	5	
Notes	Ab	F	С	С	G	E	
Chords	F - Minor			C - Major			

Levy added a provocative idea by advocating the formation of a natural dominant seventh chord (C7) within the positive series and a minor six chord (Fm6), which characterizes it as a subdominant, in the negative series. However, there are a few problems that arise with this theory.

It is to be noted at this point, that the tones located in index seven (1/7 = Bb and 7/1 = D) are technically not used in our twelve tone system since their pitch is either 31 cents higher (>) or lower (<) than the seventh in our current tone system (Swoger-Ruston 2006, 51). For this reason, the lowered seventh ratio has led to a dispute denouncing this formation as the basis for the dominant seventh chord. Rameau even went as far as calling it a dissonance that "cannot give a pleasant interval" (Rameau 1971,

6). He skipped the seventh partial in his writings and replaced is by the eighth partial, the third octave. Though he deviated from explaining the seventh interval at first, he proposed in a later chapter that the seventh chord is born from stacking a third on top of a major chord (Rameau 1971, 42).¹⁵ This idea was adopted by German theorist Paul Hindemith who disregarded polarity altogether. In his book *The Craft of Musical Composition* he writes, "The seventh overtone in the series based upon C (-bb1) does not make the triad into a dominant seventh chord such as we know in practice. It is flatter than the bb that we are used to hearing as the seventh of c" (Hindemith 1942, 24).

Levy was aware of the difficulty that comes with presenting the minor seventh as the seventh partial, especially since our tone system excludes it (Levy 1985, 46).¹⁶ He argued for a new norm that recognizes the natural seventh rather than the diatonic seventh belonging to the triad. Not only does it characterize a major triad as a dominant chord, but also, as a characteristic dissonance, it becomes an integral part of the chord that resolves into another chord. In the negative series, the minor seventh below the fundamental added to the minor triad results in a minor six chord, which characterizes it as a subdominant. Levy continued to support his argument by pointing out the position of the seventh in the positive series. Being the seventh partial, it also appears within the same octave as the triad is completed. He stated, "the seventh tone within the last of three senaric octaves would indicate that in a certain measure it belongs to the triad" (Levy 1985, 46). After hearing the natural, not diatonic, minor seventh by experimenting on a

¹⁵ Rameau writes, "it is preferable to present immediately the chord which is the most perfect of all dissonant chords, even though the false fifth occurs there in the upper position. This chord seems to exist in order to make the perfection of consonant chords more wonderful, for it always presides them, or rather should always be followed by the perfect chord or its derivatives."

¹⁶ Levy writes, "At the point where the overtone series transgresses the senarius, the flat seventh appears: it has no place in our tone system, nor do any of the subsequent partial tones which occupy the position of a prime number."

sonometer, Levy was convinced that it melts into the chord and should have greater influence in our tone system in the future.

2.4 A Theory of Harmonic Polarity

What marks the difference between Riemann's dualistic approach and Levy's theory apart from their philosophical standpoint is their approach to a tonality center. Whereas Riemann's musical thought "signifies the unified relationship of chords to a central tonic," Levy viewed chord relationships as being centered around generators (Dahlhaus and Gjerdingen 1990, 625). Tonality is no longer interpreted on a linear basis which allows notes to gravitate towards the fundamental or tonic (*telluric gravity*), but rather it suggests that chords spin around a tonal center, the generator, from two opposing directions - positive and negative. Chords from the positive side (major chords) are generated upwards and therefore gravitate down. Chords from the negative side (minor chords) are generated downwards and gravitate up.

In order to visualize the concept, Levy painted a picture that resembles a planetary system: Chords revolve around a generator and by keeping the same intervallic distance, they form major seventh chords on one side and minor sixth chords in the opposite direction (see Figure 2.4.1). The major seventh chord is generated by its root. The minor sixth chord is generated from above by its dominant. Viewing harmonic relations from a polar perspective of major and minor chords being generated in opposite directions is called *absolute conception* (Levy 1985, 13-15).

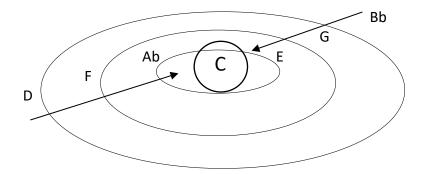


Figure 2.4.1 Absolute Conception of tonality (Polar opposites: Fm6 chord on the left and C7 on the right).

Thus, we have established a system of polar opposites which does not limit chord structures to be formed upwards from a fundamental bass only. Though this approach (telluric gravity) can be applied to the major triad in root position since the fundamental bass note and generator are the same, the polar theorist uses the opposite formula for minor (absolute conception). Whereas a major chord is perceived by the ear from a fundamental upwards, the minor chord is perceived from the generating tone downwards. Gravity is no longer a one-way street, which led Levy to the following conclusion:

The influence of gravity does not affect the major triad, for the generator C is also the fundamental of the chord. But in the other member of the pair, in the minor triad, the generator and the fundamental become divorced...we ought to hear the minor chord generated by C as C minor, but "tellurically" we do hear it as F minor. This inner schism between structure and apperception is based on polarity. (Levy 1985, 15)

From this standpoint we apply the polarity principle to the cadence of three tonal functions: Tonic, Subdominant, and Dominant. From these functions we derive the notes of the major scale. Mirroring this particular chord progression (I, IV, V, I) downwards

from each individual generator would obtain the minor chord progression -I, -V, -IV, -IV, which projects the natural minor scale (see Example 2.4.1).¹⁷

Example 2.4.1 Absolute Conception of C major and F minor as polar opposites through harmonic projection.

(a) Absolute Conception: symmetrical (b) Telluric Gravity: Non-Symmetrical

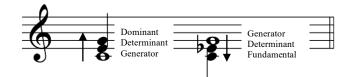


This example shows that the results of a perfect minor cadence as a symmetrical mirrored image of major can only be observed through absolute conception. The C major scale, for example, mirrors a descending C Phrygian scale which cannot be harmonized tellurically since the dominant chord would be diminished. Also, in absolute conception the polar opposite of C major is F minor, not C Phrygian. Levy provided a solution to the dilemma by taking the tonal content of C Phrygian but shifting the starting point to the tonic F, obtaining a prefect F minor scale. Why is this essential? Levy showed with this example that a major and minor scale differ slightly in their intervallic symmetry if they are viewed tellurically, meaning from the starting point of the tonic (Example 2.4.1. b). But if we broaden our spectrum by mirroring the content of the major cadence as well as tonal scale content and consider the right starting point, we can see the symmetrical relationship between major and minor.

According to the theory of polarity we assess an important correlation between fundamental and dominant: Both can function as generators. The strong magnetic force

¹⁷ The minus in front of the roman numeral (such as -V) in Levy's analysis has the same meaning as Riemann's circle in front of a chord (for example, °C). It signifies to a minor chord and the generating tone.

between those intervals is naturally given from the position they maintain in the overtone and undertone series. For example, the fundamental C (1/1) generates the fifth G in the second octave ($\frac{1}{3}$). In absolute conception, G ($\frac{1}{3}$) generates C (3/3) in the opposite direction (see Pythagorian Table 2.3.1). The direction of the magnetic force can only be determined by adding the third. It is only through the addition of this interval that the direction of musical motion can be directed. For this reason, Levy labeled the third interval *determinant* because not only does it define the gender of the chord, but it also magnetizes the current to a specific direction (Levy 1985, 23). Example 2.4.2 Magnetic force of major and minor triads.



Since dominants can function as new generators Levy constructed tonality in a system of dominant relations within absolute conception. It is illustrated in a grid of generating tones with major chords to their right and minor chords to their left side.

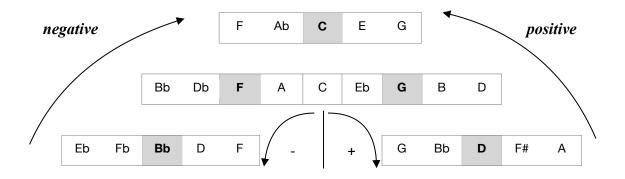
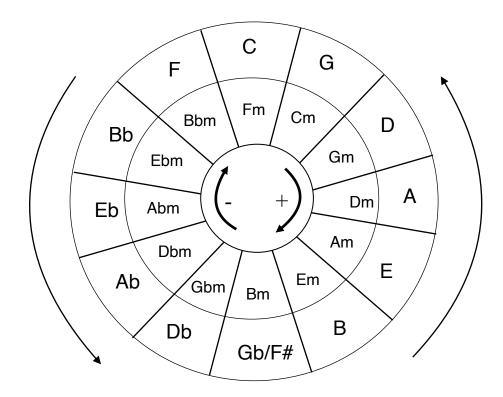
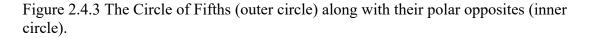


Figure 2.4.2 Generators producing major and minor chords within dominants according to Levy (arrows visualize gravity within circle of fifths).

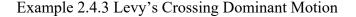
Levy categorized the chords resulting from this organization according to their gravity which produces two currents flowing in opposite directions. The major chords on the positive side (C, G, D, etc.) and the minor chords on the negative side (Fm, Bbm, Ebm, etc.) are magnetically drawn towards the main generator (C). All remaining chords, which are minor chords from the positive side (Cm, Gm, etc.) and the major chords from the negative side (F, Bb, etc.) provide a contrary motion leading the ear away from the tonal center (Figure 2.4.2). Levy's results of polar opposites within the circle of fifths are visualized in Figure 2.4.3.

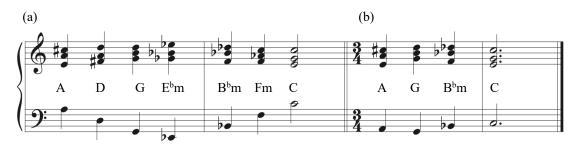




Analyzing how chords gravitate and relate to one another becomes a helpful tool for the composer in the process of establishing motion with elements of expectation and surprise. Throughout a musical work the ear can be led into a labyrinth of transitional chords and substitutions that prepare the listener for arriving at a tonal center. The point of resolution becomes even more satisfying when a chord progression expresses urgency and tension as it prepares the listener for arrival. Levy's illustration of cadences within dominants become useful in creating this exact motion and energy in a musical piece.

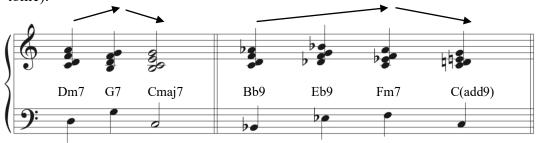
A practical application of this principle can be demonstrated in a chord progression that approaches the generator C from two sides (Example 2.4.3). It starts with a movement from the right side of the circle of fifths towards the main generator C. Levy's predictable dominant motion A - D - G is followed with an element of surprise; Ebm (negative of Bb). Instead of leading the ear towards C, Levy crosses towards the minor zone and prepares the listener for Bbm (-F) in the second measure, followed by Fm (-C). Even though Levy combines chords from different tonalities, they work together and lead the listener back to the main tonal center. Levy added a shorter chord progression (Example 2.4.3.b) to his example that reflects the same motion (Levy 1985, 30).





The table can also serve in the process of substituting chord progressions with chords that carry the same gravitating force as the original. For example, the progression ii - V - I uses the supertonic to provide a motion away from the tonal center, followed by the dominant which leads the ear back to the tonic. According to Levy's principle, we may substitute the supertonic minor chord with a major chord from the opposite side of the current leading away from the tonic. To provide an example, I have chosen to substitute the supertonic chord Dm7 in the key of C with Bb9, the subtonic of its parallel minor. Both chords lie within a distance of a whole tone from the key center and carry the same gravity towards C. I chose to continue this motion further away from C by moving towards Eb, which carries the same gravity as Dm (Example 2.4.4). Adding the seventh and ninth provides tension as well as a chromatic approach to Fm7 (-C), which shares the same generator as the final tonic chord C major.

