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Beauty in Mathematics: Symmetry and Fractality

Vladimir A. Testov

Abstract

The most important concepts underlying beauty are the concepts of symmetry and fractality, but the relationship of these concepts has not yet remained clear. For centuries, beauty was understood only as a stable order and symmetry. Synergetic worldview allows us to give a new assessment: beauty can be seen as an attractor, the result of self-organization of nature, or the flight of human thought. On the one hand, fractality can be considered one of the manifestations of symmetry in an expansive sense. On the other hand, symmetry can be considered a manifestation of fractality with a finite number of iterations. Thus, the concepts of symmetry and fractality are closely interrelated. Symmetry reveals in beauty a stable order, and fractality reflects in beauty the result of the self-organization of the chaos of nature or the freedom of human thought. Symmetry and fractality are two opposites, mutually complementing each other, aesthetically and mathematically mutually passing into each other. Thus, symmetry and fractals are the most important concepts for the disclosure of the beauty of the universe, which determine their importance for learning. The concept of self-similarity can serve as a basis for acquaintance with fractals.

Keywords: symmetry, self-similarity, fractality, chaos, order, golden ratio, Fibonacci numbers, neutrosophy

1. Introduction

Since ancient times, it has been realized that the beauty of the world exists independently of human consciousness. The sense of beauty is a product of reflection in human consciousness of really existing aesthetic properties of objects. Thanks to this feeling, a person develops an aesthetic culture, the ability to create, and an idea of the beauty of the surrounding world is formed. The beauty of the universe is revealed in a special way in every science. Mathematics is not only a coherent system of knowledge and tasks, but also a unique means of understanding the beauty of the world. Studying mathematics, a person discovers new fragments of beauty, moving to the understanding and then to the creation of beauty and harmony.

Beauty confirms and complements the universality of mathematical theorems and formulas that work equally effectively in living and inanimate nature, in atoms and in the universe, in scientific discoveries, and in works of art. Beauty helps to accept the world around with admiration, and mathematics makes it possible to realize the perceived phenomena and objects and deepen knowledge about the harmony of the world.

Scientists have proved that behind various artistic, architectural, linguistic, and musical creations and natural phenomena, there are general mathematical regularities. Special attention is paid to symmetry, the Golden ratio, and in recent decades also to fractality. The process of human cognition of beauty, self-organization of his knowledge, is a very complex, yet little studied procedure. In this process, the scientific concepts of symmetry and fractality are decisive.

Symmetry is an ancient universal symbol, which from generation to generation forms in the consciousness of man the idea of harmony of the universe. The awareness of the order and beauty of the universe is the meaning of symmetry in science, and the meaning of symmetry in art is the product of beauty and perfection. In culture, the idea of symmetry goes from a visual natural symmetry to a scientific concept. Currently, the concept of symmetry is widely used in a variety of sciences: physics, chemistry, crystallography, psychology, etc.

2. Literature review

A large number of works are devoted to beauty in mathematics. The most fundamental works in this direction are the books of Voloshinov [1], Varga et al. [2], Tarasov [3], etc. A number of scientists-mathematicians tried to reveal the concept of beauty of a mathematical object, paying special attention to the presence of a measure of order in a mathematical object. In particular, Henri Poincaré noted that "he sees the properties of beauty and grace in the elements, harmoniously arranged in such a way that the mind can easily capture them completely, guessing the details." In his opinion, the system of mathematical knowledge brings order symmetry, understood as the harmony of the individual components of this system, their happy balance, giving its components an internal meaningful unity [4].

According to V.G. Boltyansky, "the beauty of a mathematical object lies in the presence of isomorphism between the object and its visual model, the simplicity of the model and the surprise of its appearance, which can be briefly written in the form of the formula: beauty = isomorphism + simplicity + surprise" [5].

The most clear formula for the beauty of a mathematical object was defined by Garrett Birkhoff: $M = O/C$, where M is a measure of the beauty of the object, O is a measure of the order in the object, and C is a measure of the effort expended to understand the essence of the object [6].

In accordance with this, the most common view, the beauty of the object will increase as the ordering of its structure. From this point of view, the most obvious form of order in nature and human creations is symmetry.

