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Mahgoub Transform on Boehmians

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Abstract

Boehmian's space is established utilizing an algebraic way that approximates identities or delta sequences and appropriate convolution. The space of distributions can be related to the proper subspace. In this paper, firstly we establish the appropriate Boehmian space, on which the Mahgoub Transformation can be described& function space K can be embedded. We add to more in this, our definitions enhance Mahgoub transform to progressively wide spaces. We additionally explain the functional axioms of Mahgoub transform on Boehmians. Lastly toward the finishing of topic, we analyze with specify axioms and properties for continuity and the enlarged Mahgoub transform, also its inverse regards to Δ-convergence and δ.

Keywords: Mahgoub Transform; The Space $\mathbb{B}(\mathfrak{X})$; The Space$\mathbb{B}(\mathfrak{X}^{\mathfrak{M}})$; Boehmian Spaces.

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1. Introduction

The Mahgoub transform [8] which is denoted by the operator $\mathfrak{M}(.)$ and Mahgoub transform of $\mathfrak{T}(t^*)$ is defined by:

$$\mathfrak{M}(\mathfrak{T}(t^*)) = \mathbb{E}(\vartheta) = \vartheta \int_0^\infty \mathfrak{T}(t^*)\, e^{-\vartheta t^*} dt^*, t^* \geq 0, \qquad (1.1)$$

and $\rho_1 \leq \vartheta \leq \rho_2$.

In a set ;

$$\mathbb{A} = \left\{ \mathfrak{T}(t^*) : \exists \mathbb{M}, \rho_1, \rho_2 > 0 \,.\, |\mathfrak{T}(t^*)| < \mathbb{M} e^{\frac{|t^*|}{\rho_j}} \right\} \;, \qquad (1.2)$$

where ρ_1 and ρ_2(may be finite or infinite), the constant $\mathbb{M}$ must be finite. An existence's Mahgoub transform of $\mathfrak{T}(t^*)$ is essential for $t^* \geq 0$, a piece wise continuous and of exponential order is required, else it does not exist.

Convolution Theorem For Mahgoub Transform [9-11]:

If $\mathfrak{M}(\mathfrak{T}(t^*)) = \mathbb{E}(\vartheta)$ and $\mathfrak{M}(\mathfrak{P}(t^*)) = \mathbb{W}(\vartheta)$ then

$$\mathfrak{M}(\mathfrak{T}(t^*) \star \mathfrak{P}(t^*)) = \frac{1}{\vartheta} \mathfrak{M}(\mathfrak{T}(t^*)) \mathfrak{M}(\mathfrak{P}(t^*)) = \frac{1}{\vartheta} \mathbb{E}(\vartheta)\mathbb{W}(\vartheta) \qquad (1.3)$$

Linearity Property Of Mahgoub Transform:

If $\mathfrak{M}(\mathfrak{T}(t^*)) = \mathbb{E}(\vartheta)$, $\mathfrak{M}(\mathfrak{P}(t^*)) = \mathbb{W}(\vartheta)$ then

$$\mathfrak{M}\{\mathfrak{a}\mathfrak{T}(t^*) + \mathfrak{b}\,\mathfrak{P}(t^*)\} = \mathfrak{a}\,\mathfrak{M}(\mathfrak{T}(t^*)) + \mathfrak{b}\,\mathfrak{M}(\mathfrak{P}(t^*)) \qquad (1.4)$$

2. Boehmian Space

Boehmians was first developed as a generalization's standard mikusinski operators [2]. The formation necessary for Boehmians satisfying the following axioms.

i. a non empty set $\mathfrak{A}$;

ii. a semi group$(\varphi, \circledast)$ which is commutative;

iii. $\otimes : \mathfrak{A} \times \varphi \to \mathfrak{A}$ s.t.$\forall\, \xi \in \mathfrak{A}$ and $\eta_1, \eta_2 \in \varphi$, $\xi \otimes (\eta_1 \circledast \eta_2) = (\xi \otimes \eta_1) \otimes \eta_2$;

iv. a collection $\Delta \subset \varphi^N$such that

a) If$\xi_1, \xi_2 \in \mathfrak{A}$, $(\eta_n) \in \Delta$, $\xi_1 \otimes \eta_n = \xi_2 \otimes \eta_n \;\forall\, n$then $\xi_1 = \xi_2$;

b) If $(\eta_n), (\tau_n) \in \Delta$, then $(\eta_n \otimes \tau_n) \in \Delta$. where elements of Δ are known as delta sequences.

Consider

$\mathcal{H} = \{(\xi_n, \eta_n) : \xi_n \in \mathfrak{A}, (\eta_n) \in \Delta, \xi_n \otimes \eta_m = \xi_m \otimes \eta_n \forall\, m, n \in N\}$,

Now if $(\xi_n, \eta_n), (\emptyset_n, \tau_n) \in \mathcal{H}$ then $\xi_n \otimes \tau_m = \emptyset_m \otimes \eta_n, \forall\, m, n \in N$.

We say that $(\xi_n, \eta_n) \sim (\emptyset_n, \tau_n)$. where $\sim$ is an equivalence relation in $\mathcal{H}$. The Set of equivalence classes in $\mathcal{H}$ is denoted as $\mathfrak{H}$. Elements of $\mathfrak{H}$ are said to be Boehmians.

We assume that there is a canonical embedding between $\mathfrak{H}$ and $\mathfrak{A}$, expressed as $\xi \to \frac{\xi_n \otimes \eta_n}{\eta_n}$, where $\otimes$ can also be extended in

$$\mathfrak{H} \times \mathfrak{A} \text{ by } \frac{\xi_n}{\eta_n} \otimes \tau = \frac{\xi_n \otimes \tau}{\eta_n}.$$

In $\mathfrak{H}$, there are two types of convergence is given by

i. if $\beth_n \to \beth$ as $n \to \infty$ which belongs to $\mathfrak{A}$, $\mathscr{k} \in \varphi$ is any fixed element, then $\beth_n \otimes \mathscr{k} \to \beth \otimes \mathscr{k}$ as $n \to \infty$ in $\mathfrak{A}$.

ii. if $\beth_n \to \beth$ as $n \to \infty$ in $\mathfrak{A}$ and $\lambda_n \in \Delta$ then $\beth_n \otimes \lambda_n \to \beth$ as $n \to \infty$ in $\mathfrak{A}$.

