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# Dynamics of Rayleigh-Taylor Instability in Plasma Fluids

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## Abstract

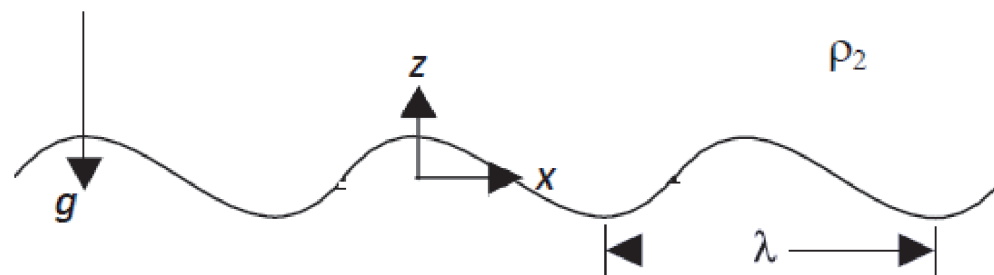
The chapter discusses the evolution of Rayleigh-Taylor instability (RTI) in ordinary fluids and in a plasma fluid. RT instability exists in many situations from overturn of the outer portion of the collapsed core of a massive star to laser implosion of deuterium-tritium fusion targets. In the mixture of fluids, the instability is triggered by the gravitational force acting on an inverted density gradient. The motivation behind the study of the instability has been explored by discussing the applications of RT instability. The basic magnetohydrodynamics equations are used to derive the dispersion relation (for an ordinary fluid and plasmas) for two fluids of unequal densities. The conditions of the growth rate of the instability and the propagating modes are obtained by linearizing the fluid equations. The perturbed potential is found to increase with the plasma parameters in a Hall thruster.

**Keywords:** instabilities, plasma, Navier-Stokes, growth rate, Hall thruster

## 1. Introduction

Flow instabilities are used to increase the heat and mass transfer rates as well as to fuse the fluids of dissimilar properties (viscosity, elasticity, density, etc.). In other technological applications, these instabilities are accountable to unstable the multilayer and free-surface flows. Multilayer flows are used in coating processes and lubricated pipeline transport. The presence of the instabilities in the system leads to nonuniform film thickness and defects, where good optical finishing and smooth edges are required by the industry, which further leads to poor product quality. Suppression of these instabilities has been a major task from a long time by the researchers to improve the product quality [1, 2]. Rayleigh-Taylor (RT) instability takes place when a lighter fluid supports a heavy fluid, then any perturbation of the interface grows and leads to spikes of the heavier fluid penetrating into the lighter one and the interface becomes unstable. The contact discontinuity between the two fluids is unstable to perturbations that grow by converting potential energy to kinetic energy, causing bubbles of the low-density fluid to rise, and spikes of the high-density fluid to sink. If the light fluid is above the heavy fluid, the interface is stable. In a magnetized plasma, the Rayleigh-Taylor instability can occur because the magnetic field acts as a light fluid supporting a heavy fluid (the plasma).

In curved magnetic fields, the centrifugal force on the plasma due to the charged particle motion along the curved field lines acts as an equivalent gravity force. When forces associated with the density gradient and gravity oppose each other, the RT instability sets in [3, 4]. The box of fluid shown in **Figure 1** is now filled with



**Figure 1.**  
Two fluids inside of a large box.

two incompressible fluids of differing densities, separated by an interface with a perturbation imposed as shown in **Figure 1**. Here, RTI is seen to play a wider role in many branches of science from astrophysical systems to industries.

## 2. Review of status of research

This instability occurs in many interesting physical situations, such as implosion of inertial confinement fusion capsules, core collapse of supernovae, or electromagnetic implosions of metal liners. The Rayleigh-Taylor problem was first studied by Lord Rayleigh in 1883 and Sir G.I. Taylor in 1950 [3]. Taylor used the theory of linearization for the small oscillations at the interface and obtained an exponential growth rate. Chandrasekhar, in 1961, studied the magnetic field case analytically for the fluids that are incompressible, inviscid, and have zero resistivity. Qin et al. [5] reported the synthesis of chains of metal nanoparticles with well-controlled particle sizes and spacing induced by the Rayleigh instability. Bychkov et al. [6] derived the dispersion relation for the internal waves and the RT instability in a nonuniform unmagnetized quantum plasma with a constant gravitational field. They have shown that the quantum effects always play a stabilizing role for the RT wave instability. Cao et al. [7] studied the RT instability incorporating the quantum magnetohydrodynamic equations and solved the second-order differential equation under different boundary conditions with quantum effects. Khomenko et al. [8] modeled the growth rate of the instability and the evolution of velocity and magnetic field vector in the prominence plasma (closer to Sun's surface) under the presence of neutral atoms. Diaz et al. [9] derived the criterion for the growth rate of the RT instability in partially ionized plasma using single fluid theory. Ibrahim and Marshall theoretically investigated the impact of velocity profile on RTI within the jet to examine the effects of its relaxation on intact length [10]. Carlyle and Hillier experimentally verified that stronger magnetic fields can suppress the growth of the rising bubbles of the RTI [11]. Litvak and Fisch derived the necessary instability conditions of azimuthally propagating perturbations in a Hall thruster plasma [12]. Recently, investigators derived the dispersion for the Rayleigh-Taylor instabilities in a Hall thruster using the two-fluid theory [13, 14]. Shorbagy and Shukla investigated the RT instability in a nonuniform multi-ion plasma in a Hall thruster to obtain the growth rate of the instability [15]. Ali et al. [16] derived the modified dispersion relation for the Rayleigh-Taylor instability under the quantum corrections incorporating the terms of Fermi pressure and the Bohm potential force.

