Using COMSOL Multiphysics To Simulate Radiation from Dipole Antenna and First Iteration Cantor Set Shape Antenna

Nasibeh Delfan[1], Amir Pishkoo[2], Mahdi Azhini[1]

[1]Department of Mathematics, Science and Research branch, Islamic Azad University, Tehran, Iran
[2]Physics and Accelerators Research School, Nuclear Science and Technology Research Institute,P.O. Box 14395-836, Tehran, Iran

apishkoo@gmail.com

Abstract

The method of electromotive force (emf) for calculation of antenna impedance is applicable to a very limited number of antenna configurations such as monopole and dipole antenna. In more general antenna structures, especially when complicated geometry such as fractals is involved numerical techniques are essential. There are Electromagnetic (EM) simulation softwares such as G-NEC, XFDTD for the modelling studies in which an understanding of the fundamental theory behind these numerical techniques in creating the antenna models is essential. In this paper, we compare the radiation from dipole antenna and the antenna with shape of the first iteration of Cantor set. The simulation of this problem has been done by COMSOL Multi-physics software.

Keywords: c.

1 Introduction

Antennas are essentially narrow-band devices. Their behaviour is highly dependent on the antenna size to the operating wavelength ratio. This means that for a fixed antenna size, the main antenna parameters (gain, input impedance, pattern shape, secondary lobe level, and distribution) will suffer strong variations when changing the operating frequency. The frequency dependence also implies that an antenna has to keep a minimum size relative to wavelength to operate efficiently. That is, given a particular frequency, the antenna cannot be made arbitrarily small: it usually has to keep a minimum size, typically on the order of a quarter wavelength [1]. The quest for smaller sized resonant antennas has been on for decades. Several of antenna design principles may have to be modified while dealing with small-sized antennas. By convention, a small antenna is defined as one occupying a fraction (typically $< \frac{1}{6}$) of the wavelength [2]. The primary concern in their design is in the impedance matching. This is better explained in the context of dipole antennas. As the length of a dipole antenna is reduced, the real part of its input impedance approaches zero, while the imaginary part tends to be an extremely large negative number. This causes major reflections at the input terminal as the transmission line connected to it generally has a standard characteristic impedance (50). For a thin cylindrical antenna of length $2l$ and radius a the current on the antenna is assumed to be [3]

$$I = I_m \sin(\beta(l - |z|), \tag{1}$$

where $\beta = \frac{2\pi}{\lambda}$ is wave number. It is also assumed that this current is distributed symmetrically around the axis of the cylinder. It leads to a current distribution of

$$J_z = \frac{I}{2\pi a}. \tag{2}$$

It can be shown that the electric field is given by [4].

$$E_z = -j\frac{\eta I_0}{4\pi}[2\frac{e^{-j\beta r}}{r}\cos\beta l - \frac{e^{-j\beta R_1}}{R_1} - \frac{e^{-j\beta R_2}}{R_2}], \tag{3}$$

where $r = \sqrt{x^2 + y^2 + z^2}$, $R_1 = \sqrt{x^2 + y^2 + (z - l)^2}$, $R_2 = \sqrt{x^2 + y^2 + (z + l)^2}$. It may be noted that when the radius of the antenna approaches zero, the resonant length of the antenna may be obtained as $2l = \frac{\lambda}{2}$.