An operation $\otimes$ can be extended in $\mathfrak{H} \times \varphi$ as per condition:
If $\left[\frac{\beth_n}{\eta_n}\right] \in \mathfrak{H}$ and $\mathscr{k} \in \varphi$ then $\left[\frac{\beth_n}{\eta_n}\right] \otimes \mathscr{k} = \left[\frac{\beth_n \otimes \mathscr{k}}{\eta_n}\right]$.
Now convergence in $\mathfrak{H}$ as following:

1. A sequence (ς_n) in $\mathfrak{H}$ is called δ– convergent to ς in $\mathfrak{H}$, i.e.
$\varsigma_n \xrightarrow{\delta} \varsigma$ if $\exists\ (\eta_n) \in \Delta$ such that $(\varsigma_n \otimes \eta_n), (\varsigma \otimes \eta_n) \in \mathfrak{A}, \forall\ n \in N$ and $(\varsigma_n \otimes \eta_{\mathscr{k}}) \to (\varsigma \otimes \eta_{\mathscr{k}})$ as $n \to \infty$ in $\mathfrak{A}, \forall\ \mathscr{k}, n \in N$.
2. A sequence (ς_n) in $\mathfrak{H}$ is said to be Δ convergent to ς in $\mathfrak{H}$ i.e. $\varsigma_n \xrightarrow{\Delta} \varsigma$, if $\exists(\eta_n) \in \Delta$ such that $(\varsigma_n - \varsigma) \to 0$ as $n \to \infty$ which belongs to $\mathfrak{A}$.

For more details, see [3-6].

3. The Boehmian Space $\mathbb{B}(\mathfrak{X})$:

Denoted by $\mathfrak{S}_+(\mathbb{R})$ and $\mathcal{C}_{0+}^{\infty}(\mathbb{R})$ are the space's smooth function over $\mathbb{R}$ and the Schwarz space's test function's compact support over $\mathbb{R}_+$ where $\mathbb{R}_+ = (0, \infty)$ respectively. We have found vital results for the structure of Boehmian space $\mathbb{B}(\mathfrak{X})$ where $\mathfrak{X} = (\mathfrak{S}_+, \mathcal{C}_{0+}^{\infty}, \Delta_+)$.

Lemma 3.1:

1) If $\mathfrak{d}_1, \mathfrak{d}_2 \in \mathcal{C}_{0+}^{\infty}(\mathbb{R})$ then $\mathfrak{d}_1 \star \mathfrak{d}_2 \in \mathcal{C}_{0+}^{\infty}(\mathbb{R})$ (Closure).

2) If $\mathfrak{F}_1, \mathfrak{F}_2 \in \mathfrak{S}_+(\mathbb{R})$ and $\mathfrak{d}_1 \in \mathcal{C}_{0+}^{\infty}(\mathbb{R})$ then $(\mathfrak{F}_1 + \mathfrak{F}_2) \star \mathfrak{d}_1 = \mathfrak{F}_1 \star \mathfrak{d}_1 + \mathfrak{F}_2 \star \mathfrak{d}_1$ (Distributive).
3) $\mathfrak{d}_1 \star \mathfrak{d}_2 = \mathfrak{d}_2 \star \mathfrak{d}_1 \forall \mathfrak{d}_1, \mathfrak{d}_2 \in \mathcal{C}_{0+}^{\infty}(\mathbb{R})$ (Commutative).

4) If $\mathfrak{F} \in \mathfrak{S}_+(\mathbb{R})$, $\mathfrak{d}_1, \mathfrak{d}_2 \in \mathcal{C}_{0+}^{\infty}(\mathbb{R})$then $(\mathfrak{F} \star \mathfrak{d}_1) \star \mathfrak{d}_2 = \mathfrak{F} \star (\mathfrak{d}_1 \star \mathfrak{d}_2)$(Associative).

Definition3.2: A sequence (η_n) of function from $\mathcal{C}_{0+}^{\infty}(\mathbb{R})$is said to be in$\Delta_+$. If

$$\Delta_+^1: \int_{\mathbb{R}^+} \eta_n(\xi) d\xi = 1.$$

$\Delta_+^2: \int_{\mathbb{R}^+} |\eta_n(\xi)| d\xi \leq m$,where m is a positive integer;

$\Delta_+^3: Supp\ \eta_n(\xi) \subset (0, \epsilon_n), \quad \epsilon_n \to 0$ as $n \to \infty$.

i.e.(η_n) shrink to zero as $n \to \infty$.every member of Δ_+ is known as an approximation identity or a delta sequences. In all manners delta sequences arise in numerous parts of Mathematics, however likely the very important application are those in the presupposition's generalized functions. The fundamental application of delta sequence is the regularization's established functions and ahead we can be utilized to characterize the convolution product and its established functions.

Lemma 3.3: If(η_n),$(\tau_n) \in \Delta_+$, then $supp(\eta_n \star \tau_n) \subset supp\eta_n + supp\tau_n$.

Lemma 3.4: If $\mathfrak{d}_1, \mathfrak{d}_2 \in \mathcal{C}_{0+}^{\infty}(\mathbb{R})$ then so is $\mathfrak{d}_1 \star \mathfrak{d}_2$ and $\int_{\mathbb{R}^+} |\mathfrak{d}_1 \star \mathfrak{d}_2| \leq \int_{\mathbb{R}^+} |\mathfrak{d}_1| \int_{\mathbb{R}^+} |\mathfrak{d}_2|$.

Theorem 3.5: Let$\mathfrak{F}_1, \mathfrak{F}_2 \in \mathfrak{S}_+(\mathbb{R})$ and $(\eta_n) \in \Delta_+$ such that

$$\mathfrak{F}_1 \star \eta_n = \mathfrak{F}_2 \star \eta_n.$$

where $n = 1,2,3, \ldots$, then $\mathfrak{F}_1 = \mathfrak{F}_2$ in $\mathfrak{S}_+(\mathbb{R})$.

Proof: To prove that $\mathfrak{F}_1 \star \eta_n = \mathfrak{F}_1$.