### 3. Basic fluid equations and Bernoulli's theorem

First, we consider the two simple fluids separated by a smooth interface to derive the dispersion relation. Let us assume that in each separate region, the density is constant. The coordinate  $x$  is in the horizontal,  $z$  in the vertical, and  $y$  is going into the page. We consider a flow in the  $x$ -direction, which in the lower half-space ( $z < 0$ ) has density  $\rho_1$ , whereas in the upper half-space ( $z > 0$ ) has density  $\rho_2$ . In addition there can be a homogeneous gravitational field  $g$  pointing into the negative  $z$ -direction. We write the basic fluid equations for the ion and electron fluids as Navier-Stokes equations for an incompressible fluid are

$$\frac{\partial \rho}{\partial t} + \vec{\nabla} \cdot (\rho \vec{v}) = 0 \quad (1)$$

$$\frac{d\vec{v}}{dt} = \frac{\partial \vec{v}}{\partial t} + (\vec{v} \cdot \vec{\nabla}) \vec{v} = -\frac{\vec{\nabla} P}{\rho} + g + \eta \nabla^2 \vec{v} \quad (2)$$

Here, we have used total time derivative. Partial time derivative keeps an eye on a point and represents the rate of velocity change at that point. Total time derivative keeps an eye on fluid element and measures its velocities at  $t$  and  $t + \Delta t$ .

Let us consider the fluid is inviscid, so that we take viscosity  $\eta = 0$ . We also assume that the fluid is irrotational, that is  $\vec{\nabla} \times \vec{v} = 0$ . Then the term  $(\vec{v} \cdot \vec{\nabla}) \vec{v}$  reduces to  $\frac{1}{2} \vec{\nabla} v^2$ . The Stokes' theorem permits us to express the velocity in terms of gradient of scalar function, that is  $\vec{v} = -\vec{\nabla} \phi$ . The variable  $\phi$  is called the scalar velocity potential of fluid. We rewrite gravity acceleration into a gradient of gravity potential  $g = -\vec{\nabla}(gz)$ . Eq. (2) can be rewritten in terms of scalar function  $\phi$  under the above assumptions:

$$\vec{\nabla} \frac{\partial \phi}{\partial t} + \frac{1}{2} \vec{\nabla} v^2 = -\frac{\vec{\nabla} P}{\rho} - \vec{\nabla}(gz) \quad (3)$$

If the density remains constant in one region, we can write Eq. (3) as

$$\vec{\nabla} \left[ \frac{\partial \phi}{\partial t} + \frac{1}{2} (\vec{\nabla} \phi)^2 + gz \right] = -\frac{\vec{\nabla} P}{\rho} \quad (4)$$

Now integrating the above equation in horizontal and vertical directions, we get unsteady equation for the Bernoulli theorem.

$$\frac{\partial \phi}{\partial t} + \frac{1}{2} (\vec{\nabla} \phi)^2 + gz + \frac{P}{\rho} = Const \quad (5)$$

That is, the total mechanical energy of the moving fluid comprising the gravitational potential energy of elevation, the energy associated with the fluid pressure, and the kinetic energy of the fluid motion remains constant.

Let  $\vec{v}_1$  and  $\vec{v}_2$  be the velocities of the fluid in the lower half-space ( $z < 0$ ) and upper half-space ( $z > 0$ ) respectively. Now, it is convenient to write velocities of fluid in terms of scalar velocity potential  $\phi$  in both regions such that

$$\vec{v}_1 = -\vec{\nabla} \phi_1 \quad (6)$$

$$\vec{v}_2 = -\vec{\nabla}\phi_2 \quad (7)$$

For the incompressible fluid, Eq. (1) yields that  $\vec{\nabla} \cdot \vec{v} = 0$ . This also states that both fluids will satisfy the Laplace equation in both regions

$$\vec{\nabla}^2 \phi = 0 \quad (8)$$

The Bernoulli theorem state that quintiles  $\rho \frac{\partial \phi}{\partial t} + \frac{1}{2} \rho (\vec{\nabla} \phi)^2 + \rho g z + P$  should be constant across the fluid, so that we have

$$\rho_1 \frac{\partial \phi_1}{\partial t} + \frac{1}{2} \rho_1 (\vec{\nabla} \phi_1)^2 + \rho_1 g z_0 + P_1 \Big|_{z=z_0} = \rho_2 \frac{\partial \phi_2}{\partial t} + \frac{1}{2} \rho_2 (\vec{\nabla} \phi_2)^2 + \rho_2 g z_0 + P_2 \Big|_{z=z_0} \quad (9)$$

To understand the interface, we must impose boundary conditions. First of all the vertical velocities of the fluids must match with the interface, so we impose the kinematic boundary condition. Now we need to introduce the location of the interface by assigning variable  $z = z_0(t)$ . Then  $\frac{dz_0}{dt}$  will represent the velocity of the interface in the  $z$ -direction. In addition, at the interface, the velocity of both fluids must be continuous.

