# Error Locating Codes Dealing with Repeated Low-Density Burst Errors

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**Abstract.** This paper presents a study of linear codes which are capable to detect and locate errors which are repeated low-density bursts of length $b$(fixed) with weight $w$ or less. An illustration for such a kind of code has also been provided.

**Keywords:** Error locating codes, burst errors, burst errors of length of $b$(fixed), repeated low-density burst errors of length $b$(fixed).

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# 1 Introduction

Burst errors are the type of errors that occur quite frequently in several communication channels. Codes developed to detect and correct such errors have been studied extensively by many authors. Abramson [1959] developed codes which dealt with the correction of single and double adjacent errors, which was extended by Fire [1959] as a more general concept called 'burst errors'. A burst of length $b$ is defined as follows:

**Definition 1.** A burst of length $b$ is a vector whose only non-zero components are among some $b$ consecutive components, the first and the last of which is non-zero.

The nature of burst errors differs from channel to channel depending upon the kind of channel. Chien and Tang [1965] proposed a modification in the definition of a burst and they defined a burst of length $b$, which shall be called as CT-burst of length $b$, as follows:

**Definition 2.** A CT-burst of length $b$ is a vector whose only non-zero components are confined to some $b$ consecutive positions, the first of which is non-zero.

Channels due to Alexander, Gryb and Nast [1960] fall in this category. This definition was further modified by Dass [1980] as follows:

**Definition 3.** A burst of length $b$(fixed) is an $n$-tuple whose only non-zero components are confined to $b$ consecutive positions, the first of which is non-zero and the number of its starting positions is among the first $n-b+1$ components.

This definition is useful for channels not producing errors near the end of a code word. In very busy communication channels errors repeat themselves. So is a situation when errors occur in the form of bursts. Dass, Garg and Zannetti [2008] studied this kind of repeated burst errors. They termed such errors as $m$-repeated burst errors of length $b$(fixed) which has been defined as follows:

**Definition 4.** An $m$-repeated bursts of length $b$(fixed) is an $n$-tuple whose only non-zero components are confined to $m$ distinct sets of $b$ consecutive digits, the first component of each set is non-zero and the number of its starting positions is among the first $n - mb + 1$ components.

In particular a 2-repeated bursts of length b(fixed) has been defined by Dass and Garg [2009(a)] as follows:

**Definition 5.** A 2-repeated bursts of length $b$(fixed) is an $n$-tuple whose only non-zero components are confined to 2 distinct sets of $b$ consecutive digits, the first component of each set is non-zero and the number of its starting positions is among the first $n - 2b + 1$ components.

During the process of transmission some disturbances cause occurrence of burst errors in such a way that over a given length, some digits are received correctly while others get corrupted i.e. not all the digits inside a burst are in error. Such bursts are termed as low-density bursts [Wyner (1963)].

A low-density burst of length $b$(fixed) with weight $w$ or less has been defined as follows:

**Definition 6.** A low-density burst of length $b$(fixed) with weight $w$ or less is an $n$-tuple whose only non-zero components are confined to some $b$ consecutive positions, the first of which is non-zero with at most $w$ ($w \leq b$) non-zero components within such $b$ consecutive digits and the number of starting positions of the burst is among the first $n-b+1$ components.

Dass and Garg [2009(b)] studied codes which are capable to detect and/or correct $m$-repeated low-density bursts of length $b$(fixed) with weight $w$ or less. They defined such codes as follows:

**Definition 7.** An $m$-repeated low-density burst of length $b$(fixed) with weight $w$ or less is an $n$-tuple whose only non-zero components are confined to $m$ distinct sets of $b$ consecutive positions, the first component of each set is non-zero where each set can have at most $w$ non-zero components $(w \leq b)$, and the number of its starting positions in an $n$-tuple is among the first $n-mb+1$ positions.

In particular, a 2-repeated low-density burst of length $b$(fixed) with weight $w$ or less has been defined as follows:

**Definition 8.** A 2-repeated low-density burst of length $b$(fixed) with weight $w$ or less is an $n$-tuple whose only non-zero components are confined to two distinct sets of $b$ consecutive positions, the first component of each set is non-zero where each set can have at most $w$ non-zero components $(w \leq b)$, and the number of its starting positions in an $n$-tuple is among the first $n-2b+1$ positions.

As an illustration, (21010000102000) is a 2-repeated low-density burst of length up to 6(fixed) with weight 3 or less over GF(3) whereas (001000011110) is a 2-repeated low-density burst of length at most 5(fixed) with weight 4 or less over GF(2).

In this paper we have presented a study of codes dealing with the location of such kind of errors occurring within a sub-block. The concept of error-locating codes, lying midway between error detection and error correction, was introduced by Wolf and Elspas [1963]. In this technique the block of received digits is to be regarded as subdivided into mutually exclusive sub-blocks and while decoding it is possible to detect the error and in addition the receiver is able to identify which particular sub-block contains error. Such codes are referred to as Error-Locating codes (EL-codes). Wolf and Elspas [1963] studied binary codes which are capable of detecting and locating a single sub-block containing random errors. A study of codes locating burst errors of length $b$(fixed) has been made by Dass and Kishanchand [1986]. Dass and Arora [2010] obtained bounds for codes capable of locating repeated burst errors of length $b$(fixed) occurring within a sub-block.

In this paper we have obtained bounds on the number of check digits required to locate 2-repeated low-density bursts of length $b$(fixed), and $m$-repeated low-density bursts of length $b$(fixed). An illustration of such a code has also been given. Development of such codes will economize in the number of parity-check digits required in comparison to the usual low-density burst error locating codes while considering such repeated bursts as single bursts.

The paper has been organized as follows. In section 2 the necessary condition for the detection and location of 2-repeated low-density burst of length $b$(fixed) with weight $w$ or less has been derived. This is followed by a sufficient condition for the existence of such a code. An illustration of 2-repeated low-density burst of length $b$(fixed) with weight $w$ or less over GF(2) has also been given. In section 3 a necessary condition for the detection and location of $m$-repeated low-density burst of length $b$(fixed) with weight $w$ or less has been given followed by a sufficient condition for the existence of such a code.

In what follows we shall consider a linear code to be a subspace of $n$-tuples over GF$(q)$. The block of $n$ digits, consisting of $r$ check digits and $k = n - r$ information digits, is considered to be divided into $s$ mutually exclusive sub-blocks. Each sub-block contains $t = n/s$ digits.

## 2 2-Repeated Low-density Burst Error Locating Codes

In this section, we consider $(n, k)$ linear codes over GF$(q)$ that are capable of detecting and locating all 2-repeated low-density burst of length $b$(fixed) with weight $w$ or less within a single sub-block.

It may be noted that an EL-code capable of detecting and locating a single sub-block containing an error which is in the form of a 2-repeated low-density bursts of length $b$(fixed) with weight $w$ or less must satisfy the following conditions:

(a) The syndrome resulting from the occurrence of a 2-repeated low-density burst of length $b$(fixed) with weight $w$ or less within any one sub-block must be distinct from the all zero syndrome.

(b) The syndrome resulting from the occurrence of any 2-repeated low-density burst of length $b$(fixed) with weight $w$ or less within a single sub-block must be distinct from the syndrome resulting likewise from any 2-repeated low-density burst of length $b$(fixed) with weight $w$ or less *within* any other sub-block.

In this section we shall derive two results. The first result derives a lower bound on the number of check digits required for the existence of a linear code over GF($q$) capable of detecting and locating a single sub-block containing errors that are 2-repeated low-density burst of length $b$(fixed) with weight $w$ or less. In the second result, an upper bound on the number of check digits which ensures the existence of such a code has been derived.