Let K be a compact support accommodating the $supp\eta_n$ for each$n \in N$. By using Δ_+^1, we write

$$\left|\xi^k D^m(\mathfrak{F}_1 \star \eta_n - \mathfrak{F}_1)(\xi)\right| \leq \int_K |\eta_n(\tau)| \left|\xi^k D^m\big(\mathfrak{F}_1(\xi - \tau) - \mathfrak{F}_1(\xi)\big)\right| d\tau \tag{3.1}$$

The mapping $\tau \to \mathfrak{F}_1^\tau$where$\mathfrak{F}_1^\tau = \mathfrak{F}_1(\xi - \tau)$, is Uniformly continuous from $\mathbb{R}^+ \to \mathbb{R}^+$. By using axiom$\Delta_+^3$that $supp\eta_n \to 0$ as $n \to \infty$, now we choose $r > 0$;$supp\eta_n \subseteq [0, r]$ for large n and $\tau < r$, that is

$$|\mathfrak{F}_1(\xi - \tau) - \mathfrak{F}_1(\xi)| = |\mathfrak{F}_1^\tau - \mathfrak{F}_1| < \frac{\epsilon_n}{M} \tag{3.2}$$

Hence using Δ_+^2 and Eq's. (3.2), (3.1) we get

$|\xi^k D^m(\mathfrak{F}_1 \star \eta_n - \mathfrak{F}_1)(\xi)| < \epsilon_n \to 0$ as $n \to \infty$.
Thus $\mathfrak{F}_1 \star \eta_n \to \mathfrak{F}_1$ in $\mathfrak{S}_+(\mathbb{R})$. Similarly, we prove that $\mathfrak{F}_2 \star \eta_n \to \mathfrak{F}_2$ in $\mathfrak{S}_+(\mathbb{R})$ ☖

Theorem 3.6: if $\mathfrak{F}_n \to \mathfrak{F}$ in $\mathfrak{S}_+(\mathbb{R})$ as $n \to \infty$ and $\mathfrak{d} \in \mathcal{C}_{0+}^{\infty}(\mathbb{R})$ then $\lim_{n\to\infty} \mathfrak{F}_n \star \mathfrak{d} = \mathfrak{F} \star \mathfrak{d}$.

Proof: Using Theorem we get

$$\left|\xi^k D^m\big((\mathfrak{F}_n \star \mathfrak{d}) - (\mathfrak{F} \star \mathfrak{d})\big)(\xi)\right| = \left|\xi^k (D^m(\mathfrak{F}_n - \mathfrak{F}) \star \mathfrak{d})(\xi)\right| \quad (3.3)$$

The equation follows from [3]

$$D^m\mathfrak{F} \star \mathfrak{d} = D^m\mathfrak{F} \star \mathfrak{d} = \mathfrak{F} \star D^m\mathfrak{d}$$

for all $\mathfrak{d} \in \mathcal{C}_{0+}^{\infty}(\mathbb{R})$, we have

$$\left|\xi^k D^m\big((\mathfrak{F}_n \star \mathfrak{d}) - (\mathfrak{F} \star \mathfrak{d})\big)(\xi)\right| \le \int_K \xi^k |D^m(\mathfrak{F}_n - \mathfrak{F})(\xi - \tau)||\mathfrak{d}(\tau)|d\tau$$

$\le M\gamma_k(\mathfrak{F}_n - \mathfrak{F})$ for some constant M$\to 0$ as $n \to \infty$. ☖

Theorem 3.7: In $\mathfrak{S}_+(\mathbb{R})$, Let $\lim_{n\to\infty} \mathfrak{F}_n = \mathfrak{F}$ and

$$(\eta_n) \in \Delta_+ \Rightarrow \lim_{n\to\infty} \mathfrak{F}_n \star \eta_n = \mathfrak{F}.$$

Proof: By the hypothesis of the Theorem 3.5, we get $\lim_{n\to\infty} \mathfrak{F}_n \star \eta_n = \mathfrak{F}_n \to \mathfrak{F}$ as $n \to \infty$.

Hence , we arrive,

$$\lim_{n\to\infty} \mathfrak{F}_n \star \eta_n = \mathfrak{F} \text{ as } n \to \infty.$$ ☖

The Canonical embedding between $\mathbb{B}(\mathfrak{X})$ and $\mathfrak{S}_+(\mathbb{R})$, defined as $\xi \to \left[\frac{\xi \star \eta_n}{\eta_n}\right]$.

The extension of $\star$ to $\mathbb{B}(\mathfrak{X}) \times \mathfrak{S}_+(\mathbb{R})$ is given by $\left[\frac{\xi_n}{\eta_n}\right] \star \tau = \left[\frac{\xi_n \star \tau}{\eta_n}\right]$.

Convergence in $\mathbb{B}(\mathfrak{X})$ is followed:

δ– Convergence: A sequence (ς_n) in $\mathbb{B}(\mathfrak{X})$ is called δ– convergent to ς in $\mathbb{B}(\mathfrak{X})$ denoted by $\varsigma_n \xrightarrow{\delta} \varsigma$ if $\exists\, (\eta_n) \in \Delta$ such that $(\varsigma_n \star \eta_n), (\varsigma \star \eta_n) \in \mathfrak{S}_+(\mathbb{R}), \forall\, n \in N$ and $(\varsigma_n \star \eta_k) \to (\varsigma \star \eta_k)$ as $n \to \infty$ in $\mathfrak{S}_+(\mathbb{R}), \forall\, k, n \in N$.

$\Delta_+ -$ Convergence: A sequence (ς_n) in $\mathbb{B}(\mathfrak{X})$ is said to be $\Delta_+ -$ convergent to ς in $\mathbb{B}(\mathfrak{X})$ i.e. $\varsigma_n \xrightarrow{\Delta} \varsigma$, if $\exists(\eta_n) \in \Delta_+$ such that $(\varsigma_n - \varsigma) \otimes \eta_n \in \mathfrak{S}_+(\mathbb{R}) \forall\, n \in N$ and $(\varsigma_n - \varsigma) \otimes \eta_n \to 0$ as $n \to \infty$ in $\mathfrak{S}_+(\mathbb{R})$.

Theorem 3.8: Define $\mathfrak{F} \to \left[\frac{\mathfrak{F}\star\eta_n}{\eta_n}\right]$ is continuous mapping which is embedding from $\mathfrak{S}_+(\mathbb{R})$ into $\mathbb{B}(\mathfrak{X})$.

Proof: To show**:** The mapping is one - one.