$$\frac{dz_0}{dt} = \left( \frac{\partial}{\partial t} + (\vec{v} \cdot \vec{\nabla}) \right) z_0 = \frac{\partial \phi_1}{\partial z} \Big|_{z=z_0} = \frac{\partial \phi_2}{\partial z} \Big|_{z=z_0} \quad (10)$$

Let us say that the pressure is continuous along the interface, that is  $P_1 = P_2$ . Then Eq. (9) leads to

$$\rho_1 \frac{\partial \phi_1}{\partial t} + \frac{1}{2} \rho_1 (\vec{\nabla} \phi_1)^2 + \rho_1 g z_0 = \rho_2 \frac{\partial \phi_2}{\partial t} + \frac{1}{2} \rho_2 (\vec{\nabla} \phi_2)^2 + \rho_2 g z_0 \quad (11)$$

#### 4. Asymptotic boundary conditions at far field

We are looking for changes only on the interface at  $z = z_0$ , therefore the velocity potentials and their derivatives must vanish at the boundaries, that is  $\phi_1 \rightarrow 0$  as  $z \rightarrow -\infty$  and  $\phi_2 \rightarrow 0$  as  $z \rightarrow \infty$ .

#### 5. Linear analysis

Eq. (9) contains a nonlinear term  $\rho_1 (\vec{\nabla} \phi_1)^2$  of the second order. If the amplitude is chosen to be much smaller than the wavelength of the instability, the equations of motion can be linearized. We assume that all the perturbed quantities and various derivatives such as  $\phi$  and  $\vec{\nabla} \phi$  are very small. In other words  $\frac{\partial \phi_1}{\partial z} \Big|_{z=z_0} \sim \frac{\partial \phi_1}{\partial z} \Big|_{z=0}$ . Hence, the difference in second order derivatives will be much smaller. Now we impose all boundary conditions at  $z = 0$ , which yields the following set of equations,

$$\vec{\nabla}^2 \phi_1 = 0 \quad (12)$$

$$\nabla^2 \phi_2 = 0 \quad (13)$$

$$\frac{dz_0}{dt} = \frac{\partial \phi_1}{\partial z} \Big|_{z=0} = \frac{\partial \phi_2}{\partial z} \Big|_{z=0} \quad (14)$$

$$\rho_1 \frac{\partial \phi_1}{\partial t} + \rho_1 g z_0 \Big|_{z=0} = \rho_2 \frac{\partial \phi_2}{\partial t} + \rho_2 g z_0 \Big|_{z=0} \quad (15)$$

## 6. Eigenvalue solution

Let us consider all the perturbed variables  $\phi$  and  $z_0$  to have oscillating behavior such that  $\phi = \phi_0 \exp [i(kx - \omega t)]$  and should satisfy the Laplace Eq. (12). This implies

$$\frac{\partial^2 \phi_1}{\partial z^2} = k^2 \phi_1 \quad (16)$$

The general solution of Eq. (16) is written as

$$\phi_1(z) = A \exp(kz) + B \exp(-kz), \quad (17)$$

The above two solutions must satisfy the boundary conditions such that  $\phi_1 \rightarrow 0$  as  $z \rightarrow -\infty$  and  $\phi_2 \rightarrow 0$  as  $z \rightarrow \infty$ . So, we need to discard the unsatisfactory part of the solutions of Eq. (16) taking into account the boundary conditions. Therefore,  $z$  dependence goes as  $\phi_1(z) \propto A \exp(kz)$  and  $\phi_2(z) \propto B \exp(-kz)$ . We note that the eigenfunction decreases exponentially on either side of the interface and the perturbation of wave number  $k$  penetrates to a depth of order  $\frac{1}{k} = \frac{\lambda}{2\pi}$ .

This solution further leads to the following form in Fourier mode,

$$\phi_1(x, z, t) = \phi_{01} \exp(kz) \exp[i(kx - \omega t)] \quad (18)$$

$$\phi_2(x, z, t) = \phi_{02} \exp(-kz) \exp[i(kx - \omega t)] \quad (19)$$

$$z_0(x, t) = z_{00} \exp[i(kx - \omega t)] \quad (20)$$

Here  $\phi_{01}$ ,  $\phi_{02}$ , and  $z_{00}$  are the amplitude of the modes. By substituting these solutions into Eq. (14), we obtain the boundary conditions at the interface.

$$k\phi_{01} = -k\phi_{02} = -i\omega z_{00} \text{ at } z = 0 \quad (21)$$

Then Eq. (20) changes into the form

$$z_0(x, t) = \frac{ik\phi_{01}}{\omega} \exp[i(kx - \omega t)] \quad (22)$$

Eq. (15) gives

$$-i\omega\rho_1\phi_{01} + \rho_1g\frac{ik\phi_{01}}{\omega} = -i\omega\rho_2\phi_{02} + \rho_2g\frac{ik\phi_{01}}{\omega} \quad (23)$$

Using Eq. (21) in Eq. (23) results in

$$-i\omega\rho_1\phi_{01} + \rho_1g\frac{ik\phi_{01}}{\omega} = i\omega\rho_2\phi_{01} + \rho_2g\frac{ik\phi_{01}}{\omega} \quad (24)$$



Since the perturbed quantity  $\phi_{01} \neq 0$ , the possible nontrivial solution of Eq. (25) gives the dispersion relation for small perturbations of the wave as below,

$$\omega^2 = \frac{g(\rho_1 - \rho_2)k}{\rho_1 + \rho_2} = A_t g k \quad (25)$$

Eq. (25) contains complete information about the linear stability of the two superposed fluid layers of different densities. The Atwood number  $A_t = \frac{(\rho_1 - \rho_2)}{\rho_1 + \rho_2}$  is a dimensionless number in fluid dynamics used to study the hydrodynamic instabilities in unequal density flows. Since the dispersion relation Eq. (25) is quadratic in  $\omega$ , it has two real or complex conjugate roots depending on the values of the densities of the fluids. Here, we will discuss different cases.

