# Tripolar fuzzy sub implicative ideals of KU-Algebras

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**Abstract**

The concept Tripolar fuzzy set is a generalization of bipolar fuzzy set, intuitionistic fuzzy set and fuzzy set. In this paper, the concept Tripolar fuzzy sub implicative ideals of KU-algebras are introduced and several properties are investigated. Also, the relations between Tripolar fuzzy sub implicative ideals and Tripolar fuzzy ideals are given. The image and the preimage of Tripolar fuzzy sub implicative ideals under homomorphism of KU-algebras are defined and how the image and the preimage of Tripolar fuzzy sub implicative ideals under homomorphism of KU-algebras become Tripolar fuzzy sub implicative ideals are studied. Moreover, the Cartesian product of Tripolar fuzzy sub implicative ideals in Cartesian product KU-algebras is established.

**Keywords:** KU-Algebras, Fuzzy Sub Implicative Ideals, Tripolar Fuzzy Sub Implicative Ideals, The Preimage of Tripolar Fuzzy Sub Implicative Ideals In KU-Algebras, Cartesian Product of Tripolar Fuzzy Sub Implicative Ideals.

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## 1. Introduction

Prabpayak and Leerawat [12, 13] introduced a new algebraic structure which is called KU-algebras. They introduced the concept of homomorphisms of KU-algebras and investigated some related properties. Zadeh [15] introduced the notion of fuzzy sets. At present this concept has been applied to many mathematical branches, such as groups, functional analysis, probability theory and topology. Bipolar-valued fuzzy sets are an extension of fuzzy sets whose membership degree range is enlarged from the interval [0, 1] to [-1, 1]. [see 3,4,5].Mostafa et al [8] introduced the notion of fuzzy KU-ideals of KU-algebras and then they investigated several basic properties which are related to fuzzy KU-ideals. Yaqoob et al [14] introduced the notion of cubic ideals in KU-algebras.

They discussed relationship between a cubic ideals a cubic KU-ideals.. Muhiuddin [11] considered the specifications of a bipolar fuzzy KU-subalgebra, a bipolar fuzzy KU-ideal in KU-algebras and discussed the relations between a bipolar fuzzy KU-subalgebra and a bipolar fuzzy KU-ideal . In [ 9 ] Mostafa and Kareem applied the bipolar-valued fuzzy set theory to n-fold KU-ideals and obtained some related properties. In [ 7 ] the notions of ku-sub implicative ideals in KU –algebras are introduced and some their related properties are investigated. In this paper, the concept Tripolar fuzzy sub implicative ideals are introduced and several properties are investigated .

Also, the relations between Tripolar fuzzy sub implicative ideals and Tripolar fuzzy KU-ideals are given .The image and the preimage of Tripolar fuzzy sub implicative ideals under homomorphism of KU-algebras are defined and how the image and the preimage of Tripolar fuzzy sub implicative ideals under homomorphism of KU-algebras become Tripolar fuzzy sub implicative ideals are studied. Moreover, the Cartesian product of Tripolar fuzzy sub implicative ideals in Cartesian product KU-algebras is established.

## 2. Preliminaries

Now we will recall some known concepts related to KU-algebra from the literature which will be helpful in further study of this article.

**Definition2.1.** [12,13] Algebra(X, $*$, 0) of type (2, 0) is said to be a KU -algebra, if it satisfies the following axioms:

$(ku_1)$ $(x*y)*[(y*z))*(x*z)]=0$,

$(ku_2)$ $x*0=0$,

$(ku_3)$ $0*x=x$,

$(ku_4)$ $x*y=\mathbf{0}$ and $y*x=0$ implies $x=y$,

$(ku_5)$ $x*x=\mathbf{0}$, for all $x, y, z \in X$ .

On a KU-algebra $(X,*,0)$ we can define a binary relation $\leq$ on $X$ by putting:

$x \leq y \Leftrightarrow y*x=0$.

Thus a KU - algebra $X$ satisfies the conditions:

$(ku_{1'})$: $(y*z)*(x*z)\leq(x*y)$

$(ku_{2'})$: $0 \leq x$

$(ku_{3'})$: $x \leq y, y \leq x$ implies $x=y$,

$(ku_{4'})$: $y*x \leq x$.

For any elements x and y of a KU-algebra, $y*x^n$ denotes by $\overbrace{(y*x)*x)\ldots\ldots*x}^{n\ times}$

**Theorem 2.2. [8]:** In a KU-algebra $X$ , the following axioms are satisfied:

For all $x, y, z \in X$ ,

(1): $x \leq y$ imply $y*z \leq x*z$,

(2): $x*(y*z)=y*(x*z)$ ,for all $x, y, z \in X$ ,

(3): $((y*x)*x)\leq y$.