We have $\left[\frac{\mathfrak{F}_1\star\eta_n}{\eta_n}\right] = \left[\frac{\mathfrak{F}_2\star\tau_n}{\tau_n}\right]$, then

$$(\mathfrak{F}_1 \star \eta_n) \star \tau_m = (\mathfrak{F}_2 \star \tau_m) \star \eta_n \,, m, n \in N.$$

$\because (\tau_n), (\eta_n) \in \Delta_+, \mathfrak{F}_1 \star (\eta_m \star \tau_n) = \mathfrak{F}_2 \star (\tau_n \star \eta_m) = \mathfrak{F}_2 \star (\eta_m \star \tau_n)$.

Using Theorem 3.5, we get $\mathfrak{F}_1 = \mathfrak{F}_2$.

To prove: The mapping is continuous.

Let $\mathfrak{F}_n \to 0$ in$\mathfrak{S}_+(\mathbb{R})$as $n \to \infty$. Then we have$\left[\frac{\mathfrak{F}_n\star\eta_m}{\eta_m}\right] \xrightarrow{\delta} 0$as $n \to \infty$.

From the Theorem 3.5, $\left[\frac{\mathfrak{F}_n\star\eta_m}{\eta_m}\right] \star \eta_m = \mathfrak{F}_n \star \eta_m \to 0$as $n \to \infty$. ☖

Theorem3.9: Let $\mathfrak{d} \in \mathcal{C}_{0+}^{\infty}(\mathbb{R})$ and $\mathfrak{F} \in \mathfrak{S}_+(\mathbb{R}) \Rightarrow \mathfrak{M}(\mathfrak{F} \star \mathfrak{d})(\xi) = \frac{1}{\xi}\mathfrak{F}^{\mathfrak{M}}(\xi)\mathfrak{d}^{\mathfrak{M}}(\xi)$.

4. TheBoehmian Space $\mathbb{B}(\mathfrak{X}^{\mathfrak{M}})$

We delineate Boehmian space as ensues. Let $\mathfrak{S}_+(\mathbb{R})$ be the space's immediately decreasing function [3]. We have

$$\mathcal{C}_{0+}^{\infty\mathfrak{M}}(\mathbb{R}) = \left\{\mathfrak{d}^{\mathfrak{M}} : \forall \mathfrak{d} \in \mathcal{C}_{0+}^{\infty}(\mathbb{R})\right\} \tag{4.1}$$

here $\mathfrak{d}^{\mathfrak{M}}$ express the Mahgoub transform of $\mathfrak{d}$ and also characterize $\mathfrak{F}\blacksquare\mathfrak{d}^{\mathfrak{M}}$ by

$$\left(\mathfrak{F}\blacksquare\mathfrak{d}^{\mathfrak{M}}\right)(\xi) = \frac{1}{\xi}\mathfrak{F}(\xi)\mathfrak{d}^{\mathfrak{M}}(\xi) \tag{4.2}$$

Lemma 4.1 Let $\mathfrak{F} \in \mathfrak{S}_+(\mathbb{R}), \mathfrak{d}^{\mathfrak{M}} \in \mathcal{C}_{0+}^{\infty\mathfrak{M}}(\mathbb{R}) \Rightarrow \mathfrak{F}\blacksquare\mathfrak{d}^{\mathfrak{M}} \in \mathfrak{S}_+(\mathbb{R})$.

Proof. Let $\mathfrak{F} \in \mathfrak{S}_+(\mathbb{R}), \mathfrak{d}^{\mathfrak{M}} \in \mathcal{C}_{0+}^{\infty\mathfrak{M}}(\mathbb{R})$, by Leibnitz' Theorem and applying the definition of $\mathfrak{S}_+(\mathbb{R})$, we found

$$\left|\xi^k D_\xi^m(\mathfrak{F}\blacksquare\mathfrak{d}^{\mathfrak{M}})(\xi)\right| \leq \left|\xi^k \sum_{j=1}^{m} D^{m-j}\left(\frac{1}{\xi}\mathfrak{F}(\xi)\right) D^j \mathfrak{d}^{\mathfrak{M}}(\xi)\right|$$

$$\leq \sum_{j=1}^{m}\left|\xi^k D^{m-j}\left(\frac{1}{\xi}\mathfrak{F}(\xi)\right)\right|\left|D^j\mathfrak{d}^{\mathfrak{M}}(\xi)\right|$$

$$= \sum_{j=1}^{m}\left|\xi^k D^{m-j}\mathfrak{F}_1(\xi)\right|\left|\vartheta\int_K \mathfrak{d}(\tau)e^{-\frac{\tau}{\vartheta}}d\tau\right|$$

Where $\mathfrak{F}_1(\xi) = \frac{1}{\xi}\mathfrak{F}(\xi) \in \mathfrak{S}_+(\mathbb{R})$ and K is a compact support accommodating the $suppu(\tau)$.

$$\left|\xi^k D_\xi^m(\mathfrak{F} \blacksquare \mathfrak{d}^{\mathfrak{M}})(\xi)\right| \leq M\gamma_{k,m-j}(\mathfrak{F}_1) < \infty,$$

for some positive constant M. ☖

Lemma 4.2 A mapping $\mathfrak{S}_+ \times \mathcal{C}_{0+}^{\infty\mathfrak{M}} \to \mathfrak{S}_+$ is defined by

$$\left(\mathfrak{F}, \mathfrak{d}^{\mathfrak{M}}\right) \to \mathfrak{F} \blacksquare \mathfrak{d}^{\mathfrak{M}}$$

Satisfying the following axioms:

(1) If $\mathfrak{d}_1^{\mathfrak{M}}, \mathfrak{d}_2^{\mathfrak{M}} \in \mathcal{C}_{0+}^{\infty\mathfrak{M}}(\mathbb{R})$, then $\mathfrak{d}_1^{\mathfrak{M}} \blacksquare \mathfrak{d}_2^{\mathfrak{M}} \in \mathcal{C}_{0+}^{\infty\mathfrak{M}}(\mathbb{R})$.