As the code is divided into several blocks of length $t$ each and we wish to detect a 2-repeated low-density burst of length $b$(fixed) with weight $w$ or less, we may come across with a situation when the difference between $2b$ and $t$ ($b+w$ and $t$) becomes narrow. We note that if $t-b+1 < b+w$ and if we consider any two 2-repeated low-density bursts $x_1$ and $x_2$ of length $b$(fixed) with weight $w$ or less such that their non-zero components are confined to first $t-b+1$ positions with $w$ components confining to some fixed $w$ positions out of first $b$ consecutive positions then their difference $x_1$ - $x_2$ is again a 2-repeated low-density burst of length $b$(fixed) with weight $w$ or less. However if we do not restrict ourselves to first $t-b+1$ positions then we may not get a 2-repeated burst of length $b$(fixed) with weight $w$ or less. This may be better understood with the help of the following examples:

**Example 1.** Let $t = 9,\ b = 4,\ w = 3$ and $q = 2$. So that $t - b + 1 = 6 < b + w(= 7)$.

Let $x_1 = (101101001)$ and $x_2 = (100101011)$.

Then $x_1$ and $x_2$ are 2-repeated low-density burst of length 4(fixed) with weight 3 or less whereas $x_1 - x_2 = (001000010)$ is not a 2-repeated burst of length 4(fixed).

**Example 2.** Let $t = 11,\ b = 5,\ w = 3$ and $q = 2$.

Let $x_1 = (10101010010)$ and $x_2 = (10101010001)$

Then $x_1$ and $x_2$ are 2-repeated low-density burst of length 5(fixed) with weight 3 or less whereas $x_1 - x_2 = (00000000011)$ which is not even a 2-repeated burst of length 4(fixed) what to talk of its weight.

So, accordingly we discuss the following cases:

*Case* 1: When $t - b + 1 \geq 2b$.

Let $X$ be the collection of all those vectors in which all the non-zero components are confined to some fixed $w$ positions out of two sets of $b$ consecutive positions each i.e. $l$-th to $(l + b)$-th position and $j$-th to $(j + b)$-th position where $j > l + b$.

We observe that the syndromes of all the elements of $X$ should be different; else for any $x_1, x_2$ belonging to $X$ having the same syndrome would imply that the syndrome of $x_1 - x_2$ which is also an element of $X$ and hence a 2-repeated low density burst of length $b$(fixed) with weight $w$ or less within the same sub-block becomes zero; in violation of condition (a). Also, since the error locates a single sub-block containing errors that are 2-repeated low-density bursts of length $b$(fixed) of weight $w$ or less, the syndromes produced by similar vectors in different sub-blocks must be distinct by condition $(b)$.

Thus the syndromes of vectors which are 2-repeated low-density burst of length $b$(fixed) with weight $w$ or less in fixed positions, whether in the same sub-block or in different sub-blocks, must be distinct. (It may be noted that the choice of different fixed components in different sub-blocks will also yield the same result).

As there are $(q^{2w}-1)$ distinct non-zero syndromes corresponding to the vectors in any one sub-block and there are $s$ sub-blocks in all, so we must have atleast $(1+s(q^{2w}-1))$ distinct syndromes counting the all zero syndrome.

As maximum number of distinct syndromes available using $r$ check bits is $q^r$, so there are $q^r$ distinct syndromes in all, therefore we must have

$$q^r \geq \{1+s(q^{2w}-1)\} \tag{1}$$

where $t-b+1 \geq 2b$.

*Case* 2: When $b+w \leq t-b+1 < 2b$.

Let $X$ be the collection of all those vectors in which all the non-zero components are confined to some $w$ fixed positions out of first $b$ components i.e first and $b$-th position and another set of $w$ fixed positions out of $(b+1)$-th to $(t-b+1)$-th positions.

As discussed in case 1 the syndromes of all the elements of $X$ is different.

In this case also, there are $(q^{2w}-1)$ distinct non-zero syndromes corresponding to the vectors in any one sub-block and there are $s$ sub-blocks in all, so we must have atleast $(1 + s(q^{2w} - 1))$ distinct syndromes counting the all zero syndrome.

So, in this case also, we must have

$$q^r \geq \{1 + s(q^{2w} - 1)\} \tag{2}$$

where $b + w \leq t - b + 1 < 2b$.

*Case* 3: When $t - b + 1 < b + w$.

In this case consider $X$ to be collection of all those vectors in which all the non-zero components are confined to some $w$ fixed positions out of first $b$ positions and $t - 2b + 1$ components from $(b+1)$-th to $(t - b + 1)$-th positions. In this case there are $(q^{w+(t-2b+1)} - 1)$ distinct non-zero syndromes corresponding to the vectors in any one sub-block. As and there are s sub-blocks in all, so we must have atleast $(1 + s(q^{w+(t-2b+1)} - 1))$ distinct syndromes counting the all zero syndrome.

Therefore in this case, we must have

$$q^r \geq \{1 + s(q^{w+(t-2b+1)} - 1)\} \tag{3}$$

where $t - b + 1 < b + w$.

From (1), (2), and (3) we have

$$r \geq \begin{cases} \log_q\{1 + s(q^{2w} - 1)\} & \text{where } \ t - b + 1 \geq 2b \\ & \text{and } \ b + w \leq t - b + 1 < 2b \\ \log_q\{1 + s(q^{w+(t-2b+1)} - 1)\} & \text{where } \ t - b + 1 < b + w. \end{cases}$$

Thus we have proved:

**Theorem 1.** *The number of parity check digits $r$ in an $(n,k)$ linear code subdivided into $s$ sub-blocks of length $t$ each, that locates a single corrupted* *sub-block containing errors that are 2-repeated low density burst of length $b$ (fixed) with weight $w$ or less is at least*

$$\begin{cases} \log_q\{1+s(q^{2w}-1)\} & \text{where } t-b+1 \geq 2b \\ & \text{and } b+w \leq t-b+1 < 2b \, . \\ \log_q\{1+s(q^{w+(t-2b+1)}-1)\} & \text{where } t-b+1 < b+w \end{cases}$$

**Remark 1.** For $w = b$, the weight consideration over the burst becomes redundant and the result coincides with Theorem 1[Dass and Arora [2010]], when the bursts considered are 2-repeated bursts of length $b$(fixed).

In the following result we derive another bound on the number of check digits required for the existence of such a code. The proof is based on the technique used to establish Varshomov-Gilbert Sacks bound by constructing a parity check matrix for such a code [refer Sacks[1958], also Theorem 4.7 Peterson and Weldon[1972]]. This technique not only ensures the existence of such a code but also gives a method for the construction of such a code.

**Theorem 2.** *An $(n, k)$ linear EL-code over* GF$(q)$ *capable of detecting a 2-repeated low density burst of length $b$ (fixed) with weight $w$ or less ($w \leq b$) within a single sub-block and of locating that sub-block can always be constructed provided that*

$$q^{n-k} > [1+(q-1)]^{(b-1,w-1)}\{1+(q-1)(t-2b+1)[1+(q-1)]^{(b-1,w-1)}\} \cdot \left\{1+(s-1)\sum_{i=1}^{2}\binom{t-ib+i}{i}(q-1)^i\{[1+(q-1)]^{(b-1,w-1)}\}^i\right\} \quad (4)$$

*where $[1+x]^{(m,r)}$ denotes the incomplete binomial expansion of $(1+x)^m$ up to the term $x^r$ in ascending power of $x$, viz.*

$$[1+x]^{(m,r)} = 1+\binom{m}{1}x+\binom{m}{2}x^2+\ldots+\binom{m}{r}x^r.$$

*Proof.* The existence of such a code will be shown by constructing an appropriate $(n-k \times n)$ parity check matrix $H$ by a synthesis procedure. For that we first construct a matrix $H_1$ from which the requisite parity check matrix $H$ shall be obtained by reversing the order of the columns of each sub-block.

