Chapter

Reconfigurable Minimum-Time Autonomous Marine Vehicle Guidance in Variable Sea Currents

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Abstract

In this chapter, we present an approach of reconfigurable minimum-time guidance of autonomous marine vehicles moving in variable sea currents. Our approach aims at suboptimality in the minimum-time travel between two points within a sea area, compensating for environmental uncertainties. Real-time reactive revisions of ongoing guidance followed by tracking controls are the key features of our reconfigurable approach. By its reconfigurable nature, our approach achieves suboptimality rather than optimality. As the basic tool for achieving minimum-time travel, a globally working numerical procedure deriving the solution of an optimal heading guidance law is presented. The developed solution procedure derives optimal reference headings that achieve minimum-time travel of a marine vehicle in any deterministic sea currents including uncertainties, whether stationary or time varying. Pursuing suboptimality, our approach is robust to environmental uncertainties compared to others seeking rigorous optimality. As well as minimizes the traveling time, our suboptimal approach works as a fail-safe or fault-tolerable strategy for its optimal counterpart, under the condition of environmental uncertainties. The efficacy of our approach is validated by simulated vehicle routings in variable sea currents.

Keywords: guidance, minimum-time, marine vehicle, sea current, suboptimal, environmental uncertainty

1. Introduction

It is well known that the sea environment contains several kinds of flows, which possibly interacts with the motion of surface or submerged vehicles. Among these, sea or ocean currents are the most significant ones, directly affecting traveling speed, power consumption, and thus the endurance and range of a vehicle. Suppose that a marine vehicle is to travel to a given destination starting from a point in the region of flow disturbance. Then, it is quite natural that the traveling time of the vehicle should change according to the selection of a specific path. In case the power consumption of a vehicle is controlled to be constant throughout the travel, the total energy consumption is directly proportional to the traveling time.

Recently, autonomous marine vehicles (AMVs) are playing important roles in diverse applications, such as oceanographic survey, marine patrol, undersea oil/gas production, and various military applications [1]. Relying on onboard energy storage as the main energy source, the endurance and moving range of an AMV are limited by its power consumption and its capacity of energy storage. Therefore, it can be said that the reduced traveling time of an AMV enhances vehicle safety and mission effectiveness [2].

Considerable research works have been done on the guidance or path planning for a mobile vehicle through varied fluid environments. Though aiming at the same objectives, the most notable difference between the guidance and the path planning of a vehicle is the consideration of its dynamical constraints. While, in general, dynamical constraints in vehicle motion are incorporated into the formulation of vehicle guidance problems [3, 4], they are ignored in most path planning problems [5, 6]. This allows great flexibility in the target path generation, enabling the use of combinatorial optimization techniques in path planning approaches. Dynamic programming (DP) might be one of the most classical and popular techniques for combinatorial optimization. In [6], the problem of minimal-time vessel routing in a region of deterministic wave environment is treated on the basis of the dynamic programming approach. In this problem, sea region is subdivided into several subregions of different sea states. The optimal path is derived by determining the sequence of subregions to be visited, which minimizes the traveling time to a given destination. Aside from the difficulty in establishing a practically available numerical procedure adjoining the formulation, the significant solution dependency on the regional subdivision is a critical issue in this approach. Some recent researches reported the application of a generic algorithm (GA) to path planning for an underwater vehicle in a variable ocean [5]. Major advantages of the GA over dynamic programming are reduced computational complexity and time, although it is susceptible to local minima, however. Also, one of its significant drawbacks is a strong constraint in generating the optimal path. In a path planning application on the basis of GA, a user-defined primary coordinate should strictly maintain a monotonic increase in the optimal path [5]. This is such a strong constraint that makes it impossible to generate the optimal path containing interim backward intervals. Minimum-time guidance of a mobile vehicle in a fluid environment of arbitrary flow field is a strongly nonlinear optimization problem, quite difficult to solve numerically as well as analytically. One of the recent approaches to treating this sort of problems is cell mapping [3]. Though it is known to be especially adequate for strongly nonlinear problems, computational demand of cell mapping for obtaining a stable solution is enormous.

Path finding or guidance algorithms can be classified into two categories according to the instant when its solution is generated. While a pregenerative one derives an unchangeable solution prior to a mission, a reactive algorithm allows revised solution during the mission [5, 7]. In this research, as a reactive strategy for achieving minimum-time travel in varied sea current environments, we propose an approach of suboptimal guidance. In our problem, minimum-time travel of a vehicle is attempted on the basis of the optimal guidance law presented by Bryson and Ho [8]. The solution of this guidance law is a time sequence of the optimal headings. In an actual field application for the minimum-time travel, obtained optimal headings are tracked by a vehicle as the reference in its heading control. Compact as it is, the optimal guidance law is derived without considering any specific dynamic constraint, like many other path planning approaches. In our suboptimal strategy, we compensate for this drawback by incorporating reactive revisions of optimal reference heading. Once there happens a failure in tracking current optimal reference attributed to the ignorance of limitations in vehicle dynamics, onboard autopilot reroutes the vehicle by reapplying the optimal guidance law.

In addition to the dynamic constraints, there are several unfavorable environmental factors that might be fatal in achieving the proposed optimal vehicle routing.

Examples of such factors are uncertainties in sea environments, severe sensor noises, or temporally faulty actuators [9, 10]. As a fail-safe or fault-tolerable strategy, our suboptimal approach can compensate for the failure in ongoing minimum-time travel due to any of the abovementioned factors. The suboptimal guidance does not achieve rigorous optimality. However, it achieves a near-optimality realized by the utmost in-situ actions as possible, which is useful and important in a practical sense.

Though provides superior adaptiveness, robustness, and more flexibility, the reactive approach in marine vehicle guidance incurs a heavy computational cost in its onboard implementation [3, 5, 10]. In this research, we present a practical solution procedure of highly reduced computational cost required for implementing our minimum-time guidance in a suboptimal manner. This is a simple procedure applicable to any sea current whether stationary or time-varying, provided that its distribution at a specified instant is deterministic. Robust global convergence is another advantage of our procedure. On the basis of the minimum principle [8, 11], our procedure realizes an efficient search space reduction, enabling optimal solution search in a global manner. Due to this algorithmic nature, our numerical procedure bears crucially lower possibility of taking local minima than other search algorithms primarily relying on initial guesses.

As mentioned previously, deterministic sea current is the prerequisite for implementing our suboptimal and optimal strategies. It is noted that, however, in many cases, it is not easy to obtain a predescribed current distribution of the sea region of interest. One of the simplest ways to build up the database of sea current distribution is direct measurement. Many governmental, public, or private institutions related to maritime affairs provide tabulated surface current distributions which are obtained by field measurements [12, 13]. The availability of these data is more or less restrictive because there are many sea regions for which the current distribution data are not built up or treated as confidential. As another source of sea current information, numerical estimation models are playing an important role. By assimilating the field measurement into them, some recent numerical models provide both forecasts and nowcasts of ocean fields with sufficiently accurate meso-scale resolution [14]. In this research, we employ two kinds of sea current data generated in totally different ways; the measurement-based stationary current distribution in Northwest Pacific, near Japan, and the sequential tidal current distribution in Tokyo Bay obtained by a numerical forecasting model.

Unlike path-planning approaches, our approach leads to simulation-based resultant optimal trajectory rather than optimal reference path. In our optimal guidance problem, not the position but the heading of a vehicle is employed as the design variable to be optimized. And, since we consider the dynamics of a specific vehicle as a constraint, optimal trajectory is to be generated by the simulated vehicle routing following the optimal reference heading. It is noted here that while the term of path is used as a route or track between one place and another, the trajectory means a curved path that an object describes on the basis of its kinematic scheme in this research [15].

2. Minimum-time guidance

2.1 Problem statement

The problem of minimum-time guidance for AMVs moving in flow fields is described in this section. Consider a marine vehicle traveling through a sea region of flows such as sea currents, whose properties are the function of space, or both space and time. The vehicle is to travel to a predetermined destination starting from the initial position at the initial time t_0. Then, it is easily anticipated that the traveling time of the vehicle varies depending on its traveling path (**Figure 1**). Furthermore, it is also anticipated that an ingenious traveling path can minimize the traveling time to the destination. In this research, we refer to the optimal path as the path of minimum traveling time to the destination. As a minimum-time problem, our problem merely takes the traveling time as the performance index. In other words, in the general form used in optimal control, the integrand of performance index takes the value of 1.

$$J = \int_{t_0}^{t_f} L d\tau = \int_{t_0}^{t_f} d\tau = t_f - t_0 \tag{1}$$

In Eq. (1), J is the performance index, and t_0 and t_f are the initial and final time of the travel.