1.1 Fractal antenna

A fractal is a mathematical form showing self-repeating patterns [5, 6]. A fractal is a shape made of parts similar to the whole in some way such as the following "Cantor mass distribution" (see Fig. 1) and "Cantor charge distribution" (see Fig. 2) which represent the 1st, 2nd and 3rd iterations. The simplest way to define a fractal is as an object which appears self-similar under varying degrees of magnification, and in effect, possessing symmetry across scale, with each small part of the object replicating the structure of the whole. In fact, self-similarity can also be understood as the property by which the fractal is found inside the fractal itself but at smaller scale [1]. The aim of this paper is to compare the radiation from an ideal dipole antenna and the radiation from two dipole antennas which their size is 1/3 (the first iteration). Our simulation has been done in COMSOL Multi-physics software. Many electrical, mechanical, fluid, and chemical simulations have been done by using COMSOL Multiphysics software. It's name is "Multi-physics" because in this software all relevant physical effects can be included and coupled. Fortunately, all industries benefit from Multiphysics simulations. COMSOL helps student to be familiar with a complete simulation environment. In fact, virtual testing is much faster than testing physical prototypes. "Electromagnetic waves, frequency Domain (emw) interface" is one of the many interfaces which can be seen and used in COMSOL. All problems are solved by the numerical method called Finite Element Method (FEM) in branch of numerical analysis. FEM is widely used as a numerical method for the solution of PDE problems. It divides the solution domain into simply shaped regions called "elements". An approximate solution for the PDE can be developed for each of these elements. Then, the total solution can be generated by linking together the individual solutions such that to be ensure continuity at the inter-element boundaries. We deal with the system matrix which is merely a set of n simultaneous equations that can be used to solve for the values of the dependent variable at the n nodes. The implementation of FEM usually follows a standard step-by-step procedure that is started with "discretization" and then "element equations". The finite element method started with significant promise in the modelling of several mechanical applications related to aerospace and civil engineering. This well-known software was chosen for our student because she could do mathematical modelling and physical modelling together. One can setup his/her model, optimize design, and even share it as a simulation application. This paper also helps one to be familiar with COMSOL's environment.

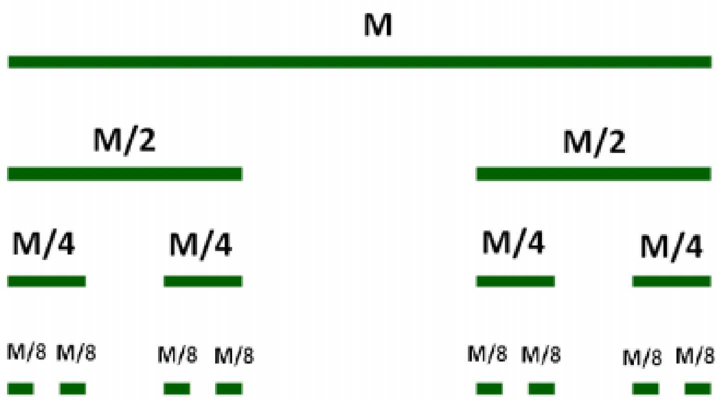

Figure 1: Cantor mass distribution

The most simple and recognizable fractal is Cantor set which demonstrates the property of self-similarity. Its fractal dimension is $d = \frac{\log 2}{\log 3} = 0.63$. It is used as the diffraction grating among notable applications in physics. The mathematical language in fractal geometry is IFS and by combining geometric transformations such as translation, rotation, and scaling, fractal structures can be formed [7, 8]. In the most comprehensive form, "fractal theory" has been formed and used for fractured material, fluids, physics, chemistry, electronics and graphics. A fractal is a shape made of parts similar to the whole in some way. In the past, antennas had simple form based on Euclidean geometry. The use of fractal shaped geometries in antennas is one of many areas of science and engineering which has been well-known. In fact, the use of fractal geometries has improved antenna features to varying extends. Depending on antenna performance, fractal shaped geometries have been proved to result in multi-band characteristics and reduction of antenna size. The space filling nature of some fractal shapes lead to significant reduction in antenna size. Recent

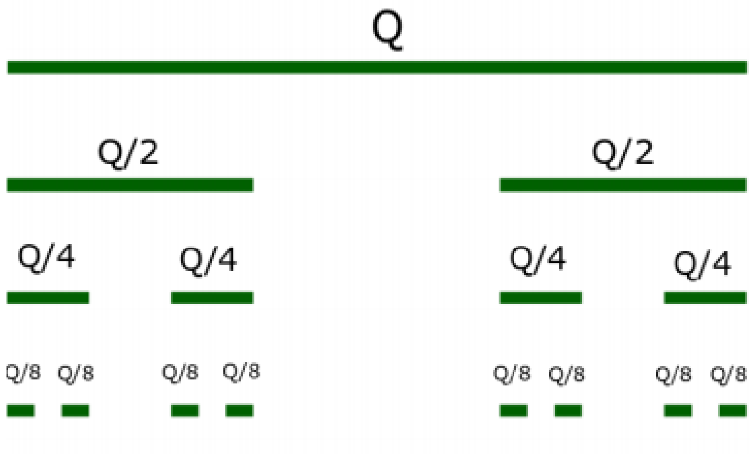

Figure 2: Cantor charge distribution

efforts by several researchers around the world to combine fractal geometry with electromagnetic theory have led to a plenty of new and innovative antenna designs. By virtue of its geometrical properties, a fractal structure can generate multiple resonances in RF applications. Fractal antennas do not follow the Euclidean geometry design [1, 3, 9–15]. One can see the form of input impedance, its real part and the reactive component in [3]. It shows that by properly choosing the antenna length, its reactance can be made zero. In this paper, the radiation from the dipole antenna and the antenna with shape of the first iteration of the Cantor set are investigated.

