

Class of Semihyperrings from Partitions of a Set

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Abstract

In this paper we show that a partition $\{P_\alpha : \alpha \in \Lambda\}$ of a non-empty set S , where Λ is an ordered set with the least element α_0 and P_{α_0} is a singleton set, induces a hyperaddition $+$ such that $(S, +)$ is a commutative hypermonoid. Also by using a collection of subsets of S , induced by the partition of the set S , we define hypermultiplication on S so that $(S, +, \cdot)$ is a semihyperring.

Key words: hypermonoid, semihyperring, *-collection.

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1 Introduction

The theory of hyperstructures has been introduced by the French Mathematician Marty [11] in 1934 at the age of 23 during the 8th congress of Scandinavian Mathematicians held in Stockholm. Since then many researchers have worked on this new area and developed it.

The theory of hyperstructure has been subsequently developed by Corsini [4, 5, 6], Mittas [13], Stratigopoulos [16] and various authors. Basic definitions and results about the hyperstructures are found in [5, 6]. Some researchers, namely, Davvaz [7], Massouros [12], Vougiouklis [18] and others developed the theory of algebraic hyperstructures.

There are different notions of hyperrings $(R, +, \cdot)$. If the addition $+$ is a hyperoperation and the multiplication \cdot is a binary operation then we say the hyperring is an Krasner (additive) hyperring [10]. Rota [15] introduced

a multiplicative hyperring, where $+$ is a binary operation and \cdot is a hyperoperation. De Salvo [8] introduced a hyperring in which addition and multiplication are hyperoperations. These hyperrings are studied by Rahnamani Barghi [14] and by Asokkumar and Velrajan [1, 2, 17]. Chvalina [3] and Hoskova [3, 9], studied $h\nu$ -groups, $H\nu$ -rings.

In this paper, by using different partitions of a set, we construct different semihyperrings $(S, +, \cdot)$ where both $+$ and \cdot are hyperoperations.

2 Preliminaries

This section explains some basic definitions that have been used in the sequel.

A *hyperoperation* \circ on a non-empty set H is a mapping of $H \times H$ into the family of non-empty subsets of H (i.e., $x \circ y \subseteq H$, for every $x, y \in H$). A *hypergroupoid* is a non-empty set H equipped with a hyperoperation \circ . For any two subsets A, B of a hypergroupoid H , the set $A \circ B$ means $\bigcup_{\substack{a \in A \\ b \in B}} (a \circ b)$.

A hypergroupoid (H, \circ) is called a *semihypergroup* if $x \circ (y \circ z) = (x \circ y) \circ z$ for all $x, y, z \in H$ (the associative axiom). A semihypergroup H is said to be regular (in the sense of Von Neumann) if $a \in a \circ H \circ a$ for every $a \in H$. A hypergroupoid (H, \circ) is called a *quasihypergroup* if $x \circ H = H \circ x = H$ for every $x \in H$ (the reproductive axiom). A reproductive semihypergroup is called a *hypergroup* (in the sense of Marty). A comprehensive review of the theory of hypergroups appears in [5].

Definition 2.1. A *semihyperring* is a non-empty set R with two hyperoperations $+$ and \cdot satisfying the following axioms:

- (1) $(R, +)$ is a commutative hypermonoid, that is,
 - (a) $(x + y) + z = x + (y + z)$ for all $x, y, z \in R$,
 - (b) there exists $0 \in R$, such that $x + 0 = 0 + x = \{x\}$ for all $x \in R$,
 - (c) $x + y = y + x$ for all $x, y \in R$.
- (2) (R, \cdot) is a semihypergroup, that is, $x \cdot (y \cdot z) = (x \cdot y) \cdot z$ for all $x, y, z \in R$.
- (3) The hyperoperation \cdot is distributive with respect to hyperoperation '+', that is, $x \cdot (y + z) = x \cdot y + x \cdot z$ and $(x + y) \cdot z = x \cdot z + y \cdot z$ for all $x, y, z \in R$.
- (4) There exists element $0 \in R$, such that $x \cdot 0 = 0 \cdot x = 0$ for all $x \in R$.