2.2 Formulation

As mentioned previously, we employ the vehicle heading as the output in our optimal guidance problem. Here, it is noted that we adopt the so-called GNC (Guidance, Navigation, and Control) system based on the hierarchical control architecture consisting of two control layers. That is, the high-level control for guidance and navigation, and the low-level control for pure tracking purpose (**Figure 2**). The optimal heading derived by solving the optimal guidance law is used as the reference output for low-level heading tracking control. In this research, we use the optimal heading guidance law presented by Bryson and Ho [8]. In deriving the optimal guidance law, two sets of coordinate systems are used: the inertial (earth-fixed) coordinate system *o-xy* and the body fixed coordinate system *o'-x'y'* (**Figure 3**).

As the marine vehicle used in our problem, we employ an autonomous underwater vehicle (AUV) "r2D4" as described by Kim and Ura [10]. In **Figure 3**, actuator inputs and kinematic variables are described. ψ is the yaw displacement of the vehicle. While δ_{pr} denotes the main thruster axis deflection, δ_{el} and δ_{er} are the deflections of elevators on left and right sides, respectively.

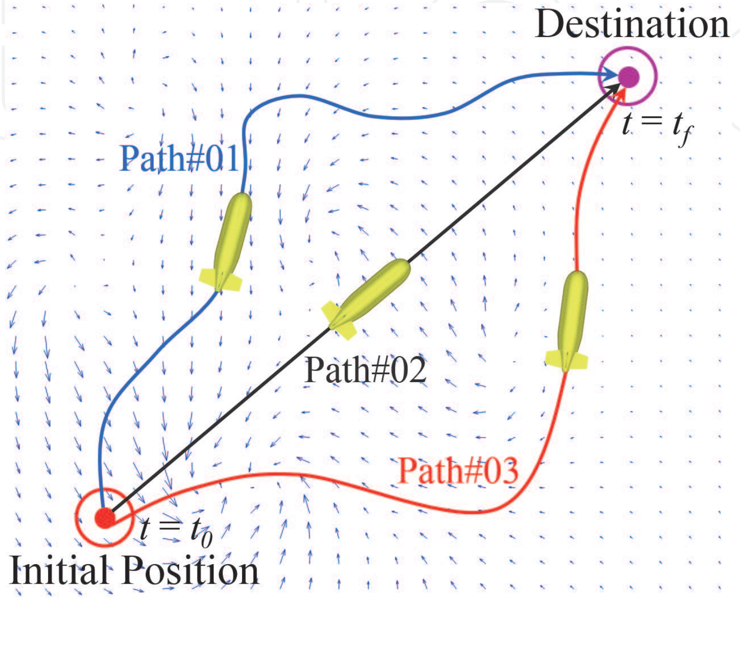

Figure 1.
Dependence of traveling time on traveling path.

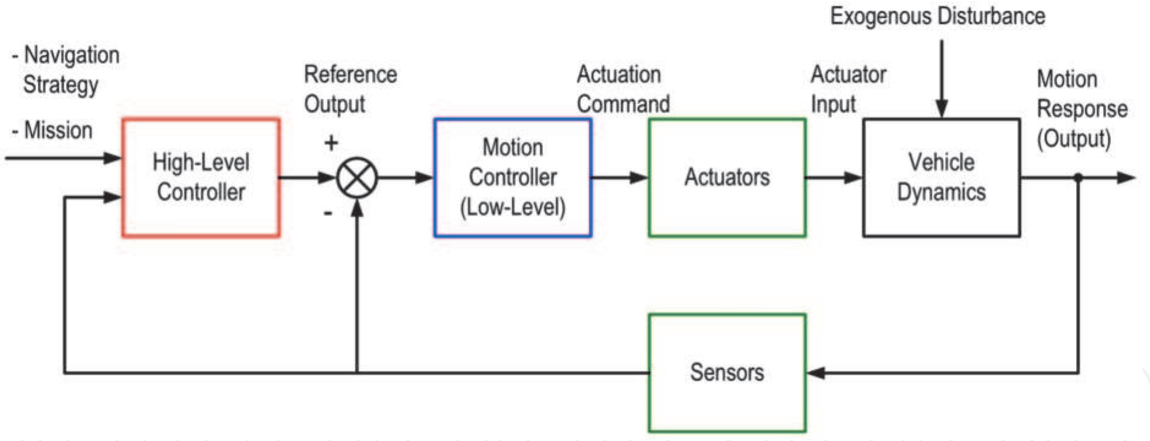

Figure 2.
Two-layer hierarchical control architecture for an AMV.

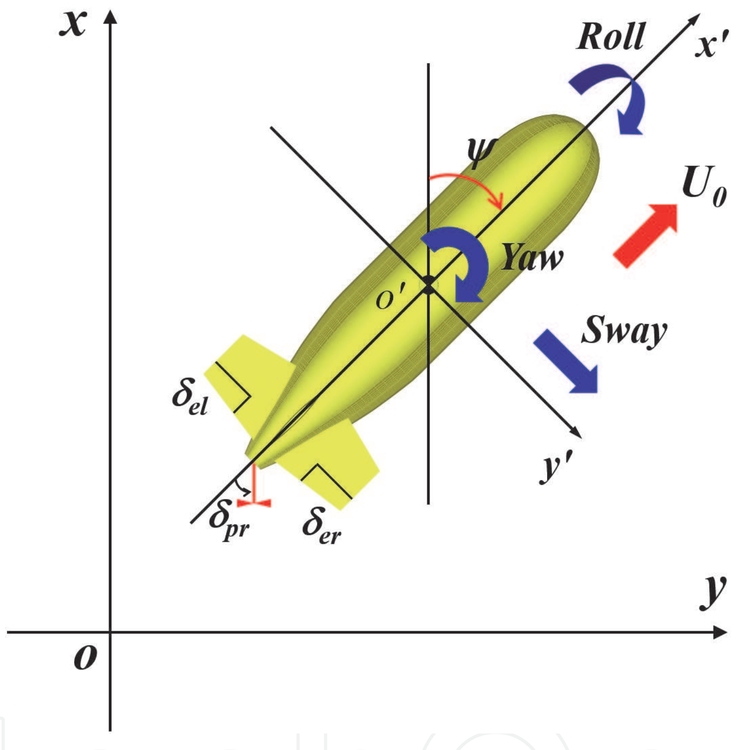

Figure 3.
Coordinate systems for optimal guidance problem formulation.

In this research, we approximate that the direction of the vehicle's advance velocity coincides with the x'-axis. It is arguable in the rigorous definition since there certainly occurs sideslip during a turning motion of an underactuated vehicle. However, as mentioned by Lewis et al. [16], the hydrodynamic sideslip induced by a low-speed, slender vehicle is bounded within a sufficiently small range, justifying our approximation. Since the distribution of a sea current is considered to be deterministic in our research, current velocity is described as a function of the position and time. Therefore, on the assumption that the advance velocity of a vehicle and the current velocity are superimposable, the resultant vehicle velocity is expressed as follows:

$$u = \dot{x} = U_0 \cos\psi + u_c(x, y, t) \tag{2}$$

$$v = \dot{y} = U_0 \sin\psi + v_c(x, y, t) \tag{3}$$

where u and v are the components of the vehicle velocity relative to the inertial frame, U_0 is the advance speed of the vehicle in still water, and u_c and v_c are the components of current velocity at a given position and time. It is noted that we assume U_0 is the constant throughout a travel, which implies the operating condition of steady cruise.