According to G.I. Sarantsev, the following can be attributed to the signs of beauty of a mathematical object: "the correspondence of a mathematical object to its standard, stereotypical image; order, logical rigor; simplicity; universality of the use of this object in various branches of mathematics; originality, surprise" ([7], p. 15).

The ancient Greek philosophers called the order in the universe as cosmos, which was opposed to chaos. They combined in the concept of "cosmos" two functions – the ordering and aesthetic functions.

The most important concept, since ancient times underlying beauty and harmony, is the concept of symmetry. According to the prominent German mathematician of the twentieth century Hermann Weyl "Symmetry ... is the idea by which man for centuries tried to comprehend and create order, beauty and perfection" ([8], p. 37).

The concepts of symmetry have existed among many peoples since ancient times, but in a broader sense as the ideas of balance and harmony. The principle of symmetry was recognized as one of the foundations of the classical scientific picture

of the world, especially after, in 1918, Emmy Noether proved the famous theorem that every continuous symmetry of a physical system corresponds to some law of conservation.

The main characteristics of the concept of symmetry are proportionality, commensurability, and invariance manifested in any transformations (**Figure 1**).

But in nature, parts like each other cannot exactly coincide, so the symmetry in nature is never absolute (**Figure 2**).

The concept of symmetry in mathematics, on the contrary, reaches absolute rigor of definitions. In particular, in geometry, symmetry is the ability of shapes and bodies to retain shape and properties under some transformations. Even at school, students are introduced to mirror and central symmetry. In addition, there is a rotary symmetry, which means that when the body rotates in space at some angles,

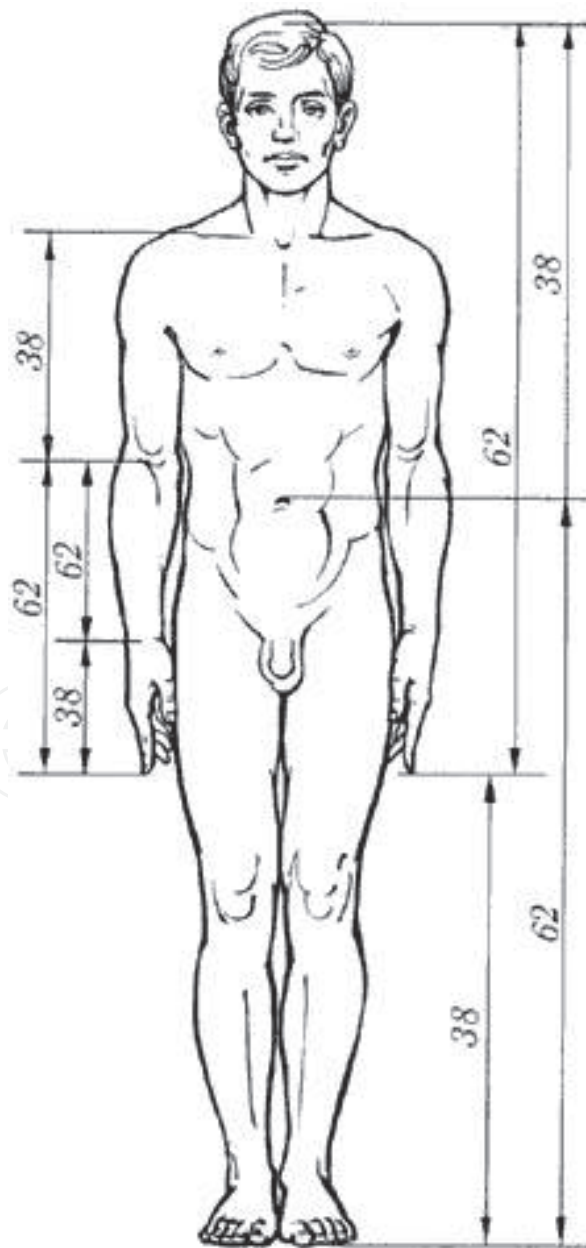
Figure 1.
Taj Mahal.

Figure 2.
Elbrus.

its appearance will not change. Considered and other types of symmetry are unlike the usual: translational (repeated by some rule pattern through the same or regular distance, for example, the pattern on the wallpaper, parquet, lace, and tiled roof, and pattern on the skin of the snake), color (mirror reflection with a change of color, for example, chess pieces arranged in the same order), screw (observed in the arrangement of leaves on the stems of many plants, so that they do not obscure each other from the light; another manifestation—the device scales pineapple).

