The Effect of Energy Transportation of High-Energy Electrons on the Electromagnetic Instability

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Abstract

The energy transportation of High-energy electrons to the compressed fuel and study of affected factors on it are the most important issues in fast ignition method. In this research, regarding the role of Weibel instability in process modulation and possible impacts of electron and plasma beam velocity to reach higher energy yields, the effect of temporal variation of particle distribution in the presence of laser electric fields on growth and condition of Weibel instability in beam-plasma medium were investigated in the form of a classic system without any Coulombic collision between the particles and magnetic fields. The results show that the time-dependent drift velocity leads to a decrease the growth rate of the Weibel , when energy transportation of energetic particles to the fusion plasma systems increases.

Keywords: Weibel Instability; Velocity Distribution Anisotropic; Vlasov Equation; Drift Velocity

1 Introduction

The presence of different instabilities and their Wrecker effects is one of important problems in fusion schemes by inertial confinement (during the compression and explosion stages). Besides the Rayleigh-Taylor instabilities and their Wrecker effects [1], another group of kinetic and reaction instabilities can be presented acting as a preclude for the transport of sufficient energy to the heart of compressed fuel and the formation of the hot spot. In inertial confinement fusion (ICF) targets, produced by an intense laser pulse, the incident laser wave [2-4] produces anisotropy in the induced plasma temperature. This is due to the fact that the plasma is preferentially heated in the direction of laser wave electric field. It has been shown that this anisotropic distribution provokes unstable Weibel electromagnetic modes [5-9]. If this instability is excited, strong (giga gauss) magnetic fields could be generated from the magnetic fields germ due to the electron thermal motion[10]. The Weibel instability is one of the most well known plasma instabilities. It arises in homogeneous plasmas possessing an electron velocity space anisotropy.The Weibel instability is an electromagnetic plasma instability driven by the presence of temperature or electron momentum anisotropy. Erich Weibel was the first who predicted the spontaneous growing of the transverse quasi-static electromagnetic waves which appear in plasma, due to an anisotropic velocity distribution of electrons[11]. This is referred to as macroscopic scale such as temperature anisotropy[11-12]. In laser fusion plasma and in particular in fast combustion, Weibel instability can occur due to presence of fast-propagating electron beams (with the energy order of several mev) formed during processes such as high amplitude plasma waves or in more general ways during instant high energy processes. Weibel instability is of crucial importance as this instability can result in negative effects on electron beam modulation and also abnormal heating of the system due to formation of highly strong magnetic fields[13]. Presence of external electric fields such as laser-produced fields, due to crucial impacts on energy distribution of the particles and therefore affecting the time-dependent drift velocities, along with other effective parameters on the particle beam transmission process, can play an important role in stabilization of the electrostatic instabilities of plasma environments[14]. Numerous studies have addressed Weibel instability[15-17]. Most of them have addressed Weibel instability in classic plasma without the presence of electric fields. Therefore, this study presents a new model by consideration of time-dependent drift velocity due to laser electric field.

In this paper, the focus is on growth rate of the Weibel instability in the beam-plasma model, where a beam of energetic relativistic electrons are penetrated into a strongly coupled plasma with non-relativistic electrons. These results highlight new terms in dispersion relation, due to the coupling between the laser electric field and the resulting magnetic field by Weibel instability. These terms contribute to the instability and the convection of Weibel modes. We consider an inhomogeneous plasma in interaction with a high frequency and a low magnitude laser field. Then the distribution function from the anisotropic Fokker-Planck equation is calculated. For this, the method of separation of time scale is used. After that, the linear part of the Fokker-Planck equation associated with the disruption of the distribution function is solved and the dispersion relation of the Weibel modes is established. It is analytically tried to calculate the growth rate of instability using two different models of distribution functions governing on each kind of electrons. Solving the dispersion relation leads to calculation of the instability growth rate.