Eq. (4) shows the minimum-time guidance law originally presented by Bryson and Ho [8]. Detailed procedures deriving Eq. (4) are well explained by Kim and Ura [10]. It is noted here that if only deterministic, there is no restriction on the type of the sea current in Eq. (4). That is, not only stationary but also time-varying sea current can be applied to Eq. (4). This leads to one of the most powerful aspects of our approach over many other path planning approaches based on combinatorial optimization.

$$\dot{\psi} = sin^2\psi \frac{\partial v_c}{\partial x} + \left(\frac{\partial u_c}{\partial x} - \frac{\partial v_c}{\partial y} \right) sin\psi cos\psi - cos^2\psi \frac{\partial u_c}{\partial y} \quad (4)$$

The optimal guidance law shown above is a nonlinear ordinary differential equation of unknown vehicle heading. The solution of optimal guidance law is used as the optimal reference heading, by tracking which a vehicle achieves the minimum-time travel to the destination, leaving the trail of optimal trajectory.

3. Numerical solution procedure

Eq. (4) is a nonlinear ordinary differential equation (ODE) for an unspecified vehicle heading $\psi(t)$. If the functions $u_c(x,y,t)$ and $v_c(x,y,t)$ describing current velocity distribution are differentiable and deterministic, the solution of Eq. (4) seems to be attainable with an initial value of $\psi(t)$, in terms of an appropriate numerical solution algorithm such as Runge-Kutta. However, in practice, with an arbitrary initial heading a vehicle following the guidance law Eq. (4) does not reach the destination, as depicted in **Figure 4**.

More precisely, consisting of a part of the solution, the initial vehicle heading is not arbitrary but is to be assigned correctly. This is because Eq. (4) is derived from

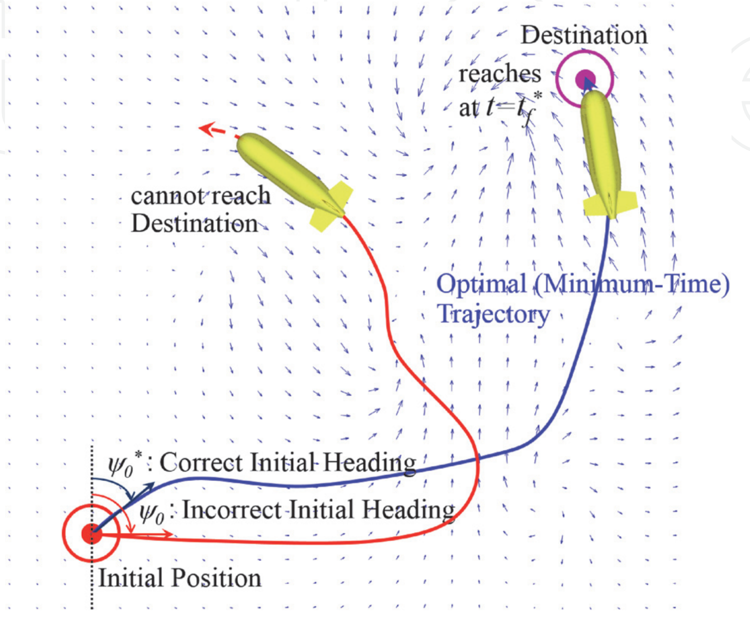

Figure 4.
Solution convergence affected by initial heading.

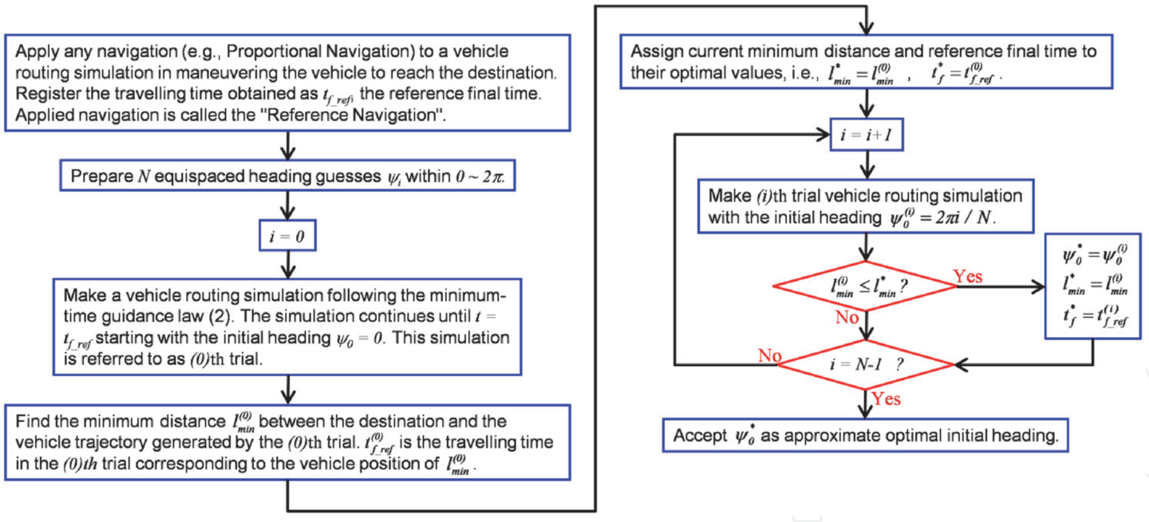

Figure 5.
Schematic of the numerical solution procedure AREN.

the Euler-Lagrange equation, which is a typical example of the two-point boundary value problem, characterized by split boundary conditions in states and costates [8, 11]. To obtain the solution of a two-point boundary value problem, an iterative solution procedure is usually required. The most famous and commonly used numerical procedures for such purpose are the shooting and relaxation methods [17]. However, direct applications of these methods to our problem have significant difficulties. In applying shooting method to a two-point boundary problem in time domain, governing ODEs with proper initial guesses should be integrated until reaching the upper limit of the boundary. However, as noticeable from its name, i.e., the minimum-time guidance, our problem is a so-called free boundary one, having unspecified upper limit in time domain. In treating a free boundary problem by relaxation method, on the other hand, the independent variable should be transformed into a new one defined between 0 and 1. Here, we can anticipate an intrinsic serious difficulty in determining the stepsize in free boundary problems. Properness of temporal grid distribution ensuring convergence is initially unknown, and to know, it is extremely difficult before the end time-marching computation. Moreover, strong initial guess dependency of the solution is another serious concern in applying the relaxation method to our problem. Inappropriate initial guess possibly leads to local optimality or divergence [17].

As a new approach for deriving the numerical solution of the optimal guidance law Eq. (4), we presented a search procedure, which determines correct initial heading. Being named AREN (Arbitrary REference Navigation), our procedure works globally on the basis of the minimum principle. **Figure 5** summarizes the algorithmic scheme of our solution procedure.

Note that in **Figure 5** and hereafter, an asterisked variable denotes the one corresponding to the optimal solution. In applying AREN, we first have to make a vehicle routing simulation in which the vehicle travels to the destination following an arbitrary guidance. It is noted here that the traveling time must be registered at the final stage of this simulation. We call the traveling time the reference final time and use it as the key criterion in seeking optimal initial heading. The navigation applied to the simulation is called reference navigation, which is arbitrary if only the vehicle's arrival at the destination is assured. Therefore, simple one such as proportional navigation (PN) based on the line-of-sight (LOS) guidance is frequently used as the reference navigation. To find the correct initial heading, the interval of 0–2π is divided by equally spaced N-1 subintervals, as represented by:

$$\psi_0^{(i)} = i\Delta\psi \quad for \quad i=0,1,...N\text{-}1 \tag{5}$$

where $\psi_0^{(i)}$ is *(i)*th initial heading guess, and $\Delta\psi$ is the increment of the guess. Next, by applying an initial heading guess $\psi_0^{(i)}$ to Eq. (4), we solve Eq. (4) in time domain. This produces a simulated vehicle routing starting from $\psi_0^{(i)}$. The routing having been produced here is called the *(i)*th trial adjoining to $\psi_0^{(i)}$. Once the vehicle passes through the destination by the *(i)*th trial, it is regarded as a possible optimal routing since the correct initial heading incorporated into the optimal guidance law lets a vehicle reach the destination. Therefore, *N* trials are the candidates for the simulated minimum-time routing. In practice, however, discretization error in the optimal initial heading causes the residual in the optimal trajectory, making the optimal solution identified in an approximate manner. For the vehicle trajectory generated by a trial, we define the "minimum distance" as the shortest distance between the destination and the trajectory. In **Figure 6**, $l_{min}^{(k-1)}$, $l_{min}^{(k)}$, and $l_{min}^{(k+1)}$ are the minimum distances corresponding to *(k-1)*th, *(k)*th, and *(k + 1)*th trials, respectively.