Example 2.4.4 Substitution of ii-V-I progression (Arrows represent chord motion: arrow pointing upwards leads away from the tonic, arrow pointing downwards leads back to the tonic).



2.5. Conclusion: It's All About Motion

Harmonic Polarity establishes chord relationships based on generators, which allows for multiple key centers within a chord progression that lies outside the diatonic scale. It provides the composer with a wide range of harmonic possibilities to create motion in a musical piece. Though they lay outside of a mode or key, they do not hinder the quality of the sound. Rather they open our imagination to new colors and contrasts that gravitate in opposite directions - either towards the tonal center or away from it. In the process of writing music and creating new chord progressions, the composer always keeps gravity in mind in order to create motion that is balanced with expectation and surprise.

Whereas Harmonic Dualism served mainly in the realm of music analysis and physics, Levy's ideas provided a gateway towards a practical implementation among instrumentalists. In the 1980s, saxophonist Steve Coleman was the first to publicize the term Negative Harmony and implemented harmonic polarity from a melodic standpoint in his music. His implementation of Levy's musical thought provided a great impact on composer, arranger, multi-instrumentalist and producer Jacob Collier, who perfected Negative Harmony in the 21st century and ignited world-wide interest in the theory of harmonic polarity among musicians. Chapter 3: The Shadow of Negative Harmony in Commercial Music

Throughout the past two chapters we have analyzed the concept of balance and gravity as a grounding philosophy for harmonic dualism and polarity. This musical gravity explains harmonic norms that revolve around a tonal key center which can be approached equally from two sides, positive and negative. This key center determines an axis point on which tones are mirrored in the opposite direction in order to produce minor chords that carry the same gravity towards the tonic as major chords. Levy fixes this axis point on the main generator and mirrors each chord individually according to its own generator. As a result, the natural minor scale is formed as the perfect symmetrical mirror opposite of major¹⁸. We shall refer to this technique as *Mirror Harmony*¹⁹. It is important to remember at this point that his goal was to establish a structure of tonality that shows the relationship between major and minor tonalities from the polarity perspective. His writings compiled in *A Theory of Harmony* are limited to major and minor only.

Negative Harmony takes Levy's approach and goes one step further: it mirrors harmonic content according to two generators rather than one. This technique is based on Levy's theory that a triad holds two potential generators located at the first and fifth scale degree. Both generators carry a magnetic force inward towards each other until the direction of gravity is determined by the third. For this reason, Negative Harmony places

¹⁸ According to Levy the F-Minor cadence Fm - Bbm - Cm - C, forming the natural minor scale, would count as the perfect polar opposite of the C-Major cadence, C - F - G - C.

¹⁹ Mirror Harmony can be defined as combining harmonic content from a major scale (for example, C) with its mirrored opposite (Fm) based on the main generator, which is the root of the major chord.

the axis point between the minor and major third of the triad and mirrors chords according to this new axis point.

In addition, this technique recognizes the intervallic relationship between the ascending Ionian scale and its mirror opposite, the descending Phrygian scale. Their symmetrical relationship makes both modes *Mirror Modal Equivalents*. and combines their harmonic content based on two generators in order to produce poly chords, chord extensions or chordal substitutions (Beato 2017, 2:51). Negative Harmony is an evolutionary form of Mirror Harmony that can be achieved by two techniques: a) through Mirror Modal Equivalents, and b) by mirroring along an axis point between two generators (Symmetrical Spiral Technique). Both techniques achieve the same results.

3.1 Mirror Modal Equivalents

As mentioned in Chapter 2, Levy affirms the symmetrical relationship between the Ionian and Phrygian mode: they are mirror opposites. This means that G major, for example, would mirror G Phrygian in the opposite direction. This mirrored Phrygian scale carries the exact same tonal content as the natural C minor scale, in fact, the only difference between those scales in the location of the starting point, which is the root.

Example 3.1.1 Mirror Modal Equivalents: G major along with its symmetrical opposite, G Phrygian, and its new telluric starting point C, producing a descending natural C minor scale.



According to this conclusion a natural minor scale (C minor) can be evaluated as the exact mirrored image of a major scale (G major). This discovery also supports the argument that a minor triad is generated from the fifth downwards. The symmetrical relation between G major and G Phrygian (or C minor) makes both scales Mirror Modal Equivalents, which allows their harmonic content to be used interchangeably. For example, the composer may choose to combine them as poly chords in order to add color and exaggerate tension towards the tonal center. Applying this technique to G major and its mirror opposite, C minor, would therefore produce the chords G/Cm, Am/Bb, Bm/Ab, C/Gm, D/Fm, Em/Eb, and F#^o/D^o.

Example 3.1.2: Mirror Harmony: Combining the harmonic content of G major with G Phrygian (or C minor) in form of poly chords.

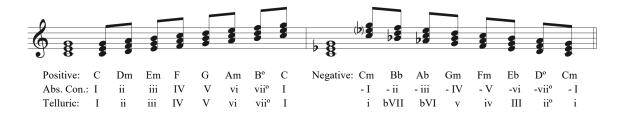


Example 3.1.2 is a classic example of Mirror Harmony since it combines tonalities formed by mirroring along one single generator according to Levy's principle. Negative Harmony on the other hand differs slightly by applying the same principle to two generators. Based on the fact that a triad has two potential generators (the first or fifth scale degree of the triad) depending on the quality of the chord,²⁰ the axis is set right in between the major and minor third of the scale. Mirroring the ascending harmonic content of a major scale, for example C major, along this axis produces its parallel minor scale, C minor. This means that the composer would no longer combine the harmonies of

²⁰ As discussed in Chapter one and two, the third decides the quality as well as gravity of the chord. C and its fifth G, for example, carry a gravity whose direction is not determined unless a third is added. A major third determines the gravity to go upwards from the generating tonic, a minor chord determines the gravity to go downwards from the generating fifth (review Example 1.3).

C major and F minor (Mirror Harmony) but C major and C minor (Negative Harmony), which is generated downwards by G (see Example 3.1.3).

Example 3.1.3: Negative Harmony: C major and C minor evaluated in form of absolute conception and tellurically according to traditional Western Music Theory.



Another way to illustrate the outcome of Negative Harmony is according to the circle of fifths. The symmetry between the right and left half of the circle allows the composer to view mirrored opposites that are spaced with an equal distance from the tonal center (see Figure 3.1.1). The rule is that every major seventh chord becomes minor sixth chord on the opposite side of the circle of fifths. Diminished chords remain diminished because they are symmetrically equal and would only mirror themselves.

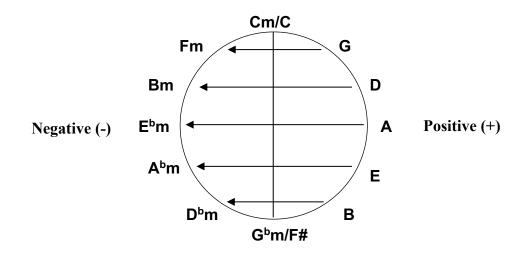


Figure 3.1.1 Negative Harmony according to the Circle of Fifths. The major triads on the right side (positive) mirror minor chords that are located on the opposite side of the circle (negative).

We can conclude that applying the mirroring technique to two generators (Negative Harmony) produces different results than applying it to one single generator (Mirror Harmony). Mirroring a major scale according to one generator produces the harmonic content of a minor scale that is the exact symmetrical opposite of the original mode (for example, C major mirrors C Phrygian or F minor). Negative Harmony introduces a new way of mirroring by pulling harmonies from the content of the original mode and the symmetrical opposite of its parallel mode (for example, C major is combined with G Phrygian, or C minor). This mirroring technique places the axis between two generators, the first and fifth degree of the original chord. The gravity shifts inwards which means that the symmetrical center between both generators becomes the major and minor third. The illustration within the circle of fifths shows the results of this technique, which transforms all dominant major chords on the positive side to plagal minor chords on the negative side (Lee 2017, 2:18).

3.2 The Symmetrical Spiral Technique

Negative Harmony applies mirroring to two generators rather than one. The symmetrical central point falls exactly between major and minor third, which provides an axis point on which tones are mirrored outwards according to the *Symmetrical Movement Concept (SMC)*, a concept introduced by Steve Coleman. This spiraling technique provides the same results as combining a major scale with the negative of its fifth (review Example 3.1.3). Both techniques determine the harmonic material from the parallel minor as the Negative Harmony of the original major mode. If we were, for example, to mirror the tonic chord C major over the axis point E – Eb, C would become G, E would become

E-flat, G would become C, resulting in a C minor chord. Adding the flat seventh (Bb) would mirror a minor sixth in the opposite direction (Cm6).

Example 3.2.1 Negative Harmony: Symmetrical Spiral Technique according to Steve Coleman's Symmetrical Movement Concept: The axis point E and E-flat is determined by two generators (C and G)



Comparing the harmonic results of Negative Harmony with Levy's way of mirroring can seem confusing at first since Negative Harmony combines harmonic content of parallel modes whereas Levy amplifies the connection between an Ionian mode with its exact mirrored opposite (Phrygian descending) in order to establish minor as the perfect symmetrical opposite of major. Nevertheless, both techniques have the same view on tonal structure and harmonic function based on gravity and symmetry. We can see an example of this by observing the harmonic function of the major dominant (V) and the minor subdominant (iv or -V) according to their balance. Whereas the major dominant chord provides a gravity upwards due to the leading tone, which is the rising third, the minor subdominant chord provides the same gravity in the opposite direction due to the falling third. The dominant chord G7, for example, gravitates to the tonic C major due to the leading tone B, whereas the minor subdominant Fm gravitates down towards the tonic because of its negative leading tone A-flat which tends to fall a half step down to G. Due to their same harmonic function, which is to lead towards the tonic, Levy categorizes the minor subdominant chord (Fm) as the negative counterpart to the positive dominant chord (G) and labels it negative dominant (-V). The results of Negative Harmony accomplished through Mirror Modal Equivalents (see Example 3.1.3) support

the same statement. It analyses the chord Fm as more than a borrowed chord from Cm: According to its harmonic function it takes the role of the negative dominant (-V) because it carries the same gravity towards the key center as the positive dominant G, which makes them symmetrically related.

The shadow of polarity becomes very visible when we compare the quality of the chords to which both dominants lead. The positive dominant gravitates to the positive tonic (G7- C), whereas the negative dominant gravitates to the same key center but parallel tonic chord (Fm6- Cm). Both dominants progress to the same chord root (C) but lead towards tonic chords with different polarities or parallel tonalities (see Example 3.2.2), (Coleman 2015).

Example 3.2.2 Levy's illustration of Harmonic Polarity: (a) Negative Dominant (Fm⁶ or -C⁷), Positive Dominant (G⁷), Positive Tonic (C), followed by its polar opposite of the same generator, (b) Positive Dominant (C⁷), Negative Dominant (Bbm⁶ or -F), Negative Tonic (Fm or -C), (adapted from Levy 1985, 49).



We can conclude that Negative Harmony approaches tonality from a standpoint of polarity which grounds major and minor chords as symmetrical opposites that equally gravitate towards the tonal key center. Whereas Levy's principle of Mirror Harmony focuses on the symmetrical relation based on mirroring along one generator (the root of the tonic), Negative Harmony differs by mirroring according to two generators. This is also why the chord labels in Levy's method differ in their meaning from those used in Negative Harmony. When Levy studied the potential sounds achieved through harmonic polarity in the early to mid 20th century, he initially intended it to be used in the realm of classical music. In the 1980s, the term Negative Harmony was born out of Steve Coleman's experiment to project Levy's concept of symmetry into his melodic ideas. Coleman's application of polarity into the realm of jazz improvisation gave way to new sounds which caught the attention of Jacob Collier, who explored the harmonic possibilities of Levy's ideas.