After adding $(s-1)t$ columns appropriately corresponding to the first $(s-1)$ sub-blocks, suppose that we have added the first $j-1$ columns $h_1, h_2, \ldots, h_{j-1}$ of the $s$-th sub-block also, out of which the first $b-1$ columns $h_1, h_2, \ldots, h_{b-1}$ may be chosen arbitrarily (non-zero). We now lay down the condition to add the $j$-th column $h_j$ to $H_1$ as follows:

According to condition (a), for the detection of 2-repeated low-density burst of length $b$(fixed) with weight $w$ or less in the $s$-th sub-block $h_j$ should not be a linear combination of any $w-1$ or fewer columns among the immediately preceding $b-1$ columns $h_{j-b+1}, h_{j-b+2}, \ldots h_{j-1}$ together with any $w$ or fewer columns from amongst some $b$ consecutive columns from the first $j-b$ columns of the $s$-th sub-block.

i.e.

$$h_j \neq (\alpha_{j_1} h_{j_1} + \alpha_{j_2} h_{j_2} + \ldots + \alpha_{j_{w-1}} h_{j_{w-1}}) + (\beta_{l_1} h_{l_1} + \beta_{l_2} h_{l_2} + \ldots + \beta_{l_w} h_{l_w}) \quad (5)$$

where $h_{j_1}, h_{j_2}, \ldots, h_{j_{w-1}}$ are any $w-1$ columns among $h_{j-b+1}, h_{j-b+2}, \ldots h_{j-1}$ and $h_l$'s are any $w$ columns from a set of $b$ consecutive columns among the first $j-b$ columns of the $s$-th sub-block such that either all the coefficients $\beta_{l_i}$'s are zero or if the $p$-th coefficient $\beta_{l_p}$ is the last non-zero coefficients then $b \leq p \leq j-b$;

$$\alpha_{j_i}\text{'s}, \beta_{l_i}\text{'s} \in \mathrm{GF}(q).$$

The number of ways in which the coefficients $\alpha_i$'s can be selected is $[1+(q-1)]^{(b-1,w-1)}$. To enumerate the coefficients $\beta_i$'s is equivalent to enumerate the number of bursts of length $b$(fixed) with weight $w$ or less in a vector of length $j-b$.

This number including the vector of all zeros [refer Theorem 1, Dass [1983]] is

$$1+(j-2b+1)(q-1)[1+(q-1)]^{(b-1,w-1)}$$

So, the number of linear combinations on the right hand side of (5) is

$$[1+(q-1)]^{(b-1,w-1)}[1+(j-2b+1)(q-1)[1+(q-1)]^{(b-1,w-1)}] \quad (6)$$

According to condition $(b)$, for the location of 2-repeated low-density bursts of length $b$(fixed) with weight $w$ or less, $h_j$ should not be a linear combination of any $w-1$ or fewer columns among the immediately preceding the $b-1$ columns and any $w$ columns from a set of $b$ consecutive columns from the remaining $j-b$ columns of the $s$-th sub-block along with any $w$ or less columns each from any of the two sets of $b$ consecutive columns out of any one of the previously chosen $s-1$ sub-blocks, the coefficient of the last column of either both or one of the sets being non-zero.

The number of 2-repeated low-density bursts of length $b$(fixed) with weight $w$ or less in a sub-block of length $t$ [refer Dass and Garg [2009(b)]] is

$$\sum_{i=1}^{2}\binom{t-ib+i}{i}(q-1)^i\{[1+(q-1)]^{(b-1,w-1)}\}^i \quad (7)$$

Since there are $(s-1)$ previous sub-blocks, therefore number of such linear combinations becomes

$$(s-1)\sum_{i=1}^{2}\binom{t-ib+i}{i}(q-1)^i\{[1+(q-1)]^{(b-1,w-1)}\}^i \tag{8}$$

So, for the location of 2-repeated low-density burst of length $b$(fixed) with weight $w$ or less the number of linear combinations to which $h_j$ can not be equal to is the product of expr.(6) and expr.(8)

$$\text{i.e. expr.(6)} \times \text{ expr.(8)} \tag{9}$$

Thus the total number of linear combinations to which $h_i$ can not be equal to is the sum of exp.(6) and exp.(9) At worst all these combinations might yield distinct sum.

Therefore $h_i$ can be added to the $s$-th sub-block provided that

$$q^{n-k} > [1+(q-1)]^{(b-1,w-1)}\{1+(q-1)(j-2b+1)[1+(q-1)]^{(b-1,w-1)}\} \cdot \left\{1+(s-1)\sum_{i=1}^{2}\binom{t-ib+i}{i}(q-1)^i\{[1+(q-1)]^{(b-1,w-1)}\}^i\right\}$$

To obtain the length of the block as $t$ we replace $j$ by $t$ in the above expression.

The required parity-check matrix $H$ can be obtained from $H_1$ by reversing the order of the columns in each sub-block.

**Remark 2.** For $w=b$, the weight consideration over the burst becomes redundant and the inequality in Theorem 2 reduces to

$$\begin{aligned} q^{n-k} &> q^{b-1}\{1+(q-1)(t-2b+1)q^{b-1}\} \\ &\times \left\{1+(s-1)\sum_{i=1}^{2}\binom{t-ib+i}{i}(q-1)^i q^{i(b-1)}\right\} \end{aligned}$$

which coincides with the condition for the location of 2-repeated burst of length $b$(fixed) [refer Theorem 2, Dass and Arora [2010]].

We conclude this section with the following example:

**Example 3.** For an (27,15) linear code over GF(2) consider the following $12 \times 27$ matrix $H$ which has been constructed by the synthesis procedure given in the proof of theorem 2 by taking $s = 3,\ t = 9,\ b = 3,\ w = 2$.

$$H = \begin{bmatrix}
0\,0\,0\,0\,0\,0\,0\,0\,1 & 1\,1\,1\,1\,1\,1\,0\,0\,0 & 1\,0\,1\,0\,0\,0\,1\,1\,1 \\
0\,0\,0\,0\,0\,0\,0\,1\,0 & 1\,1\,0\,1\,1\,1\,0\,0\,0 & 1\,0\,0\,0\,1\,0\,1\,1\,0 \\
0\,0\,0\,0\,0\,0\,1\,0\,0 & 1\,1\,1\,0\,0\,0\,0\,0\,0 & 1\,0\,0\,0\,1\,1\,1\,1\,0 \\
0\,0\,0\,0\,0\,1\,0\,0\,0 & 1\,0\,0\,0\,0\,0\,0\,0\,0 & 1\,0\,0\,0\,1\,0\,0\,0\,0 \\
0\,0\,0\,0\,1\,0\,0\,0\,0 & 1\,1\,1\,1\,0\,0\,0\,0\,0 & 1\,0\,1\,0\,0\,1\,1\,0\,0 \\
0\,0\,0\,1\,0\,0\,0\,0\,0 & 1\,1\,0\,1\,0\,0\,0\,0\,0 & 1\,0\,0\,0\,0\,0\,1\,0\,0 \\
0\,0\,1\,0\,0\,0\,0\,0\,0 & 0\,1\,1\,0\,0\,0\,0\,0\,0 & 1\,0\,0\,0\,0\,0\,1\,0\,0 \\
0\,1\,0\,0\,0\,0\,0\,0\,0 & 0\,0\,0\,0\,0\,0\,0\,0\,0 & 1\,0\,1\,1\,0\,1\,1\,0\,0 \\
1\,0\,0\,0\,0\,0\,0\,0\,0 & 0\,1\,1\,1\,0\,0\,0\,0\,0 & 1\,1\,1\,1\,1\,0\,0\,0\,0 \\
0\,0\,0\,0\,0\,0\,0\,0\,0 & 0\,1\,0\,1\,0\,0\,0\,0\,1 & 0\,0\,1\,1\,1\,1\,1\,1\,0 \\
0\,0\,0\,0\,0\,0\,0\,0\,0 & 0\,1\,1\,0\,1\,0\,0\,1\,0 & 0\,1\,1\,1\,1\,0\,1\,1\,0 \\
0\,0\,0\,0\,0\,0\,0\,0\,0 & 0\,0\,0\,0\,1\,0\,1\,0\,0 & 0\,0\,1\,1\,0\,0\,1\,1\,1
\end{bmatrix}$$