2 Main Results

The dipole antenna is one of the most straightforward antenna configurations. It can be realized with two thin metallic rods that have a sinusoidal voltage difference applied between them. The length of the rods is chosen such that they are quarter wavelength elements at the operating frequency. Such an antenna has a well-known torus-like radiation pattern.
In high-frequency systems, the wave length is comparable to the size of the system. Here $\lambda_0 = 4m$ and height of dipole antenna is $1m$. Using COMSOL Multiphysics software, in this paper one of the “Radio Frequency” interfaces is used to compute the electric field in high-frequency system. This interface is called “Frequency Domain” which is used to solve for time-harmonic electromagnetic field fractal distribution (Cantor set distribution). This type of physics interface solves time-harmonic wave equation for electric field $\overrightarrow{E}$. The governing equation is

$$\frac{1}{\mu_r}\overrightarrow{\nabla}(\times\overrightarrow{\nabla}\times\overrightarrow{E}) - \kappa_0^2(\varepsilon_r - \frac{j\sigma}{\omega\epsilon_0})\overrightarrow{E} = 0, \tag{4}$$

for the time-harmonic and eigen frequency problems. In the above equation μ_r, σ, and ε_r are relative permeability, electrical conductivity and relative permittivity, respectively. The wave number of free space namely κ_0 is defined as $\kappa_0 = \frac{\omega}{c}$, where c is the speed of light in vacuum.
Mathematical and physical modelling in COMSOL (Model builder) is performed in the following steps:

1. To define “parameters”
2. To draw “geometry”
3. To specify “materials”
4. To choose “interface(s)” and type of study
5. To choose “mesh” size
6. To compute “numerical calculations”

7. To choose "results" and postprocessing

In "Model Builder", air and copper have been chosen for materials. Under a "Geometry node" one can define and create the geometry sequence for the mode component.

In COMSOL Multiphysics Finite Element Method (FEM) is used to do numerical calculations. As it is shown in

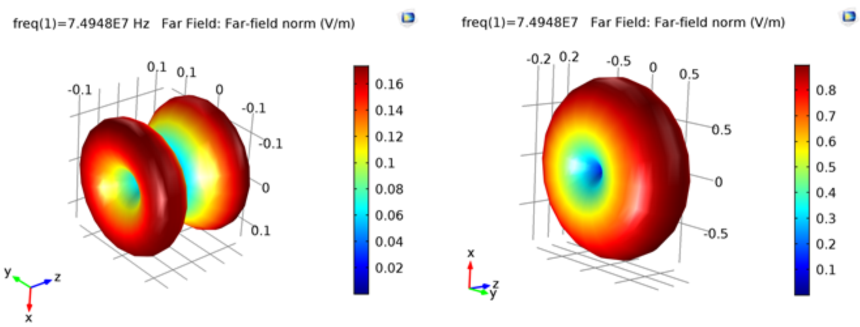

Figure 3: Far field norm for the dipole antenna (right) and the antenna with shape of first iteration Cantor set (left)

Fig. 3, we have used "Far Field" plot for its electromagnetic field for which $f = 7.4948 \times 10^7 Hz$. In Fig. 3 (right hand side) three dimensional radiation pattern of an ideal dipole can be seen. If middle third of an antenna is removed at the first iteration, then two smaller dipole antennas are made. Their radiation pattern in Fig. 3 (left hand side) has different graphical representation. Far fields are computed in an effort to plot the radiation pattern and gain of the antenna. One of the variables in the RF Module is the far field norm which is demonstrated in Fig. 4. "Far Field"