### 6.1 First case: capillary-gravity waves ( $\rho_2 = 0$ )

$$\text{Hence } \omega = \sqrt{gk} \text{ and } V_{ph} = \sqrt{\frac{g}{k}} \quad (26)$$

It is classical dispersion relation for gravity-capillary waves in deep water [17, 18]. These are also called short gravity waves. In this category the longer waves travel faster. Any initial disturbance may be regarded as the superposition of waves of a broad spectrum of lengths. The above relation then says that waves of different lengths will eventually separate, that is, disperse. This phenomenon is called dispersion, hence above relations are also known as the dispersion relation.

### 6.2 Second case: propagating modes ( $\rho_1 > \rho_2$ )

If the lighter fluid is supported by heavier fluid, that is,  $\rho_1 > \rho_2$ , then solutions of the equation leads to two waves with constant amplitude propagating in opposite directions with phase velocity  $\frac{\omega_{\pm}}{k}$  with  $\omega_{\pm} = \sqrt{\frac{g(\rho_1 - \rho_2)k}{\rho_1 + \rho_2}}$ . Then the interface is stable and will only oscillate when perturbed. The phase velocity is given by

$$V_{ph} = \frac{\omega_{\pm}}{k} = \sqrt{\frac{g \left( \frac{\rho_1}{\rho_2} - 1 \right)}{k \left( \frac{\rho_1}{\rho_2} + 1 \right)}} \quad (27)$$

**Figure 2** shows the variations of phase velocity of RT instability with (a) density ratio and (b) wave number respectively.

### 6.3 Third case: Rayleigh-Taylor instability ( $\rho_2 > \rho_1$ )

The frequency of oscillations will be negative imaginary and unstable if  $\rho_2 > \rho_1$ , that is, when heavier fluid is supported by lighter fluid. Writing  $\omega = i\gamma$  where  $\gamma$  is real and positive gives

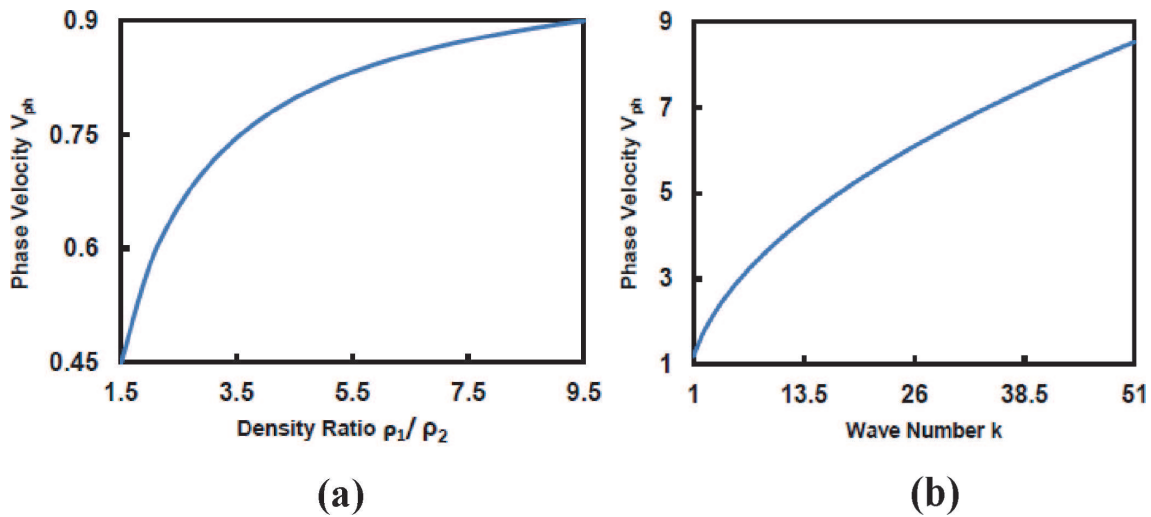
$$\gamma = \pm \sqrt{\frac{g(\rho_2 - \rho_1)k}{\rho_1 + \rho_2}} \quad (28)$$

$$\gamma = \pm \sqrt{\frac{g \left( \frac{\rho_2}{\rho_1} - 1 \right) k}{1 + \frac{\rho_2}{\rho_1}}} \quad (29)$$