The null space of this matrix can be used as a code to locate a sub-block of length $t = 9$ containing 2-repeated burst of length 3(fixed). From the error pattern syndrome Table 1 we observe that:

The syndromes of 2-repeated burst of length 3(fixed) within any sub-block are all non-zero showing thereby that the code detects all 2-repeated low-density bursts of length 3(fixed) with weight 2 or less occurring within a sub-block.

It has been verified through MS-Excel program that the syndromes of the 2-repeated bursts of length 3(fixed) with weight 2 or less in any sub-block is different from the syndrome of a 2-repeated burst of length 3(fixed) with weight 2 or less within any other sub-block.

**Table 1**

| | Low density 2-repeated bursts of length 3(fixed) | | | Syndromes |
|---|---|---|---|---|
| | | Sub-block - 1 | | |
| 1 | 100100000 | 000000000 | 000000000 | 0000 0100 1000 |
| 2 | 100101000 | 000000000 | 000000000 | 0001 0100 1000 |
| 3 | 100110000 | 000000000 | 000000000 | 0000 1100 1000 |
| 4 | 101100000 | 000000000 | 000000000 | 0000 0110 1000 |
| 5 | 101101000 | 000000000 | 000000000 | 0001 0110 1000 |
| 6 | 101110000 | 000000000 | 000000000 | 0000 1110 1000 |
| 7 | 110100000 | 000000000 | 000000000 | 0000 0101 1000 |
| 8 | 110101000 | 000000000 | 000000000 | 0001 0101 1000 |
| 9 | 110110000 | 000000000 | 000000000 | 0000 1101 1000 |
| 10 | 100010000 | 000000000 | 000000000 | 0000 1000 1000 |
| 11 | 100010100 | 000000000 | 000000000 | 0010 1000 1000 |
| 12 | 100011000 | 000000000 | 000000000 | 0001 1000 1000 |
| 13 | 101010000 | 000000000 | 000000000 | 0000 1010 1000 |
| 14 | 101010100 | 000000000 | 000000000 | 0010 1010 1000 |
| 15 | 101011000 | 000000000 | 000000000 | 0001 1010 1000 |
| 16 | 110010000 | 000000000 | 000000000 | 0000 1001 1000 |
| 17 | 110010100 | 000000000 | 000000000 | 0010 1001 1000 |
| 18 | 110011000 | 000000000 | 000000000 | 0001 1001 1000 |
| 19 | 100001000 | 000000000 | 000000000 | 0001 0000 1000 |
| 20 | 100001010 | 000000000 | 000000000 | 0101 0000 1000 |
| 21 | 100001100 | 000000000 | 000000000 | 0011 0000 1000 |
| 22 | 101001000 | 000000000 | 000000000 | 0001 0010 1000 |
| 23 | 101001010 | 000000000 | 000000000 | 0101 0010 1000 |
| 24 | 101001100 | 000000000 | 000000000 | 0011 0010 1000 |
| 25 | 110001000 | 000000000 | 000000000 | 0001 0001 1000 |
| 26 | 110001010 | 000000000 | 000000000 | 0101 0001 1000 |
| 27 | 110001100 | 000000000 | 000000000 | 0011 0001 1000 |
| 28 | 100000100 | 000000000 | 000000000 | 0010 0000 1000 |
| 29 | 100000101 | 000000000 | 000000000 | 1010 0000 1000 |

| Low density 2-repeated bursts of length 3(fixed) | | | | Syndromes |
|---|---|---|---|---|
| | | Sub-block - 1 | | |
| 30 | 100000110 | 000000000 | 000000000 | 0110 0000 1000 |
| 31 | 101000100 | 000000000 | 000000000 | 0010 0010 1000 |
| 32 | 101000101 | 000000000 | 000000000 | 1010 0010 1000 |
| 33 | 101000110 | 000000000 | 000000000 | 0110 0010 1000 |
| 34 | 110000100 | 000000000 | 000000000 | 0010 0001 1000 |
| 35 | 110000101 | 000000000 | 000000000 | 1010 0001 1000 |
| 36 | 110000110 | 000000000 | 000000000 | 0110 0001 1000 |
| 37 | 010010000 | 000000000 | 000000000 | 0000 1001 0000 |
| 38 | 010010100 | 000000000 | 000000000 | 0010 1001 0000 |
| 39 | 010011000 | 000000000 | 000000000 | 0001 1001 0000 |
| 40 | 010110000 | 000000000 | 000000000 | 0000 1101 0000 |
| 41 | 010110100 | 000000000 | 000000000 | 0010 1101 0000 |
| 42 | 010111000 | 000000000 | 000000000 | 0001 1101 0000 |
| 43 | 011010000 | 000000000 | 000000000 | 0000 1011 0000 |
| 44 | 011010100 | 000000000 | 000000000 | 0010 1011 0000 |
| 45 | 011011000 | 000000000 | 000000000 | 0001 1011 0000 |
| 46 | 010001000 | 000000000 | 000000000 | 0001 0001 0000 |
| 47 | 010001010 | 000000000 | 000000000 | 0101 0001 0000 |
| 48 | 010001100 | 000000000 | 000000000 | 0011 0001 0000 |
| 49 | 010101000 | 000000000 | 000000000 | 0001 0101 0000 |
| 50 | 010101010 | 000000000 | 000000000 | 0101 0101 0000 |
| 51 | 010101100 | 000000000 | 000000000 | 0011 0101 0000 |
| 52 | 011001000 | 000000000 | 000000000 | 0001 0011 0000 |
| 53 | 011001010 | 000000000 | 000000000 | 0101 0011 0000 |
| 54 | 011001100 | 000000000 | 000000000 | 0011 0011 0000 |
| 55 | 010000100 | 000000000 | 000000000 | 0010 0001 0000 |
| 56 | 010000101 | 000000000 | 000000000 | 1010 0001 0000 |
| 57 | 010000110 | 000000000 | 000000000 | 0110 0001 0000 |
| 58 | 010100100 | 000000000 | 000000000 | 0010 0101 0000 |
| 59 | 010100101 | 000000000 | 000000000 | 1010 0101 0000 |
| 60 | 010100110 | 000000000 | 000000000 | 0110 0101 0000 |
| 61 | 011000100 | 000000000 | 000000000 | 0010 0011 0000 |
| 62 | 011000101 | 000000000 | 000000000 | 1010 0011 0000 |
| 63 | 011000110 | 000000000 | 000000000 | 0110 0011 0000 |
| 64 | 001001000 | 000000000 | 000000000 | 0001 0010 0000 |
| 65 | 001001010 | 000000000 | 000000000 | 0101 0010 0000 |
| 66 | 001001100 | 000000000 | 000000000 | 0011 0010 0000 |