(4) $(y*x^3)=(y*x)$

We will refer to $X$ is a KU-algebra unless otherwise indicated.

**Definition2.3. [**12,13**]** Let $I$ be a non empty subset of a KU-algebra $X$. Then $I$ is said to be an ideal of $X$, if

$(I_1)$ $0 \in I$

$(I_2)$ $\forall y, z \in X$, if $(y * z) \in I$ and $y \in I$, imply $z \in I$.

**Definition2.4. [8]** Let $I$ be a non empty subset of a KU-algebra $X$. Then $I$ is said to be an KU- ideal of $X$, if

$(I_1)$ $0 \in I$

$(I_3)$ $\forall x, y, z \in X$, if $x * (y * z) \in I$ and $y \in I$, imply $x * z \in I$.

**Definition2.5.[ 7 ]**. KU- algebra X is said to be implicative if it satisfies $(x * y^2) = (x * y) * (y * x^2)$

**Definition2.6.[7]** . KU- algebra X is said to be commutative if it satisfies

$$x \leq y \text{ implies} (x * y^2) = x$$

**Lemma 2.7.[ 7 ]** .Let $X$ be a KU-algebra. $X$ is **ku** -implicative iff $X$ is **ku** -positive implicative and **ku** – commutative.

**Definition2.8[7 ].** A non empty subset $A$ of a KU-algebra $X$ is called a **sub** implicative ideal of $X$, if $\forall x, y, z \in X$,

**(1)** $\mathbf{0} \in A$

(2) $z * ((x * y) * ((y * x^2)) \in A$ and $z \in A$, imply $(x * y^2) \in A$.

**Definition2.9[7].** Let $(X, *, 0)$ be a KU-algebra, a nonempty subset $A$ of $X$ is said to be a **ku** - positive implicative ideal if it satisfies, for all $x, y, z$ in $X$,

(1) $0 \in A$,

(2) $z * (x * y) \in A$ and $z * x \in A$ imply $z * y \in A$.

**Definition2.10[7].** A non empty subset $A$ of a KU-algebra $X$ is called a **ku –** sub commutative ideal of $X$, if

**(1)** $\mathbf{0} \in A$

(2) $z * \{((y * x^2)) * y^2)\} \in A$ and $z \in A$, imply $(y * x^2) \in A$.

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**Definition 2.11.**[7] A nonempty subset $A$ of a KU-algebra X is called a kp-ideal of X if it

satisfies

**(1)** $\mathbf{0} \in A$ ,

(2) $(z * y) * (z * x) \in A$ , $y \in A \Rightarrow x \in A$

**Remark2 12:** If $\mu$ fuzzy set , we denotes $\mu(x) \wedge \mu(y)$ = $\min\{\mu(x), \mu(y)\}$ and

$\mu(x) \vee \mu(y)$ = $\max\{\mu(x), \mu(y)\}$

**Definition2 13[8].** A fuzzy set $\mu$ in a KU-algebra $X$ is called a fuzzy sub -algebra of $X$ if

$$\mu(x * y) \geq \mu(x) \wedge \mu(y) \quad \forall x, y \in X ,$$

**Definition 2.14[8].** Let X be a KU-algebra, a fuzzy set $\mu$ in $X$ is called a fuzzy ideal of $X$ if it satisfies the following conditions:

$(F_1)$ $\mu(\mathbf{0}) \geq \mu(x)$ for all $x \in X$ .

$(F_2)$ $\forall x, y \in X,$ $\mu(y) \geq \mu(x * y) \wedge \mu(x)$.

**Definition2.15. [4]** A bipolar valued fuzzy subset $\mathrm{B}$ in a nonempty set $X$ is an object having the form $\mathrm{B} = \{(x, \mu^N(x), \mu^P(x)) \mid x \in X\}$ where $\mu^N : X \to [-1,0]$ and $\mu^P : X \to [0,1]$ are mappings. The positive membership degree $\mu^P(x)$ denotes the satisfaction degree of an element $x$ to the property corresponding to a bipolar-valued fuzzy set $\mathrm{B} = \{(x, \mu^N(x), \mu^P(x)) \mid x \in X\}$, and the negative membership degree $\mu^N(x)$ denotes the satisfaction degree of $x$ to some implicit counter-property of a bipolar-valued fuzzy set $\mathrm{B} = \{(x, \mu^N(x), \mu^P(x)) \mid x \in X\}$.