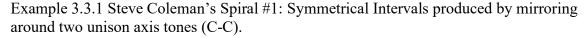
3.3 Steve Coleman's Improvisational Use of Polarity

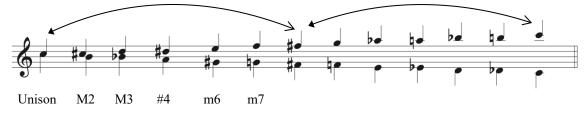
Surrounded by the music scene in Chicago and New York, Steve Coleman (born 1956) continuously sought to expand his knowledge about advanced musical concepts in the genre of Jazz and Bebop. With a focus on improvisation on the alto saxophone his early stages were mostly impacted by tenor saxophonist Van Freeman, saxophonist Sam Rivers, and drummer Doug Hammond, who inspired Coleman's conceptual thinking. In the 1980s, his musical ideas shifted from western music to African music traditions as his interest grew in the philosophical and spiritual ideas behind music. Inspired by his findings Coleman's improvisation style began being shaped by those ideas as they reflected musically achieved balance through symmetry (Coleman 2015a).

When I play, I'm not thinking scales. A lot of younger players are so locked into scales that they can't think of anything else. I've often wondered why the two modes, major and minor, dominate out of the countless diatonic structures. After studying African, Eastern and Bulgarian folk music, I decided that it wasn't necessary to use just major and minor, and consciously abandoned it. Now I'm working with cells. There's nothing mystic about that - they're just small musical constructs which I manipulate to get two types of sound motion, stationary and in transit. I've also been looking for geometric ways of doing progression rather than thinking in standard terms. (Hrebeniak 1991)

Coleman's musical ideas that are essential to the theory of Negative Harmony are comprised in the Symmetrical Movement Concept (SMC), which involves a melodic system in which tones spiral outwards from a tonal center. This tonal center is established by two tone unisons (for example, C-C) or two different axis tones (C-Db). The motion of mirroring notes around one or two axis tones creates two spirals which form two separate sets of intervals: symmetrical and non-symmetrical.

The first spiral mirrors the chromatic scale on the axis point of the unison. If we apply the axis of the first spiral to C, we form the unison as the first interval and move out each tone in the opposite direction within half steps: C will move up to C-sharp whereas the second voice will move down to B, creating the interval of a major second. Coleman calls the intervals that are resulting through this process Symmetrical Intervals, which are Unison, M2, M3, Tritone, m6, and m7. The center of the spiral is located a tritone above the unison. Since both spirals form the interval of an octave it qualifies it as a new beginning of the spiral (Coleman 2015b).

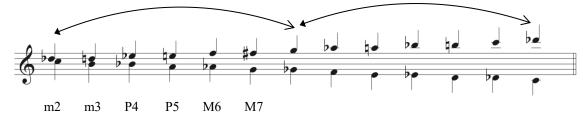




The second spiral (Example 3.3.2) mirrors the chromatic scale on two different tones, for example C-Db. Spiraling out chromatically results in the formation of Non-Symmetrical Intervals, which are m2, m3, P4, P5, M6, and M7. Though the axis point is

shifted, the center of the spiral continues to be located a tritone above the axis tones. For example, the spiral of C-Db produces a second center point or axis G-Gb.

Example 3.3.2 Steve Coleman's Spiral #2: Non-Symmetrical Intervals produced by mirroring around two different tones (C-Db).



Originally this theory started out as a melodic concept. Coleman states that it "can also be adapted to deal with the gravity of other types of tonality such as cells or the traditional dominant-tonic harmonic system" (Coleman, 2015b). Within an improvisational Coleman centers a melodic motion around an axis point. Moving in one direction requires the completion of the symmetrical motion by moving the same distance in the opposite direction. For example, if a melody moves a minor third upwards it needs to follow with the same interval in the opposite direction. Coleman applies this rule of mirroring only to non-symmetrical intervals from the second spiral in order to gain symmetrical completion. Since the sum of non-symmetrical intervals results in a symmetrical interval, Coleman achieves a completed symmetrical motion through this technique. That means that following a minor third in the upward direction with a minor third below the axis creates a diminished fifth, which is equivalent to a tritone belonging to the first spiral. Since the goal is to achieve symmetrical intervals either directly or by summing up intervals the rule does not require symmetrical intervals to be mirrored. Instead they can be followed by any other tone.

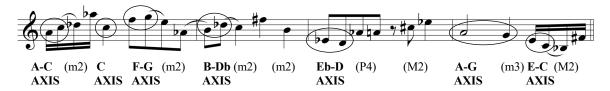
Coleman provides examples of an atonal melodic lines centered around axis points as a guide for applying his theory to improvisation. Example 3.3.3 provides a melodic line centered around one axis tone. It begins with axis tone C that is followed with an interval of a M2 from the first spiral. Though it does not have to be followed with the same interval in the opposite direction, Coleman follows with a B-flat in order to approach the axis tone C from below, creating a perfectly symmetrical double appoggiatura. He follows with the same notes but projects them into a different octave since that does not affect the tone character. The second measure introduces axis tone G that is followed by a minor third above and below the axis. This melodic movement completes the symmetrical motion.

Example 3.3.3 Coleman's example of symmetrical motion around unison axis tones (see circled notes), (Coleman 2015b).



The next example provides a variation to the rule by spacing the axis tones more than a minor second apart.

Example 3.3.4 Analysis of Coleman's example of symmetrical motion around two axis tones (Coleman 2015b).



The first beat in measure one presents a symmetrical variation that is centered around two axis tones: A and C (Example 3.3.4). This is noticeable since Coleman does not follow this interval with G-flat but rather progresses with a minor second above axis tone C (D-flat) and a minor second below the first axis tone A (A-flat) that completes the symmetrical motion. Coleman projects A-flat in a different octave for artistic purposes.

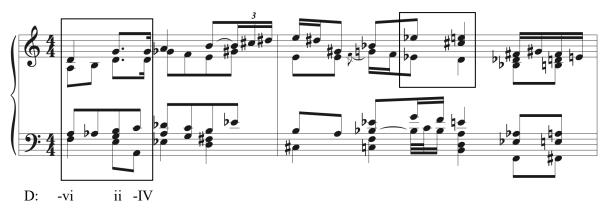
He continues with axis tone C on beat two and follows with a completed symmetrical motion by moving up a perfect fourth to F leading towards G, which lies a perfect fourth below C but is transposed into a different octave. These two notes, F and G, serve as the new axis tones on beat three that determine the minor second (E below F, A-flat above G within a different octave).

3.4 Jacob Collier's Harmonic Implementation of Polarity

Upon receiving his first Grammy for "Best Arrangement, Instrumental or A Capella" in 2017, Jacob Collier (born 2 August 1994) startled the creative minds of many composers by saying, "if we're encouraged to invest in our imaginations, we will do no wrong" (Jones 2017, 1:40). This statement embodies Collier's core belief that has inspired him on his journey to a successful career as a composer, arranger, producer, singer, and multi-instrumentalist in the genres of iazz, funk, pop, soul, and a cappella. Fusing elements of those genres along with experimenting with the building stones of music itself such as microtones, scales, poly-chords, rhythms, harmony, etc., Collier has gained world-wide attention as a "mad scientist" of music responsible for tone colors beyond any mortal imagination.

Growing up in London Collier was exposed to music from an early age. Being emerged in the music world through his mother Suzie Collier, who currently teaches at the Royal Academy of music in London, Collier's interest in arranging was inspired by composers such as Benjamin Britten. Fascinated by the harmonies in Britten's *The Turn* *of the Screw*, Collier explored harmonies of his own and began arranging music on computer-sequencing programs by the age of seven. As a teenager, his stylistic interest in music expanded to Stevie Wonder, Joni Mitchell, The Beatles, Sting, Radiohead, Michael Jackson, etc. Dropping out of college after studying two years of jazz piano at the Royal Academy of music, Collier continued to grow his skills as a self-taught musician. He believed that his education was teaching him much about technique but nothing about texture, which he crafts entirely from his imagination (Lewis, 2016).

His sense of experimenting with sounds and tonalities is beautifully displayed in the chorus of "Hideaway." Whereas the main tonality in the verses is D-Major, the elements of surprise are reserved for the harmonies in the chorus of the song, which perfectly paint the lyrics: "Whether you be lost or found, darling, if you've gone astray, I'm on my way to my hideaway." The tonal journey itself contain examples of Negative Harmony. The very beginning of the first harmonic progression F⁶ - F⁰⁷ - Em⁷ - Am⁷ can be interpreted as Negative Harmony.



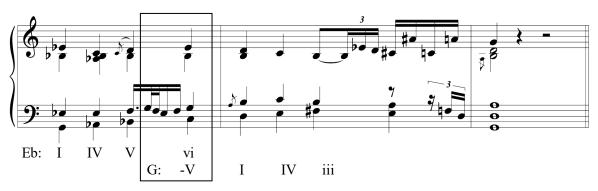
Example 3.4.1 "Hideaway" (first two measures of second chorus), (Lee 2018, 19:22).

Collier prepares for a chromatic descending line that can be seen in the bass line of the first two measures by continuing with F^{o7} in order to approach Em (ii). Before

continuing the chromatic descend, Collier allows for a falling leap towards Am7 – the negative subdominant of D (since it technically belongs to the tonality of its polar opposite, Dm).

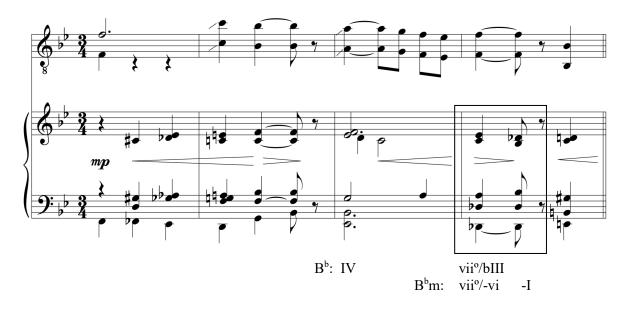
A form of outward melodic spiraling can be seen in the second measure of the chorus. Collier approaches Bm^{11(omit5)} chromatically with Cm^{7(add11)} and provides a form of voicing in the soprano and alto section that reflects mirror opposites. Whereas the upper voice ascends a half-step from E-flat to E, the lowest voicing descends from E-flat to D.

Another example of Negative Harmony can be found in the form of the negative dominant or, in other words, minor subdominant (Example 3.4.2). In the third measure of the second chorus, the guitars play a harmonic progression starting with Eb/G. The tonality is set on Eb-Major and continues to be reaffirmed by the harmonic progression of Ab⁹ - Bb - Cm (IV - V - vi). Collier uses the last chord, Cm, as a negative dominant of G and leads the listener to the final "hideaway," which is G major. The contrast between the descending motion of the first two measures of the chorus and the ascending motion in the second half of the chorus towards arriving at G major amplifies the tension for arrival and gives this particular section an appropriate sound color.



Example 3.4.2 Negative Dominant in "Hideaway" (final three measures of second chorus), (Lee 2018, 19:32).

"Hideaway" gives the composer a more complex example of Negative Harmony. That does not mean that it always has to be used in a tangled web of tonalities. An example of a very subtle use of Negative Harmony can be found in "Moon River." Example 3.4.3 "Moon River": Polarity in form of a Negative Tonic.



The chorus of "Moon River" is set in the tonality of B-flat major. Polarity is introduced in the moment Collier ends a four-measure chord progression with Bbm, the negative tonic of Bb (see Example 3.4.3). He approaches the negative tonic with two chords; the subdominant chord Eb (IV) leading towards A^o (vii^o) over pedal tone D-flat. Whereas the diminished chord belongs to the harmonies B-flat major, the tone D-flat belongs to generator F. It is part of the descending F Phrygian scale which has the same tonal content as Bbm. According to Negative Harmony, the chord Db is analyzed as the negative submediant (-vi) of Bb. This pedal tone serves the purpose of introducing polarity by approaching the negative tonic Bbm.