(2) If $\mathfrak{F}_1, \mathfrak{F}_2 \in \mathfrak{S}_+(\mathbb{R}), \mathfrak{d}^{\mathfrak{M}} \in \mathcal{C}_{0+}^{\infty\mathfrak{M}}(\mathbb{R})$, then $(\mathfrak{F}_1 + \mathfrak{F}_2) \blacksquare \mathfrak{d}^{\mathfrak{M}} = \mathfrak{F}_1 \blacksquare \mathfrak{d}^{\mathfrak{M}} + \mathfrak{F}_2 \blacksquare \mathfrak{d}^{\mathfrak{M}}$.

(3) For $\mathfrak{d}_1^{\mathfrak{M}}, \mathfrak{d}_2^{\mathfrak{M}} \in \mathcal{C}_{0+}^{\infty\mathfrak{M}}(\mathbb{R}), \mathfrak{d}_1^{\mathfrak{M}} \blacksquare \mathfrak{d}_2^{\mathfrak{M}} = \mathfrak{d}_2^{\mathfrak{M}} \blacksquare \mathfrak{d}_1^{\mathfrak{M}}$.

(4) For $\mathfrak{F} \in \mathfrak{S}_+(\mathbb{R}), \mathfrak{d}_1^{\mathfrak{M}}, \mathfrak{d}_2^{\mathfrak{M}} \in \mathcal{C}_{0+}^{\infty\mathfrak{M}}$ $(\mathbb{R})$then$\left(\mathfrak{F} \blacksquare \mathfrak{d}_1^{\mathfrak{M}}\right) \blacksquare \mathfrak{d}_2^{\mathfrak{M}} = \mathfrak{F} \blacksquare \left(\mathfrak{d}_1^{\mathfrak{M}} \blacksquare \mathfrak{d}_2^{\mathfrak{M}}\right)$.

Proof .Axioms of above lemma as follows:

(1) Let $\mathfrak{d}_1, \mathfrak{d}_2 \in \mathcal{C}_{0+}^{\infty}$ $(\mathbb{R})$ then $\mathfrak{d}_1 \blacksquare \mathfrak{d}_2 \in \mathcal{C}_{0+}^{\infty}(\mathbb{R})$.

$$\Rightarrow (\mathfrak{d}_1 \blacksquare \mathfrak{d}_2)^{\mathfrak{M}} \in \mathcal{C}_{0+}^{\infty\mathfrak{M}}(\mathbb{R})$$

By using Theorem (3.9) implies $\mathfrak{d}_1^{\mathfrak{M}} \blacksquare \mathfrak{d}_2^{\mathfrak{M}} \in \mathcal{C}_{0+}^{\infty\mathfrak{M}}$ $(\mathbb{R})$.

(2) Proof is straightforward.

(3) We have

$$\begin{aligned}\left(\mathfrak{d}_1^{\mathfrak{M}} \blacksquare \mathfrak{d}_2^{\mathfrak{M}}\right)(\xi) &= \frac{1}{\xi}\mathfrak{d}_1^{\mathfrak{M}}(\xi)\mathfrak{d}_2^{\mathfrak{M}}(\xi) \\ &= \frac{1}{\xi}\mathfrak{d}_2^{\mathfrak{M}}(\xi)\mathfrak{d}_1^{\mathfrak{M}}(\xi) \\ &= \left(\mathfrak{d}_2^{\mathfrak{M}} \blacksquare \mathfrak{d}_1^{\mathfrak{M}}\right)(\xi).\end{aligned}$$

$$\left(\mathfrak{d}_1^{\mathfrak{M}} \blacksquare \mathfrak{d}_2^{\mathfrak{M}}\right) = \left(\mathfrak{d}_2^{\mathfrak{M}} \blacksquare \mathfrak{d}_1^{\mathfrak{M}}\right)$$

(4)Let $\mathfrak{F} \in \mathfrak{S}_+(\mathbb{R})$and $\mathfrak{d}_1^{\mathfrak{M}}, \mathfrak{d}_2^{\mathfrak{M}} \in \mathcal{C}_{0+}^{\infty\mathfrak{M}}(\mathbb{R})$, then

$$\begin{aligned}\left(\left(\mathfrak{F} \blacksquare \mathfrak{d}_1^{\mathfrak{M}}\right) \blacksquare \mathfrak{d}_2^{\mathfrak{M}}\right)(\xi) &= \frac{1}{\xi}\left(\mathfrak{F} \blacksquare \mathfrak{d}_1^{\mathfrak{M}}\right)(\xi)\mathfrak{d}_2^{\mathfrak{M}}(\xi) \\ &= \frac{1}{\xi}\left\{\frac{1}{\xi}\mathfrak{F}(\xi)\mathfrak{d}_1^{\mathfrak{M}}(\xi)\right\}\mathfrak{d}_2^{\mathfrak{M}}(\xi) \\ &= \frac{1}{\xi}\mathfrak{F}(\xi)\frac{1}{\xi}\mathfrak{d}_1^{\mathfrak{M}}(\xi)\,\mathfrak{d}_2^{\mathfrak{M}}(\xi)\end{aligned}$$

$$= \frac{1}{\xi}\mathfrak{F}(\xi)\left(\mathfrak{d}_1^{\mathfrak{M}} \blacksquare \mathfrak{d}_2^{\mathfrak{M}}\right)(\xi)$$
$$= \left(\mathfrak{F} \blacksquare \left(\mathfrak{d}_1^{\mathfrak{M}} \blacksquare \mathfrak{d}_2^{\mathfrak{M}}\right)\right)(\xi),$$
$$\left(\mathfrak{F} \blacksquare \mathfrak{d}_1^{\mathfrak{M}}\right) \blacksquare \mathfrak{d}_2^{\mathfrak{M}} = \mathfrak{F} \blacksquare \left(\mathfrak{d}_1^{\mathfrak{M}} \blacksquare \mathfrak{d}_2^{\mathfrak{M}}\right)$$ ☖

Denote by$\Delta_+^{\mathfrak{M}}$ where$\Delta_+^{\mathfrak{M}}$ is the collection of all Mahgoub transform's delta sequence inΔ_+. i.e.,

$$\Delta_+^{\mathfrak{M}} = \left\{\left(\eta_n^{\mathfrak{M}}\right) : (\eta_n) \in \Delta_+, \forall\, n \in N\right\}. \tag{4.3}$$

Lemma 4.3 Let $\mathfrak{F}_1, \mathfrak{F}_2 \in \mathfrak{S}_+(\mathbb{R}), \left(\eta_n^{\mathfrak{M}}\right) \in \Delta_+^{\mathfrak{M}}$ such that $\mathfrak{F}_1 \blacksquare \eta_n^{\mathfrak{M}} = \mathfrak{F}_2 \blacksquare \eta_n^{\mathfrak{M}}$, $\forall n$, then $\mathfrak{F}_1 = \mathfrak{F}_2$ in $\mathfrak{S}_+(\mathbb{R})$.