In his book, Hermann Weyl understood symmetry as the immutability of any properties of an object under some kind of transformations. These transformations may not only be movements. H. Weil devoted one of the chapters of his book to ornamental symmetry. In patterns and ornaments, orderliness and subordination to a certain set of rules can also be found. In the case of potentially infinite patterns, as H. Weyl notes, "the operation with respect to which this pattern remains unchanged does not necessarily have to be a movement, it can also be a similarity" ([8], p. 93). Next, he considers one kind of symmetry, defined by a group of extensions, the real embodiment of which in nature is the shell Turritella duplicata.

Figure 3.
Golden ratio in the human body.

Special attention from the point of view of the laws of beauty in addition to symmetry attracts the "Golden ratio." The ratio of the Golden ratio is now often used in a variety of spheres of life. But the history of this concept goes back to ancient times when such Sciences as mathematics and philosophy were just emerging. As a scientific concept, the Golden ratio came into use in the time of Pythagoras, namely in the sixth century BC. But even before the knowledge of such a ratio, in practice, it was used in Ancient Egypt and Babylon. A striking evidence of this is the pyramids, for the construction of which used just such a Golden proportion. Apparently the term "Golden ratio" was introduced by Leonardo da Vinci. This term denotes the division of a segment, in which one part of it is as many times larger than the other, as many times smaller than the whole segment. If you make the necessary calculations, you can find the ratio of the greater part of the segment to the smaller. This constant number in the middle ages was called the divine proportion, and is now in our day called the Golden ratio, the Golden mean, or the Golden proportion. This number is usually denoted by the Greek letter Φ; it is approximately equal to 1,61803 .... Roughly speaking, the most part of the segment in this division is 62%, and the smaller 38% of the length of the entire segment.

The Golden ratio is a truly incredible concept, and throughout history, we can find many interesting facts about this proportion. Over time, the Golden ratio rule became an academic routine, but the German scientist Adolf Zeising in 1855 gave it a second life. He published his work entitled "Aesthetic studies." In his work, he presented the Golden ratio as an absolute concept that is universal for all phenomena both in nature and in art. A. Zeising was able to prove that the Golden ratio; in fact, it is the average law for the human body. From the study of A. Zeising, it follows that the main indicator of the Golden ratio is the division of the body by the navel point. And the male body is a little closer to the Golden ratio than the female. Also, the Golden proportion can be observed in other parts of the body, such as, for example, the hand (**Figure 3**).

The Golden ratio is used in painting, sculpture, in the construction of temples, and is found in music and poetry.

3. Methodology and research methods

The study used the following methods: analysis of pedagogical and methodological literature, comparative, historical, and logical types of analysis of the problem of beauty in nature, culture, and science. The methodological basis of the study is a post-non-classical methodology based on synergetic worldview. In post-non-classical science, Trinitarian methodology has also been increasingly used. This methodology assumes the presence of a third element besides two binary oppositions, which is necessary to solve the problem of contradiction of binary oppositions, their integration into a single whole, as a condition of their coexistence. In a sense, a generalization of Trinitarian methodology is a new direction in philosophy-neutrosophy, created recently by Florentin Smarandache, which instead of one third element considers a whole set of neutral elements [9]. The advantage of this philosophical direction is its reliance on a number of new mathematical theories, which are based on non-standard analysis, created by Abraham Robinson [10].

4. Main results

The end of the twentieth century brought a new understanding of beauty in the universe. The synergetic worldview that emerged during this period allows us to

give a new aesthetic assessment of the creative role of chaos, its ability under certain conditions to self-organization. Self-organization is characteristic of many complex systems. It consists in the fact that very often a large or even infinite number of quantities or variables characterizing an object "obey" only a few variables, the so-called parameters of order. All these processes are described in synergetics. Traditional science was rejecting the existence of a certain role of chaos in the process of knowledge, referring it to disorganizing factors. However, its constructive role is becoming increasingly apparent nowadays.