When the minimum distance of *(k)*th trial is smaller than any other one, satisfying:

$$l_{min}^{(k)} \le l_{min}^{(i)} \; for \quad i=0,1,...N\text{-}1 \tag{6}$$

we choose the *(k)*th trial as the optimal routing because the vehicle approaches the destination marking the smallest deviation. In determining the optimal routing among the trials, however, there still remains a serious drawback. We have no idea how long we have to continue a trial not to miss the true minimum distance of the trial. We settle this problem by exploiting the result of reference navigation. The reference navigation is apparently a nonoptimal one based on an arbitrary guidance only assuring the arrival at the destination. Therefore, the reference final time t_{f_ref} must be larger or equal to that of the optimal routing as follows:

$$0 < t_f^* \le t_{f\_ref} \tag{7}$$

where t_f^* represents the traveling time of the optimal routing. It should be noted here that by the minimum principle [8, 11], we can set up a sufficient condition for

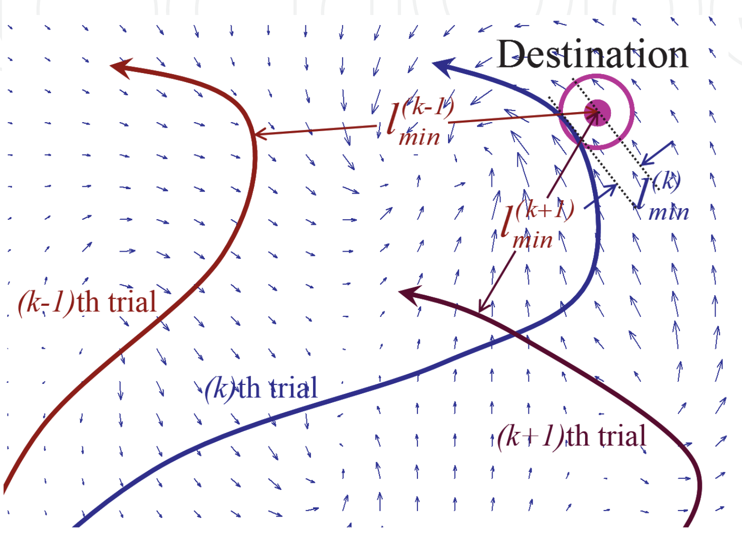

Figure 6.
Minimum distances of trials.

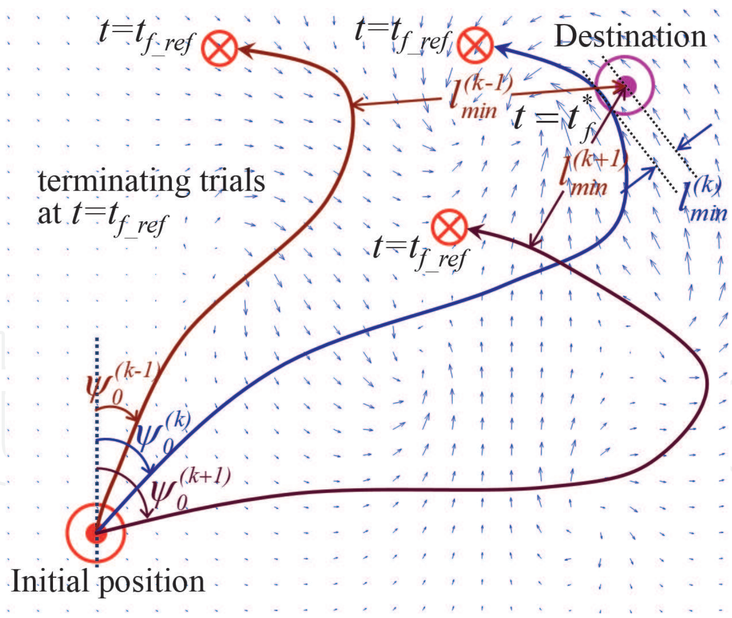

Figure 7.
Determining optimal routing among trials.

seeking the optimal solution. By the minimum principle, once a trial has started with an initial heading sufficiently close to the optimal value, the vehicle obviously passes by the vicinity of the destination at the traveling time smaller than t_{f_ref}. In other words, the reference final time qualifies as the upper limit of the necessary simulation time of any trial, which assures the convergence to the vicinity of the destination in case the trial is near optimal. In **Figure 7**, *(k)*th trial is selected as the optimal routing among all trials terminated at t_{f_ref} since $l_{min}^{(k)}$ is the smallest minimum distance.

The minimum distance of the optimal routing is to be interpreted as the residual error in the converged solution. Therefore, it can be said that the smaller the minimum distance is, the better the convergence is. When $l_{min}^{(k)}$ is still unacceptably large though the *(k)*th trial has been accepted as the optimal routing, the initial heading interval of $\psi_0^{(k-1)} \sim \psi_0^{(k+1)}$ is subdivided, and the trials are repeated starting from these subdivisions pursuing finer convergence.

4. Dynamic constraint

As mentioned previously, we adopt GNC system based on the hierarchical control architecture consisting of two control layers. In the high-level control layer,

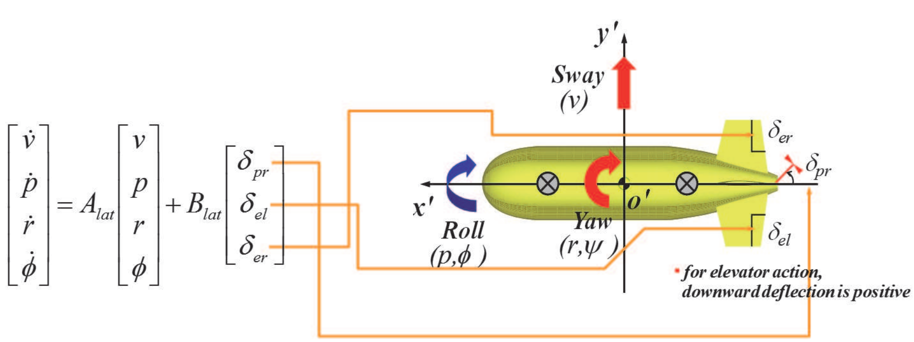

Figure 8.
Graphical description of the equation of motion for lateral dynamics.

i.e., the optimal guidance, the guidance law derived irrespective of specific vehicle dynamics is used. In the low-level control layer, however, vehicle dynamics is implemented as an implicit constraint. When conducting the trials explained in previous section, closed-loop dynamics of a specific vehicle is used. Therefore, vehicle trajectories generated by the trials are feasible ones subject to the dynamic constraint of a specific vehicle. Eq. (8) is the state-space model of the lateral dynamics of r2D4 describing its sway, roll, and yaw responses. In **Figure 8**, kinematic variables and actuations appearing in Eqs. (8)–(10) are described graphically. By solving Eq. (8) in time domain, velocities and attitudes of the vehicle are obtained.

$$\begin{bmatrix} \dot{v} \\ \dot{p} \\ \dot{r} \\ \dot{\phi} \end{bmatrix} = A_{lat} \begin{bmatrix} v \\ p \\ r \\ \phi \end{bmatrix} + B_{lat} \begin{bmatrix} \delta_{pr} \\ \delta_{el} \\ \delta_{er} \end{bmatrix} \tag{8}$$

where

$$A_{lat} = \begin{bmatrix} -0.5266 & 0.0029 & -0.7754 & -0.0052 \\ -3.5308 & -4.7898 & 7.9422 & -10.5536 \\ -0.1472 & -0.0334 & -0.5742 & -0.0722 \\ 0.0 & 1.0 & -0.0129 & 0.0 \end{bmatrix} \tag{9}$$

$$B_{lat} = \begin{bmatrix} -0.0335 & -0.0011 & 0.0011 \\ 0.0244 & -2.1704 & 2.1704 \\ 0.0486 & -0.0149 & 0.0149 \\ 0.0 & 0.0 & 0.0 \end{bmatrix} \tag{10}$$

5. Minimum-time guidance in stationary current flows

5.1 Reference navigation

As mentioned previously, in order to practice the numerical procedure AREN, a vehicle routing simulation by the reference navigation has to be conducted beforehand. Among several strategies for mobile vehicle navigation, the simplest one ensuring arrival at the destination might be the PN based on LOS guidance. In all of our optimal guidance examples presented in this paper, we employ PN as the reference navigation.