3.5 Negative Harmony in Commercial Music: Paul McCartney, Joni Mitchell, and Herbie Hancock

Although Collier and Coleman have specifically built out strategies on how to use Negative Harmony it is certainly not always used intentionally. Many artists in the genre of pop, rock, folk, jazz, and funk reflect this technique in their music without even knowing it. The simplest and most common use is in the form of the negative dominant (or minor subdominant in a telluric sense) leading towards the tonic (-V or iv to I). Traditional music theory would simply interpret this chord as borrowed from minor, but from the perspective of polarity one cannot help but see the symmetrical relationship towards the positive dominant that produces the same gravity towards the tonic but from the opposite direction. Another common way Negative Harmony is used is in the form of switching from the positive tonic to the negative tonic (I, -I), which is a popular feature in Joni Mitchell's music.

Song and Artist	Section and Negative Harmony Implementation	Time Location on Record (Minutes)	
"Dancing in the Dark" by Frank Sinatra	Verse: Fm^6 - G^7 - C_{MA}^7 (Negative Dominant, Positive Dominant, Tonic)	0:20 - 0:26	
"This Never Happened Before" by Paul McCartney	Chorus: Am - E (Negative Dominant to Tonic)	0:42 - 0:45, 0:48 - 0:53	
"Bohemian Rhapsody" by Queen	Verse 1: Fm - C (Negative Dominant to Tonic)	1:41 - 1:44	
"You and I" by Stevie Wonder	Intro: F _{MA} ⁷ - Bb/F - B^bm/F (Tonic, Subdominant, Negative Dominant)	0:00 - 0:38	
"I wish" by Stevie Wonder	Verse: $\mathbf{D}^{\#}\mathbf{m} - \mathbf{G}^{\#7}$ (Negative Tonic, Positive Subdominant) Bridge: $\mathbf{A}^{\#7} - \mathbf{C}^7 - \mathbf{Fm}^7 - \mathbf{G}^{\#}\mathbf{m}^6$ (Positive Dominant, V/ii which is Mirror Harmony of Fm, followed by Negative Dominant)	0:18 - 0:41	

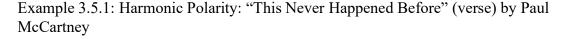
Table 3.5.1 Examples of Polarity in form of Negative Harmony in Commercial Music.

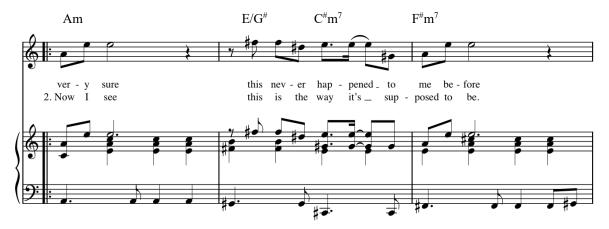
Song and Artist	Section and Negative Harmony Implementation	Time Location on Record (Minutes)
"Knocks Me Off My Feet" by Stevie Wonder	Verse: C _{MA} ⁷ - Gm - C - A ^{b6} - Bb ⁷ - C (Positive Tonic, Negative Subdominant, Positive Tonic, Negative Mediant, Negative Supertonic, Positive Tonic)	0:20 - 0:30
"You Still Believe in Me" by The Beach Boys	Verse ending: Em/G - B (Negative Dominant Function, Positive Tonic	0:50 - 0:53
"Space Between" by SIA	Verse 1 and 2: C - Fm (Tonic to Negative Dominant)	0:00 - 0:28
"The Voice Within" by Christina Aguilera	Chorus: G - Cm (Tonic to Negative Dominant)	1:02 - 1:05
"The Dawntreader" by Joni Mitchell	Intro: Dm - G - D (Negative Tonic, Subdominant, Positive Tonic)	0:00 - 0:10
"Help Me" by Joni Mitchell	Verse: Asus2 - Em⁷ - G_{MA}⁷ (Positive Tonic, Negative Subdominant, Negative Submediant)	0:06 - 0:10
"Court and Spark" version by Herbie Hancock (feat. Norah Jones), originally written by Joni Mitchell	Verse beginning in Fm , ending in F. (Negative Tonic – Positive Tonic)	1:14 (Fm), 1:35 (F)
"Butterfly" by Herbie Hancock	Intro: Fm ⁷ - Am ⁷ over melody in Cm which resolves to C-Major (Negative Dominant, Positive Submediant)	0:00 - 0:16
"Us and Them" by Pink Floyd	Intro and Verse: D - Bm/D - Dm(maj7) - G (Positive Tonic, Positive Submediant, Negative Tonic, Positive Subdominant)	0:33 - 0:55, 1:40 - 2:05
"Black Hole Sun" by Soundgarden	Verse: G ⁶ - Bb ⁶ - F5 - Em - Eb - Dsus - G ⁶ - Dm ⁶ / F - Ab (Positive Tonic, Negative Submediant, Negative Supertonic, Positive Submediant, Negative Mediant, Positive Tonic, Negative Subdominant Function, Tritone Substitution	0:15 - 0:35

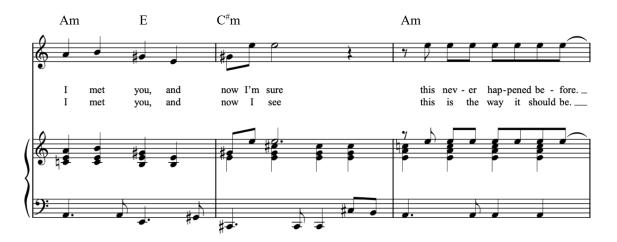
The search for Negative Harmony in Commercial Music reveals that its use is never only bound to a certain genre or time period. It surpasses its barriers and becomes completely independent from musical expectations by allowing the freedom of imagining new tone colors. Those do not necessarily have to be complex, but they need to make musical sense. The way to find the use of Negative Harmony is to search for symmetry in the process of analyzing unordinary chord progressions. In order to demonstrate this search, I shall present a view musical examples of polarity in depth.

"This Never Happened Before" (Paul McCartney)

This particular song was published on Paul McCartney's album *Chaos and Creation in the Backyard* in 2005. When I first heard this song in the movie "The Lake House," I recognized Paul McCartney's efforts to establish A minor as the tonal key center at the beginning of the song. It starts out with a piano playing the progression of Am to E, it's dominant chord. Transitioning to the verse, the song continues with Am but continues with a chord progression that establishes a new key – E major.







From the perspective of polarity, a key center is never really established until the arrival on E-Major at the end of the verse. Remembering Riemann's dual "Klang" (review Example 1.4.4) in reference to Levy's statement that both major and minor spring from one generator, a key center can technically not be established without all three harmonic pillars: tonic, subdominant, and dominant. The same principle applies to the introduction of "It Never Happened Before." The rotation between E and Am, which spring from the exact same generator, reflect a harmonic circle that leave the listener uncertain about the tonality and key center. Both chords form a cadence that is never completely closed. Additionally, the ear is deceived by starting the intro and verse with Am rather than E major. The establishment of the tonality of E major begins with introducing its harmonic content; G[#]m7 - C[#]m7 - F[#]m7 (iii7 - vi7 - ii7). The chord Am takes the role of the negative dominant (-V) leading the listener to the true tonic, E major. At this point, the ear finally settles. Though Paul McCartney did not intentionally think about polarity when he introduced the song, he chose chords that make you "feel like you're going somewhere" (McCartney 2005). This strategy was made possible by not

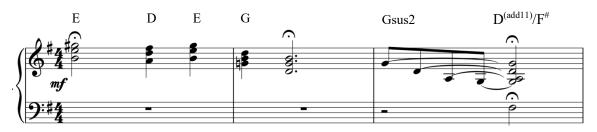
establishing a key center at the beginning but leaving it open according to polarity until the ear settles at the end of the verse.

Another shift in tonality can be heard in the chorus section of the song. The chord progression goes back to the negative dominant Am but continues in form of the chord progression ii - V - I (Am - D - G) to G major. But there's more behind the reason why the tonalities of E and G can be connected. Since G is considered the relative major key of E minor, it allows us to consider its harmonic content as Negative Harmony. What would be considered a ii-V-I progression in G major has the same tonal content as -V, -ii, and -vi in E major (see Appendix A for sheet music).

"Court and Spark" (Joni Mitchell)

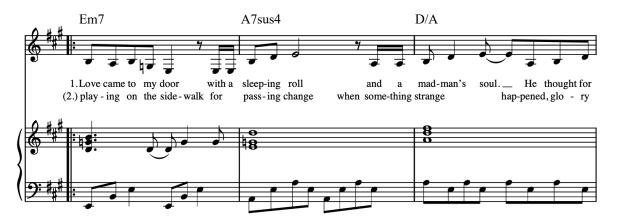
The title of Mitchell's song is the same as the title of the album she published in on in 1974, which counts by far as her best-selling album in her whole career. Winning double platinum, Mitchell's crossover between folk and jazz received a public response that can be summarized in the words of New York Times journalist Loraine Alterman: "…the drama of the soul that unfolds in her lyrics is now matched by the drama of the music" (Alterman 1974).

Mitchell's songs have always been marked by a musical accompaniment that journeys through diverse tonalities and color combinations (see Appendix B for sheet music). "Court and Spark" is but one of many examples of this particular writing style. The beginning of the song starts with the chord progression E - D - E - G which is continued by a falling arpeggio starting out as a Gsus2 developing into D^(add11)/F[#].

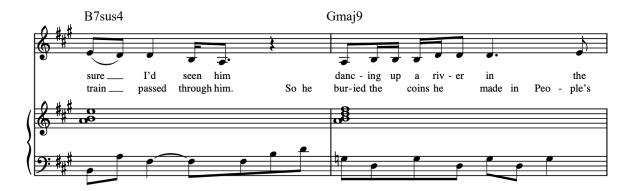


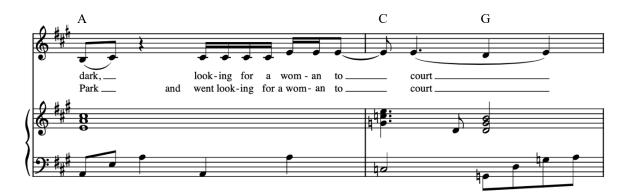
Example 3.5.2: Harmonic Polarity: "Court and Spark" (introduction)

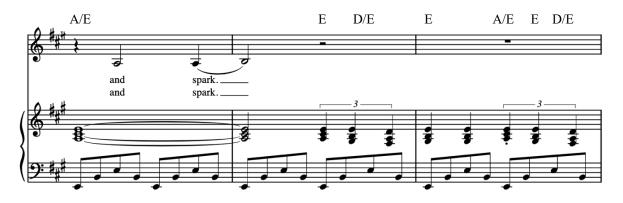
At this point tonality is not yet established. Mitchell continues the chord progression with Em7 for four measures before she starts out the verse with the same chord. The first verse, which is thirteen measures long, takes the listener on a journey that ends up establishing E major as the tonal key center. It begins with a chord progression that is entirely in the key of E minor, but it changes character in the sixth measure of the verse represented by the positive subdominant A. From then on, the progression C - G -A/E follows. Belonging to the tonality of E minor, the chord C can be considered a negative mediant (-iii), which would make G the negative submediant (-vi). Polarity is established when those chords from the negative side land on the positive tonic E rather than the negative tonic Em.



Example 3.5.3 Harmonic Polarity: "Court and Spark" (verse).





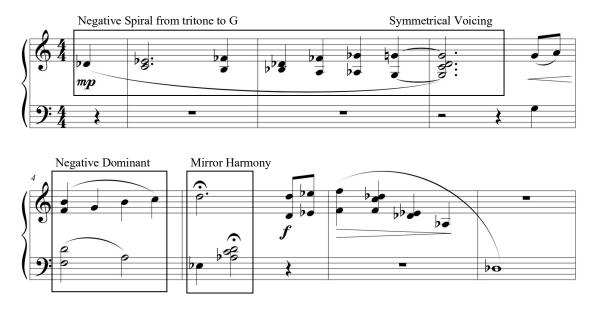


The shadow of polarity lies in the contrasting elements of a minor tonic (Em7) and a major tonic (E). Keeping in mind that Em is the Negative Harmony of E, we can look back at the intro of "Court and Spark" and interpret the very first chord progression as a positive tonic (E) leading to the negative supertonic chord (D) and the negative subdominant (G), (I, -ii, -vi). That means that this chord progression carries the same gravity towards the tonal key center as its positive, E - F#m - E - C#m.