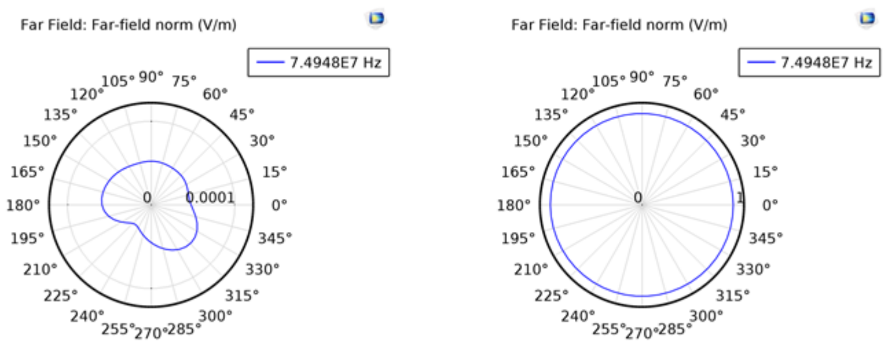

Figure 4: Far field plotted for a selected number of angles on a circle

plots are used to plot the value of a global variable which is here far field of an electromagnetic field. This variable is plotted for a selected number of angles on a circle (in 2D). An ideal dipole as an isotropic radiator radiates power with unit gain uniformly in all directions (right hand side in Fig. 4). However, when middle third of an antenna is removed some lobes are formed (left hand side in Fig. 4). Convergence plots use graphics to show how an error estimate or time step evolves during the solution process for nonlinear, time-dependent, and parametric solvers. These plots show an error estimate against the iteration number. Fig. 5 gives convergence plot which shows how an error estimate evolves during the solution process. The numbers appear in the Log window. For our configuration, solution time was $68s$ while number of degrees of freedom solved for 712192. In the "parameters" section we enter parameters manually. Furthermore, definitions which in someway refer to domains in a geometry are under the component branch (see Fig. 6). Under a "Geometry node" we define and create the geometry sequence for the model component which includes one sphere and two cylinders (see right side in Fig. 7). In Model Builder the "Materials column" lists our materials which includes "air" and "copper" (see left side in Fig. 7). When adding a physics interface, we choose the "Electromagnetic waves, frequency Domain (emw) interface" (see left side in Fig. 8). When a new component is

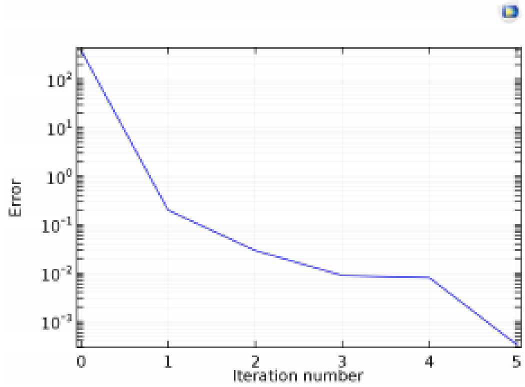

Figure 5: Convergence plot shows how an error estimate evolves during the solution process