5.2 Linear shearing flow

The first example of the optimal guidance in this paper is the minimum-time routing in a current disturbance of linear shearing flow, taken from Bryson and Ho [8]. The current velocity in this problem is described by:

$$u_c(x, y) = 0 \tag{11}$$

$$v_c(x, y) = -U_c x/h \tag{12}$$

In Eq. (12), U_c and h are constants whose numerical values are set to be 1.54 m/s and 100 m, respectively. In this example, the vehicle is to travel to the destination located at the origin, starting from the initial position at (−186 m, 366 m). As an operating condition, the vehicle is assumed to maintain its thrust power constant throughout its travel, producing a constant advance speed of 1.54 m/s. This is an important operating condition applied to all examples presented thereafter. With the current distribution given as Eqs. (11) and (12), we can derive the analytic optimal guidance law in a closed form, shown as follows:

$$\frac{x}{h} = csc\psi - csc\psi_f \tag{13}$$

$$\frac{y}{h} = \frac{1}{2}\left[csc\psi_f(cot\psi - cot\psi_f) + cot\psi(csc\psi_f - csc\psi) + log\frac{csc\psi_f - cot\psi_f}{csc\psi - cot\psi}\right] \tag{14}$$

where ψ_f represents the final vehicle heading taken at the destination. The analytic optimal guidance law shown above is similar to that found by Bryson and Ho [8] but has some differences due to the switched x and y.

Vehicle trajectories are shown in **Figure 9**. In **Figure 10**, vehicle headings obtained by the routings conducted by PN, that is, the reference navigation, and by optimal guidance are shown.

During the vehicle routing by PN, significant adverse drift happens at the initial stage. This is because the speed of current flow exceeds the advance speed of the vehicle in the region $|x| > 100$ m, as noticeable from Eqs. (11) and (12). On the other hand, optimal guidance detours the vehicle across the upper half plane of the flow region on purpose, taking advantage of the strong current flowing to favorable direction. This leads to the dramatic decrease in traveling time. The traveling times by PN and optimal guidance are 353.8 and 739.2 s, respectively, implying a 52% reduction in the optimal guidance. As seen in **Figure 9**, the optimal trajectory

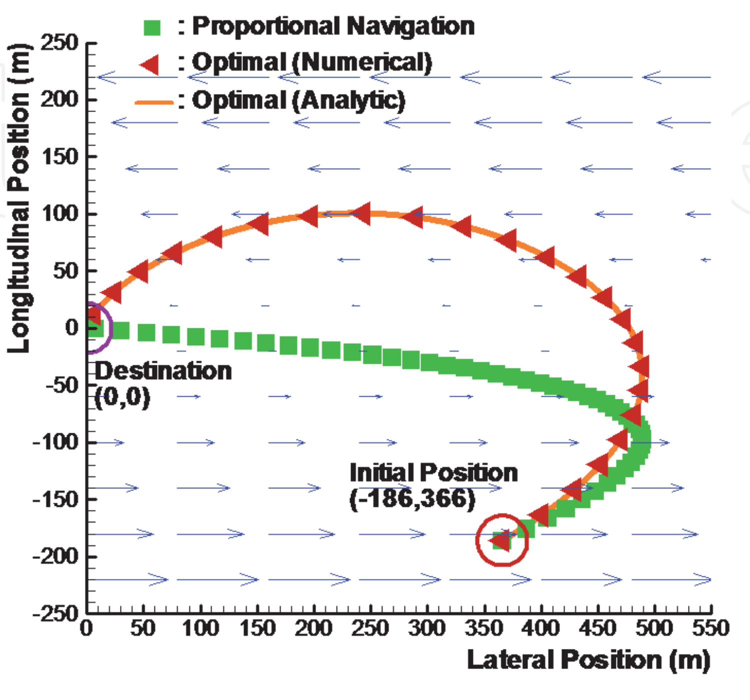

Figure 9.
Vehicle trajectories in a stationary linear shearing flow.

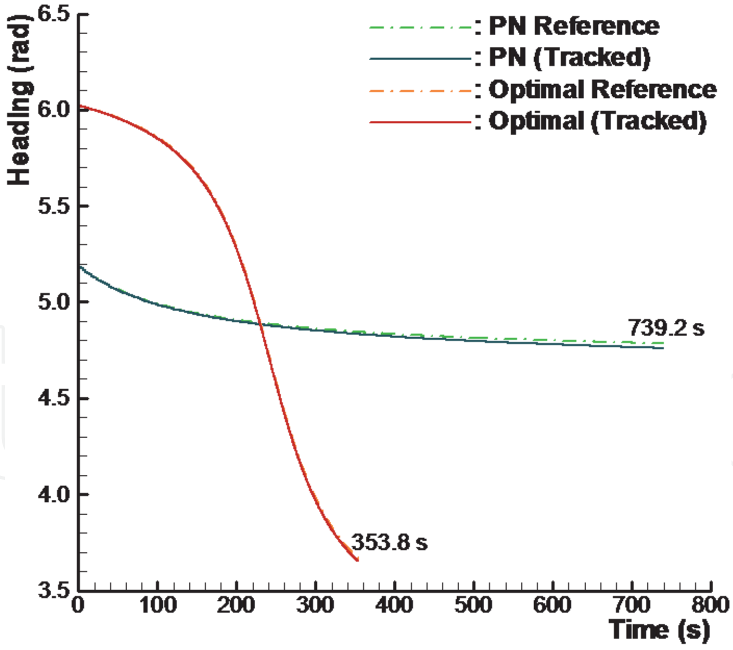

Figure 10.
Vehicle headings during travels in a stationary linear shearing flow.

obtained by the numerical solution shows extremely good agreement with the analytic one.

As a criterion for evaluating the reference tracking performance in our two-layer control architecture, we employ Normalized Root-Mean Square Error (NRMSE) fit defined as follows:

$$fit = 1 - \frac{\left\| \xi_{ref} - \xi \right\|}{\left\| \xi_{ref} - \bar{\xi} \right\|} \tag{15}$$

where $\boldsymbol{\xi}_{ref}$ and $\boldsymbol{\xi}$ are the vectors of output reference and output, and $\bar{\xi}$ represents the mean value of $\boldsymbol{\xi}$. The value of NRMSE fit varies between $-\infty$ to 1, implying full decorrelation to perfect fit between the output reference and the output. In this simulation, NMRES fit between the optimal reference heading and the actually tracked one has marked 0.993. This means highly good heading tracking result, which is also found in **Figure 10**. The numerical solution approximates the analytic solution with extremely high accuracy (**Figure 9**), validating AREN as an effective numerical procedure for the optimal guidance law Eq. (4). As shown in this example, our approach based on the numerical procedure AREN and two-layer control architecture works properly achieving minimum-time AMV routing in a given sea current field.