**Definition2.16[10 ].** A non empty subset $\mu$ of a KU-algebra $X$ is called a fuzzy **ku -sub** implicative ideal of $X$ , if $\forall x, y, z \in X$,

$(FI)$ $\mu(0) \geq \mu(x)$

$(FI_1)$ $\mu(x * y^2) \geq \min\{\mu(z * ((x * y) * ((y * x^2)), \mu(z)\}$

**Example 2.17.** Let $X = \{0,1,2,3,4\}$ in which the operation $*$ is given by the table 1

Table 1

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| ∗ | 0 | 1 | 2 | 3 | 4 |
|---|---|---|---|---|---|
| 0 | 0 | 1 | 2 | 3 | 4 |
| 1 | 0 | 0 | 1 | 3 | 4 |
| 2 | 0 | 0 | 0 | 3 | 4 |
| 3 | 0 | 0 | 0 | 0 | 4 |
| 4 | 0 | 0 | 0 | 0 | 0 |

Then $(X,*,0)$ is a KU-Algebra. Define a fuzzy set $\mu : X \to [0,1]$ by $\mu(0) = t_0$,

$\mu(1) = \mu(2) = t_1$, $\mu(3) = \mu(4) = t_2$, where $t_0, t_1, t_2 \in [0,1]$ with $t_0 > t_1 > t_2$ .

Routine calculation gives that $\mu$ is a fuzzy **ku -sub** implicative ideal of KU- algebra

## **3.** Tripolar fuzzy ku-sub-implicative ideal

**Definition3.1** Let (X, ∗, 0) be a **KU**-algebra. A Tripolar fuzzy set on X is an object having the form $B = \{(x, \mu^P(x), \mu^N(x), \lambda^P(x)) : x \in X\}$, where $\mu^P : X \to [0,1]$, $\mu^N : X \to [-1,0]$

and $\lambda^P : X \to [0,1]$ are the mappings such that $0 \le \mu^P + \lambda^P \le 1, -1 \le \mu^N \le 0$.We use the positive membership degree $\mu^P(x)$ to denote the satisfaction degree of an element x to the property corresponding to a Tripolar fuzzy set B and the negative membership degree $\mu^N(x)$ to denote the satisfaction degree of an element x to some implicit counter property corresponding to a Tripolar fuzzy set B. Similarly we use the positive nonmember ship degree $\lambda^P(x)$ to denote satisfaction degree of an element x to the property corresponding to a Tripolar fuzzy set B.

**Definition3.2.(a)** A Tripolar fuzzy set $B = \{(x, \mu^P(x), \mu^N(x), \lambda^P(x)) : x \in X\}$in $X$ is called a Tri-bipolar fuzzy sub-algebras of $X$ if it satisfies the following condition: for all $x, y \in X$

($a_1$) $\mu^P(0) \ge \mu^P(x)$ and $\mu^N(0) \le \mu^N(x)$,

($a_2$) $\mu^P(x*y) \ge \min\{\mu^P(x), \mu^P(y)\}$, $\mu^N(x*y) \le \max\{\mu^N(x), \mu^N(y)\}$ ,

($a_3$) $\lambda^P(0) \le \lambda^P(x)$, $\lambda^P(x*y) \le \max\{\lambda^P(x), \lambda^P(y)\}$ .

**Definition3.2.(b)** A Tripolar fuzzy set $B = \{(x, \mu^P(x), \mu^N(x), \lambda^P(x)) : x \in X\}$in $X$ is called a Tripolar fuzzy ideal of $X$ if it satisfies the following condition: for all $x, y \in X$

(b1) $\mu^P(0) \geq \mu^P(x)$ and $\mu^N(0) \leq \mu^N(x)$,

(b2) $\mu^P(y) \geq \min\{\mu^P(x*y), \mu^P(x)\}$, $\mu^N(y) \leq \max\{\mu^N(x*y), \mu^N(x)\}$ ,

(b3) $\lambda^P(0) \leq \lambda^P(x)$, $\lambda^P(y) \leq \max\{\lambda^P(x*y), \lambda^P(x)\}$ .