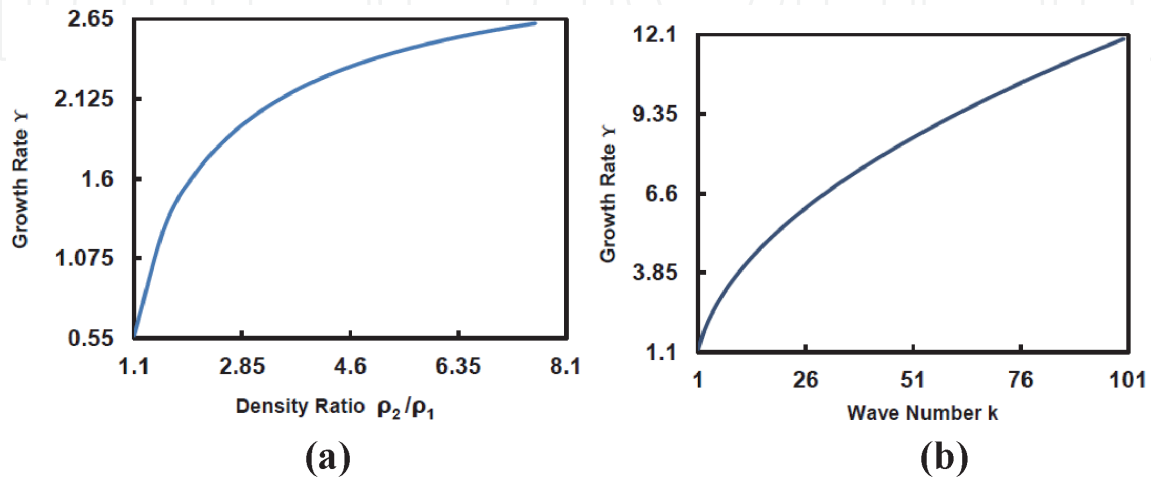
Substituting the value of  $\omega = i\gamma$  into Eq. (18), the amplitude grows exponentially with the perturbation and is given by

$$\phi_1(x, z, t) = \phi_{01} \exp(kz) \exp(ikx) \exp(\pm\gamma t) \quad (30)$$

The term  $\exp(\gamma t)$  increases the amplitude of the oscillation exponentially as time progress. **Figure 3(a)** and **(b)** shows the variations of growth rate of RT instability with (a) density ratio and (b) wave number. The inverse of the growth rate  $\gamma^{-1} = t_{char}$  is called the linear characteristic timescale of the RTI. In other words, characteristic timescale has to be the order of the lifetime of the plasma oscillations to observe the RT instability. On the other hand, if the linear characteristic time-scale is much larger than the oscillation lifetime, the plasma instability would not be observed.



**Figure 2.** Variation of phase velocity of RT instability with (a) density ratio and (b) wave number respectively.



**Figure 3.** Variation of growth rate of RT instability with (a) density ratio and (b) wave number respectively.



## 7. Rayleigh-Taylor instability in plasma thruster

In the previous section, the general idea of RT instability has been explored. Here we have derived the RT equation for a plasma fluid using two fluid theory. In a Hall thruster, the propellant (plasma) is ionized and then accelerated by electrostatic forces. It has high thrust resolution, so it is best suited for the adjustment of the location of the satellite onboard [19–27]. Let us consider a plasma with nonuniform density confined under the crossed electric and magnetic fields.

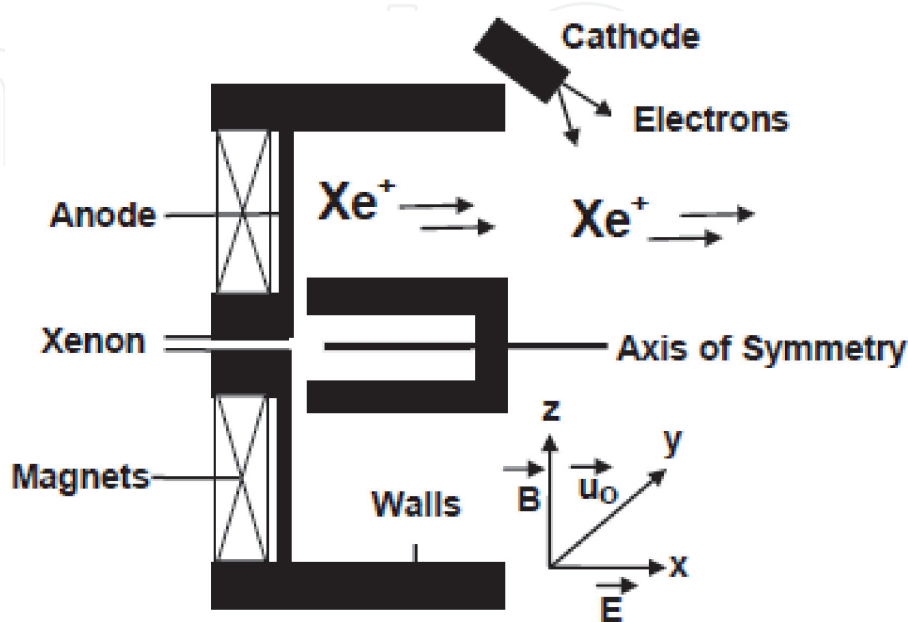
**Figure 4** shows the typical diagram of a Hall plasma thruster [26]. RT instability is common in Hall thrusters. Studies show that Rayleigh instability is driven by the presence of gradients in axial density, magnetic field, and velocity of the plasma species. Here we deduce a Rayleigh equation under the presence of ion temperature and check the variations of perturbed potential with plasma parameters.