Definition 2.2. Let S be a semihyperring, An element $a \in S$ is said to be regular if there exists an element $y \in S$ such that $a \in a y a$. A semihyperring S is said to be regular if each element of S is regular.

3 Semihyperring constructed from a $*$ -collection.

In this section, for a given commutative hypermonoid $(S, +)$, we define hyperoperation \cdot on S suitably so that $(S, +, \cdot)$ is a regular semihyperring.

Definition 3.1. Let S be a commutative hypermonoid. A collection of non-empty subsets $\{S_a : a \in S\}$ of S satisfying the following conditions is called a $*$ -collection if (i) $S_a = \{0\}$ if and only if $a = 0$, (ii) if $a \neq 0$ then $\{0, a\} \subseteq S_a$, (iii) $\bigcup_{x \in S_a} S_x = S_a$ for every $a \in S$, (iv) $S_a + S_a = S_a$ for every $a \in S$ and (v) $\bigcup_{x \in a+b} S_x = S_a + S_b$ for every $a, b \in S$.

Example 3.2. Consider the set $S = \{0, a, b\}$. If we define a hyperoperation $+$ on S as in the following table, then $(S, +)$ is a commutative hypermonoid.

$+$	0	a	b
0	0	a	b
a	a	$\{a, b\}$	$\{a, b\}$
b	b	$\{a, b\}$	$\{a, b\}$

Now it is easy to see that $S_0 = \{0\}; S_a = S; S_b = S$ is a $*$ -collection.

Example 3.3. Consider the set $S = \{0, a, b\}$. If we define a hyperoperation $+$ on S as in the following table, then $(S, +)$ is a commutative hypermonoid.

$+$	0	a	b
0	0	a	b
a	a	$\{a\}$	$\{a, b\}$
b	b	$\{a, b\}$	$\{b\}$

Now it is easy to see that $S_0 = \{0\}; S_a = S; S_b = S$ is a $*$ -collection. Now we show that $S_0 = \{0\}; S_a = \{a, 0\}; S_b = \{b, 0\}$ is another $*$ -collection.

For each $a \in S$, $\bigcup_{x \in S_a} S_x = \bigcup_{x \in \{a, 0\}} S_x = S_a \cup S_0 = \{a, 0\} \cup \{0\} = \{a, 0\} = S_a$. Also $S_0 + S_0 = \{0\} + \{0\} = \{0\} = S_0; S_a + S_a = \{0, a\} + \{0, a\} = \{0, a\} = S_a; S_b + S_b = \{0, b\} + \{0, b\} = \{0, b\} = S_b$. Further, for $a, b \in S$, we get $\bigcup_{x \in a+b} S_x = \bigcup_{x \in \{a, b\}} S_x = S_a \cup S_b = \{0, a, b\} = S_a + S_b$.

Example 3.4. Consider the set $S = \{0, a, b\}$. If we define a hyperoperation $+$ on S as in the following table, then $(S, +)$ is a commutative hypermonoid.

$+$	0	a	b
0	0	a	b
a	a	$\{0, a\}$	S
b	b	S	$\{0, b\}$

It is easy to show that $S_0 = \{0\}$; $S_a = S$ for every $a \neq 0 \in S$, is a $*$ -collection and $S_0 = \{0\}$; $S_a = \{a, 0\}$ for every $a \neq 0 \in S$ is another $*$ -collection

Example 3.5. Consider the set $S = \{0, a, b, c\}$. If we define a hyperoperation $+$ on S as in the following table, then $(S, +)$ is a commutative hypermonoid.

$+$	0	a	b	c
0	$\{0\}$	$\{a\}$	$\{b\}$	$\{c\}$
a	$\{a\}$	$\{a\}$	$\{a, b\}$	$\{a, c\}$
b	$\{b\}$	$\{a, b\}$	$\{b\}$	$\{b, c\}$
c	$\{c\}$	$\{a, c\}$	$\{b, c\}$	$\{c\}$

In this commutative hypermonoid, each one of the following is a $*$ -collection.