"Both Sides Now" and "Cantaloupe Island" (Herbie Hancock)

The list of Collier's inspirations carries many names such as the Beatles, Stevie Wonder, and Joni Mitchell. It is of no surprise that Herbie Hancock counts as another one of His role models. Hancock's skills to produce extraordinary harmonic colors are achieved by coupling chords from different tonalities that are related in symmetry and mediant relationships, which he mends often times through chromaticism. Traces of symmetry can be tracked harmonically and melodically. A perfect example of existing symmetry in a melodic line can be found in the piano introduction of "Both Sides Now," which is an original tune by Joni Mitchell. Hancock's exemplary jazz version of this song was published on his album *River: The Joni Letters* in 2007 as a tribute to Joni Mitchell (Peacock 2017). The special part about this album was that the driving force of the music were the lyrics. Hancock admits this to be a unique approach compared to his usual compositions, in which he focuses more on orchestration and texture (Hancock 2017, 1:23).

Text painting in form of symmetry becomes very vivid in "Both Sides Now" (see Example 3.5.4). The piano introduction starts out with a symmetrical spiraling from axis tone Db. Whereas the lower voice spirals out chromatically, the upper voice forms a melody that slowly forms the chord Db^o. Both voices spiral out to G – the symmetrical center of the D-flat unison spiral – and land within a distance of an octave. Hancock adds more symmetry in the third measure by filling the octave with two more notes, G and D, that carry a distance of a half step towards the symmetrical unison D-flat. In addition, both notes carry the same distance of a fourth towards the outer voicings.



Example 3.5.4 "Both Sides Now" (introduction) by Herbie Hancock.

Arriving at a voicing of Gsus4 that is perfectly symmetrical through a spiraling approach, Hancock continues to form a Negative Subdominant of G, which is Dm, over bass tone F. The chord progression continues with Eb in the bass, which belongs to the tonality of G minor. The addition, the inner voicings in measure five form Ab (add #11) in its second inversion, which represents a tritone substitution in the key of G. The shadow of polarity becomes visible when we consider that this chord belongs to G Phrygian, which is the mirror modal equivalent of G major. Whereas the left hand descends and takes harmonic material from G Phrygian in measure five, the right hand ascends with a melody based on G major. This type of mirroring would not be considered Negative Harmony but rather Mirrored Harmony since it is based on one single generator, G. From this point forward, Hancock completes the phrase by landing back on Db, which is the same note that started out the song. The last chord, Dbsus2, is approached by the melody line G, E-flat, and F, which belong to the tonality of G minor, the negative of G major. Another example of Negative Harmony can be seen in Hancock's famous piece "Cantaloupe Island." The chord progression used for section A is Fm - Db7 - Dm - Fm. The first two chords establish F minor as the main tonality. The ear is pulled into a different tonality with the chord Dm, which adds a moment of surprise, before it returns back to Fm. This particular chord belongs to the tonality of F major, which is the polar opposite of F minor according to Negative Harmony. It carries the same gravity towards the tonal key center F as Ab, a chord that would qualify as its positive mirrored opposite on the axis Ab-A. For this reason, the entire chord progression of Fm - Db7 - Dm - Fmcan be analyzed as negative tonic, negative mediant, positive submediant, and negative tonic (-I, -iii, vi, -I).

Positive	F	Gm	Am	Bb	С	Dm	E°
	Ι	ii	iii	IV	V	vi	vii
Negative	Fm	Eb	Db	Cm	Bbm	Ab	G°
	-I	-ii	-iii	-IV	-V	-vi	-viiº

Table 3.5.2 Negative Harmony in "Cantaloupe Island" by Herbie Hancock.

Now, what have we learned through taking a closer look at these examples? It has become clear that the shadow of harmonic polarity can be found throughout many genres of Commercial Music. It is reflected in the symmetry of tonal structure that produces unordinary chord progressions that take the listener to a whole new level of anticipation. This symmetry does not necessarily mean to limit polarity as atonal as in Coleman's compositions. Our examples from compositions within Commercial Music show that polarity in form of Mirror Harmony and Negative Harmony can also be used to produce multi-tonalities that long for resolution and arrival in a key center. This technique allows the composer to create musical colors that are symmetrically connected through harmonic function. Rather than being classified as random chords, this symmetrical connection allows those progressions to make musical sense.

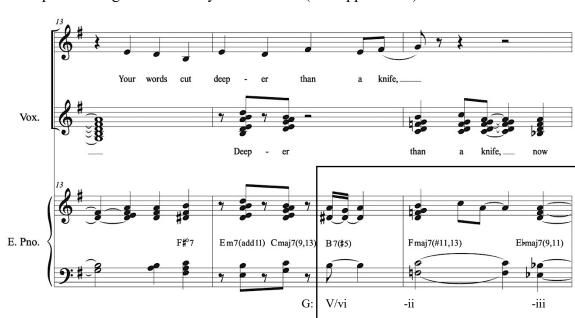
In the next chapter I will dive in the practical application of Negative harmony in form of reharmonization and composition. For this purpose, I will provide excerpts of my own musical repertoire in order to demonstrate different ways polarity can be achieved through chord substitutions, poly chords, mirrored melodic lines, and chord extensions. Chapter 4: Guidelines to a Practical Application of Polarity

The past three chapters we have reviewed the historical pillars of harmonic polarity, rooted in mathematics, psychology, and philosophy; and we have explored its practicality in commercial music of the twentieth and twenty-first century. The application of harmonic polarity can be summarized in the following techniques: a) Mirror Harmony, which centers the axis point on one single generator, and b) Negative Harmony, which centers the axis point between two generators. Both techniques produce harmonic content that carries the same gravity towards the key center as the original harmonies due to tonal symmetry. In my personal application I have used polarity in form of mirroring chords, mirroring melodies, applying multi-tonalities related through symmetry, and moving voicings according to the symmetrical spiral technique. In this chapter I shall refer to excerpts of my own compositions and arrangements for this project in order to demonstrate my own form of application of polarity.

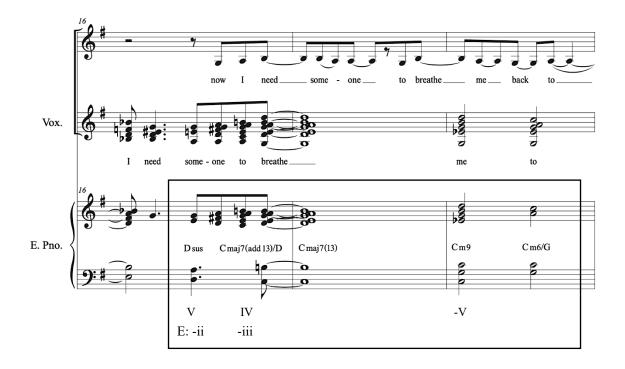
Stitches (Arrangement for Vocals and Rhythm Section)

Before I arranged "Stitches" I developed a concept about how to approach polarity. Rather than simply mirroring each original chord from the song I created new chords for almost every melodic note and changed its rhythmic and stylistic features to the genre of funk. These new chords reflect harmonic paths outside the key that reflect polarity without losing the listener to a new tonality since they happen very quickly. In this arrangement I intentionally kept coming back frequently to the keycenter Em or its relative, G major.

One harmonic path I want to draw attention to reflects the use of Negative Harmony by combining tonal content of G major, or E minor, with its negative, G minor (see Example 4.1). Polarity is introduced in form of the dominant B7 falling within the interval of the fourth towards Fmaj7 (-ii of G), rather than rising towards the tonic as the listener would expect. The motion towards Fmaj7 is elevated by the top voice in the electric piano staff. The melody moves from A a step downwards to G and continues with a motion in the opposite direction towards B before it completes an appoggiatura through C towards A.



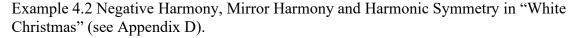
Example 4.1. Negative Harmony in "Stitches" (see Appendix C).

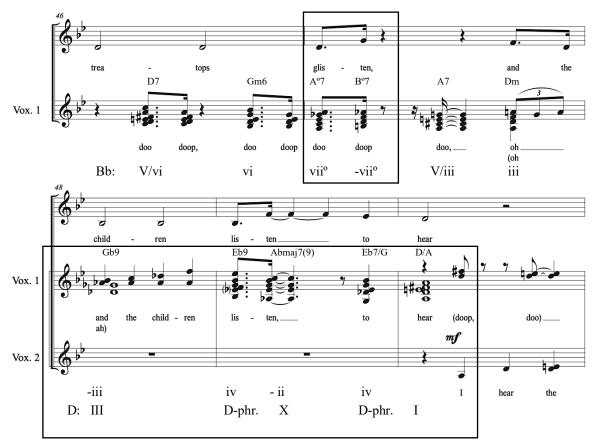


The harmonic motion continues with Ebmaj7 (-iii of G), and repeats the same motion based on the key center E by approaching the chords Dsus (-ii of E), and Cmaj7 (-iii of E). These chords also belong to the original key. The harmonic progression adds one last element of polarity by following with Cm9 (-V of G), which leads the listener back to the tonic, Gmaj7, in measure nineteen.

"White Christmas" (Arrangement for Female Vocals)

When I arranged "White Christmas" for vocals only I chose to apply polarity more aggressively than in "Stitches." Not only did I use it to add color but to jump between different tonalities more frequently. Since the voices stood alone in this arrangement I allowed for more harmonic colors. "Moon River" by Jacob Collier was a big inspiration behind this song. The harmonic progression I want to elaborate on is located at the end of the second verse (m. 46 – 50, see Example 4.2). This section is in the key of B-flat major but prepares to shift to the tonality of D major in measure 50 by means of polarity and chromaticism. The harmonic progression in measure 46 starts with a dominant function (D7) in order to approach Gm7 (vi of B-flat major), which is still in the key of B-flat major. Both chords, D and Gm, are generated by the same fundamental (G) but in opposite directions. The same motion is repeated in measure 47 with A7 and Dm (iii of B-flat major). Both functions are connected by a short moment of polarity with A°7 and B°7. Whereas the first diminished chord (A°7) is located a half step below the tonic B-flat, the second diminished chord (B°7) is located a half step above it.



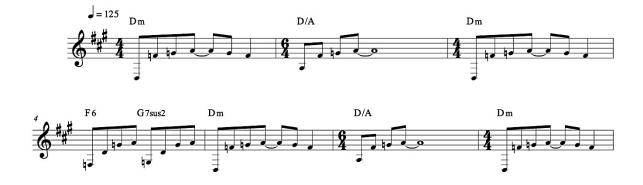


Polarity and symmetry continue in measure 48 by approaching Gb9 (-iii of B-flat major) followed by Eb9 (iv of B-flat major). Technically only Gb9 counts as Negative Harmony, but the element of symmetry and melodic mirroring is reflected in the location of both chords, Gb9 and Eb9, as their root tones are a result of a chromatic spiraling inwards from the first two chords, Gm and Dm.

Measure 48 starts preparing the listener for a new tonality. Instead of leading the listener back to B-flat, the shift towards D major is made by approaching the symmetrical center of the chromatic spiral based on D, which is A-flat ("X" for symmetrical center). Due to their symmetry, which we explored in chapter three (review Example 3.3.1), they can be interchangeably used. Measure 49 continues to introduce a new tonality by drawing upon chords such as Eb9 belonging to the Mirror Harmony of D major (not Negative Harmony), which is D Phrygian. Based on mirroring along one single generator, D, we establish the chord Eb9 as the mirrored version of C#°. This means that Eb7/G in measure 49 carries a mirrored leading tone function towards D major. In measure 50 the switch to the key center D is completed.