**Definition3.3** A Tribipolar fuzzy set $B = \{(x, \mu^P(x), \mu^N(x), \lambda^P(x)) : x \in X\}$ in $X$ is called a Tripolar fuzzy ku-sub-implicative ideal of $X$ if it satisfies the following condition: for all $x, y, z \in X$

(Tb1) $\mu^P(0) \geq \mu^P(x)$ and $\mu^N(0) \leq \mu^N(x)$

(Tb2) $\mu^P(x*y^2) \geq \min\{\mu^P(z*((x*y)*((y*x^2)), \mu^P(z)\}$ ,

$\mu^N(x*y^2) \leq \max\{\mu^N(z*((x*y)*((y*x^2)), \mu^N(z)\}$

(Tb3) $\lambda^P(0) \leq \lambda^P(x)$, $\lambda^P(x*y^2) \leq \max\{\lambda^P(z*((x*y)*((y*x^2)), \lambda^P(z)\}$

**Example 3.4** Let $X = \{0,1,2,3,4\}$ in which the operation $*$ is given by the Table 1

Define Tripolar fuzzy sets as follows :

Table 2

| | 0 | 1 | 2 | 3 | 4 |
|---|---|---|---|---|---|
| $\mu^N$ | **-0.8** | **-0.8** | **- 0.7** | **- 0.5** | **-0.3** |
| $\mu^P$ | **0.7** | **0.6** | **0.3** | **0.2** | **0.1** |
| $\lambda^P$ | **0.1** | **0.2** | **0.4** | **0.5** | **0.6** |

By routine calculations ,we can prove , that $B = (x, \mu^N, \mu^P, \lambda^P)$ is a Tripolar fuzzy **ku -sub** implicative ideal of KU- algebra X.

**Proposition 3.5** Every a Tripolar fuzzy ku-sub-implicative ideal is a a Tripolar fuzzy ideal of $X$ .

proof . Let $B = (x, \mu^N, \mu^P, \lambda^P)$ be Tri-bipolar fuzzy ku- sub-implicative ideal of X; put x=y in $(Tb_2)$ and $(Tb_3)$ Defintion3.3 , we get

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$$\mu^P(\overbrace{x*x^2}) \geq \min\{\mu^P(z*((x*x)*((x*x^2)),\mu^P(z)\},\ \textit{then}$$

$$\mu^P(x) \geq \min\left\{\mu^P(z*((\overbrace{x*x}^{0})*((\overbrace{x*x^2})),\mu^P(z)\right\} = \min\{\mu^P(z*x),\mu^P(z)\},$$

$$\mu^N(\overbrace{x*x^2}) \leq \max\{\mu^N(z*((x*x)*((x*x^2)),\mu^N(z)\},\ \textit{hence}$$

$$\mu^N(x) \leq \max\left\{\mu^N(z*((\overbrace{x*x}^{0})*((\overbrace{x*x^2})),\mu^N(z)\right\} = \max\{\mu^N(z*x),\mu^N(z)\},\ \textit{and}$$

$$\lambda^P(\overbrace{x*x^2}) \leq \max\{\lambda^P(z*((x*x)*((x*x^2)),\lambda^P(z)\},\ \textit{then we get}$$

$$\lambda^P(x) \leq \max\left\{\lambda^P(z*((\overbrace{x*x}^{0})*((\overbrace{x*x^2})),\lambda^P(z)\right\} = \max\{\lambda^P(z*x),\lambda^P(z)\},$$

for all x, z ∈ X . Hence $\mathrm{B} = (x,\mu^N,\mu^P,\lambda^P)$ is Tripolar fuzzy ideal of X .

The proof is complete.

The following example shows that the converse of Theorem 3.5 may not be true.

**Example 3.6.** Let $X = \{0,1,2,3,4\}$ in which the operation $*$ is given by the Table 1

Define Tripolar fuzzy set $\mathrm{B} = (x,\mu^N,\mu^P,\lambda^P)$ by

Table 3

| | 0 | 1 | 2 | 3 | 4 |
|---|---|---|---|---|---|
| $\mu^N$ | **-0.8** | **-0.3** | **- 0.3** | **- 0.3** | **-0.3** |
| $\mu^P$ | **0.6** | **0.1** | **0.1** | **0.1** | **0.1** |
| $\lambda^P$ | **0.2** | **0.3** | **0.3** | **0.3** | **0.3** |

Then $\mathrm{B} = (x,\mu^N,\mu^P,\lambda^P)$ is a Tri-polar fuzzy ideal of X ,but it not Tripolar fuzzy ku -sub implicative ideal of KU-algebra X, since ,for z = 0 , x = 1 and y = 2 we get that,

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(L.H.S of Definition3.3 $(Tb_2)$): $\mu^P((1*2)*2)=\mu^P(1)=0.1$

$(R.H.S\ of\ \text{Definition}3.3\,(Tb_2):(\min\left\{\mu^P(0*((\overbrace{1*2}^{1})*((\overbrace{2*1}^{0})*1),\mu^P(0)\right\}=\mu^P(0)=0.6$, in this case $\mu^P(x*y^2)\ngeq\ \min\{\mu^P(z*((x*y)*((y*x^2)),\mu^P(z)\}$,