### 7.1 Theoretical model for RTI in plasma

We consider plasma comprising of ions and electrons immersed in a magnetic field  $\vec{B} = B\hat{z}$ . The magnetic field is strong enough so that only electrons get magnetized, but the ions remain unaffected due to their Larmor radius being much larger than the dimension of the thruster. These trapped electrons (due to crossed fields) drift in azimuthal direction along the annular channel [24]. The applied electric field  $\vec{E}$  is along the  $x$ -axis (axis of the thruster) and the magnetic field  $\vec{B}$  is taken along the  $z$ -axis (along the radius of the thruster). Hence, the azimuthal dimension is along the  $y$ -axis. We use  $\Omega_z = \frac{eB}{m}$  as the electron gyro frequency and  $u_0 = -\frac{E_0}{B}\hat{y}$  as the initial drift of the electrons [14–17] and write the continuity equation and equation of motion for plasma species as

$$\frac{\partial n_i}{\partial t} + \vec{\nabla} \cdot (\vec{v}_i n_i) = 0 \quad (31)$$

$$\left( \frac{\partial}{\partial t} + \vec{v}_i \cdot \vec{\nabla} \right) \vec{v}_i = \frac{e\vec{E}}{M} - \frac{\vec{\nabla} p_i}{Mn_i} \quad (32)$$



**Figure 4.**  
Typical diagram of a Hall plasma thruster.

$$\frac{\partial n_e}{\partial t} + \vec{\nabla} \cdot (\vec{v}_e n_e) = 0 \quad (33)$$

$$\left(\frac{\partial}{\partial t} + \vec{v}_e \cdot \vec{\nabla}\right) \vec{v}_e = \frac{\vec{\nabla} \phi}{m} - (\vec{v}_e \times \vec{\Omega}_z) \quad (34)$$

We use the linearized form of the above equations for small perturbations of the ion and electron densities, their velocities, and electric field. We write perturbed densities of ions (electrons) by  $n_{i1}$  ( $n_{e1}$ ) velocities by  $\vec{v}_{i1}$  ( $\vec{v}_{e1}$ ). The unperturbed electrons' drift is  $u_0$  in the  $y$ -direction. The unperturbed density (electric field) is taken as  $n_0(E_0)$  and the perturbed value of the electric field is taken as  $\vec{E}$  (corresponding potential  $\phi$ ). Hence, the linearized form of Eqs. (31)–(34) reads

$$\frac{\partial n_{i1}}{\partial t} + v_{ix1} \frac{\partial n_0}{\partial x} + n_0 (\vec{\nabla} \cdot \vec{v}_{i1}) = 0 \quad (35)$$

$$\frac{\partial \vec{v}_{i1}}{\partial t} = \frac{e}{M} \vec{E} - \frac{\vec{\nabla} p_{i1}}{n_0 M} \quad (36)$$

$$\frac{\partial n_{e1}}{\partial t} + u_0 \frac{\partial n_{e1}}{\partial y} + n_0 (\vec{\nabla} \cdot \vec{v}_{e1}) + v_{ex1} \frac{\partial n_0}{\partial x} = 0 \quad (37)$$

$$\frac{\partial \vec{v}_{e1}}{\partial t} + u_0 \frac{\partial \vec{v}_{e1}}{\partial y} = \vec{\nabla} \phi - (\vec{v}_{e1} \times \vec{\Omega}_z) \quad (38)$$

The unperturbed ions' velocity  $v_0$  is taken zero here for the case of simplification. We are looking for oscillating solution of the above equations that should vary as  $f = f_0 \exp(i\omega t - ik_y y)$ . The ion thermal velocity can be written as  $V_{thi}^2 = \frac{Y_i T_i}{M}$ . With the help of Eqs. (35) and (36) we obtain the following expression for the perturbed ion density in terms of the perturbed electric potential  $\phi$ :

$$n_{i1} = \frac{en_0}{M(\omega^2 - V_{thi}^2 k_y^2)} \left( k_y^2 \phi - \frac{\partial^2 \phi}{\partial x^2} - \frac{Y_i T_i}{en_0} \frac{\partial^2 n_0}{\partial x^2} \right) \quad (39)$$

Eq. (38) provides the velocity components of electron

$$i(\omega - k_y u_0) v_{ex1} = \frac{e}{m} \frac{\partial \phi}{\partial x} - \Omega_z v_{ey1} \quad (40)$$

$$i(\omega - k_y u_0) v_{ey1} + v_{ex1} \frac{\partial u_0}{\partial x} = -ik_y \frac{e}{m} \phi + \Omega_z v_{ex1} \quad (41)$$

In the above equations, the coordinate  $x$  lies in the interval  $0 < x < d$ , where  $d$  is the channel length. Let us define  $\omega - k_y u_0$  by  $\hat{\omega}$  in the above set of expressions. Further we readily obtain from the above equations

$$v_{ex1} = \frac{\frac{e}{m} i \hat{\omega} \frac{\partial \phi}{\partial x} + \frac{e}{m} i k_y \Omega_z \phi}{\Omega_z^2 - \hat{\omega}^2 - \Omega_z \frac{\partial u_0}{\partial x}} \quad (42)$$

$$v_{ey1} = \frac{e}{m \Omega_z} \frac{\partial \phi}{\partial x} + \frac{\frac{e}{m} \hat{\omega}^2 \frac{\partial \phi}{\partial x} + \frac{e}{m} \hat{\omega} k_y \Omega_z \phi}{\Omega_z (\Omega_z^2 - \hat{\omega}^2 - \Omega_z \frac{\partial u_0}{\partial x})} \quad (43)$$