Proof. Let$\mathfrak{F}_1, \mathfrak{F}_2 \in \mathfrak{S}_+(\mathbb{R}), \left(\eta_n^{\mathfrak{M}}\right) \in \Delta_+^{\mathfrak{M}}$. Since $\mathfrak{F}_1 \blacksquare \eta_n^{\mathfrak{M}} = \mathfrak{F}_2 \blacksquare \eta_n^{\mathfrak{M}}$, using Eq.(4.2)

$\Rightarrow \frac{1}{\xi}\mathfrak{F}_1(\xi)\eta_n^{\mathfrak{M}}(\xi) = \frac{1}{\xi}\mathfrak{F}_2(\xi)\eta_n^{\mathfrak{M}}(\xi)$

Hence $\mathfrak{F}_1(\xi) = \mathfrak{F}_2(\xi)$ for allξ. ☖

Lemma 4.4 For all $(\tau_n), (\eta_n) \in \Delta_+$, $\left(\eta_n^{\mathfrak{M}} \blacksquare {\tau_n}^{\mathfrak{M}}\right) \in \Delta_+^{\mathfrak{M}}$.

Proof. Since$(\tau_n), (\eta_n) \in \Delta_+$,$\eta_n \star \tau_n \in \Delta_+$, $\forall n$ hence from Theorem 3.9, we get

$\mathfrak{M}(\eta_n \star \tau_n)(\xi) = \frac{1}{\xi}\eta_n^{\mathfrak{M}}(\xi){\tau_n}^{\mathfrak{M}}(\xi) = \eta_n^{\mathfrak{M}} \blacksquare {\tau_n}^{\mathfrak{M}} \in \Delta_+^{\mathfrak{M}}$, for each n. ☖

Lemma 4.5 Let $\lim_{n\to\infty} \mathfrak{F}_n = \mathfrak{F}$in $\mathfrak{S}_+(\mathbb{R})$, $\mathfrak{d}^{\mathfrak{M}} \in C_{0+}^{\infty\mathfrak{M}}(\mathbb{R})$ then $\mathfrak{F}_n \blacksquare \mathfrak{d}^{\mathfrak{M}} \to \mathfrak{F} \blacksquare \mathfrak{d}^{\mathfrak{M}}$in $\mathfrak{S}_+(\mathbb{R})$.

Proof.we know that $\mathfrak{d}^{\mathfrak{M}}$ is bounded in $C_{0+}^{\infty\mathfrak{M}}(\mathbb{R})$ we have

$$(\mathfrak{F}_n \blacksquare \mathfrak{d}^{\mathfrak{M}})(\xi) \to \frac{1}{\xi}\mathfrak{F}(\xi)\mathfrak{d}^{\mathfrak{M}}(\xi)$$

$\to \left(\mathfrak{F} \blacksquare \mathfrak{d}^{\mathfrak{M}}\right)(\xi)$.

Hence $\mathfrak{F}_n \blacksquare \mathfrak{d}^{\mathfrak{M}} \to \mathfrak{F} \blacksquare \mathfrak{d}^{\mathfrak{M}}$. ☖

Lemma 4.6 Let $\lim_{n\to\infty} \mathfrak{F}_n = \mathfrak{F}$ in $\mathfrak{S}_+(\mathbb{R})$, $\left(\eta_n^{\mathfrak{M}}\right) \in \Delta_+^{\mathfrak{M}}$ then $\mathfrak{F}_n \blacksquare \eta_n^{\mathfrak{M}} \to \mathfrak{F}$ in $\mathfrak{S}_+(\mathbb{R})$.

Proof.Let $(\eta_n) \in \Delta_+$, $\eta_n^{\mathfrak{M}}(\xi) \to \xi$ is uniformly on compact subsets of $\mathbb{R}_+$. Hence

$$\left|\xi^k D_\xi^m\left(\mathfrak{F}_n \blacksquare \eta_n^{\mathfrak{M}} - \mathfrak{F}\right)(\xi)\right| = \left|\xi^k D_\xi^m\left(\frac{1}{\xi}\mathfrak{F}_n(\xi)\eta_n^{\mathfrak{M}}(\xi)\right) - \mathfrak{F}(\xi)\right|$$

$\to \left|\xi^k D_\xi^m(\mathfrak{F}_n - \mathfrak{F})(\xi)\right|$ as $n \to \infty$

Thus $\left|\xi^k D_\xi^m\left(\mathfrak{F}_n \blacksquare \eta_n^{\mathfrak{M}} - \mathfrak{F}\right)(\xi)\right| \to 0$ as $n \to \infty$.

This yield $\mathfrak{F}_n \blacksquare \eta_n^{\mathfrak{M}} \to \mathfrak{F}$ in $\mathfrak{S}_+(\mathbb{R})$. ☖

Lemma 4.7 Define $\mathfrak{F} \to \left[\frac{\mathfrak{F}\blacksquare\eta_n^{\mathfrak{M}}}{\eta_n^{\mathfrak{M}}}\right]$ is a continuous mapping which is embedding from $\mathfrak{S}_+(\mathbb{R})$ into $\mathbb{B}(\mathfrak{X}^{\mathfrak{M}})$. (4.4)

Proof. Let $\frac{\mathfrak{F}\blacksquare\eta_n^{\mathfrak{M}}}{\eta_n^{\mathfrak{M}}}$ is a quotient of sequences where $\mathfrak{F} \in \mathfrak{S}_+(\mathbb{R}), \eta_n^{\mathfrak{M}} \in \Delta_+^{\mathfrak{M}}$. We have $(\mathfrak{F}\blacksquare\eta_n^{\mathfrak{M}})\blacksquare\eta_m^{\mathfrak{M}} = \mathfrak{F}\blacksquare(\eta_m^{\mathfrak{M}}\blacksquare\eta_n^{\mathfrak{M}})$.We show that the map (4.3) is one - to - one.