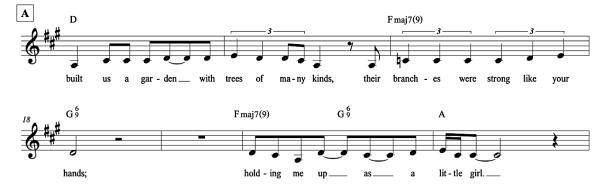
"Paradise" (Composition for Voice and Guitar)

The application of harmonic polarity in a simpler format can be heard in my composition called "Paradise" (see Appendix F). I imitated Joni Mitchell's technique of switching between parallel keys (inspired by songs like "I had a king") as well as Paul McCartney's harmonic art of deception that leaves the listener unclear about the tonality until a firm resolution is made (such as in "It never happened before"). Example 4.3 Negative Harmony in the introduction of "Paradise" (Negative and Positive Tonic, leading to Negative Submediant, F6, and Positive Subdominant, G7).



The song starts out with polarity by switching between D minor and D major in the guitar accompaniment (see Example 4.3). The chords surrounding them, which are F6 (-vi) and G7sus2 (iv) also belong to both tonalities and thus create an equal balance between them.

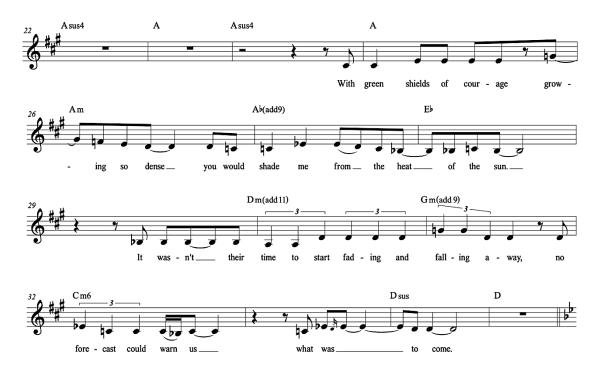
Example 4.4 Negative Harmony and Polarity in "Paradise" (m. 15–21).



Whereas the introduction starts with D minor, the verse begins with D major. It repeats the harmonic progression towards Fmaj7 (-vi of D) and G (iv of D) before using both chords to establish A as the key-center. Thus, both chords preceding A don't only work in the realm of the generator D, but they also carry the harmonic function of -III and -II in A major. The tonality of A major and its polar opposite, A minor, is foreshadowed already in the melody of the verse. The deception takes place by sustaining D and Dm throughout the introduction and the first half of the verse. The listener has no clue that the song is technically in the tonality of A major and its polar opposite.

Polarity continues by switching between A major and A minor (negative tonic), (see Example 4.5). The melody reflects polarity as well by combining both scales (see m. 25 and 26 in Example 4.5). This can already be observed in the melodic contrast between m. 15 in A major, and m. 17 in A minor. The harmonic progression that follows upon Am is based on minor plagal cadences (Dm, Gm, Cm). The flow of those cadences is shortly interrupted in m. 27 and 28 by Ab and Eb; two chords that provide symmetry as they are bookended within a distance of a half-step by Am and Dm. There is also another reason why those chords work so well in this spot: Ab belongs to the Negative Harmony of C major and functions as its negative mediant (-III). This particular tonality is foreshadowed in the melody a measure ahead of the chord (m. 26). With a melody based on C major the progression towards its negative mediant Ab (-III) is prepared. What follows is a merely a symmetrical completion of this cycle by stepping to Eb, which gives the listener a new sense of arrival before the progression continues with a series of minor plagal cadences (Dm, Gm, Cm), which function as the Negative Harmony of G major (-IV, -I, -V).

The tonality of A major and its negative is starting to shift towards G major and its negative. The switch is visualized in the melody of measure 30, which is symmetrically organized by placing perfect fourths around D, the first initial key center of the song. Those fourths, A and G, symbolize the contrast of tonalities of the verse and the chorus.



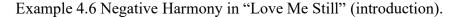
Example 4.5 Negative Harmony and Polarity in "Paradise" (m. 22 - 35).

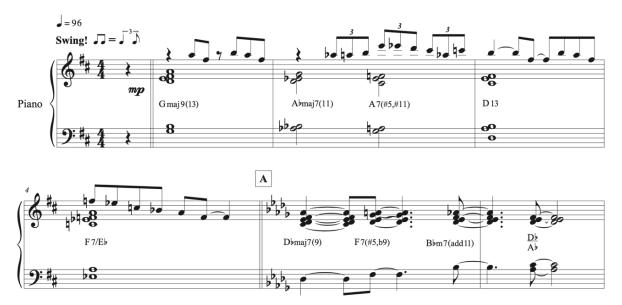
Love Me Still (Composition for Jazz Band)

When I wrote "Love Me Still" (see Appendix E) it was my goal to combine compositional features of Herbie Hancock, the vocal style of Norah Jones, and the poetic elements of Joni Mitchell's lyrics. I set the song in a genre of jazz and gave it colorful harmony and placed every section in a new tonality. Since the tonalities are related and close bye, they don't seem overbearing throughout the song. In this chapter I will analyze two sections in order to demonstrate harmonic polarity: the introduction and bridge.

The idea behind the introduction was to set it in the tonality of D major and provide a contrast with the verse, which is in D-flat major (see Example 4.6). Since the lyrics talk about someone obsessing about their imperfections in terms of "falling," I decided to text paint the idea by letting the tonality fall a half step into the verse. Rather than giving away the tonality of the introduction right away, I decided to open with the cadence IV - V - I and added extensions as well as chromaticism (see measure two) in order to provide more colors. However, the idea of D major is hinted on in the piano melody in measure one.

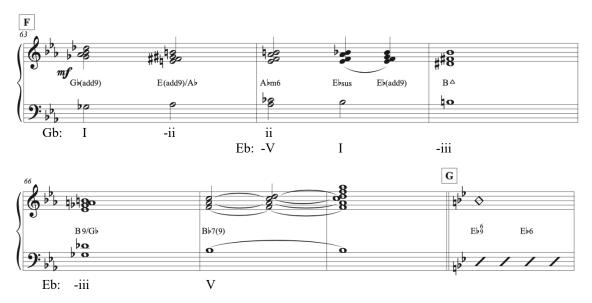
Connecting both tonalities, D major and D-flat major (see measure four) required the use of a chord that works in both tonalities. For this purpose, I chose to connect them with F7/Eb. This chord belongs to the negative harmony of D major (-vi) and is related to D-flat major according to Riemann's theory of third relationships. Technically, only the triad and not its seventh extension belongs to the chords exact mirrored opposite of the submediant in Negative Harmony, but I chose to give the triad a positive dominant function.





The chord progression in the bridge provides a good example of rich harmonic color achieved through polarity in form of Negative Harmony (see Example 4.7). The

first measure of the bridge is preceded by the tonality of A major (m. 62). I chose to apply Levy's technique of primary successions (Levy 1985, 34), which forms cadences between two triads by keeping one tone the same while moving the other two a step up or down to form a new chord. Thus, I took the triad A major and while keeping its third Csharp constant, I spiraled the root tone A and the fifth, E, a half step outward, resulting in F#-Major. I planned on using this chord as a steppingstone to the tonality of E-flat major, which is why I labeled it enharmonically (Gb).



Example 4.7 Negative Harmony and Harmonic Polarity in "Love Me Still" (bridge).

The bridge section gives the listener the feeling of arriving at a new tonic, G-flat major, in measure 63. This tonic continues to be surrounded by its negative supertonic, E, while carrying the bass note of its positive supertonic, Abm, to which the chord progression continues in the next measure (m. 64). This particular chord is used as the negative dominant of Eb in order to switch to a new tonality which is establishes in measure 64. The chord progression continues with the negative mediant, B (-iii), before it chromatically descends to Bb7, the dominant of Eb. The song continues with sustaining

the tonality of E-flat major in the chorus, but as the section continues it turns out that the chorus belongs to the tonality of B-flat major. The ear was simply deceived as the chord Eb changed its tonic function to a dominant function belonging to B-flat major.

La Vie En Rose (String Arrangement)

In choosing to write for strings, I decided to take a different approach on polarity. Rather than composing complex chord clusters such as in "Stitches" and "White Christmas," I decided to stick to a simpler approach by using less extensions, simpler harmonies, and melodic mirroring. An example of melodic mirroring can be found right at the beginning of the song, in which the first violin plays the mirrored opposite – with minor modifications – of the chorus melody.

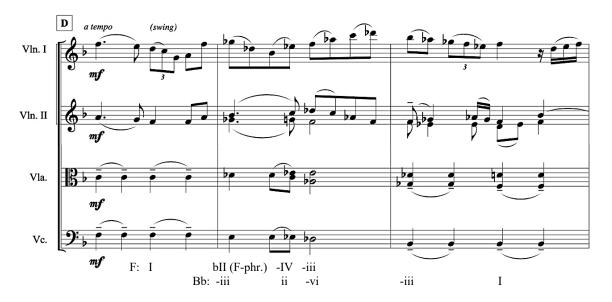
Example 4.8. Melodic Mirroring and Negative Harmony in "La Vie En Rose" (see Appendix H for full score).

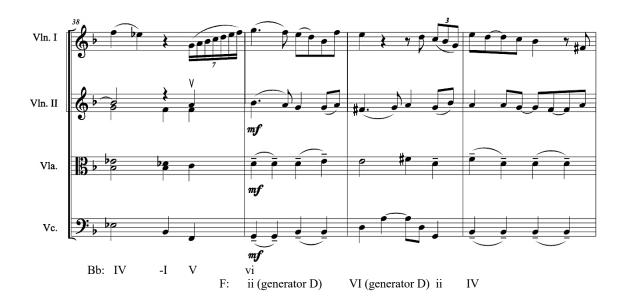


The introduction of "La Vie En Rose" (see Example 4.8) starts harmonically with the negative tonic, A minor, which is the Negative Harmony of A major. The original melody of the chorus that is mirrored in this section starts with the tonic A (see m. 9). Keeping in mind that the positive series is generated from the tonic whereas the negative series is generated from the fifth scale degree downwards, I established the first note of the mirrored melody as E. From then, I chose the exact intervallic opposite with a small exception on beat 4 of measure 1 (D instead of C) to keep the melody line smoother for the violin. The chord progression continues from the negative tonic Am to the negative dominant Dm in the second measure and follows with E7 (V), D (IV), Dm (-V), and E in order to lead the listener back to the tonality of A major.

The next section I want to elaborate on is the instrumental section (Example 4.9), in which the first violin improvises upon the melody of the chorus. I chose for this section to be the most harmonically colorful part of the entire arrangement since the strings are in the spotlight.







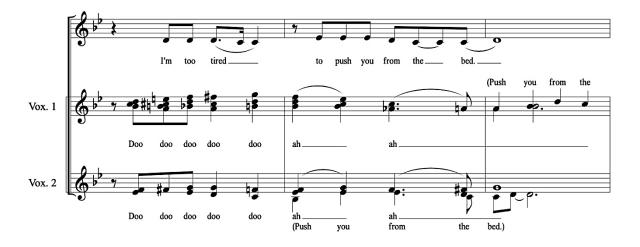
The harmonic analysis of "section D" begins with establishing F as the key center. In the second measure the chord progression continues with F#7/E, which leads the upper voicing a half step up whereas the bass moves in the opposite direction. This symmetrical motion is well displayed between the Viola and Cello parts. The chord itself belongs to the mirror harmony of F major, which is F Phrygian and functions as a Neapolitan chord (bII). At the same time the harmonic progression prepares for the establishment of a new key-center, B-flat major. The chord progression continues with F#7/E leading to Cm (ii of B-flat) in order to approach Dbsus (-vi), which belongs to the negative series of B-flat major. This chord is followed by Gb7/Bb (-iii) in the third measure and finds resolution in the new tonic, B-flat major (I). An appearance of the negative tonic can be seen in the second staff system (m. 38) as the phrase is completed with B-flat minor.