5.3 Sea current in Northwest Pacific near Japan

In response to the successful result obtained from the benchmark example shown in previous section, we apply our optimal guidance to the minimum-time routing problem in an actual sea region of stationary sea current. The sea region selected is located in the Northwest Pacific Ocean near Japan. The daily updated sea current data of this region are available from [18] presented by the Japan Meteorological Agency. The most notable environmental characteristic in this sea region is the current field dominated by Kuroshio. The Kuroshio is a strong western

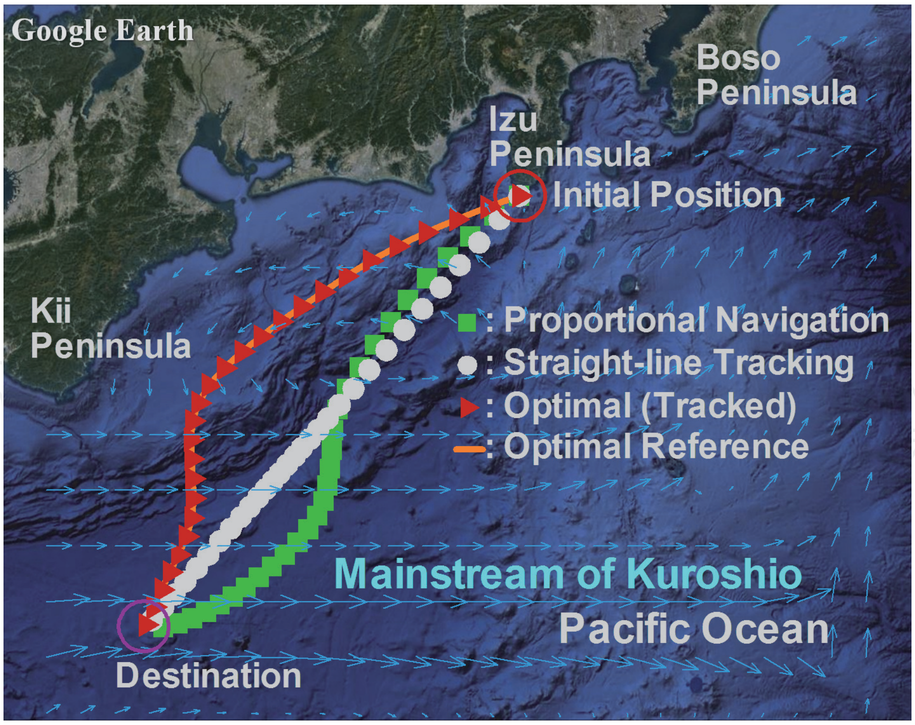

Figure 11.
Vehicle trajectories in a sea current in Northwest Pacific near Japan.

boundary current flowing northeastward along the coast of Japan. In the sea current data from [18], current velocity is defined only on the grid nodes covering the region. As noticeable from Eq. (4), however, in order to derive the optimal heading reference, current velocity and its gradient at every vehicle position have to be available. In the previous example, their exact values are easily obtained by analytic formulae. In this example, however, since they are defined only on the grid nodes, current velocity and its gradient are estimated by interpolating the predefined values on grid nodes surrounding the present vehicle position. In applying the current velocity interpolation, the grid node nearest the present vehicle position is identified first. Then, the current velocity at the present vehicle position is estimated by 2-D bi-quadratic interpolation utilizing the values on the nearest node and

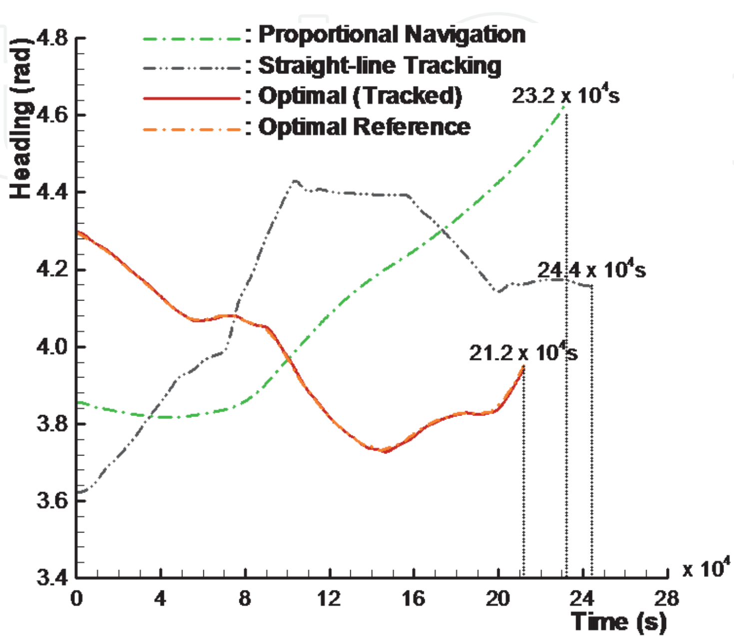

Figure 12.
Time sequences of vehicle headings during the travel.

eight nodes surrounding current vehicle position. Gradients of current velocities are obtained by the same manner. Since the velocity gradients are not provided from the database, however, prior to the interpolation, we calculate their nodal values by finite difference approximation.

Figure 11 shows the vehicle trajectories obtained by the guidance of three different objectives already explained in the previous example. Time sequences of the vehicle headings corresponding to the vehicle trajectories shown in **Figure 11** are depicted in **Figure 12**.

As shown in the figure, like the preceding examples in which the current velocities and their gradients are analytically available anywhere in the region, optimal reference trajectory has successfully been derived by interpolation-based current velocities and gradients. Moreover, subject to its dynamic constraint, the vehicle tracks the optimal reference trajectory with a negligibly small deviation, resulting in the NMRES fit to be 0.986. This demonstrates the validity of our optimal guidance strategy in any actual sea currents, if only their distribution is deterministic.

6. Suboptimal strategy

6.1 Environmental uncertainty

In the following example, we apply our optimal guidance strategy to a vehicle routing in the same sea region shown in the preceding example. The only thing different from the preceding example is that we consider uncertainty in our sea current data in this example. The uncertainty components in sea current velocities are expressed as additive white Gaussian noise (AWGN). Taking the sea current velocities in the Northwest Pacific Ocean used beforehand as the mean values, the on-site current velocity including uncertainty is given by:

$$u_{cs}(x,y) = u_c(x,y) + e_u(\sigma) \tag{16}$$

$$v_{cs}(x,y) = v_c(x,y) + e_v(\sigma) \tag{17}$$

where u_{cs} and v_{cs} are the components of onsite current velocity, u_c and v_c are the components of deterministic current velocity taken from the database, and $e_u(\sigma)$ and $e_v(\sigma)$ are the AWGNs with standard deviation σ. As the parameter for specifying the value of σ, we introduce the regional mean current speed U_{cm} defined as follows:

$$U_{cm} = \frac{\sum_{i=1}^{N} \sqrt{u_{ci}^2 + v_{ci}^2}}{N} \tag{18}$$

where i represents the index covering all grid nodes on which the database-based current velocities are defined.

Vehicle trajectories by optimal vehicle routings conducted on two different level uncertainties are shown in **Figure 13**. When the level uncertainty is such that $\sigma = 2U_{cm}$, optimal heading reference derived without considering any uncertainty still seems acceptable. As a result, though slightly deviating from the destination, the final position of the vehicle remains in the vicinity of the destination. When the level of uncertainty increases up to $\sigma = 4U_{cm}$, however, the vehicle following the

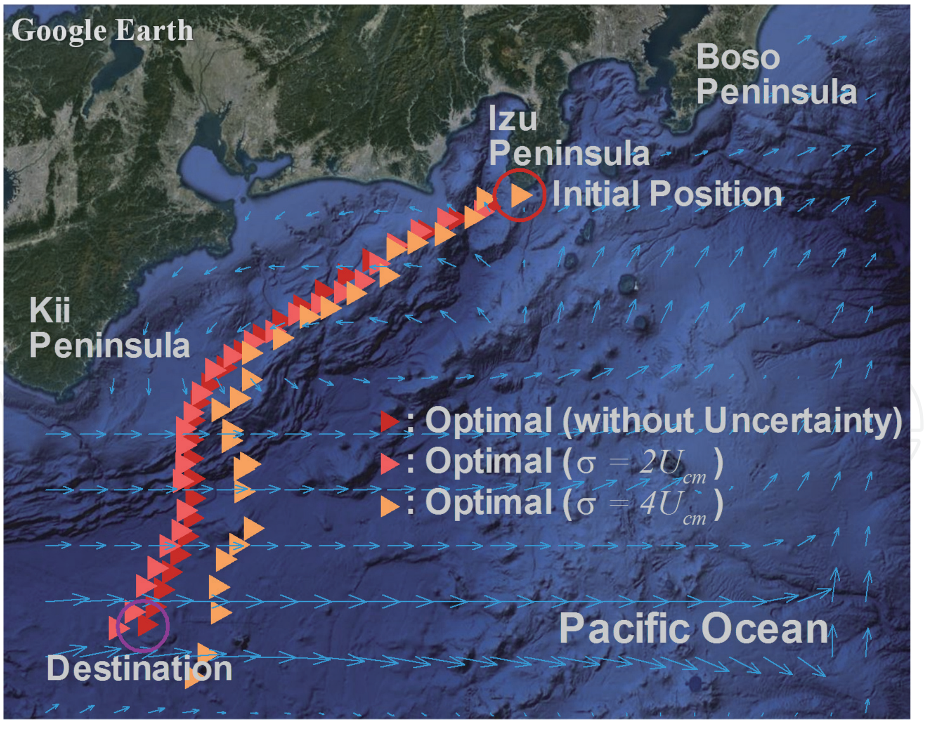

Figure 13.
Vehicle trajectories in Northwest Pacific near Japan. The sea current velocities in this example include uncertainties modeled by AWGN.

optimal reference heading can no longer approach the destination, which means the failure in accomplishing the minimum-time travel to the destination.