Chaos throughout the history of world culture had a negative connotation, and the harmony of the universe was understood solely as overcoming the original chaos with the help of order. In the synergetic worldview, chaos appears as a mechanism of access to attractor structures. To fight against chaos is pointless, because the presence of chaos is a hallmark of complex open systems; you just need to learn how to use its constructive role.

Beauty, since the time of Socrates, in accordance with the prevailing views for centuries, was understood only as a stable order and symmetry. In particular, Socrates argued that beauty is expediency. However, some other philosophers of the past held a different point of view; they saw the product of free thought in beauty. In particular, I. Kant believed that beauty is expediency without purpose; it expresses the ability of man to think of nature according to the laws of freedom. Using modern terminology, this idea can be reformulated as follows: beauty is an attractor, the result of the self-organization of nature, or the flight of free human thought. Synergetic paradigm opened a new vision of beauty as the interaction of space and chaos, their harmonic balance.

As Y.V. Tabakova and A.V. Voloshinov noted: "the Cosmos is beauty relevant, whereas chaos is beauty potential. Again, it is easy to see how the properties of actual and potential beauty in the philosophy of beauty—aesthetics—echo the properties of actual and potential infinity in the philosophy of mathematics, which drew attention to Aristotle" [11].

As it was mentioned above, one of the most common types of symmetry is the similarity transformation. But the similarity transformation is also at the heart of another concept now widely used in mathematics—the concept of fractality. This most important concept underlying beauty and harmony arose at the end of the twentieth century in connection with the development of synergetic worldview, chaos theory, as well as computer technology. Today, this concept as well as the concepts of fractal geometry and fractal graphics have become common among mathematicians and computer artists.

The word fractal was proposed in 1975 by an American mathematician Benoît Mandelbrot. He identified with this word the structures to which his research was devoted. There are several definitions of fractality. One of them is based on such an important property of the fractal as self-similarity. An object is called self-similar when the enlarged parts of the object resemble the object itself and each other. Moreover, self-similarity is understood not only in the classical sense, when the part is an exact copy of the whole, but also in the nonclassical nonlinear sense, when the part is "similar" to the whole. Paraphrasing this definition, we can say that in the simplest case, a small part of the fractal is a reduced copy of the whole (at least approximately), i.e., it contains information about the whole fractal. For example, a snowflake carries information about a snowdrift, and a rock has the same shape as a mountain range.

Theoretically, the self-similarity of the fractal parts is infinite, but visually the human eye is able to distinguish no more than 5–7 fractal self-similarity. The greater number of self-similar parts can be distinguished only by computer magnification.

In the unexplored elements of such structures, the whole mysterious world of the universe can be represented.

The first fractal sets appeared in mathematics long before the works of B Mandelbrot, in the late nineteenth—early twentieth century, but fractals originally, in contrast to symmetry, caused much dislike and bewilderment of many mathematicians of the time. One of the founders of set theory, Georg Cantor, was the first to construct a fractal set from a segment by dividing the segment and throwing out an infinite number of intervals of different lengths from this segment. The result is a fractal object—Cantor dust (**Figure 4**).

Later, other fractal sets were constructed (the Serpinski triangle, the Koch snowflake, etc.), which attracted attention for their aesthetic appeal (**Figure 5**).

Thanks to the works of Mandelbrot [12] in addition to the beauty of symmetry, the aesthetic natural beauty of fractals was discovered. This phenomenon is closely related to the laws of beauty and is often found in living and inanimate nature, as well as in art, architecture, and other spheres of human activity. Fractals give rise to really colorful, original paintings, not inferior to the works of abstract painting (**Figure 6**).