2 Mathematical method

Electric and magnetic fields presented in plasma media can be presented as: $\mathbf{E} = \mathbf{E}_h + \mathbf{E}_s$ and $\mathbf{B} = \mathbf{B}_h + \mathbf{B}_s$, where $\mathbf{E}_h$ and $\mathbf{B}_h$ represent the high-frequency fields associated with the laser wave, and $\mathbf{E}_s$ and $\mathbf{B}_s$ are low-frequency fields associated with the disturbance in plasma. The contribution of high-frequency laser wave magnetic field $\mathbf{B}_h$ can be neglected compared with the contribution of the laser wave high-frequency electric field, $\mathbf{E}_h$. Value of the time dependent $\mathbf{E}_h$ is supposed to follow a normal mode

$$E_h = E_x Re[\exp(iw_l t)] \tag{1}$$

where E_x and w_l are the complex magnitude and the frequency of the laser wave respectively[15]. Cycloidal trajectories are described by ions and electrons in crossed electric and magnetic fields. The electric field $\mathbf{E}$ acting together with the magnetic flux density $\mathbf{B}$ gives rise to a drift velocity in the direction given by $\mathbf{E} \times \mathbf{B}$[16].

Let us choose a Cartesian coordinate system with the z, x axis pointing in the direction of $\mathbf{B}$ and $\mathbf{E}$ respectively, so that

$$\upsilon = \upsilon_0 + \frac{e}{m}\int_0^t E_h dt \tag{2}$$

The unperturbed velocity υ is driven by the time-dependent externally applied electrical field E_h. This setup has been experimentally demonstrated by Decker and Levin[17].

Weibel instability problem can be solved by two different models to describe the distribution of electrons. In both models, the plasma electron velocity distribution is considered as a bi-Maxwell distribution, while two different modes of bi-Maxwell distribution and Delta distribution will be used for beam electrons. The anisotropy created in the system will be due to the arrival of the relativistic beam electron, which is due to the appearance of recurring flows due to the linear response of the plasma to the presence of an electron beam as a result of quasi-neutral definitions. A system involving a relativistic beam electron and cold plasma containing non- relativistic electrons is considered. The distribution function is a bi-maxwell distribution:

$$\begin{aligned} f_0(p_x,p_y) = {} & \frac{1}{2\pi m(T_x^p T_y^p)^{\frac{1}{2}}}\exp[-\frac{(p_x + p\star_d^p)^2}{2mT_x^p} - \frac{p_y^2}{2mT_y^p}] \\ & + \frac{1}{2\pi m\gamma(T_x^b T_y^b)^{\frac{1}{2}}}\exp[-\frac{(p_x - p\star_d^b)^2}{2m\gamma T_x^b} - \frac{p_y^2}{2m\gamma T_y^b}] \end{aligned} \tag{3}$$

where the velocity of leakage of the laser field for two electron species is applied using Eqs. (1 and 2) $p_d^{\star j} = p_d^j + (eE_x)/\omega sin(w_l t)$, $j = p, b$. By linearization of relativistic and non- relativistic Vlasov equation for each of the different types of plasma, the beginning of the dispersion relation can be presented as [18-19]:

$$\begin{aligned} & \omega^2 - c^2k^2 - \frac{4\pi n_p e^2}{k}\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty}\frac{p_x}{p_y}\left[(\omega - \frac{kp_y}{m})(\frac{\partial f_0^p}{\partial p_x}) + \frac{kp_x}{m}(\frac{\partial f_0^p}{\partial p_y})\right]dp_x dp_y \\ & -\frac{4\pi n_b e^2}{m}\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty}\frac{p_x}{(\frac{kp_y}{m} - 2\omega)}\left[(\omega - \frac{kp_y}{\gamma m})(\frac{\partial f_0^b}{\partial p_x}) + \frac{kp_x}{\gamma m}(\frac{\partial f_0^b}{\partial p_y})\right]dp_x dp_y = 0 \end{aligned} \tag{4}$$