6.2 Suboptimal guidance

The suboptimal guidance proposed in this research is a fail-safe or fault-tolerable strategy toward robust field implementation of our optimal guidance strategy. The optimal reference heading obtained by our approach is the one derived without considering the dynamics of a specific vehicle. This means the optimal trajectory may not be realized by a specific vehicle. Hence, we note that the dynamic constraint is one possible source of the failure in putting our approach into practice for an actual field application. Another significant source of the failure is the environmental uncertainty, as already shown above. It is easily expected that as a vehicle progresses following the optimal heading in the sea region of environmental uncertainty, due to the interaction with the current flow different from that was used in deriving the optimal heading, its actual trajectory deviates away from the optimal reference trajectory, and eventually, it might fail in reaching the destination. The basic idea of our suboptimal approach is rather simple. Let d_1 denotes the deviation distance between the present vehicle position and the preassigned one on the optimal reference trajectory obtained by AREN. When d_1 exceeds a prescribed acceptable limit d_a, the high-level controller in autopilot is activated to reroute the vehicle by reapplying AREN. This rerouting is repeated whenever d_1 exceeds a predefined acceptable limit. The resulting vehicle routing is not rigorously optimal, since it includes past nonoptimal travels. However, Bellman's principle of optimality [8, 11] states that it is evidently the best strategy we can take under the condition we are faced with. We, therefore, call this approach the suboptimal guidance. **Figure 14** depicts the schematic of our suboptimal guidance explained thus far.

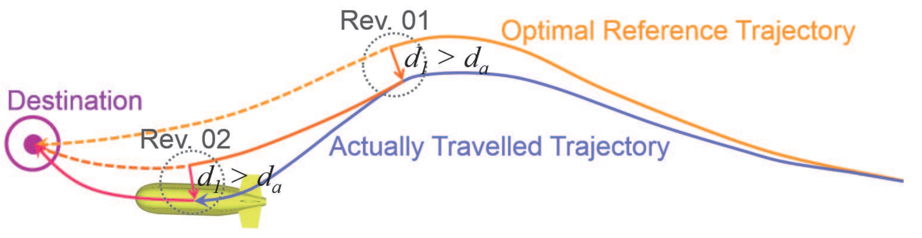

Figure 14.
Schematic of the vehicle routing by suboptimal guidance.

6.3 Suboptimal vehicle routing in a stationary sea current

Here we show another example of vehicle routing in the Northwest Pacific Ocean containing the environmental uncertainty of $\sigma = 4U_{cm}$. Vehicle trajectories are shown in **Figure 15**.

In this example, a vehicle does not merely track the pregenerated optimal heading reference throughout but regenerates and follows new ones whenever necessary. In other words, the vehicle follows the optimal heading references revised repeatedly on the basis of our suboptimal strategy. In this example, we set d_a, the acceptable limit of position deviation, to be 12,500 m. As noted in the figure, optimal vehicle routing is revised five times, making the vehicle to reach the destination, at last. Repeating the vehicle rerouting five times, our suboptimal guidance has succeeded in taking the vehicle to the destination. Traveling times of vehicle routings are summarized in **Table 1**. It is noted here that, while the results of vehicle routing by PN, straight-line tracking, and optimal guidance are deterministic, the result by suboptimal guidance is not since it is derived using the model including environmental uncertainties. Therefore, care should be taken in interpreting the suboptimal result. The result of suboptimal vehicle routing is event-dependent, so that it differs in every event. In **Table 1**, we find that the traveling time of suboptimal vehicle routing is 208372.0 s. Notably, it is even shorter than that of the

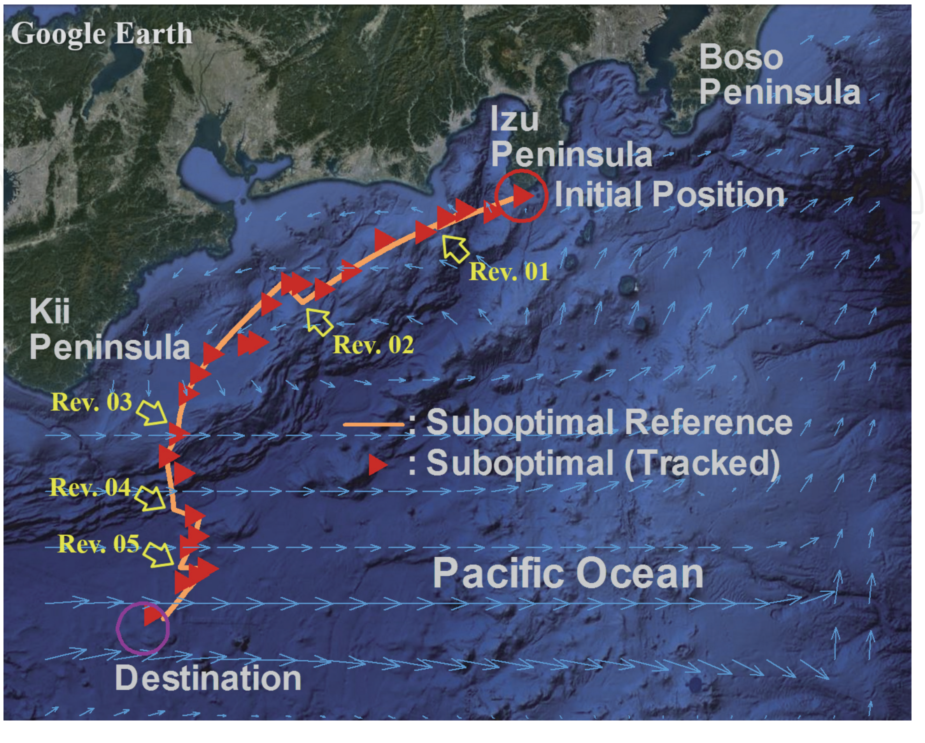

Figure 15.
Vehicle trajectories in NW Pacific generated by suboptimal routing.

			Traveling time
Guidance	Nonoptimal	PN	232198.0 s
		Straight-line tracking	244312.0 s
	Optimal		212006.5 s
	Suboptimal		208372.0 s

Table 1.
Traveling times of vehicle routings in NW Pacific.

optimal routing in the sea current without uncertainty. This implies that in total, the uncertainties have affected the travel in a favorable manner, which may not be the case in other events, however.

6.4 Suboptimal vehicle routing in a time-varying tidal flow

The last example presented in this chapter is an underwater vehicle routing in Tokyo Bay. In this example, we consider the mission of minimum-time homing to a destination. Due to its narrow entrance and shallow depth, sea currents in Tokyo Bay are hardly affected by the outer ocean currents such as Kuroshio. Instead, like many other littoral zones, currents in Tokyo Bay are dominated by the tidal flow. In this research, we use the time-varying sea current distribution data in Tokyo Bay, generated by a numerical tidal flow simulation model by Kitazawa et al. [19]. As was the case in the previous examples, we first conduct optimal vehicle routing without considering environmental uncertainties. In **Figure 16**, vehicle trajectories generated by the routings of different guidance strategies are shown. As found in the figure, our approach of optimal guidance successfully accomplishes the minimum-time homing mission even the sea current is time varying. Note that the current distribution shown in the figure is the one taken at the traveling time of straight-line tracking, which makes the last arrival at the destination.