Fractal sets contribute to the emergence of a new look at the aesthetic appeal of mathematics and contribute to the creation of the human ability to "see" the mathematical in the nonmathematical, which is the defining role of fractal geometry for
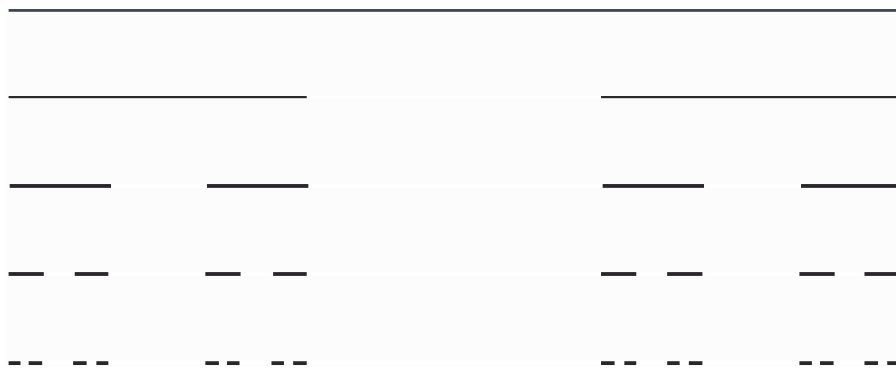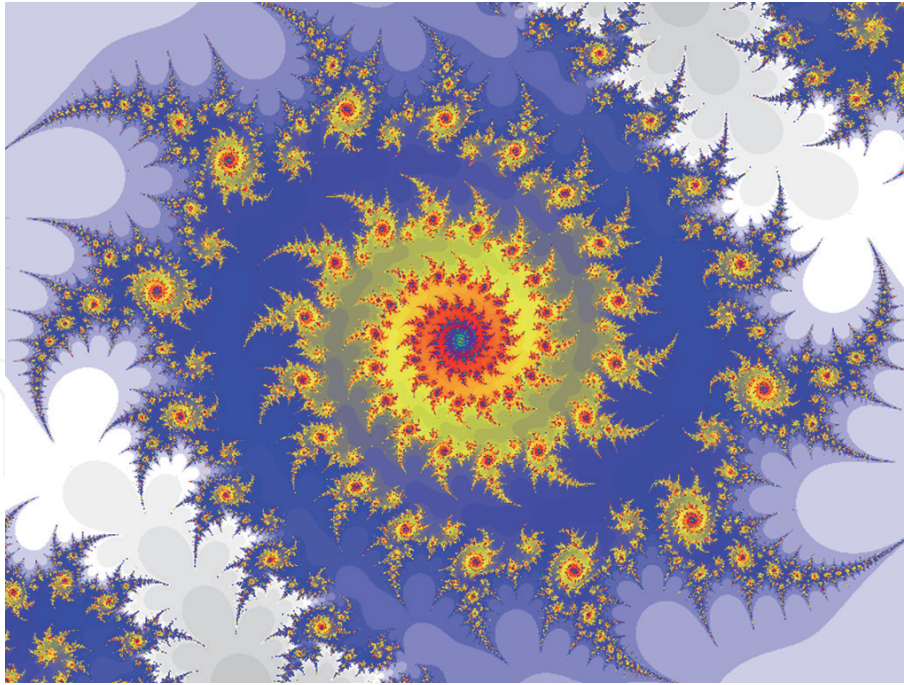

Figure 4.
Cantor Dust.
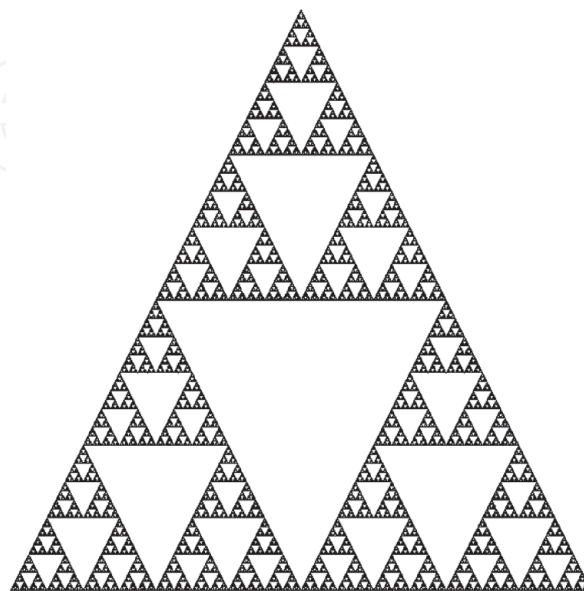
Figure 5.
Sierpinski Triangle.

Figure 6.
The beauty of fractals.

the future of science and education. In fact, with the advent of fractals, a new kind of computer graphics is born—fractal graphics. With its help, you can create a planar set and the surface of a very complex shape.

Fractal graphics attract our eyes, and software tools for its creation can be the step that will allow us to get closer to this fractal creativity. Familiarity with the elements of fractal geometry contributes to the formation and development of human creativity and artistic (aesthetic) component of his personality.