Let $\left[\frac{\mathfrak{F}_1\blacksquare\eta_n^{\mathfrak{M}}}{\eta_n^{\mathfrak{M}}}\right] = \left[\frac{\mathfrak{F}_2\blacksquare\tau_n^{\mathfrak{M}}}{\tau_n^{\mathfrak{M}}}\right]$, then $(\mathfrak{F}_1\blacksquare\eta_n^{\mathfrak{M}})\blacksquare\tau_m^{\mathfrak{M}} = (\mathfrak{F}_2\blacksquare\tau_m^{\mathfrak{M}})\blacksquare\eta_n^{\mathfrak{M}}, m, n \in N$. Now using of Lemma 4.2& 4.3, we get $\mathfrak{F}_1 = \mathfrak{F}_2$.

To establish the continuity of Eq.(4.4), let $\mathfrak{F}_n \to 0$ as $n \to \infty$ in $\mathfrak{S}_+(\mathbb{R})$. Then $\mathfrak{F}_n\blacksquare\eta_n^{\mathfrak{M}} \to 0$ as $n \to \infty$ by Lemma 4.6, and hence

$\left[\frac{\mathfrak{F}\blacksquare\eta_n^{\mathfrak{M}}}{\eta_n^{\mathfrak{M}}}\right] \to 0$, as $n \to \infty$ in $\mathbb{B}(\mathfrak{X}^{\mathfrak{M}})$. ☖

5. The Mahgoub transform of Bohemians

Let $\mathfrak{H} = \left[\frac{\mathfrak{F}_n}{\eta_n}\right] \in \mathbb{B}(\mathfrak{X})$, then we delineate the Mahgoub transform of $\mathfrak{H}$ by the relation

$$\mathfrak{H}_1^{\mathfrak{M}} = \left[\frac{\mathfrak{F}_n^{\mathfrak{M}}}{\eta_n^{\mathfrak{M}}}\right] \text{in } \mathbb{B}(\mathfrak{X}^{\mathfrak{M}}). \qquad (5.1)$$

Theorem 5.1 $\mathfrak{H}_1^{\mathfrak{M}}: \mathbb{B}(\mathfrak{X}) \to \mathbb{B}(\mathfrak{X}^{\mathfrak{M}})$ is well defined.

Proof. Let $\mathfrak{H}_1 = \mathfrak{H}_2 \in \mathbb{B}(\mathfrak{X})$, where $\mathfrak{H}_1 = \left[\frac{\mathfrak{F}_n}{\eta_n}\right], \mathfrak{H}_2 = \left[\frac{g_n}{\tau_n}\right]$Then the concept of quotients yields $\mathfrak{F}_n \star \tau_m = g_m \star \eta_n$. Applying Theorem 3.9, we get $\frac{1}{\xi}\mathfrak{F}_n^{\mathfrak{M}}(\xi)\tau_m^{\mathfrak{M}}(\xi) = \frac{1}{\xi}g_m^{\mathfrak{M}}(\xi)\eta_n^{\mathfrak{M}}(\xi)$,

$i.e. \mathfrak{F}_n^{\mathfrak{M}}\blacksquare\tau_m^{\mathfrak{M}} = g_m^{\mathfrak{M}}\blacksquare\eta_n^{\mathfrak{M}} \Rightarrow \frac{f_n^{\mathfrak{M}}}{\eta_n^{\mathfrak{M}}} \sim \frac{g_n^{\mathfrak{M}}}{\tau_n^{\mathfrak{M}}}$. Thus $\mathfrak{H}_1^{\mathfrak{M}} = \mathfrak{H}_2^{\mathfrak{M}}$. ☖

Theorem 5.2 $\mathfrak{H}^{\mathfrak{M}}: \mathbb{B}(\mathfrak{X}) \to \mathbb{B}(\mathfrak{X}^{\mathfrak{M}})$ is continuous regards to δ-convergence.

Proof. Let $\mathfrak{H}_n \to 0$ in $\mathbb{B}(\mathfrak{X})$as $n \to \infty$. using [4] we get, $\mathfrak{H}_n = \left[\frac{\mathfrak{F}_{n,k}}{\eta_k}\right]$ and $\mathfrak{F}_{n,k} \to 0$ in $\mathfrak{S}_+(\mathbb{R})$ as $n \to \infty$ in $\mathfrak{S}_+(\mathbb{R})$. Now we apply the Mahgoub transform to both sides revenue $\mathfrak{F}_{n,k}^{\mathfrak{M}} \to 0$ as $n \to \infty$. Hence

$\mathfrak{H}_n^{\mathfrak{M}} = \left[\frac{\mathfrak{F}_{n,k}^{\mathfrak{M}}}{\eta_k^{\mathfrak{M}}}\right] \to 0$ as $n \to \infty$ in $\mathbb{B}(\mathfrak{X}^{\mathfrak{M}})$. ☖

Theorem 5.3 $\mathfrak{H}^{\mathfrak{M}} : \mathbb{B}(\mathfrak{X}) \rightarrow \mathbb{B}(\mathfrak{X}^{\mathfrak{M}})$is one-to-one mapping.

Proof**.** Let $\mathfrak{H}_1^{\mathfrak{M}} = \left[\frac{\mathfrak{F}_n^{\mathfrak{M}}}{\eta_n^{\mathfrak{M}}}\right] = \left[\frac{g_n^{\mathfrak{M}}}{\tau_n^{\mathfrak{M}}}\right] = \mathfrak{H}_2^{\mathfrak{M}}$,then $\mathfrak{F}_n^{\mathfrak{M}} \blacksquare \tau_m^{\mathfrak{M}} = g_m^{\mathfrak{M}} \blacksquare \eta_n^{\mathfrak{M}}$.

Hence

$$(\mathfrak{F}_n \star \tau_m)^{\mathfrak{M}} = (g_m \star \eta_n)^{\mathfrak{M}}.$$

Since the Mahgoub transform is one to one, we get $\mathfrak{F}_n \star \tau_m = g_m \star \eta_n$.Thus

$$\frac{\mathfrak{F}_n}{\eta_n} \sim \frac{g_n}{\tau_n}.$$

Hence $\left[\frac{\mathfrak{F}_n}{\eta_n}\right] = \mathfrak{H}_1 = \left[\frac{g_n}{\tau_n}\right] = \mathfrak{H}_2$. ☖

Theorem 5.4 Let $\mathfrak{H}_1, \mathfrak{H}_2 \in \mathbb{B}(\mathfrak{X})$, then

(1) $(\mathfrak{H}_1 + \mathfrak{H}_2)^{\mathfrak{M}} = \mathfrak{H}_1^{\mathfrak{M}} + \mathfrak{H}_2^{\mathfrak{M}}$;

(2) $(\mathscr{k}\mathfrak{H})^{\mathfrak{M}} = \mathscr{k}\, \mathfrak{H}^{\mathfrak{M}}, \mathscr{k} \in \mathbb{C}$.