- $S_0 = \{0\}$; $S_a = \{a, 0\}$ for every $a \neq 0 \in S$,
- $S_0 = \{0\}$; $S_a = S$ for every $a \neq 0 \in S$,
- $S_0 = \{0\}$; $S_a = \{0, a\}$; $S_b = \{0, b, a\}$; $S_c = \{0, c, a\}$,
- $S_0 = \{0\}$; $S_a = \{0, a, b\}$; $S_b = \{0, b\}$; $S_c = \{0, c, b\}$,
- $S_0 = \{0\}$; $S_a = \{0, a, c\}$; $S_b = \{0, b, c\}$; $S_c = \{0, c\}$.

Theorem 3.6. Let S be a commutative hypermonoid with the additive identity 0 with the condition that $x + y = \{0\}$ for $x, y \in S$ implies either $x = 0$ or $y = 0$. Let $\{S_a : a \in S\}$ be a $*$ -collection on S . For $a, b \in S$, if we define a hypermultiplication on S as

$$a \cdot b = \begin{cases} S_a & \text{if } a \neq 0, b \neq 0, \\ 0 & \text{otherwise} \end{cases}$$

then $(S, +, \cdot)$ is a regular semihyperring.

Proof. From the definition of the hypermultiplication, $a \cdot 0 = 0 \cdot a = 0$ for all $a \in S$. Let $a, b, c \in S$. If any one of a, b, c is 0, then $a \cdot (b \cdot c) = \{0\} = (a \cdot b) \cdot c$. If $a \neq 0, b \neq 0$ and $c \neq 0$, then $a \cdot (b \cdot c) = a \cdot S_b = S_a$. Also, $(a \cdot b) \cdot c = S_a \cdot c = \bigcup_{x \in S_a} (x \cdot c) = \bigcup_{x \in S_a} S_x = S_a$. Thus $(a \cdot b) \cdot c = a \cdot (b \cdot c)$. Therefore, (S, \cdot) is a semihypergroup.

Let $a, b, c \in S$. If $a = 0$ or $b = 0$ or $c = 0$, then it is obvious that $a \cdot (b + c) = a \cdot b + a \cdot c$. Suppose $a \neq 0, b \neq 0$ and $c \neq 0$. If $0 \in b + c$, then $a \cdot (b + c) = S_0 \cup S_a = S_a = S_a + S_a = a \cdot b + a \cdot c$. If $0 \notin b + c$, then $a \cdot (b + c) = S_a = S_a + S_a = a \cdot b + a \cdot c$. Thus $a \cdot (b + c) = a \cdot b + a \cdot c$.

Now we prove $(a + b) \cdot c = a \cdot c + b \cdot c$. For, $(a + b) \cdot c = \bigcup_{x \in a+b} x \cdot c = \bigcup_{x \in a+b} S_x = S_a + S_b = a \cdot c + b \cdot c$. Therefore, $(a + b) \cdot c = a \cdot c + b \cdot c$. Thus $(S, +, \cdot)$ is a semihyperring.

Let $x \neq 0 \in S$. Now, for any $y \neq 0 \in S$, we have $x \in S_x = x \cdot y \subseteq x \cdot S_y = x \cdot (y \cdot x)$. Hence the semihyperring is regular. \square

Class of Semihyperrings from Partitions of a Set

Example 3.7. The semihyperring obtained by using the Theorem 3.1 in the Example 3.1 is as follows.

+	0	a	b
0	0	a	b
a	a	{a,b}	{a,b}
b	b	{a,b}	{a,b}

.	0	a	b
0	0	0	0
a	0	S	S
b	0	S	S

Example 3.8. The semihyperrings obtained by using the Theorem 3.1 in the Example 3.2 are as follows.