where k and ω are wave number and frequency of wave of instability, respectively. Calculating each of the quantities $(\frac{\partial f_0}{\partial p_x})$ and $(\frac{\partial f_0}{\partial p_y})$ relevant to both kinds of electrons and replacing the obtained results in Eq. (4), the dispersion relation will be rewritten as;

$$1 - \frac{c^2k^2}{\omega^2} - \frac{\omega_p^2}{\omega^2} + \frac{(T_x^p + \frac{(p\star_d^p)^2}{m})}{T_y^p}\frac{\omega_p^2}{\omega^2} + \frac{(T_x^b + \frac{(p\star_d^b)^2}{m\gamma})}{T_y^b}\frac{\omega_b^2}{\omega^2}\xi Z(\xi)$$

$$+\frac{(T_x^b+\frac{(p\star_d^b)^2}{m\gamma})}{T_y^b}\frac{\omega_b^2}{\omega^2}-\frac{\omega_b^2}{\omega^2}\xi Z(\xi)-\frac{\omega_b^2}{\omega^2}+\frac{\omega_b^2}{2\omega^2}\xi Z(\xi)=0 \quad (5)$$

where ω_p is the non-relativistic electron frequency of plasma and ω_b is the relativistic electron beam frequency. Here, function $Z(\xi)$ is the dispersion function of relation plasma with $x=\frac{p_y}{\sqrt{(2m\gamma T_y^b)}}$ and $\xi=\frac{\omega}{k}\sqrt{\frac{2m\gamma}{T_y^b}}$ is defined as;

$$Z(\xi)=\frac{1}{\sqrt{\pi}}\int_{-\infty}^{+\infty}\frac{\exp(-x^2)}{(x-\xi)}dx \quad (6)$$

Introducing the new quantities, a new image of the dispersion relationship can be displayed. For this $A=\frac{(T_x^p+\frac{(p\star_d^p)^2}{m})}{T_y^p}$ and $D=\frac{(T_x^b+\frac{(p\star_d^b)^2}{m\gamma})}{T_y^b}$, whereby the dispersion relation can be rewritten as:

$$1+\frac{\omega_p^2}{\omega^2}[A-1]+\frac{\omega_b^2}{\omega^2}\left[(1+\xi Z(\xi))(D-1)+\frac{\xi Z(\xi)}{2}\right]-\frac{c^2k^2}{\omega^2}=0 \quad (7)$$

A wave propagates in plasma environment when its frequency is higher than the frequency of the plasma. The Weibel unstable wave is a low-frequency wave ($|\omega|\ll ck$), therefore, the specific limiting condition, $|\xi|\gg 1$, can not be suitable and the limiting $|\xi|\ll 1$ is suitable for discussion. For the limiting condition $|\xi|\ll 1$, the corrected configuration of the plasma dispersion function can be presented as:

$$Z(\xi)=-2\xi+\ldots+i\sqrt{\pi}\exp(-\xi^2) \quad (8)$$

Since the growth rate instability is defined as $\delta=Im\omega$, Eq. (7) is corrected as follows, by considering that in the limit $\xi^2\ll 1$, ω^2 can be neglected against c^2k^2. δ can be obtained as;

$$\delta_{bi-Max}=k[\omega_p^2(A-1)+\omega_b^2(D-1)-c^2k^2]\left(\omega_b^2(D-\frac{1}{2})\sqrt{\frac{2\pi m\gamma}{T_y^b}}\right)^{-1} \quad (9)$$