Theorem5.5 $\mathfrak{H}^{\mathfrak{M}} : \mathbb{B}(\mathfrak{X}) \rightarrow \mathbb{B}(\mathfrak{X}^{\mathfrak{M}})$ is continuous regards to Δ_+ -convergence.

Proof**.** Let $\mathfrak{H}_n \xrightarrow{\Delta} \mathfrak{H}$ in $\mathbb{B}(\mathfrak{X})$ as $n \rightarrow \infty$ Then $\exists \mathfrak{F}_n \rightarrow 0\ \mathfrak{S}_+(\mathbb{R})$ and $(\eta_n) \in \Delta_+$ such that $(\mathfrak{H}_n - \mathfrak{H}) * \eta_n = \left[\frac{\mathfrak{F}_n * \eta_{\mathscr{k}}}{\eta_{\mathscr{k}}}\right]$ and $\mathfrak{F}_n \rightarrow 0$ as $n \rightarrow \infty$.Applying in Eq.(5.1) , we get

$$\mathfrak{M}\big((\mathfrak{H}_n - \mathfrak{H}) * \eta_n\big) = \left[\frac{\mathfrak{M}(\mathfrak{F}_n * \eta_{\mathscr{k}})}{\eta_{\mathscr{k}}^{\mathfrak{M}}}\right].$$

Hence we have $\mathfrak{M}\big((\mathfrak{H}_n - \mathfrak{H}) * \eta_n\big) = \left[\frac{\mathfrak{F}_n^{\mathfrak{M}} \eta_{\mathscr{k}}^{\mathfrak{M}}}{\xi \eta_{\mathscr{k}}^{\mathfrak{M}}}\right] \rightarrow 0$ as $n \rightarrow \infty$ in $\mathbb{B}\big(\mathfrak{X}^{\mathfrak{M}}\big)$. therefore

$\mathfrak{M}\big((\mathfrak{H}_n - \mathfrak{H}) * \eta_n\big) = \frac{1}{\xi}\big(\mathfrak{H}_n^{\mathfrak{M}} - \mathfrak{H}^{\mathfrak{M}}\big)\eta_n^{\mathfrak{M}} \rightarrow 0$ as $n \rightarrow \infty$. ☖

Theorem5.6 Let $\mathfrak{H}^{\mathfrak{M}} : \mathbb{B}(\mathfrak{X}) \rightarrow \mathbb{B}(\mathfrak{X}^{\mathfrak{M}})$ is onto.

Proof. Let $\left[\frac{\mathfrak{F}_n^{\mathfrak{M}}}{\eta_n^{\mathfrak{M}}}\right] \in \mathbb{B}\big(\mathfrak{X}^{\mathfrak{M}}\big)$ be arbitrary then $\mathfrak{F}_n^{\mathfrak{M}} \blacksquare \eta_m^{\mathfrak{M}} = \mathfrak{F}_m^{\mathfrak{M}} \blacksquare \eta_n^{\mathfrak{M}}$ for each $m, n \in N$.Then

$\mathfrak{F}_n \star \eta_m = \mathfrak{F}_m \star \eta_n$.That is, $\frac{\mathfrak{F}_n}{\eta_n}$is the corresponding quotient of sequences of $\frac{\mathfrak{F}_n^{\mathfrak{M}}}{\eta_n^{\mathfrak{M}}}$. Thus $\frac{\mathfrak{F}_n}{\eta_n} \in \mathbb{B}(\mathfrak{X})$ is such that $\mathfrak{M}\left[\frac{\mathfrak{F}_n}{\eta_n}\right] = \left[\frac{\mathfrak{F}_n^{\mathfrak{M}}}{\eta_n^{\mathfrak{M}}}\right]$ in $\mathbb{B}\big(\mathfrak{X}^{\mathfrak{M}}\big)$.

Let $\mathfrak{H}^{\mathfrak{M}} = \left[\frac{\mathfrak{F}_n^{\mathfrak{M}}}{\eta_n^{\mathfrak{M}}}\right] \in \mathbb{B}(\mathfrak{X}^{\mathfrak{M}})$, then we express the inverse of Mahgoub transform of $\mathfrak{H}^{\mathfrak{M}}$ given by

$\mathfrak{H}^{\mathfrak{M}^{-1}} = \left[\frac{\mathfrak{F}_n}{\eta_n}\right]$ in the space $\mathbb{B}(\mathfrak{X})$. ☖

Theorem5.7 Let $\left[\frac{\mathfrak{F}_n^{\mathfrak{M}}}{\eta_n^{\mathfrak{M}}}\right] \in \mathbb{B}(\mathfrak{X}^{\mathfrak{M}})$ and $\mathfrak{d}^{\mathfrak{M}} \in C_{0+}^{\infty\mathfrak{M}}(\mathbb{R}), \mathfrak{d} \in C_{0+}^{\infty}(\mathbb{R})$.

$\mathfrak{H}\left(\left[\frac{\mathfrak{F}_n}{\eta_n}\right] \star \mathfrak{d}\right) = \left[\frac{\mathfrak{F}_n^{\mathfrak{M}}}{\eta_n^{\mathfrak{M}}}\right] \blacksquare \mathfrak{d}^{\mathfrak{M}}and\mathfrak{H}^{\mathfrak{M}^{-1}}\left(\left[\frac{\mathfrak{F}_n^{\mathfrak{M}}}{\eta_n^{\mathfrak{M}}}\right] \blacksquare \mathfrak{d}^{\mathfrak{M}}\right) = \left[\frac{\mathfrak{F}_n}{\eta_n}\right] \star \mathfrak{d}$.

We can easily proof from the definitions.

Conflict of Interests

The authors declare that there is no conflict of interests regarding the publication of this paper.

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