A form of mirror harmony based on one generator is located in measure 40. As the section is returning to the tonality of F major, the chord progression continues with Gm (ii) leading to D major before returning back to Gm in order to approach the positive subdominant, Bb. Both chords, Gm and D, are generated by D, which makes them exact symmetrical opposites according to Levy.

Space Between (Vocal Arrangement)

The challenge of reharmonizing Sia Furler's "Space Between" (see Appendix G) came with the fact that the original chords of the song already contained a lot of elements of Negative Harmony. The verse, for example, starts with the chord progression C - Fm, which can be interpreted as tonic and negative dominant of C. The verse ends with Am (vi) in order to chromatically approach Ab in the pre-chorus, the subdominant chord of a new tonality (E-flat major) that continues throughout the chorus.

I approached the reharmonization of "Space Between" with a plan to add harmonic color and variety through Negative Harmony, Mirror Harmony, combining tonalities that are related by third relationships, as well as chromaticism. I focused on keeping the voice leading as smooth as possible and gave priority to the horizontal spectrum rather than the vertical. I started the first verse with familiar harmonies but added little moments of harmonic surprise to keep the listener interested. For example, in the second half of the verse that originally starts out with the tonic (in this case B-flat major), I chose to begin with the subdominant Ebmaj7(9,13) and allowed some of the upper voices to climb up chromatically while the other voices either fall by step or leap (see Example 4.10).



Example 4.10 Chromatic Motions and Polarity in "Space Between."

With a short moment of landing on A9(add13)/F#, the symmetrical center (X) of the E-flat major scale, the chord progression continues with a new voicing of the tonic, Eb9(add13). At this point the voicing begins to stretch outwards to Dmaj7(add11) followed by Cmaj7(11, omit3), which are major chords built on the melodic line (III, II of B-flat major). These chords would technically be minor in the key of B-flat major but shifting the generator from the root to the fifth allows the composer to change the quality of the chord. At this point the choir expresses the emotion of the lyrics that reflect tiredness by stretching the voicings outwards, leading to a yawning moment in the next measure.

The chord progression continues with finishing up the verse with Ebmaj7(9, omit3)/Bb – Eb6 - Fm6 - D7 - Gm (see second measure of Example 4.10). The first chord (Eb) fulfills the subdominant function (IV) and follows with the negative subdominant of Bb (Fm6). The next chord, D7 (V/vi), fulfills a dominant function towards the last chord of the first verse, Gm.

The pre-chorus that follows immediately introduces polarity by centering the tonality in B-flat negative. The chord progression moves chromatically towards Gb (-iii of B-flat major) and moves upwards to Bbm, which provides a stark contrast with the previous verse that is centered around B-flat major.

A point of experimenting with Mirror Harmony appears with the start of the second verse. Rather than combining the harmonic content of a major scale with the descending phrygian scale of its fifth scale degree (as is done in Negative Harmony), I combined the melodic content of the tonic centered on B major with the harmonies of descending phrygian scale of its third scale degree (D-sharp Phrygian). Since fifth relationships (dominants) can be replaced by thirds according to Hugo Riemann, I applied his principle to Mirror Harmony by combining the melody in B major with the harmonies of D-sharp Phrygian.

Example 4.11. Mirror Modal Equivalents in "Space Between" (second verse)



Conclusion

I want to express my conclusion in the words of Öttingen, who commented upon his discoveries: "...the theory of harmony in its development is nowhere near the end, but that it rings with new forms on and on" (Öttingen, 1866). My research has taught me much on how harmonies can be combined due to symmetry and tonal structure. The efforts of dualists such as Riemann and Levy laid the foundation for a theory that attempts to explain the mathematical and symmetrical structure of harmonic paths which equip the composer with new strategies to create motion within music. Reflecting on my writing and arranging, it has definitely changed my musical approach by expanding the horizon of harmonic possibilities. Discovering those possibilities made me realize how much more there is yet to find in the area of harmonic polarity. Seeing how this study has influenced my own writing, I believe that a deeper insight and interest in harmonic polarity encourages composers to create something new and out of the ordinary. For this reason, a general understanding of this topic is very beneficial to the music community as it spikes new ways of approaching harmony and tonal structure that could open doors to new music styles and genres.

Negative Harmony is only the newest branch among many within harmonic polarity that has led many composers around the world back to exploring symmetry and polarity in music. I have spent the past year devoting myself to dive into the musical universe of polarity and can confirm that its realm is infinite and indeed nowhere near the end.

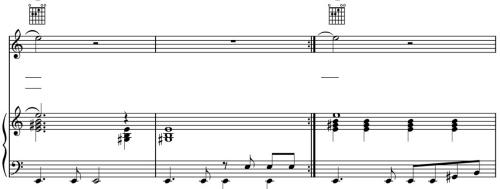
Appendix A: "It Never Happened Before" by Paul McCartney

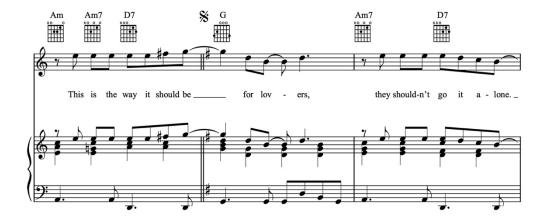


This Never Happened Before

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Appendix B" "Court and Spark" by Joni Mitchell









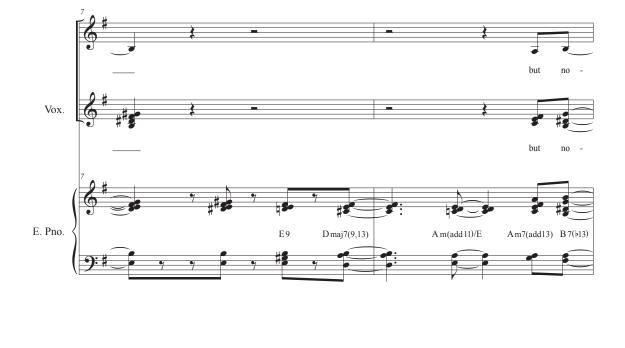
Appendix C: "Stitches"

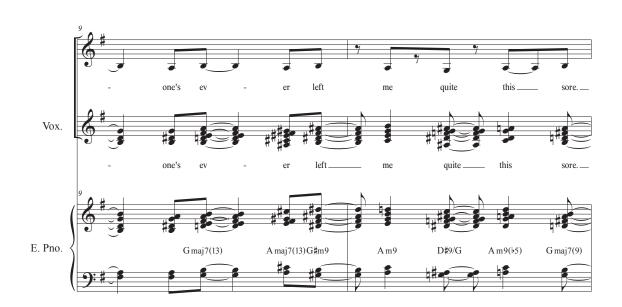
Stitches by Shawn Mendez



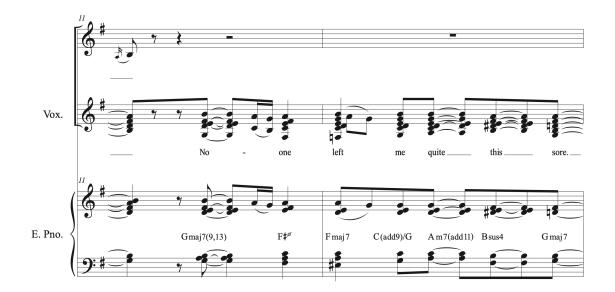
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Stitches

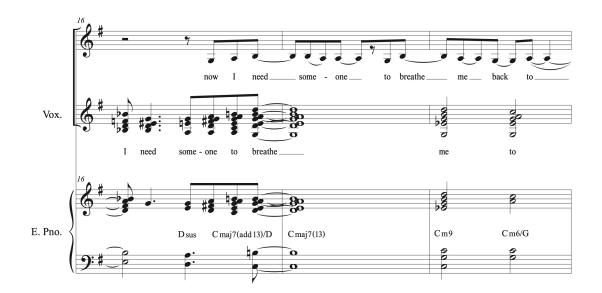


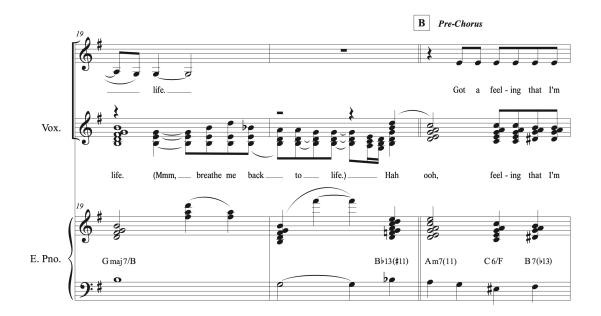




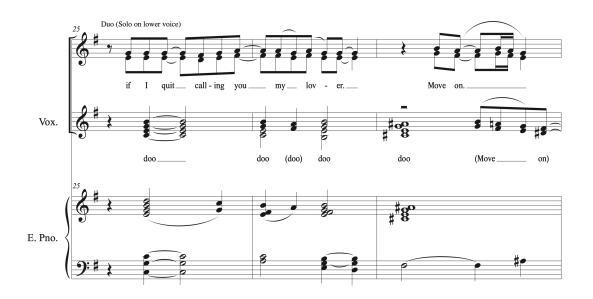












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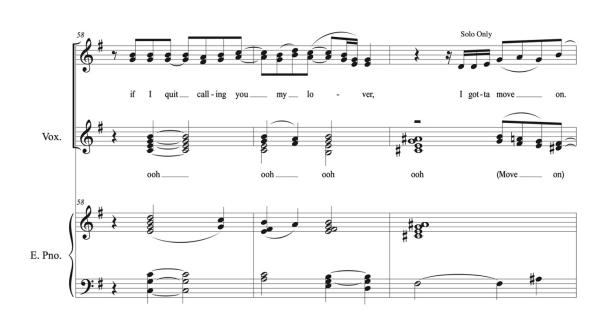




















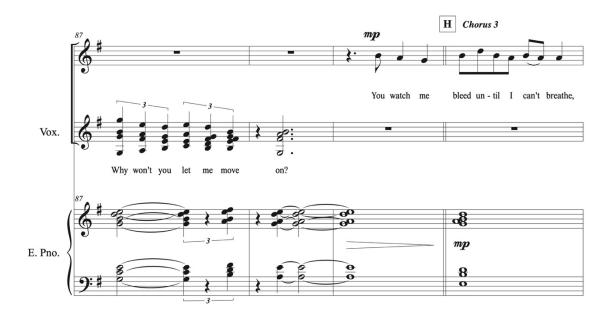








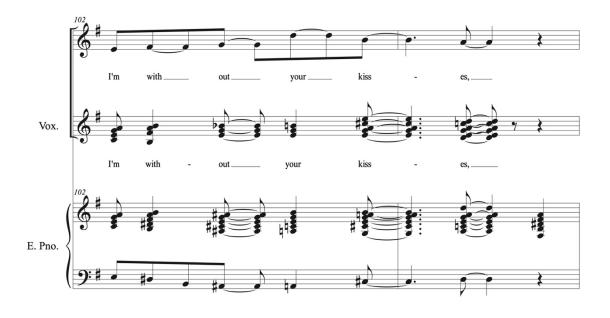
















Appendix D: "White Christmas"

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White Christmas

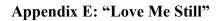


















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Love Me Still





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Love Me Still













Appendix F: "Paradise"

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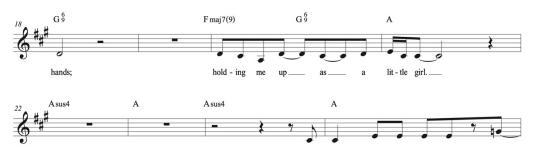












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Paradise





Paradise













Appendix G: "Space Between"