| Low density 2-repeated bursts of length 3(fixed) | | | | Syndromes |
|---|---|---|---|---|
| | | Sub-block - 1 | | |
| 67 | 001011000 | 000000000 | 000000000 | 0001 1010 0000 |
| 68 | 001011010 | 000000000 | 000000000 | 0101 1010 0000 |
| 69 | 001011100 | 000000000 | 000000000 | 0011 1010 0000 |
| 70 | 001101000 | 000000000 | 000000000 | 0001 0110 0000 |
| 71 | 001101010 | 000000000 | 000000000 | 0101 0110 0000 |
| 72 | 001101100 | 000000000 | 000000000 | 0011 0110 0000 |
| 73 | 001000100 | 000000000 | 000000000 | 0010 0010 0000 |
| 74 | 001000101 | 000000000 | 000000000 | 1010 0010 0000 |
| 75 | 001000110 | 000000000 | 000000000 | 0110 0010 0000 |
| 76 | 001010100 | 000000000 | 000000000 | 0010 1010 0000 |
| 77 | 001010101 | 000000000 | 000000000 | 1010 1010 0000 |
| 78 | 001010110 | 000000000 | 000000000 | 0110 1010 0000 |
| 79 | 001100100 | 000000000 | 000000000 | 0010 0110 0000 |
| 80 | 001100101 | 000000000 | 000000000 | 1010 0110 0000 |
| 81 | 001100110 | 000000000 | 000000000 | 0110 0110 0000 |
| 82 | 000100100 | 000000000 | 000000000 | 0010 0100 0000 |
| 83 | 000100101 | 000000000 | 000000000 | 1010 0100 0000 |
| 84 | 000100110 | 000000000 | 000000000 | 0110 0100 0000 |
| 85 | 000101100 | 000000000 | 000000000 | 0011 0100 0000 |
| 86 | 000101101 | 000000000 | 000000000 | 1011 0100 0000 |
| 87 | 000101110 | 000000000 | 000000000 | 0111 0100 0000 |
| 88 | 000110100 | 000000000 | 000000000 | 0010 1100 0000 |
| 89 | 000110101 | 000000000 | 000000000 | 1010 1100 0000 |
| 90 | 000110110 | 000000000 | 000000000 | 0110 1100 0000 |
| 91 | 100000000 | 000000000 | 000000000 | 0000 0000 1000 |
| 92 | 101000000 | 000000000 | 000000000 | 0000 0010 1000 |
| 93 | 110000000 | 000000000 | 000000000 | 0000 0001 1000 |
| 94 | 010000000 | 000000000 | 000000000 | 0000 0001 0000 |
| 95 | 010100000 | 000000000 | 000000000 | 0000 0101 0000 |
| 96 | 011000000 | 000000000 | 000000000 | 0000 0011 0000 |
| 97 | 001000000 | 000000000 | 000000000 | 0000 0010 0000 |
| 98 | 001010000 | 000000000 | 000000000 | 0000 1010 0000 |
| 99 | 001100000 | 000000000 | 000000000 | 0000 0110 0000 |
| 100 | 000100000 | 000000000 | 000000000 | 0000 0100 0000 |
| 101 | 000101000 | 000000000 | 000000000 | 0001 0100 0000 |
| 102 | 000110000 | 000000000 | 000000000 | 0000 1100 0000 |

| Low density 2-repeated bursts of length 3(fixed) | | | | Syndromes |
|---|---|---|---|---|
| | | Sub-block - 1 | | |
| 103 | 000010000 | 000000000 | 000000000 | 0000 1000 0000 |
| 104 | 000010100 | 000000000 | 000000000 | 0010 1000 0000 |
| 105 | 000011000 | 000000000 | 000000000 | 0001 1000 0000 |
| 106 | 000001000 | 000000000 | 000000000 | 0001 0000 0000 |
| 107 | 000001010 | 000000000 | 000000000 | 0101 0000 0000 |
| 108 | 000001100 | 000000000 | 000000000 | 0011 0000 0000 |
| 109 | 000000100 | 000000000 | 000000000 | 0010 0000 0000 |
| 110 | 000000101 | 000000000 | 000000000 | 1010 0000 0000 |
| 111 | 000000110 | 000000000 | 000000000 | 0110 0000 0000 |

| Low density 2-repeated bursts of length 3(fixed) | | | | Syndromes |
|---|---|---|---|---|
| | | Sub-block - 2 | | |
| 112 | 000000000 | 100100000 | 000000000 | 0011 0000 1100 |
| 113 | 000000000 | 100101000 | 000000000 | 1111 0000 1100 |
| 114 | 000000000 | 100110000 | 000000000 | 1111 0000 1111 |
| 115 | 000000000 | 101100000 | 000000000 | 1001 1010 0110 |
| 116 | 000000000 | 101101000 | 000000000 | 0101 1010 0110 |
| 117 | 000000000 | 101110000 | 000000000 | 0101 1010 0101 |
| 118 | 000000000 | 110100000 | 000000000 | 1101 1110 0010 |
| 119 | 000000000 | 110101000 | 000000000 | 0001 1110 0010 |
| 120 | 000000000 | 110110000 | 000000000 | 0001 1110 0001 |
| 121 | 000000000 | 100010000 | 000000000 | 0011 1100 0011 |
| 122 | 000000000 | 100010100 | 000000000 | 0011 1100 0010 |
| 123 | 000000000 | 100011000 | 000000000 | 1111 1100 0011 |
| 124 | 000000000 | 101010000 | 000000000 | 1001 0110 1001 |
| 125 | 000000000 | 101010100 | 000000000 | 1001 0110 1000 |
| 126 | 000000000 | 101011000 | 000000000 | 0101 0110 1001 |
| 127 | 000000000 | 110010000 | 000000000 | 1101 0010 1101 |
| 128 | 000000000 | 110010100 | 000000000 | 1101 0010 1100 |
| 129 | 000000000 | 110011000 | 000000000 | 0001 0010 1101 |
| 130 | 000000000 | 100001000 | 000000000 | 0011 1100 0000 |
| 131 | 000000000 | 100001010 | 000000000 | 0011 1100 0010 |
| 132 | 000000000 | 100001100 | 000000000 | 0011 1100 0001 |
| 133 | 000000000 | 101001000 | 000000000 | 1001 0110 1010 |
| 134 | 000000000 | 101001010 | 000000000 | 1001 0110 1000 |
| 135 | 000000000 | 101001100 | 000000000 | 1001 0110 1011 |
| 136 | 000000000 | 110001000 | 000000000 | 1101 0010 1110 |