The electron cyclotron frequency is almost  $\Omega_z \sim 10^8$ /s (corresponding to 200 Gauss magnetic field). Generally,  $\Omega_z$  is much larger than the frequency of the

oscillations. Therefore, under the condition  $\Omega_z \gg \omega, k_y u_0, \frac{\partial u_0}{\partial x}$ , the velocity components of electrons are reduced into the form

$$v_{x1} = \frac{ie}{m\Omega_z^2} \left[ \hat{\omega} \frac{\partial \phi}{\partial x} + \Omega_z k_y \phi + \frac{\partial u_0}{\partial x} k_y \phi \right] \quad (44)$$

$$v_{y1} = \frac{e}{m\Omega_z} \left[ \frac{\partial \phi}{\partial x} + \frac{k_y \hat{\omega} \phi}{\Omega_z} + \frac{k_y \hat{\omega} \phi \partial u_0}{\Omega_z^2 \partial x} \right] \quad (45)$$

The electron continuity equation gives the perturbed electron density  $n_e$  with the help of Eqs. (44) and (45)

$$n_{e1} = \frac{en_0}{m\Omega_z^2} \left[ \left( k_y^2 \phi - \frac{\partial^2 \phi}{\partial x^2} \right) + \frac{k_y}{\omega - k_y u_0} \left( \Omega_z \frac{\partial}{\partial x} \ln \frac{B}{n_0} - \frac{\partial^2 u_0}{\partial x^2} \right) \phi \right] \quad (46)$$

The plasma frequency of oscillations for ion (electron) is defined as

$$\omega_i (\omega_e) = \sqrt{\frac{n_0 e^2}{M(m) \epsilon_0}} \quad (47)$$

$$\text{The Poisson's equation } \epsilon_0 \nabla^2 \phi = e(n_{e1} - n_{i1}) \quad (48)$$

Using Eqs. (39) and (46) in Eq. (48) gives the perturbed potential in the following form:

$$\frac{\partial^2 \phi}{\partial x^2} - k_y^2 \phi - \frac{k_y \phi \left( \Omega_z \frac{\partial}{\partial x} \ln \frac{B}{n_0} - \frac{\partial^2 u_0}{\partial x^2} \right)}{(\omega - k_y u_0) \left( 1 + \Omega_z^2 / \omega_e^2 - \frac{\Omega_z^2 \omega_i^2}{\omega_e^2 (\omega^2 - k_y^2 V_{thi}^2)} \right)} = 0 \quad (49)$$

In the case of high frequency of oscillations and in the absence of ion thermal pressure, Eq. (49) turns into Rayleigh's equation of fluid dynamics as below

$$\left( \frac{\partial^2 \phi}{\partial x^2} - k_y^2 \phi \right) + \frac{\phi k_y}{(\omega - k_y V_y)} \frac{\partial^2 V_y}{\partial x^2} = 0 \quad (50)$$

Here  $V_y$  is the flow velocity in the y-direction and  $\phi$  is called the flow function related to  $V_y = \vec{\nabla} \phi$ . The analytical eigenvalue solution of Eq. (49) is given in Ref. [12].

### Resonance condition for the RT instability

From Eq. (49), it is clear that propagating mode may lead to instability if parameter  $\Omega_z \frac{\partial}{\partial x} \ln \frac{B}{n_0} - \frac{\partial^2 u_0}{\partial x^2} = 0$  at some point inside the Hall thruster.

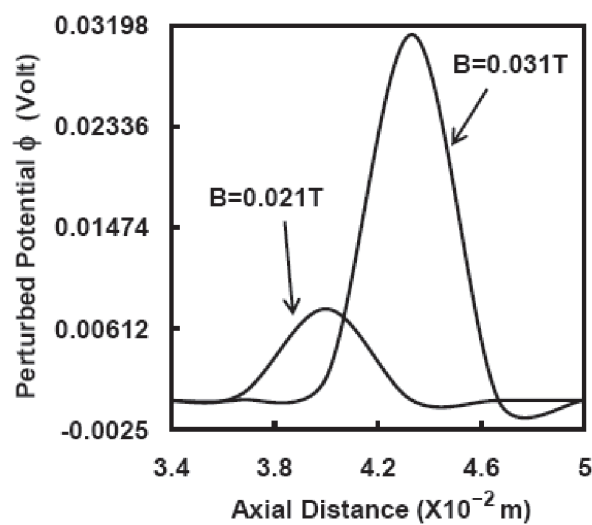
## 7.2 Variations of perturbed potential

The RT Eq. (49) is solved numerically for the perturbed potential  $\phi$  along with the boundary conditions such that  $\phi(0) = \phi(d) = 0$ . We plot perturbed potential of the instability with magnetic field  $B$ , initial drift of the electrons  $u_0$ , channel length  $d$ , and ion temperature  $T_i$ . These parameters can have values as  $B = (100 - 250)$  G,  $n_0 = 5 \times 10^{17} - 10^{18} / \text{m}^3$ ,  $T_i = 0.1 - 5$  eV, and  $u_0 \sim 10^6$  m/s [13, 15].