+	0	a	b
0	0	a	b
a	a	{a}	{a,b}
b	b	{a,b}	{b}

.	0	a	b
0	0	0	0
a	0	S	S
b	0	S	S

.	0	a	b
0	0	0	0
a	0	{0, a}	{0,a}
b	0	{0,b}	{0,b}

Example 3.9. The semihyperrings obtained by using the Theorem 3.1 in the Example 3.3 are as follows.

+	0	a	b
0	0	a	b
a	a	{0,a}	S
b	b	S	{0,b}

.	0	a	b
0	0	0	0
a	0	S	S
b	0	S	S

.	0	a	b
0	0	0	0
a	0	{0, a}	{0,a}
b	0	{0,b}	{0,b}

Theorem 3.10. Let S be a commutative hypermonoid with the additive identity 0 with the condition that $x + y = 0$ for $x, y \in S$ implies either $x = 0$ or $y = 0$. Let $\{S_a : a \in S\}$ be a $*$ -collection on S . For $a, b \in S$, if we define a hypermultiplication on S as

$$a \cdot b = \begin{cases} S_b & \text{if } a \neq 0, b \neq 0, \\ 0 & \text{otherwise} \end{cases}$$

then $(S, +, \cdot)$ is a regular semihyperring.

Proof. The proof follows by the same lines as in the Theorem 3.1. Let $x \neq 0 \in S$. Now, for any $y \neq 0 \in S$, we have $x \in S_x = y \cdot x \subseteq S_y \cdot x = (x \cdot y) \cdot x$. Hence the semihyperring is regular. \square

Theorem 3.11. *Let S be a commutative hypermonoid with the additive identity 0 such that $x + y = 0$ for $x, y \in S$ implies either $x = 0$ or $y = 0$. Let $\{S_a : a \in S\}$ be a $*$ -collection on S such that $S_a \cap S_b = X$ for all $a \neq 0, b \neq 0 \in S$ where X is a subset of S such that $X + X = X$. For $a, b \in S$, if we define a hypermultiplication on S as*

$$a \cdot b = \begin{cases} S_a \cap S_b = X & \text{if } a \neq 0, b \neq 0, \\ 0 & \text{otherwise} \end{cases}$$

then $(S, +, \cdot)$ is a regular semihyperring.

Proof. Since $0 \in S_a$ and $0 \in S_b$, we get $0 \in S_a \cap S_b$. This implies that the set X is non-empty. From the definition of hypermultiplication, $a \cdot 0 = 0 \cdot a = 0$ for all $a \in S$. Let $a, b, c \in S$. If any one of a, b, c is 0, then $a \cdot (b \cdot c) = \{0\} = (a \cdot b) \cdot c$. If $a \neq 0, b \neq 0$ and $c \neq 0$, then $a \cdot (b \cdot c) = X = (a \cdot b) \cdot c$. Thus $(a \cdot b) \cdot c = a \cdot (b \cdot c)$. Therefore, (S, \cdot) is a semihypergroup.

If $a = 0$ or $b = 0$ or $c = 0$, then it is obvious that $a \cdot (b + c) = a \cdot b + a \cdot c$. Suppose $a \neq 0, b \neq 0$ and $c \neq 0$ then, $a \cdot (b + c) = X = X + X = a \cdot b + a \cdot c$. Similarly we have $(a + b) \cdot c = X = a \cdot c + b \cdot c$. Thus $(S, +, \cdot)$ is a semihyperring. Let $x \neq 0 \in S$. Since $x \in S_x$, we have $x \in S_x = x \cdot x \subseteq x \cdot S_x = x \cdot (x \cdot x)$. Hence the semihyperring is regular. \square

Example 3.12. Using the Theorem 3.3 in the commutative hypermonoid given in the Example 3.4 and by using the following each $*$ -collection

$S_0 = \{0\}; S_a = \{0, a\}; S_b = \{0, b, a\}; S_c = \{0, c, a\}$ with $X = \{0, a\}$,
 $S_0 = \{0\}; S_a = \{0, a, b\}; S_b = \{0, b\}; S_c = \{0, c, b\}$ with $X = \{0, b\}$,
 $S_0 = \{0\}; S_a = \{0, a, c\}; S_b = \{0, b, c\}; S_c = \{0, c\}$ with $X = \{0, c\}$, we get three hypermultiplications so that we get three semihyperrings.