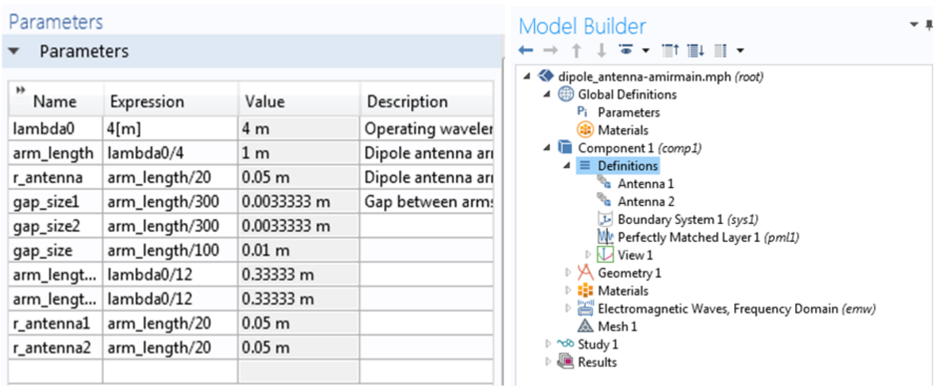

Figure 6: To introduce "parameters" and "definitions" in Model Builder

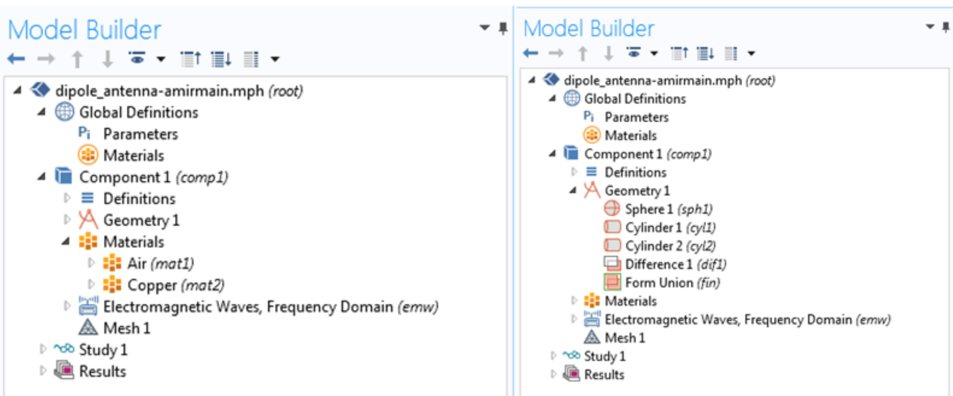

Figure 7: Used materials (air and copper) and defined geometry in "Model Builder"

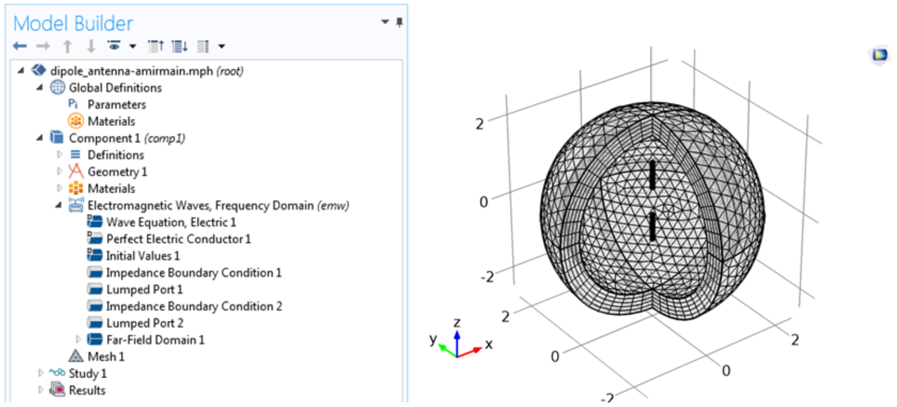

Figure 8: The RF module and used Mesh in Finite Element Method (FEM)

inserted to the Model Builder, a meshing sequence is added by default in the node “Mesh1” (see right side in Fig. 6). Finite Element Method (FEM) as other numerical methods are used to provide engineering analysis. It divides the body into smaller pieces. A solution can be computed depending on the sequence implemented for our model. The

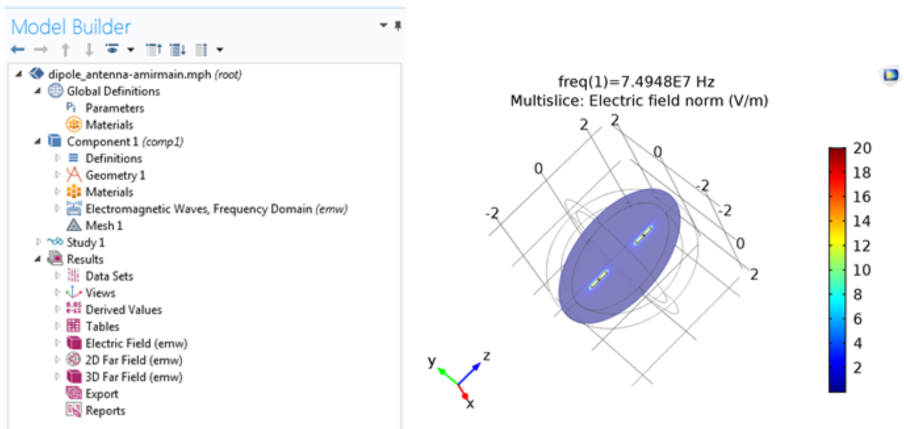

Figure 9: Obtained “Results” for the second configuration antenna (the first iteration of Cantor set)

“Results branch” in the COMSOL Multiphysics model tree contain tools for postprocessing and analyzing the results from our simulations (see Fig. 9).