| | Low density 2-repeated bursts of length 3(fixed) | | | Syndromes |
|---|---|---|---|---|
| | | Sub-block - 2 | | |
| 137 | 000000000 | 110001010 | 000000000 | 1101 0010 1100 |
| 138 | 000000000 | 110001100 | 000000000 | 1101 0010 1111 |
| 139 | 000000000 | 100000100 | 000000000 | 1111 1100 0001 |
| 140 | 000000000 | 100000101 | 000000000 | 1111 1100 0101 |
| 141 | 000000000 | 100000110 | 000000000 | 1111 1100 0011 |
| 142 | 000000000 | 101000100 | 000000000 | 0101 0110 1011 |
| 143 | 000000000 | 101000101 | 000000000 | 0101 0110 1111 |
| 144 | 000000000 | 101000110 | 000000000 | 0101 0110 1001 |
| 145 | 000000000 | 110000100 | 000000000 | 0001 0010 1111 |
| 146 | 000000000 | 110000101 | 000000000 | 0001 0010 1011 |
| 147 | 000000000 | 110000110 | 000000000 | 0001 0010 1101 |
| 148 | 000000000 | 010010000 | 000000000 | 0010 1110 1101 |
| 149 | 000000000 | 010010100 | 000000000 | 0010 1110 1100 |
| 150 | 000000000 | 010011000 | 000000000 | 1110 1110 1101 |
| 151 | 000000000 | 010110000 | 000000000 | 1110 0010 0001 |
| 152 | 000000000 | 010110100 | 000000000 | 1110 0010 0000 |
| 153 | 000000000 | 010111000 | 000000000 | 0010 0010 0001 |
| 154 | 000000000 | 011010000 | 000000000 | 1000 0100 0111 |
| 155 | 000000000 | 011010100 | 000000000 | 1000 0100 0110 |
| 156 | 000000000 | 011011000 | 000000000 | 0100 0100 0111 |
| 157 | 000000000 | 010001000 | 000000000 | 0010 1110 1110 |
| 158 | 000000000 | 010001010 | 000000000 | 0010 1110 1100 |
| 159 | 000000000 | 010001100 | 000000000 | 0010 1110 1111 |
| 160 | 000000000 | 010101000 | 000000000 | 1110 0010 0010 |
| 161 | 000000000 | 010101010 | 000000000 | 1110 0010 0000 |
| 162 | 000000000 | 010101100 | 000000000 | 1110 0010 0011 |
| 163 | 000000000 | 011001000 | 000000000 | 1000 0100 0100 |
| 164 | 000000000 | 011001010 | 000000000 | 1000 0100 0110 |
| 165 | 000000000 | 011001100 | 000000000 | 1000 0100 0101 |
| 166 | 000000000 | 010000100 | 000000000 | 1110 1110 1111 |
| 167 | 000000000 | 010000101 | 000000000 | 1110 1110 1011 |
| 168 | 000000000 | 010000110 | 000000000 | 1110 1110 1101 |
| 169 | 000000000 | 010100100 | 000000000 | 0010 0010 0011 |
| 170 | 000000000 | 010100101 | 000000000 | 0010 0010 0111 |
| 171 | 000000000 | 010100110 | 000000000 | 0010 0010 0001 |
| 172 | 000000000 | 011000100 | 000000000 | 0100 0100 0101 |
| 173 | 000000000 | 011000101 | 000000000 | 0100 0100 0001 |

| Low density 2-repeated bursts of length 3(fixed) | | | | Syndromes |
|---|---|---|---|---|
| | | Sub-block - 2 | | |
| 174 | 000000000 | 011000110 | 000000000 | 0100 0100 0111 |
| 175 | 000000000 | 001001000 | 000000000 | 0110 1010 1010 |
| 176 | 000000000 | 001001010 | 000000000 | 0110 1010 1000 |
| 177 | 000000000 | 001001100 | 000000000 | 0110 1010 1011 |
| 178 | 000000000 | 001011000 | 000000000 | 1010 1010 1001 |
| 179 | 000000000 | 001011010 | 000000000 | 1010 1010 1011 |
| 180 | 000000000 | 001011100 | 000000000 | 1010 1010 1000 |
| 181 | 000000000 | 001101000 | 000000000 | 1010 0110 0110 |
| 182 | 000000000 | 001101010 | 000000000 | 1010 0110 0100 |
| 183 | 000000000 | 001101100 | 000000000 | 1010 0110 0111 |
| 184 | 000000000 | 001000100 | 000000000 | 1010 1010 1011 |
| 185 | 000000000 | 001000101 | 000000000 | 1010 1010 1111 |
| 186 | 000000000 | 001000110 | 000000000 | 1010 1010 1001 |
| 187 | 000000000 | 001010100 | 000000000 | 0110 1010 1000 |
| 188 | 000000000 | 001010101 | 000000000 | 0110 1010 1100 |
| 189 | 000000000 | 001010110 | 000000000 | 0110 1010 1010 |
| 190 | 000000000 | 001100100 | 000000000 | 0110 0110 0111 |
| 191 | 000000000 | 001100101 | 000000000 | 0110 0110 0011 |
| 192 | 000000000 | 001100110 | 000000000 | 0110 0110 0101 |
| 193 | 000000000 | 000100100 | 000000000 | 1100 1100 1101 |
| 194 | 000000000 | 000100101 | 000000000 | 1100 1100 1001 |
| 195 | 000000000 | 000100110 | 000000000 | 1100 1100 1111 |
| 196 | 000000000 | 000101100 | 000000000 | 0000 1100 1101 |
| 197 | 000000000 | 000101101 | 000000000 | 0000 1100 1001 |
| 198 | 000000000 | 000101110 | 000000000 | 0000 1100 1111 |
| 199 | 000000000 | 000110100 | 000000000 | 0000 1100 1110 |
| 200 | 000000000 | 000110101 | 000000000 | 0000 1100 1010 |
| 201 | 000000000 | 000110110 | 000000000 | 0000 1100 1100 |
| 202 | 000000000 | 100000000 | 000000000 | 1111 1100 0000 |
| 203 | 000000000 | 101000000 | 000000000 | 0101 0110 1010 |
| 204 | 000000000 | 110000000 | 000000000 | 0001 0010 1110 |
| 205 | 000000000 | 010000000 | 000000000 | 1110 1110 1110 |
| 206 | 000000000 | 010100000 | 000000000 | 0010 0010 0010 |
| 207 | 000000000 | 011000000 | 000000000 | 0100 0100 0100 |
| 208 | 000000000 | 001000000 | 000000000 | 1010 1010 1010 |
| 209 | 000000000 | 001010000 | 000000000 | 0110 1010 1001 |
| 210 | 000000000 | 001100000 | 000000000 | 0110 0110 0110 |
| 211 | 000000000 | 000100000 | 000000000 | 1100 1100 1100 |

| Low density 2-repeated bursts of length 3(fixed) | | | | Syndromes |
|---|---|---|---|---|
| | | Sub-block - 2 | | |
| 212 | 000000000 | 000101000 | 000000000 | 0000 1100 1100 |
| 213 | 000000000 | 000110000 | 000000000 | 0000 1100 1111 |
| 214 | 000000000 | 000010000 | 000000000 | 1100 0000 0011 |
| 215 | 000000000 | 000010100 | 000000000 | 1100 0000 0010 |
| 216 | 000000000 | 000011000 | 000000000 | 0000 0000 0011 |
| 217 | 000000000 | 000001000 | 000000000 | 1100 0000 0000 |
| 218 | 000000000 | 000001010 | 000000000 | 1100 0000 0010 |
| 219 | 000000000 | 000001100 | 000000000 | 1100 0000 0001 |
| 220 | 000000000 | 000000100 | 000000000 | 0000 0000 0001 |
| 221 | 000000000 | 000000101 | 000000000 | 0000 0000 0101 |
| 222 | 000000000 | 000000110 | 000000000 | 0000 0000 0011 |

| Low density 2-repeated bursts of length 3(fixed) | | | | Syndromes |
|---|---|---|---|---|
| | | Sub-block - 3 | | |
| 223 | 000000000 | 000000000 | 100100000 | 1111 1110 0111 |
| 224 | 000000000 | 000000000 | 100101000 | 1101 0111 0011 |
| 225 | 000000000 | 000000000 | 100110000 | 1000 1110 1001 |
| 226 | 000000000 | 000000000 | 101100000 | 0111 0111 1000 |
| 227 | 000000000 | 000000000 | 101101000 | 0101 1110 1100 |
| 228 | 000000000 | 000000000 | 101110000 | 0000 0111 0110 |
| 229 | 000000000 | 000000000 | 110100000 | 1111 1110 1101 |
| 230 | 000000000 | 000000000 | 110101000 | 1101 0111 1001 |
| 231 | 000000000 | 000000000 | 110110000 | 1000 1110 0011 |
| 232 | 000000000 | 000000000 | 100010000 | 1000 1111 0110 |
| 233 | 000000000 | 000000000 | 100010100 | 0110 0000 0001 |
| 234 | 000000000 | 000000000 | 100011000 | 1010 0110 0010 |
| 235 | 000000000 | 000000000 | 101010000 | 0000 0110 1001 |
| 236 | 000000000 | 000000000 | 101010100 | 1110 1001 1110 |
| 237 | 000000000 | 000000000 | 101011000 | 0010 1111 1101 |
| 238 | 000000000 | 000000000 | 110010000 | 1000 1111 1100 |
| 239 | 000000000 | 000000000 | 110010100 | 0110 0000 1011 |
| 240 | 000000000 | 000000000 | 110011000 | 1010 0110 1000 |
| 241 | 000000000 | 000000000 | 100001000 | 1101 0110 1100 |
| 242 | 000000000 | 000000000 | 100001010 | 0011 0110 1011 |
| 243 | 000000000 | 000000000 | 100001100 | 0011 1001 1011 |
| 244 | 000000000 | 000000000 | 101001000 | 0101 1111 0011 |