Then, the dispersion relation can be rewritten as;

$$\delta_{bi-Max}=\sqrt{\frac{T_y^b}{2\pi m\gamma}}k\left(\eta-\frac{(1+\frac{c^2k^2}{\omega_p^2})}{\mu}\right) \quad (10)$$

where, η and μ functions are defined as;

$$\mu=(D-\frac{1}{2})\frac{(\omega_b^2)}{(\omega_p^2)};\quad \eta=1-\frac{[\frac{1}{2}\frac{\omega_b^2}{\omega_p^2}]-A}{(D-\frac{1}{2})\frac{\omega_b^2}{\omega_p^2}} \quad (11)$$

wherer reagent parameters are temperature dependent anisotropy fractions. Then, the condition of the growth rate for the Weibel instability can be obtained as;

$$\eta\gg\frac{(1+\frac{c^2k^2}{\omega_p^2})}{\mu} \quad (12)$$

Similar to bi-Maxwellian distribution, the Weibel instability growth rate in the presence of a delta-like distribution for the beam electrons is calculated as;

$$1-\frac{c^2k^2}{\omega^2}-\frac{\omega_p^2}{\omega^2}+\frac{(T_x^p+\frac{(p\star_d^p)^2}{m})}{T_y^p}\frac{\omega_p^2}{\omega^2}-\frac{(p\star_d^b)^2}{mT_y^b}\frac{\omega_b^2}{\omega^2}\xi Z(\xi)$$
$$+\frac{(p\star_d^b)^2}{m\gamma T_y^b}\frac{\omega_b^2}{\omega^2}-\frac{\omega_b^2}{\omega^2}\xi Z(\xi)-\frac{\omega_b^2}{\omega^2}+\frac{\omega_b^2}{2\omega^2}\xi Z(\xi)=0 \quad (13)$$

With the introduction of new quantity, $D' = \frac{(p\star_d^b)^2}{m\gamma T_y^b}$ which can be rewritten as;

$$1 + \frac{\omega_p^2}{\omega^2}[A-1] + \frac{\omega_b^2}{\omega^2}\left[(1+\xi Z(\xi))(D'-1) + \frac{\xi Z(\xi)}{2}\right] - \frac{c^2k^2}{\omega^2} = 0 \tag{14}$$

Similarly, the growth rate is corrected as;

$$\delta_{delta} = k[\omega_p^2(A-1) + \omega_b^2(D'-1) - c^2k^2]\left(\omega_b^2(D'-\frac{1}{2})\sqrt{\frac{2\pi m\gamma}{T_y^b}}\right)^{-1} \tag{15}$$

and

$$\delta_{delta} = \sqrt{\frac{T_y^b}{2\pi m\gamma}}k\left(\eta' - \frac{(1+\frac{c^2k^2}{\omega_p^2})}{\mu'}\right) \tag{16}$$

with

$$\mu = (D'-\frac{1}{2})(\frac{\omega_b^2}{\omega_p^2}); \quad \eta = 1 - \frac{(\frac{1}{2}\frac{\omega_b^2}{\omega_p^2}) - A}{(D'-\frac{1}{2})(\frac{\omega_b^2}{\omega_p^2})} \tag{17}$$

In Delta function, each of the quantities μ' and η' are different and, in fact, smaller than the corresponding values in the bi-Maxwell function, which itself is derived from the structure of the distribution function.

$$\mu' = \mu - (\frac{T_x^b}{T_y^b})(\frac{\omega_b^2}{\omega_p^2})$$
$$(\mu' < \mu \quad and \quad \eta' < \eta) \Rightarrow \delta_{delta} < \delta_{bi-Max} \tag{18}$$

Based on each of the analytic relationships obtained, considering energetic electrons in the dominant part of a bi-maxwell distribution, in comparison with the fear of single-energy electrons following the delta function, the underlying cause of the growth is appropriate and more than the weibel instability (Figure (4)).

3 Conclusion

The aim of the present study is to investigate the effect of time-dependent electric fields on growth rates of Weibel instabilities in diluted plasma systems at the presence of relativistic electrons and also non-relativistic background electrons, where pseudo-Maxwellian distribution functions were used for both electrons in kinetic theory. The obtained results show that presence of time-dependent electric fields may result in reduction of instability growth rate and its approach to a stable state in comparison with a sample lacking the mentioned fields (Figure (1)).