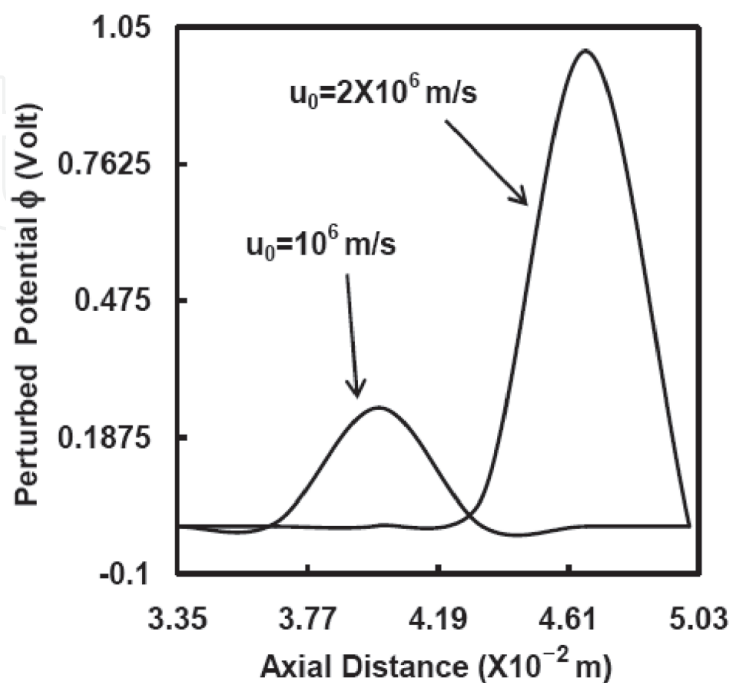
**Figure 5** shows the variation of the perturbed potential with the magnetic field and it has been observed that the potential increases with the increasing magnetic

field. These results are consistent with Keidar and Boyd model [28] and that other investigators [13, 14] for the potential of plasma plume. This situation is correspond to the plasma jet enters a transverse magnetic field with a high velocity under the condition that the magnetic field is relatively weak so that only the electrons are magnetized whereas the ions move out of the effect of magnetic field. However, ambipolar (both electrons and ions moving in opposite directions) plasma flow across the magnetic field may require an electric field to appear under the above conditions. Therefore, we can expect the potential to increase across the magnetic field.

The perturbed potential gets increased with the higher value of electron's initial drift velocity (shown in **Figure 6**). Similar behavior of the potential was reported experimentally by King et al. [29] for the potential of plasma plume. Similar results are also reported in Refs. [13, 14]. The enhanced perturbed potential  $\phi$  with the ion temperature is shown in **Figure 7** which is consistent with an experiment [30].



**Figure 5.**  
Effect of magnetic field on the perturbed potential  $\phi$ .



**Figure 6.**  
Dependence of perturbed potential  $\phi$  on the drift velocity of the electrons.

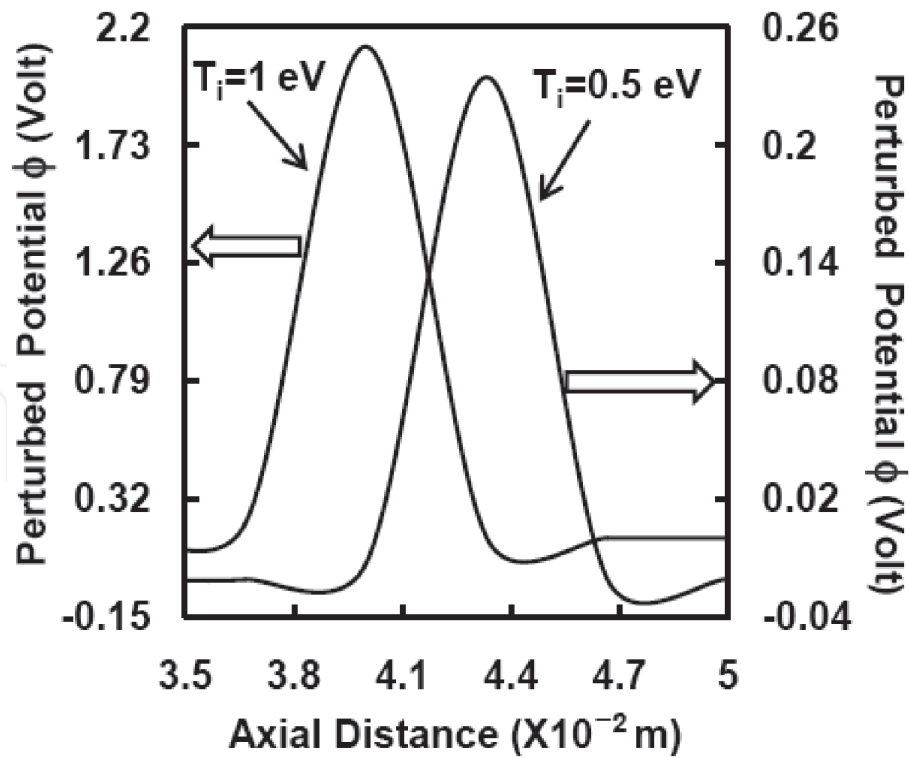


Figure 7.  
Variation of perturbed potential  $\phi$  with the ion temperature.

## 8. Discussions and summary

In conclusion, we can say that short-wavelength perturbations blow up exponentially much more quickly in RTI. The primary source by which this instability is triggered is the gravitational force acting on an inverted density gradient (e.g., a heavy fluid supported by a light fluid). Stable and steady flows may become unstable depending on the ranges of the flow parameters. The instability takes free energy from the mean flow or externally supplied heat and the amplitude of waves grows exponentially. The instabilities exist in all natural and artificial phenomena (in smoke from chimneys, in rivers, in flickering flames) and their effects result in turbulence or random waves. The presence of plasma density and magnetic field gradients is one of the main sources for plasma instabilities in Hall thrusters. It is found that perturbed potential increases with the higher value of electrons' drift velocity, magnetic field, and ion temperature.

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