| | Low density 2-repeated bursts of length 3(fixed) | | | Syndromes |
|---|---|---|---|---|
| | | Sub-block - 3 | | |
| 245 | 000000000 | 000000000 | 101001010 | 1011 1111 0100 |
| 246 | 000000000 | 000000000 | 101001100 | 1011 0000 0100 |
| 247 | 000000000 | 000000000 | 110001000 | 1101 0110 0110 |
| 248 | 000000000 | 000000000 | 110001010 | 0011 0110 0001 |
| 249 | 000000000 | 000000000 | 110001100 | 0011 1001 0001 |
| 250 | 000000000 | 000000000 | 100000100 | 0001 0000 1111 |
| 251 | 000000000 | 000000000 | 100000101 | 1001 0000 1110 |
| 252 | 000000000 | 000000000 | 100000110 | 1111 0000 1000 |
| 253 | 000000000 | 000000000 | 101000100 | 1001 1001 0000 |
| 254 | 000000000 | 000000000 | 101000101 | 0001 1001 0001 |
| 255 | 000000000 | 000000000 | 101000110 | 0111 1001 0111 |
| 256 | 000000000 | 000000000 | 110000100 | 0001 0000 0101 |
| 257 | 000000000 | 000000000 | 110000101 | 1001 0000 0100 |
| 258 | 000000000 | 000000000 | 110000110 | 1111 0000 0010 |
| 259 | 000000000 | 000000000 | 010010000 | 0111 0000 0100 |
| 260 | 000000000 | 000000000 | 010010100 | 1001 1111 0011 |
| 261 | 000000000 | 000000000 | 010011000 | 0101 1001 0000 |
| 262 | 000000000 | 000000000 | 010110000 | 0111 0001 1011 |
| 263 | 000000000 | 000000000 | 010110100 | 1001 1110 1100 |
| 264 | 000000000 | 000000000 | 010111000 | 0101 1000 1111 |
| 265 | 000000000 | 000000000 | 011010000 | 1111 1001 1011 |
| 266 | 000000000 | 000000000 | 011010100 | 0001 0110 1100 |
| 267 | 000000000 | 000000000 | 011011000 | 1101 0000 1111 |
| 268 | 000000000 | 000000000 | 010001000 | 0010 1001 1110 |
| 269 | 000000000 | 000000000 | 010001010 | 1100 1001 1001 |
| 270 | 000000000 | 000000000 | 010001100 | 1100 0110 1001 |
| 271 | 000000000 | 000000000 | 010101000 | 0010 1000 0001 |
| 272 | 000000000 | 000000000 | 010101010 | 1100 1000 0110 |
| 273 | 000000000 | 000000000 | 010101100 | 1100 0111 0110 |
| 274 | 000000000 | 000000000 | 011001000 | 1010 0000 0001 |
| 275 | 000000000 | 000000000 | 011001010 | 0100 0000 0110 |
| 276 | 000000000 | 000000000 | 011001100 | 0100 1111 0110 |
| 277 | 000000000 | 000000000 | 010000100 | 1110 1111 1101 |
| 278 | 000000000 | 000000000 | 010000101 | 0110 1111 1100 |
| 279 | 000000000 | 000000000 | 010000110 | 0000 1111 1010 |
| 280 | 000000000 | 000000000 | 010100100 | 1110 1110 0010 |
| 281 | 000000000 | 000000000 | 010100101 | 0110 1110 0011 |

| | Low density 2-repeated bursts of length 3(fixed) | | | Syndromes |
|---|---|---|---|---|
| | | Sub-block - 3 | | |
| 282 | 000000000 | 000000000 | 010100110 | 0000 1110 0101 |
| 283 | 000000000 | 000000000 | 011000100 | 0110 0110 0010 |
| 284 | 000000000 | 000000000 | 011000101 | 1110 0110 0011 |
| 285 | 000000000 | 000000000 | 011000110 | 1000 0110 0101 |
| 286 | 000000000 | 000000000 | 001001000 | 1010 0000 1011 |
| 287 | 000000000 | 000000000 | 001001010 | 0100 0000 1100 |
| 288 | 000000000 | 000000000 | 001001100 | 0100 1111 1100 |
| 289 | 000000000 | 000000000 | 001011000 | 1101 0000 0101 |
| 290 | 000000000 | 000000000 | 001011010 | 0011 0000 0010 |
| 291 | 000000000 | 000000000 | 001011100 | 0011 1111 0010 |
| 292 | 000000000 | 000000000 | 001101000 | 1010 0001 0100 |
| 293 | 000000000 | 000000000 | 001101010 | 0100 0001 0011 |
| 294 | 000000000 | 000000000 | 001101100 | 0100 1110 0011 |
| 295 | 000000000 | 000000000 | 001000100 | 0110 0110 1000 |
| 296 | 000000000 | 000000000 | 001000101 | 1110 0110 1001 |
| 297 | 000000000 | 000000000 | 001000110 | 1000 0110 1111 |
| 298 | 000000000 | 000000000 | 001010100 | 0001 0110 0110 |
| 299 | 000000000 | 000000000 | 001010101 | 1001 0110 0111 |
| 300 | 000000000 | 000000000 | 001010110 | 1111 0110 0001 |
| 301 | 000000000 | 000000000 | 001100100 | 0110 0111 0111 |
| 302 | 000000000 | 000000000 | 001100101 | 1110 0111 0110 |
| 303 | 000000000 | 000000000 | 001100110 | 1000 0111 0000 |
| 304 | 000000000 | 000000000 | 000100100 | 1110 1110 1000 |
| 305 | 000000000 | 000000000 | 000100101 | 0110 1110 1001 |
| 306 | 000000000 | 000000000 | 000100110 | 0000 1110 1111 |
| 307 | 000000000 | 000000000 | 000101100 | 1100 0111 1100 |
| 308 | 000000000 | 000000000 | 000101101 | 0100 0111 1101 |
| 309 | 000000000 | 000000000 | 000101110 | 0010 0111 1011 |
| 310 | 000000000 | 000000000 | 000110100 | 1001 1110 0110 |
| 311 | 000000000 | 000000000 | 000110101 | 0001 1110 0111 |
| 312 | 000000000 | 000000000 | 000110110 | 0111 1110 0001 |
| 313 | 000000000 | 000000000 | 100000000 | 1111 1111 1000 |
| 314 | 000000000 | 000000000 | 101000000 | 0111 0110 0111 |
| 315 | 000000000 | 000000000 | 110000000 | 1111 1111 0010 |
| 316 | 000000000 | 000000000 | 010000000 | 0000 0000 1010 |
| 317 | 000000000 | 000000000 | 010100000 | 0000 0001 0101 |
| 318 | 000000000 | 000000000 | 011000000 | 1000 1001 0101 |