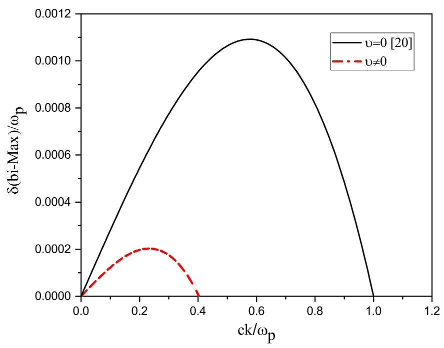

Figure 1: Variation of the growth rate of the Weibel instability normalized to ω_p, $\frac{\delta_{bi-Max}}{\omega_p}$, according to ck/ω_p by consideration of time-dependent drift velocity($\nu \neq 0$) and ($\nu = 0$)

Further decrease in temperature of electron beam will lead to further decrease in growth rate, while, increase of temperature anisotropy fraction can be an important factor in increase of instability growth rate (Figures (2 and 3)).

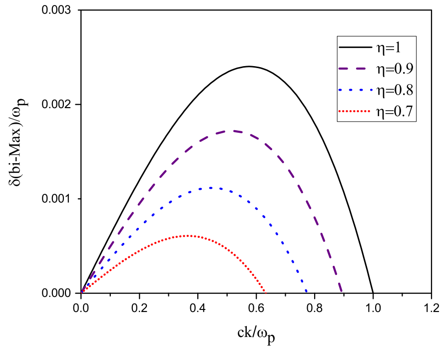

Figure 2: Variation of the growth rate normalized to ω_p, $\frac{\delta_{bi-Max}}{\omega_p}$, according to ck/ω_p for different η in limit $\mu = 2$.

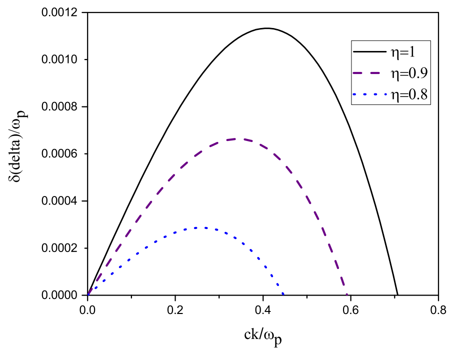

Figure 3: Variation of the growth rate normalized to ω_p, $\frac{\delta_{delta}}{\omega_p}$, according to ck/ω_p for different η in limit $\mu = 2$.

In this content, control of temperature anisotropy in the system along with the effects of laser electric fields can be the effective factors in transmission of high-energy particles in plasma fusion systems. In other words, electric field of the laser can be regarded as an effective factor in return of the system from instable to stable state. The obtained equations show effect of system distribution function shape on reduction of growth rate. Therefore, it can be said that even at the presence of electric fields which will result in reduction of growth rate, Weibel instability with pseudo-Maxwellian distribution, in comparison with a delta function, has higher growth rates (Figure 4).

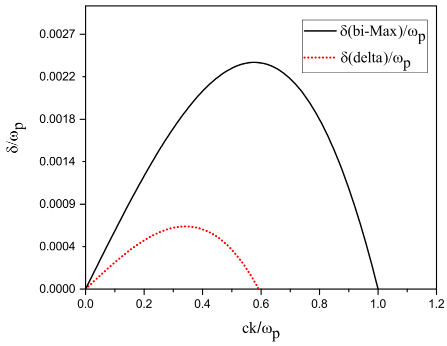

Figure 4: Variation of the growth rate normalized to ω_p ,($\frac{\delta_{bi-Max}}{\omega_p}$ and $\frac{\delta_{delta}}{\omega_p}$), according to ck/ω_p.

This can be clearly observed in the definitions presented for μ' and η' quantities.

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