Space Between



Composed by SIA FURLER Arranged by Debora Cahoon

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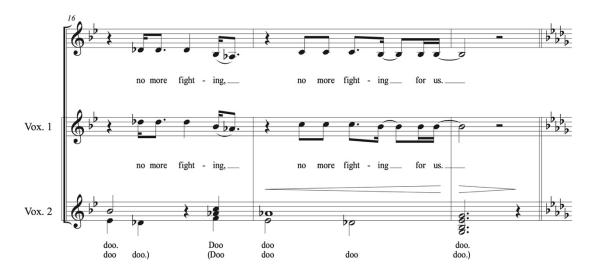
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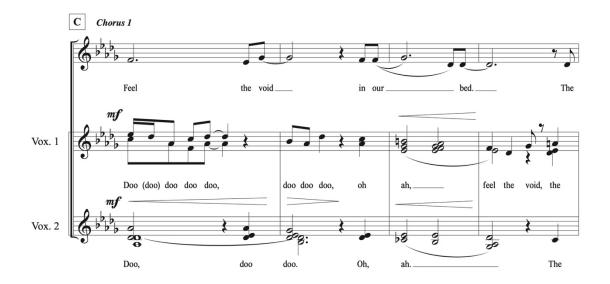
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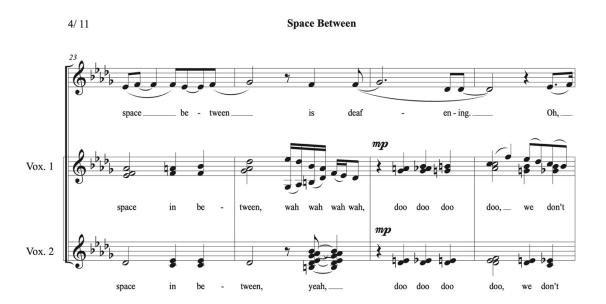
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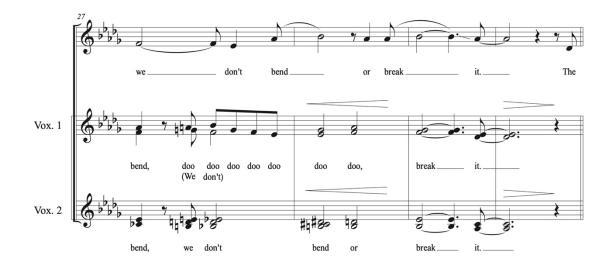
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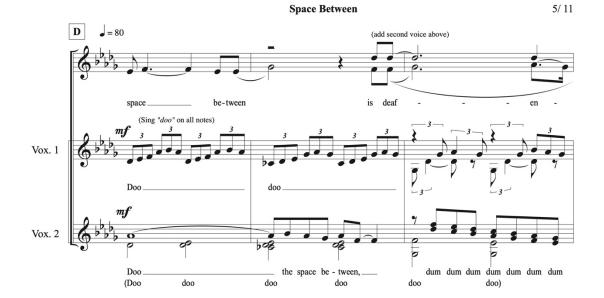
Space Between

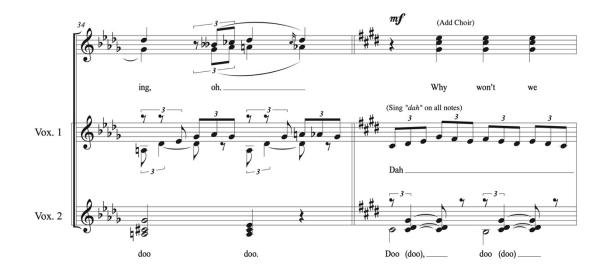


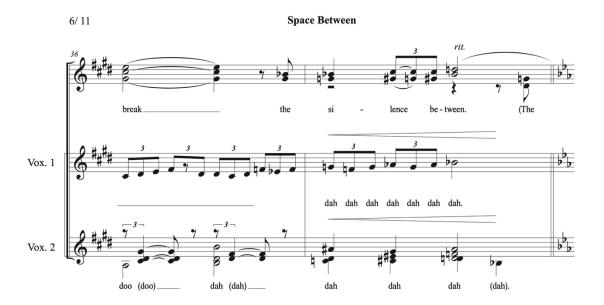


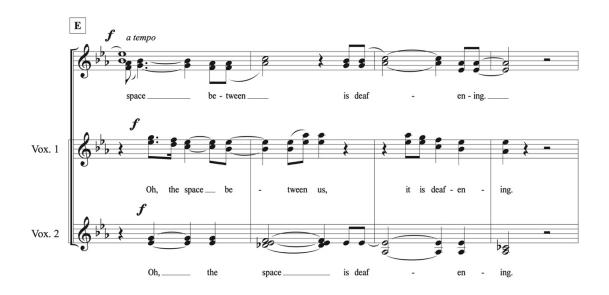


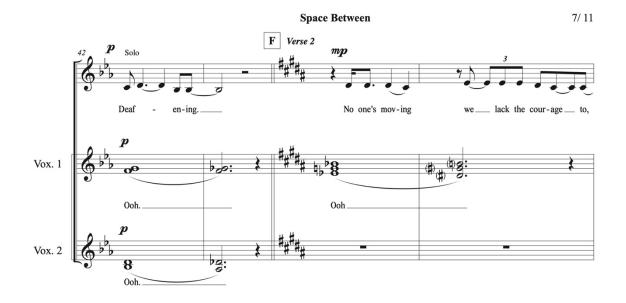


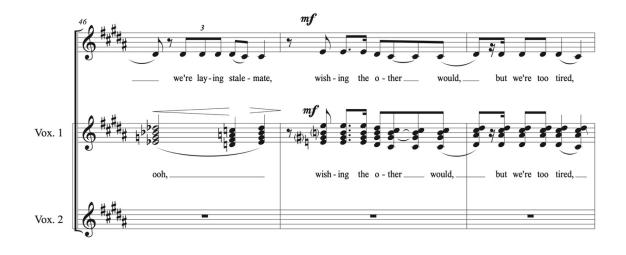




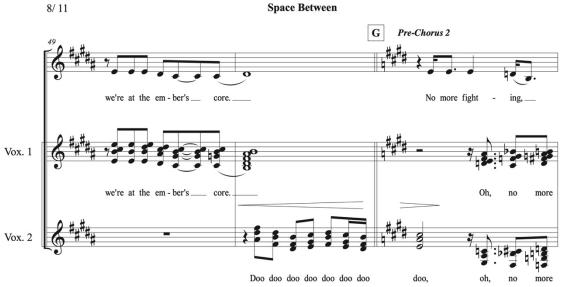


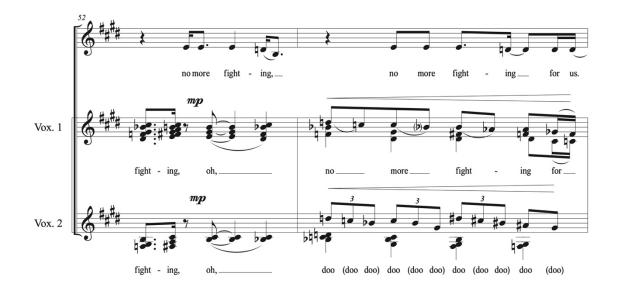


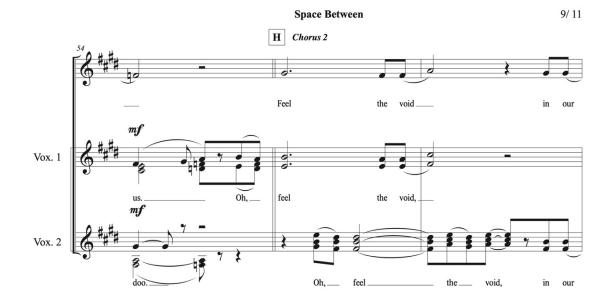


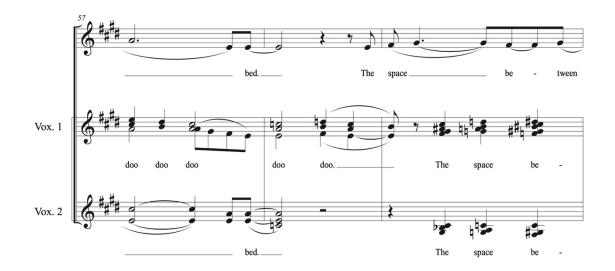


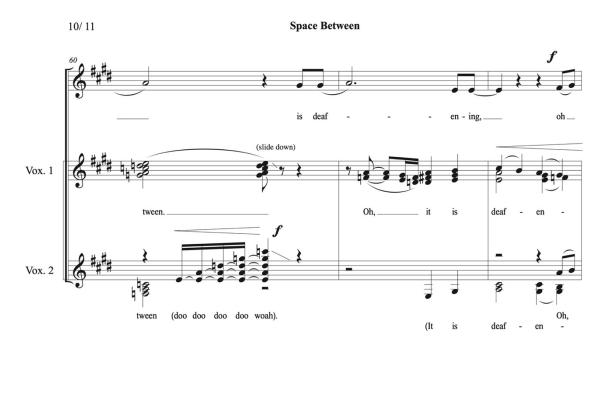
Space Between

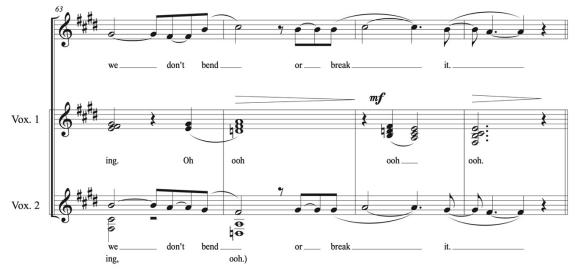


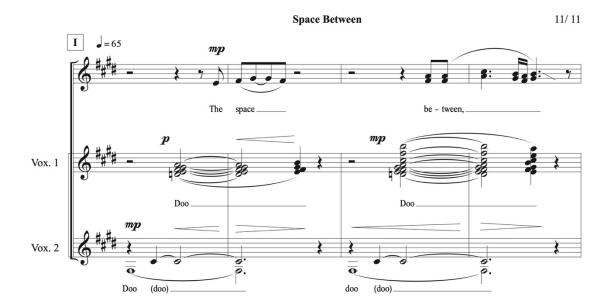


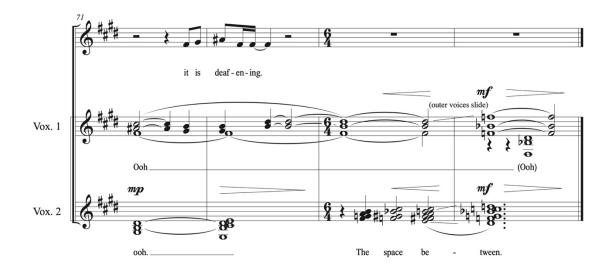












Appendix H: "La Vie En Rose"

Score

La Vie En Rose





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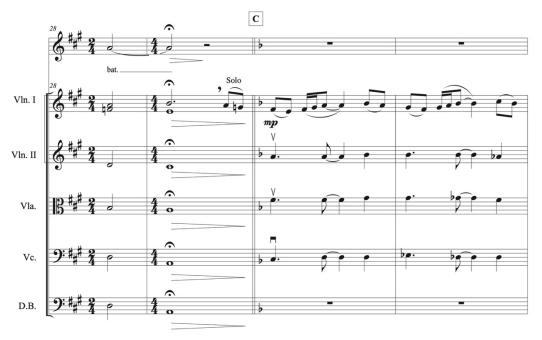




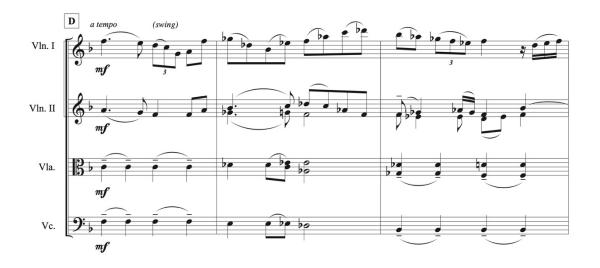








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