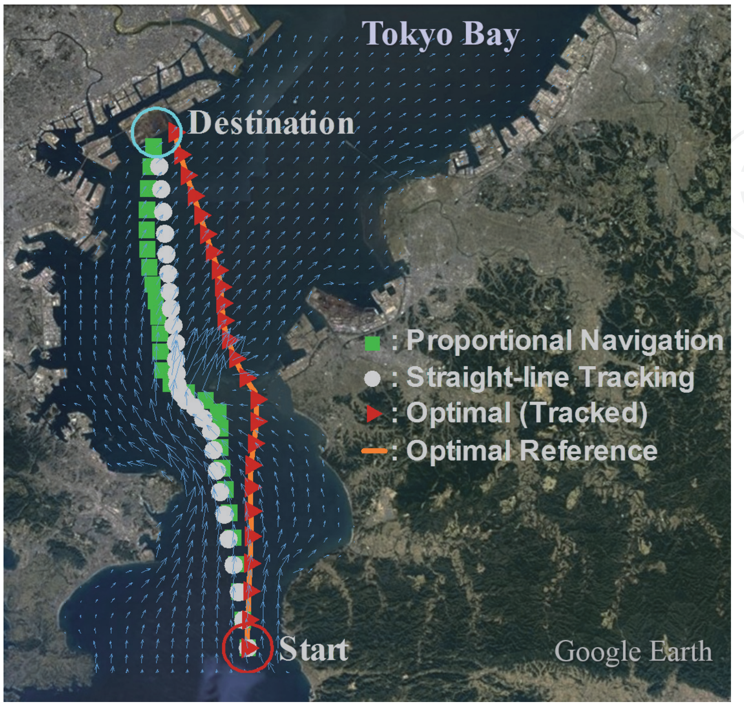

Figure 16.
Vehicle trajectories in the tidal flow in Tokyo Bay.

Next, on the basis of our suboptimal strategy, we conduct the vehicle routing simulation, incorporating uncertainties into the time-varying tidal flow in Tokyo Bay. The standard deviation of the uncertainties is set to be $4U_{cm}$, as was the case in the preceding examples. **Figure 17(a)–(f)** shows sequential vehicle trajectories created by our suboptimal vehicle routing. The traveling time of the suboptimal

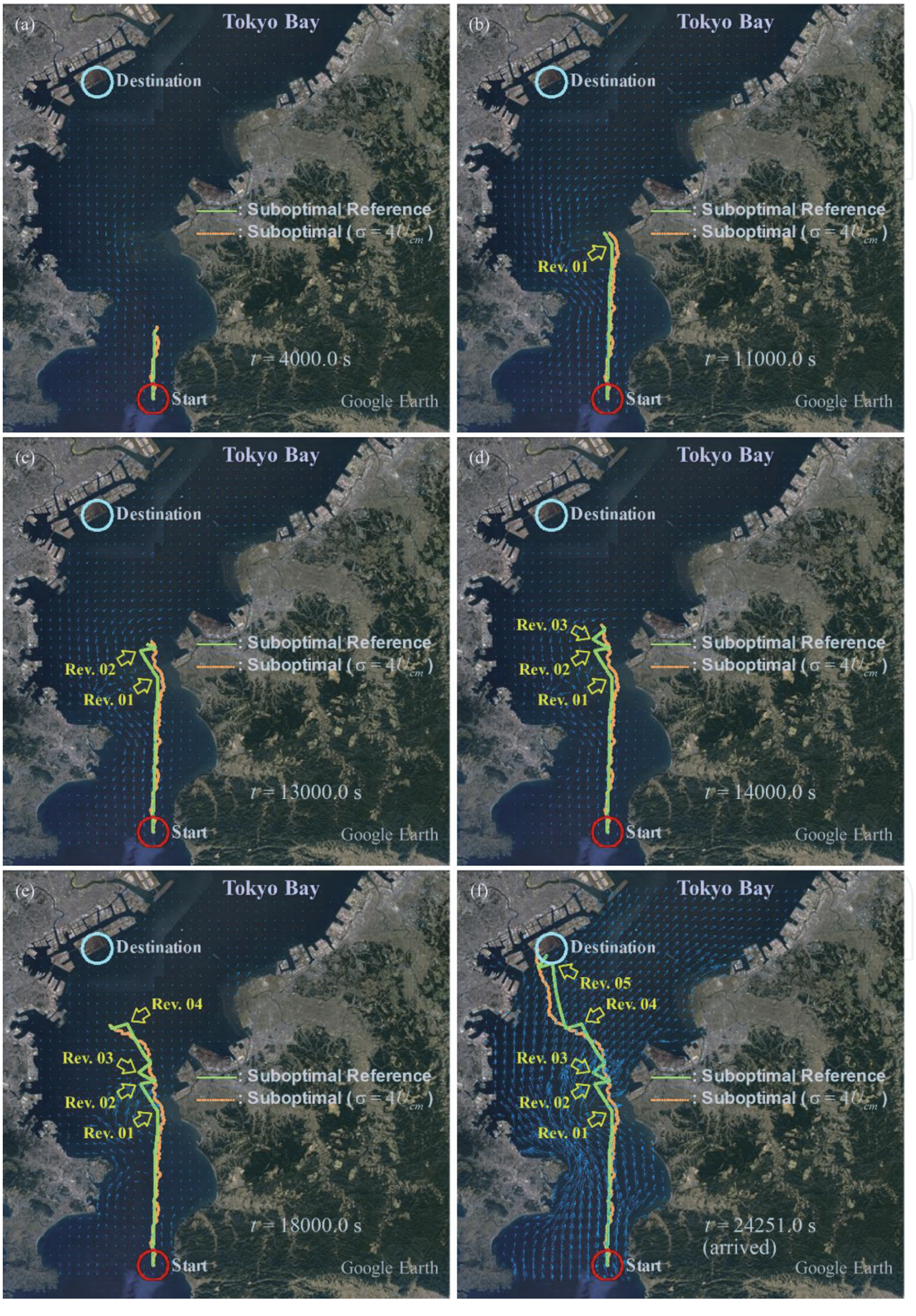

Figure 17.
Sequential vehicle trajectories in Tokyo Bay generated by suboptimal routing (a) t = 4000 s (b) t = 11000 s (c) t = 13000 s (d) t = 14000 s (e) t = 18000 s (f) t =24251 s.

			Traveling time
Guidance	Nonoptimal	PN	25848.0 s
		Straight-line tracking	27744.5 s
	Optimal		24002.0 s
	Suboptimal		24251.0 s

Table 2.
Traveling times of vehicle routings in Tokyo Bay.

vehicle routing and those of the optimal, PN, and straight-line tracking obtained in the previous example are summarized in **Table 2**.

In this example, we set d_a, the acceptable limit of deviation distance, to be 1800 m. As seen in the figures, the vehicle has successfully accomplished its homing mission by the suboptimal routing, repeating five revised travels. In view of the results obtained by this example, we find that our suboptimal approach works effectively even in a time-varying environment including uncertainties.

7. Conclusion

In this chapter, a systematic procedure for obtaining the numerical solution of the optimal guidance law to achieve the minimum-time routing in a region of sea current has been presented. The optimal heading is obtained as the solution of the optimal guidance law, which is fed to the heading control system as the reference.

Reduced computational cost is one of the outstanding features of the proposed procedure. While linearly proportional to the area of a search region in DP, the computational time in our procedure exhibits square root dependence. Moreover, unlike the other path finding algorithms such as DP or GA, when applied to a time-varying environment, our procedure does not increase the search space, resulting in the same computational cost as required in the time-invariant ones.

The performance of the optimal guidance has strong dependency on the current distribution. While an extremely simple configuration, such as uniform flow, hardly allows navigation time reduction by the optimal guidance, a multi-directional complicated flow distribution enhances the potential efficacy of the optimal guidance.

As a fail-safe or fault-tolerable strategy in optimal guidance, the concept of suboptimal guidance has been proposed. The fact that there actually are several possible actions lessening the chance of optimality emphasizes the practical importance of our suboptimal strategy.

We have not considered the problem of unknown or nondeterministic currents. Our approach cannot be applied to an entirely unknown environment. For a sea region with partially or coarsely defined current flow, however, an estimated distribution can be built by means of interpolation and extrapolation. As has already been shown in the optimal and suboptimal vehicle routing examples in actual sea regions, spatiotemporal interpolation of the current velocity successfully derives the converged solution.

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