4 Semihyperrings induced by a Partition.

In this section we show that a partition of a non-empty set S induces a hyperaddition $+$ such that, $(S, +)$ is a commutative hypermonoid and also the partition induces a $*$ -collection. Using this $*$ -collection, we define hypermultiplication \cdot on the set S , so that $(S, +, \cdot)$ a regular semihyperring.

Theorem 4.1. *Let S be any non-empty set and $\{P_\alpha\}_{\alpha \in \Lambda}$ be a partition of S , where Λ is an ordered set with the least element $\alpha_0 \in \Lambda$ and P_{α_0} be a singleton set, say, $\{0\}$. Define a hyperaddition " $+$ " on S as follows: For all $a \in S$, $0 + a = a + 0 = \{a\}$. For $a \neq 0, b \neq 0 \in S$, suppose $a \in P_\alpha$ and $b \in P_\beta$ and $\gamma = \max\{\alpha, \beta\}$,*

$$a + b = \begin{cases} P_\gamma & \text{if } \alpha \neq \beta, \\ P_\alpha = P_\beta & \text{if } \alpha = \beta \end{cases}$$

Class of Semihyperrings from Partitions of a Set

Then (i) $(S, +)$ is a commutative monoid and (ii) the partition $\{P_\alpha\}_{\alpha \in \Lambda}$ induces a $*$ -collection.

Proof. It is clear that $a + b = b + a$ for all $a, b \in S$. Let $a, b, c \in S$. Suppose that $a \in P_\alpha$, $b \in P_\beta$ and $c \in P_\gamma$, where $\alpha, \beta, \gamma \in \Lambda$.

Case 1 : Suppose $\alpha < \beta < \gamma$.
Then $a + (b + c) = a + P_\gamma = P_\gamma$. Also, $(a + b) + c = P_\beta + c = P_\gamma$. Therefore, $a + (b + c) = (a + b) + c$.

Case 2 : Suppose $\beta < \alpha < \gamma$.
Then $a + (b + c) = a + P_\gamma = P_\gamma$. Also, $(a + b) + c = P_\alpha + c = P_\gamma$. Therefore, $a + (b + c) = (a + b) + c$.

Case 3 : Suppose $\alpha < \gamma < \beta$.
Then $a + (b + c) = a + P_\beta = P_\beta$. Also, $(a + b) + c = P_\beta + c = P_\beta$. Therefore, $a + (b + c) = (a + b) + c$.

Case 4 : Suppose $\gamma < \alpha < \beta$.
Then $a + (b + c) = a + P_\beta = P_\beta$. Also, $(a + b) + c = P_\beta + c = P_\beta$. Therefore, $a + (b + c) = (a + b) + c$.

Case 5 : Suppose $\gamma < \beta < \alpha$.
Then $a + (b + c) = a + P_\beta = P_\alpha$. Also, $(a + b) + c = P_\alpha + c = P_\alpha$. Therefore, $a + (b + c) = (a + b) + c$.

Case 6 : Suppose $\beta < \gamma < \alpha$.
Then $a + (b + c) = a + P_\gamma = P_\alpha$. Also, $(a + b) + c = P_\alpha + c = P_\alpha$. Therefore, $a + (b + c) = (a + b) + c$.

Case 7 : Suppose $\alpha = \beta < \gamma$.
Then $a + (b + c) = a + P_\gamma = P_\gamma$. Also, $(a + b) + c = P_\beta + c = P_\gamma$. Therefore, $a + (b + c) = (a + b) + c$.

Case 8 : Suppose $\gamma < \alpha = \beta$.
Then $a + (b + c) = a + P_\alpha = P_\alpha$. Also, $(a + b) + c = P_\alpha + c = P_\alpha$. Therefore, $a + (b + c) = (a + b) + c$.