The idea of "self-similarity" expresses the fact that the hierarchical principle of the organization of fractal structures does not undergo significant changes when viewed through a microscope with different magnification. As a result, these structures look on average the same on small scales as they do on large ones. B. Mandelbrot on this occasion notes that the word "similar" does not always have the classical meaning of "linearly increased or decreased", but is always in agreement with a convenient and broad interpretation of the word "similar". Self-similarity is one of the fundamental principles of the organization of the structure of the world. Information presented in the form of a visual self-similar model is quite simply perceived, reflecting the essential aspects of the object of study in self-similarity and, therefore, visual for the subject working with the model.

Therefore, in creating artificial visual self-similar models, we essentially use one of the fundamental properties of the geometry of nature to organize the perception of the knowledge we are interested in and then we are perceived by this knowledge through the usual visual structure that surrounds us everywhere. In fact, there are more self-similar objects in the real world than we imagine. These include trees and the human circulatory system, galaxies, and snowflakes and these are just the simplest examples.

Under the definition of a fractal, as a structure with self-similar parts, not only designs created with the help of computer graphics are suitable, but also long-known children's pyramids, matryoshkas, and works of art, in which there is a uniformity of repetitions, which is especially the characteristic of folk tales and songs.

In mathematics, fractal (self-similar) structures are not only in geometry. Algebra and number theory also have examples of self-similar structures.

Self-similarity is clearly seen in the very first numerical system with which mankind was acquainted in its historical development—natural numbers. The first natural number 1 is represented by one dash |, the number 2-by two dashes ||, the number 3-by three dashes |||, etc.

In other cases, self-similarity is a little more difficult to see. For example, in decimal fractions 2,1451454514545451..., the principle of self-similarity in the sequence of digits is as follows: after the first unit should be a combination of digits "45," after the second unit "4545", after the third "454545", etc. In the recording of a given fraction, you can find a cut of numbers of the form "4545...45" of any finite length which reflected an even number of digits; moreover, none of these segments of the digits does not contain a unit. It is not difficult to see that this decimal fraction is infinite nonperiodic and represents an irrational number.

For algebraic problems with self-similarity of expressions, there are usually simple and beautiful solutions.

For example, calculate the product:

$$\left(\sqrt{2}-1\right)\left(\sqrt{2}+1\right)\left(\sqrt{3}-\sqrt{2}\right)\left(\sqrt{3}+\sqrt{2}\right)\cdot\ldots\cdot\left(\sqrt{99}-\sqrt{98}\right)\left(\sqrt{99}+\sqrt{98}\right)$$

In this example, self-similarity is manifested in the fact that the product consists of pairs of multipliers of the form $\left(\sqrt{n+1}-\sqrt{n}\right)\left(\sqrt{n+1}+\sqrt{n}\right)$, which are arranged in the usual order of natural numbers. In solving this problem, it is important to see that the product of such a pair of factors is 1. Then, distributing the factors in pairs: $\left(\sqrt{2}-1\right)\left(\sqrt{2}+1\right)=1$, $\left(\sqrt{3}-\sqrt{2}\right)\left(\sqrt{3}+\sqrt{2}\right)=1$, ..., we find that the desired product is 1.

Self-similar structures in algebra and number theory can be represented by beautiful multi-story radicals and chain fractions.

$$\sqrt{a+\sqrt{a+\sqrt{a+\sqrt{a+\cdots}}}}$$

$$a+\cfrac{1}{a+\cfrac{1}{a+\cfrac{1}{a+\cdots}}}$$

The most famous of the numerical self-similar structures is the Fibonacci numbers' sequence structure. These numbers were discovered by an Italian mathematician of the middle ages Leonardo of Pisa, better known as Fibonacci. After his discovery, these numbers began to be called in the name of a famous mathematician. The amazing essence of the Fibonacci number sequence is that each number in this sequence, starting with the third, is obtained from the sum of the previous two numbers. These numbers form an infinite sequence:

$$0, 1, 1, 2, 3, 5, 8, 13, 21, 34, 55, 89, 144, 233, 377, 610, 987, 1597, 2584, \ldots$$

which is called the Fibonacci sequence.