3 Conclusion

In this paper, the rule of forming Cantor set (for just the first iteration) is used to compare the radiation of single ideal dipole and two smaller ideal dipoles made in the first iteration. The physical and mathematical modelling and simulation has been done in COMSOL Multiphysics software (Model builder) which is very convenient for solving many kinds of problems in engineering, physics, and mathematics has been explained and demonstrated by figures. Further research in this field can be done for Cantor set or other type of fractals such as Koch curve, complex tree fractals, etc. and for second, third , and upper iterations in future by applying COMSOL.

4 Acknowledgments

The authors are grateful to the referees and the editor for valuable remarks which contributed to the improvement of the paper. Miss Nasibeh Delfan is Ph.D. student in mathematics whose supervisor is Dr. Amir Pishkoo and her advisor is Dr. Mahdi Azhini.

5 Conflicts of Interest

The authors declare that there are no conflicts of interest regarding the publication of this paper.

References

[1] A., Pishkoo, M., Darus, R., Pashaei, M. S., Asgari, From calculating fractal dimension to fabrication of fractal antennas, *Journal of Computational and Theoretical Nanoscience*, 13, pp. 4323-4330, 2016.

[2] H.A. Wheeler, "Small Antennas," *IEEE Trans. Ant. Propagat*, vol. AP-23, pp. 462-469, 1975.

[3] K. J. Vinoy, Fractal shaped antenna elements for wide- and multi- band wireless applications, Ph.D. thesis,2002.

[4] R.E. Collin and F.J. Zucker, Antenna Theory Pt. 1, New York: McGraw-Hill, 1969.

[5] Mandelbrot, B.B. *The Fractal Geometry of Nature*. W. H. Freeman and Company, 1977.

[6] Falconer, K. *Fractal Geometry: Mathematical Foundations and Applications*. Wiley; 2^{nd} ed., 2007.

[7] W.J. Krzysztofik. Fractal geometry in electromagnetics applications-from antenna to metamaterials. *Microwave Review*, 19(2), 2013.

[8] W.J. Krzysztofik and M. Baranski. Fractal structures in antenna technology. *Scientific Books of Gdansk University of Technology*, 1, pp.381-384, 2007.

[9] M. A. Dorostkar, M. T. Islam, and R. Azim, Design of a novel super wide band circular-hexagonal fractal antenna, *Progress in Electromagnetics Research*, 139, pp. 229-245, 2013.

[10] M. A., Dorostkara, R. Azima, M. T., Islama, A novel Γ-shape fractal antenna for wideband communications, *Procedia Technology*, 11, pp.1285-1291, 2013.

[11] N. K. Darimireddy, R. R. Reddy, and A. M. Prasad, A miniaturized hexagonal-triangular fractal antenna for wide-band applications, *IEEE Antennas & Propagation Magazine*, pp. 104-110, April 2018.

[12] F. Mokhtari-Koushyar, P. M. Grubb, M. Y. Chen, and R. T. Chen, A miniaturized tree-shaped fractal antenna printed on a flexible substrate, *IEEE Antennas & Propagation Magazine*, June 2019.

[13] S. Singhal, Elliptical ring terahertz fractal antenna,*Optik*, Vol. 194, 2019, https://doi.org/10.1016/j.ijleo.2019.163129.

[14] T. R. S. Kumar, K.J. Vinoy, Tri-band band-stop frequency selective surface using tortuous jerusalem cross with angularly stable response, 2018 *IEEE Indian Conference on Antennas and Propogation (InCAP)*, https://doi.org/10.1109/INCAP.2018.8770705.

[15] P. S. Nalumakkal, K. M. Reddy, K.J. Vinoy, S. Shukla, Wideband stripline fed tapered slot antenna with integral coupler for wide scan angle active phased array, *IET Microwaves, Antennas & Propagation*, Vol. 12 Iss. 9, pp. 1487-1493, 2018, https://doi.org/10.1049/iet-map.2017.0784.