Case 9 : Suppose $\alpha = \gamma < \beta$.
Then $a + (b + c) = a + P_\beta = P_\beta$. Also, $(a + b) + c = P_\beta + c = P_\beta$. Therefore, $a + (b + c) = (a + b) + c$.

Case 10 : Suppose $\beta < \alpha = \gamma$.
Then $a + (b + c) = a + P_\alpha = P_\alpha$. Also, $(a + b) + c = P_\alpha + c = P_\alpha$. Therefore, $a + (b + c) = (a + b) + c$.

Case 11 : Suppose $\beta = \gamma < \alpha$.
Then $a + (b + c) = a + P_\gamma = P_\alpha$. Also, $(a + b) + c = P_\alpha + c = P_\alpha$. Therefore, $a + (b + c) = (a + b) + c$.

Case 12 : Suppose $\alpha < \beta = \gamma$.
Then $a + (b + c) = a + P_\gamma = P_\gamma$. Also, $(a + b) + c = P_\beta + c = P_\gamma$. Therefore, $a + (b + c) = (a + b) + c$.

Case 13 : Suppose $\alpha = \beta = \gamma$.
 Then $a+(b+c) = P_\alpha = P_\beta = P_\gamma = (a+b)+c$. Therefore, $a+(b+c) = (a+b)+c$.
 Thus the hyperoperation $+$ is associative. So, $(S, +)$ is a commutative hypermonoid. Let $S_0 = P_{\alpha_0} = \{0\}$. For $a \neq 0 \in S$, then $S_a = \bigcup_{\alpha_0 \leq t \leq \alpha} P_t$ where $a \in P_\alpha$. It is clear that $S_a = \bigcup_{x \in S_a} S_x$. For $a \neq 0 \in S$, and $a \in P_\alpha$, then $S_a + S_a = \bigcup_{\alpha_0 \leq t \leq \alpha} P_t + \bigcup_{\alpha_0 \leq t \leq \alpha} P_t = \bigcup_{\alpha_0 \leq t \leq \alpha} P_t = S_a$. Also $S_0 + S_0 = \{0\} + \{0\} = \{0\} = S_0$. If either $a = 0$ or $b = 0$, then $\bigcup_{x \in a+b} S_x = S_a + S_b$. Let $a \neq 0, b \neq 0 \in S$. Then $a \in P_\alpha$ and $b \in P_\beta$ for some $\alpha, \beta \in \Lambda$.

Case 1 : Suppose $\alpha \neq \beta$, say $\alpha < \beta$, then $a + b = P_\beta$. Now $x \in a + b$ implies $x \in P_\beta$. Therefore, $S_x = \bigcup_{\alpha_0 \leq t \leq \beta} P_t$. Hence

$$\bigcup_{x \in a+b} S_x = \bigcup_{x \in a+b} \left(\bigcup_{\alpha_0 \leq t \leq \beta} P_t \right) = \bigcup_{\alpha_0 \leq t \leq \alpha} P_t = \bigcup_{\alpha_0 \leq t \leq \alpha} P_t + \bigcup_{\alpha_0 \leq t \leq \beta} P_t = S_a + S_b.$$

Case 2 : Suppose $\alpha = \beta$ then $a + b = P_\alpha$. Therefore, $\bigcup_{x \in a+b} S_x = \bigcup_{x \in P_\alpha} S_x = S_a + S_b$. Therefore, $\bigcup_{x \in a+b} S_x = S_a + S_b$. Thus $\{S_a : a \in S\}$ is a $*$ -collection. □

Remark 4.2. Let S be any non-empty set and $x_0 \in S$. Let $P_0 = \{x_0\}$ and $\{P_1, P_2, P_3, \dots, P_n, \dots\}$ be a partition of $S \setminus \{x_0\}$. Then the partition $\{P_0, P_1, P_2, \dots, P_n, \dots\}$ of S induces a hyperoperation $+$ on S so that $(S, +)$ is a commutative hypermonoid and $\{P_0, P_1, P_2, \dots, P_n, \dots\}$ induces a $*$ -collection.