| Low density 2-repeated bursts of length 3(fixed) | | | | Syndromes |
|---|---|---|---|---|
| | | Sub-block - 3 | | |
| 319 | 000000000 | 000000000 | 001000000 | 1000 1001 1111 |
| 320 | 000000000 | 000000000 | 001010000 | 1111 1001 0001 |
| 321 | 000000000 | 000000000 | 001100000 | 1000 1000 0000 |
| 322 | 000000000 | 000000000 | 000100000 | 0000 0001 1111 |
| 323 | 000000000 | 000000000 | 000101000 | 0010 1000 1011 |
| 324 | 000000000 | 000000000 | 000110000 | 0111 0001 0001 |
| 325 | 000000000 | 000000000 | 000010000 | 0111 0000 1110 |
| 326 | 000000000 | 000000000 | 000010100 | 1001 1111 1001 |
| 327 | 000000000 | 000000000 | 000011000 | 0101 1001 1010 |
| 328 | 000000000 | 000000000 | 000001000 | 0010 1001 0100 |
| 329 | 000000000 | 000000000 | 000001010 | 1100 1001 0011 |
| 330 | 000000000 | 000000000 | 000001100 | 1100 0110 0011 |
| 331 | 000000000 | 000000000 | 000000100 | 1110 1111 0111 |
| 332 | 000000000 | 000000000 | 000000101 | 0110 1111 0110 |
| 333 | 000000000 | 000000000 | 000000110 | 0000 1111 0000 |

**Remark 3.** The space visible between vectors in the first column in Table 1 has been given to distinguish between different sub-blocks whereas the space given in the syndrome vector is for convenience.

**Observation.** Syndromes of some of the 2-repeated bursts of length 3(fixed) occurring within the second sub-block are same. For coding efficiency it is desired that the syndromes of the error patterns within any sub-block is identical whenever possible.

# 3 Location of $m$-Repeated Low-density burst of length $b$(fixed)

In this section a necessary and sufficient condition for the location of an $m$-repeated low-density burst of length $b$(fixed) with weight $w$ or less has been given.

It may be noted that an EL-code capable of detecting and locating a single sub-block containing an error which is in the form of an $m$-repeated low-density burst of length $b$(fixed) with weight $w$ or less $(w \leq b)$ must satisfy the following conditions:

(c) The syndrome resulting from the occurrence of an $m$-repeated low-density burst of length $b$(fixed) with weight $w$ or less within any one sub-block must be distinct from the all zero syndrome.

(d) The syndrome resulting from the occurrence of any $m$-repeated low-density burst of length $b$(fixed) with weight $w$ or less within a single sub-block must be distinct from the syndrome resulting likewise from any $m$-repeated low-density burst of length $b$(fixed) with weight $w$ or less *within* any other sub-block.

In this section we shall derive two results. The first result gives a lower bound on the number of check digits required for the existence of a linear code over GF$(q)$ capable of detecting and locating a single sub-block containing errors that are $m$-repeated low-density bursts of length $b$(fixed) with weight $w$ or less. In the second result, we derive an upper bound on the number of check digits which ensures the existence of such a code.

**Theorem 3.** *The number of parity check digits $r$ in an $(n, k)$ linear code subdivided into $s$ sub-blocks of length $t$ each, that locates a single corrupted sub-block containing errors that are 2-repeated low density bursts of length $b$ (fixed) with weight $w$ or less is at least*

$$\begin{cases} \log_q\{1 + s(q^{mw} - 1)\} & \textit{where } \; t - b + 1 \geq mb \\ & \textit{and } \; (m-1)b + w \leq t - b + 1 < mb \\ \log_q\{1 + s(q^{(m-1)w+(t-mb+1)} - 1)\} & \textit{where } \; t - b + 1 < (m-1)b + w. \end{cases} \tag{10}$$

The proof of this result is on the similar lines as that of proof of Theorem 1 so we omit the proof.

**Remark 4.** For $m = 2$ the result coincides with that of Theorem 1 when 2-repeated low-density bursts of length $b$(fixed) with weight $w$ or less are considered.

**Remark 5.** For $m = 1$, the result obtained in (10) reduces to

$$\begin{cases} \log_q\{1 + s(q^w - 1)\} & \text{where } \ t - b + 1 \geq b \\ & \text{and } \ w \leq t - b + 1 < b \ . \\ \log_q\{1 + s(q^{(t-b+1)} - 1)\} & \text{where } \ t - b + 1 < w \end{cases}$$

which is a case of detecting and locating a sub-block containing errors which are usual low-density bursts of length $b$(fixed) with weight $w$ or less.

**Remark 6.** For $w = b$, the result obtained in (10) reduces to

$$r \geq \begin{cases} \log_q\{1 + s(q^{mb} - 1)\} & \text{where } \ t - b + 1 \geq mb \\ \log_q\{1 + s(q^{(t-b+1)} - 1)\} & \text{where } \ t - b + 1 < mb \end{cases}$$

which coincides with the result due to Dass and Arora [Theorem 3, 2010].

In the following result we derive another bound on the number of check digits required for the existence of such a code. As earlier the proof is based on the technique used to establish Varshomov-Gilbert Sacks bound by constructing a parity check matrix for such a code (refer Sacks, Theorem 4.7 Peterson and Weldon(1972)).

**Theorem 4.** *An* $(n, k)$ *linear EL-code over* GF$(q)$ *capable of detecting an* $m$*-repeated low density burst of length* $b$ *(fixed) with weight* $w$ *or less*

*$(w \le b)$ within a single sub-block and of locating that sub-block can always be constructed provided that*

$$\begin{aligned} q^{n-k} > {} & [1+(q-1)]^{(b-1,w-1)} \\ & \cdot \left\{ \sum_{i=0}^{m-1} \binom{t-(i+1)b+i}{i} (q-1)^i [1+(q-1)]^{(b-1,w-1)} \right\} \\ & \cdot \left\{ 1+(s-1) \sum_{i=1}^{m} \binom{t-ib+i}{i} (q-1)^i \{[1+(q-1)]^{(b-1,w-1)}\}^i \right\} \quad (11) \end{aligned}$$

*where $[1+x]^{(m,r)}$ denotes the incomplete binomial expansion of $(1+x)^m$ up to the term $x^r$ in ascending power of $x$, viz.*

$$[1+x]^{(m,r)} = 1 + \binom{m}{1} x + \binom{m}{2} x^2 + \ldots + \binom{m}{r} x^r.$$

As in theorem 3 we omit the proof because proof of this result is on the similar lines as that of proof of Theorem 2.

**Remark 7.** For $m = 2$ the result coincides with that of Theorem 2 when 2-repeated low-density bursts of length $b$(fixed) with weight $w$ or less are considered.

**Remark 8.** For $m = 1$, the result obtained in (11) reduces to

$$q^{n-k} > [1+(q-1)]^{(b-1,w-1)} \{1+(s-1)(t-b+1)(q-1)[1+(q-1)]^{(b-1,w-1)}\}$$

which is a necessary condition for detecting and locating a sub-block containing errors which are usual low-density bursts of length $b$(fixed) with weight $w$ or less.

**Remark 9.** For $w = b$, the result obtained in (11) reduces to

$$q^{n-k} > q^{b-1}\left\{\sum_{i=0}^{m-1}\binom{j-(i+1)b+i}{i}(q-1)^i q^{i(b-1)}\right\} \cdot \left\{1+(s-1)\sum_{i=1}^{m}\binom{j-(i+1)b+i}{i}(q-1)^i q^{i(b-1)}\right\}$$

which coincides with the result due to Dass and Arora [Theorem 4, 2010].

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