There is one very interesting feature in Fibonacci numbers. When dividing any number from a sequence by the number in front of it in a row, the result will always be a value that fluctuates around the already familiar number Φ from the Golden section Φ = 1.61803398875... and through times the a bit more, then a bit less his. But for sufficiently distant two adjacent numbers in the sequence, this result of division becomes almost constant and equal to the number Φ.

It turns out that this remarkable number Φ can be represented as two different self-similar structures.

$$\Phi = \sqrt{1 + \sqrt{1 + \sqrt{1 + \cdots}}} \tag{1}$$

$$\Phi = 1 + \cfrac{1}{1 + \cfrac{1}{1 + \cfrac{1}{1 + \cdots}}} \tag{2}$$

There are examples of isomorphic fractal structures based on a number of Golden sections, which allow us to talk about a single fractal grammar of art and attribute the property of fractality to the metalanguage of different arts in nature. Moreover, according to A.V. Voloshinov, there is every reason to say that "the book of nature is written in the language of fractals"; in addition, there are attempts to prove that the book of art is written in the language of fractals.

As we noted above, the transformation of similarity, in particular self-similarity, is a special case of symmetry; then, on the one hand, fractality can be considered one of the manifestations of symmetry. A.V. Voloshinov adheres to an equally wide understanding of symmetry: "since symmetry today is understood expansively as the preservation (invariance) of a certain characteristic, then the varieties of symmetry should include proportion as an invariant of growth, and the Golden ratio as a geometric proportion that has an additive property, and rhythm as a portable symmetry in space or time, and, finally, fractals as self-similar structures that have invariant morphology at different scales" [13].

On the other hand, almost all the types of symmetry we have identified can be considered as special cases of similarity or a combination of similarities, that is, we can consider symmetry as a manifestation of fractality with a finite number of iterations. Thus, the concepts of symmetry and fractality are closely interrelated.

As time goes on, it becomes increasingly clear that the relentless interest in fractals is due not so much to a peculiar fashion and novelty, as to the new opportunities that are opened up to modern Sciences thanks to fractality.

According to recent physical representations, the universe consists of an infinite number of nested fractal levels of matter with similar characteristics. Fractality, according to some philosophers, is one of the universal fundamental properties of being. With the advent of fractals, the limitation of the description of nature with the help of Euclidean geometry was clearly manifested. The world around us is much more diverse than the classical description; it turned out to be a lot of objects described using fractals.

Fractal geometry is not only a new direction in mathematics. Fractal theory is used in geology, geochemistry, hydrodynamics, oceanology, biology, and hydrology. Fractal sets have found application in animated films. But perhaps the main application of fractals is modern computer graphics.

Fractals as well as symmetry have aesthetic appeal; it does not require additional knowledge and skills to feel their natural beauty, to experience aesthetic pleasure from this beauty. It is also worth getting acquainted with fractals in order to understand the beauty of chaos, to learn a new nonlinear world, and awareness of the process of scientific knowledge of the world is one of the most important qualities of a cultural person. Therefore, this new direction in mathematics has great methodological, developmental, and applied potential and requires its gradual introduction, both in the university and in the school curriculum in mathematics [14, 15].

Trinitarian methodology and neutrosophy play an important role in understanding the role of beauty in the universe. From the standpoint of these methodologies, symmetry and fractality are two opposites in the beauty of the universe, mutually complementing each other, aesthetically and mathematically mutually passing into each other. Symmetry reveals in beauty a stable order, and fractality

reflects in beauty the result of the self-organization of the chaos of nature or the freedom of human thought.

5. Conclusions

Since the time of the ancient Greeks, space and chaos have been considered oppositions in the concept of beauty. Beauty is determined by the interaction, the harmonic balance of these two oppositions. Symmetry embodies the cosmos, which reveals in beauty a stable order, and fractality penetrates into the second opposition, which reflects in beauty the result of the self-organization of the chaos of nature or the freedom of human thought. Thus, symmetry and fractality are two sides of beauty, aesthetically complementing each other. The synergetic worldview opens to humanity a new vision of beauty—beauty as a synthesis of symmetry and fractality, the most important components of the scientific picture of the world. The interrelated study of symmetry and fractal geometry contributes both to the increase of interest in the study of mathematics and aesthetic education.

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