Theorem 4.3. Let S be any non-empty set and $\{P_\alpha\}_{\alpha \in \Lambda}$ be a partition of S , where Λ is an ordered set with the least element α_0 and P_{α_0} is a singleton set. Then the partition induces a semihyperring.

Proof. By the Theorem 4.1, the partition induces a hyperaddition $+$ such that $(S, +)$ is a commutative hypermonoid and it also induces a $*$ -collection. Hence by the Theorem 3.1, we get a regular semihyperring. □

Example 4.4. We illustrate the construction of semihyperrings from the following examples. Let $S = \{0, a, b\}$. Consider a partition $P_1 = \{0\}, P_2 = \{a\}, P_3 = \{b\}$ of S . Here, the indexing set is $\Lambda = \{1, 2, 3\}$ which is an ordered set. The commutative hypermonoid induced by this partition is given by the following Caley table.

$+$	0	a	b
0	0	a	b
a	a	$\{a\}$	$\{b\}$
b	b	$\{b\}$	$\{b\}$

Class of Semihyperrings from Partitions of a Set

The $*$ -collection induced by this partition is $S_0 = \{0\}$, $S_a = \{0, a\}$, $S_b = \{0, a, b\}$ and the hypermultiplication induced by the $*$ -collection is given in the Cayley table.

.	0	a	b
0	0	0	0
a	0	$\{0, a\}$	$\{0, a\}$
b	0	$\{0, a, b\}$	$\{0, a, b\}$

Example 4.5. Let $S = \{0, a, b\}$. Consider a partition $P_1 = \{0\}$, $P_2 = \{b\}$, $P_3 = \{a\}$ of S . Here, the indexing set is $\Lambda = \{1, 2, 3\}$ which is an ordered set. The commutative hypermonoid induced by this partition is given by the following Cayley table.

+	0	a	b
0	0	a	b
a	a	$\{a\}$	$\{a\}$
b	b	$\{a\}$	$\{b\}$

The $*$ -collection induced by this partition is $S_0 = \{0\}$, $S_a = \{0, a, b\}$, $S_b = \{0, b\}$ and the hypermultiplication induced by the $*$ -collection is given in the Cayley table.

.	0	a	b
0	0	0	0
a	0	$\{0, a, b\}$	$\{0, a, b\}$
b	0	$\{0, b\}$	$\{0, b\}$

Example 4.6. Let $S = \{0, a, b\}$. Consider a partition $P_1 = \{0\}$, $P_2 = \{a, b\}$ of S . Here, the indexing set is $\Lambda = \{1, 2\}$ which is an ordered set. The commutative hypermonoid induced by this partition is given by the following Cayley table.

+	0	a	b
0	0	a	b
a	a	$\{a, b\}$	$\{a, b\}$
b	b	$\{a, b\}$	$\{a, b\}$

The $*$ -collection induced by this partition is $S_0 = \{0\}$, $S_a = \{0, a, b\}$, $S_b = \{0, a, b\}$ and the hypermultiplication induced by the $*$ -collection is given in the Cayley table.

.	0	a	b
0	0	0	0
a	0	$\{0, a, b\}$	$\{0, a, b\}$
b	0	$\{0, a, b\}$	$\{0, a, b\}$

Thus we have a regular semihyperring.

Conclusion : In the section 3 of this paper, for the given commutative hypermonoid, given $*$ -collection, we construct three semihyperrings. In the section 4, by the Theorem 4.1, a partition of a set S induces a hyperaddition $+$ such that $(S, +)$ is a commutative hypermonoid and it also induces a $*$ -collection. Hence by the Theorem 3.1, we get a semihyperring. Thus we get semihyperrings depending on the partitions of the set satisfies the conditions of the Theorem 4.2. All the semihyperrings so constructed are regular.

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Class of Semihyperrings from Partitions of a Set

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