(L.H.S of Definition3.3 $(Tb_2)$ ): $\mu^N((1*2)*2)=\mu^N(1)=-0.3$

$R.H.S\ of\ \text{Definition}3.3\,(Tb_2):\max\left\{\mu^N(0*((\overbrace{1*2}^{1})*((\overbrace{2*1}^{0})*1),\mu^N(0)\right\}=\mu^N(0)=-0.8$, in this case $\mu^N(x*y^2)\nleq\ \max\{\mu^N(z*((x*y)*((y*x^2)),\mu^N(z)\}$

Finally $L.H.S\ of\ Definition\ 3.3(Tb_3):\ \lambda^P((1*2)*2)=\lambda^P(1)=0.3$

$(R.H.S\ of\ \text{Definition}3.3\,(Tb_3)):\max\left\{\lambda^P(0*((\overbrace{1*2}^{1})*((\overbrace{2*1}^{0})*1),\lambda^P(0)\right\}=\lambda^P(0)=0.2$,

in this case $\lambda^P(x*y^2)\nleq\ \max\{\lambda^P(z*((x*y)*((y*x^2)),\lambda^P(z)\}$.

**Proposition 3.7.** If $\mathrm{B}=(x,\mu^N,\mu^P,\lambda^P)$ is a Tripolar fuzzy **ku -sub** implicative ideal of $X$ and $x\le z$, then $\mu^N(x)\le\mu^N(z)$ , $\mu^P(x)\ge\mu^P(z)$ and $\lambda^P(x)\le\lambda^P(z)$.

**Proof:**

If $x\le z$, then $z*x=0$, since $\mathrm{B}=(x,\mu^N,\mu^P,\lambda^P)$ is a Tripolar fuzzy **ku -sub** implicative ideal of $X$ , put x=y in ($Tb_2$) and ($Tb_3$) Defintion3.3 we get

$$\mu^N(x*x^2)=\mu^N(x)\le\max\{\mu^N(z*((x*x)*((x*x^2)),\mu^N(z)\}=$$
$$\max\left\{\mu^N(\overbrace{z*x}^{0}),\mu^N(z)\right\}=\max\{\mu^N(0),\mu^N(z)\}=\mu^N(z)$$

$$\mu^P(x*x^2)=\mu^P(x)\ge\min\{\mu^P(z*((x*x)*((x*x^2)),\mu^P(z)\}=$$
$$\min\left\{\mu^P(\overbrace{z*x}^{0}),\mu^P(z)\right\}=\min\{\mu^N(0),\mu^P(z)\}=\mu^P(z)$$

finally ,we have

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$$\lambda^P(x*x^2)=\lambda^P(x)\leq \max\{\lambda^P(z*((x*x)*((x*x^2)),\lambda^P(z)\}=$$
$$\max\left\{\lambda^P(\overbrace{z*x}^{0}),\lambda^P(z)\right\}=\max\{\lambda^N(0),\lambda^P(z)\}=\lambda^P(z)$$

This completes the proof.

**Lemma 3.8.** Tripolar fuzzy **ku -sub** implicative ideal of KU-algebra is an Tripolar fuzzy ku-sub-algebra of X.

Proof. Let $B=(x,\mu^N,\mu^P,\lambda^P)$ is a Tri-polar fuzzy **ku -sub** implicative ideal of KU-algebra X, then by ($Tb_2$) Defintion3.3 , we get

$\mu^P(x*y^2)\geq \min\{\mu^P(z*((x*y)*((y*x^2)),\mu^P(z)\}$ .Let y = x

$\mu^P(x*x^2)\geq \min\{\mu^P(z*((x*x)*((x*x^2)),\mu^P(z)\}$

, $\mu^P(x)\geq \min\{\mu^P(z*x),\mu^P(z)\}$ and hence

$\mu^P(z*x)\geq \min\{\mu^P(z*(z*x)),\mu^P(z)\}=\min\{\mu^P(x),\mu^P(z)\}$,

Now $\mu^N(x*y^2)\leq \max\{\mu^N(z*((x*y)*((y*x^2)),\mu^N(z)\}$ .Let y = x

$\mu^N(x*x^2)\leq \max\{\mu^N(z*((x*x)*((x*x^2)),\mu^N(z)\}$ i.e

, $\mu^N(x)\leq \max\{\mu^N(z*x),\mu^N(z)\}$ and hence

$\mu^N(z*x)\leq \max\{\mu^N(z*(z*x)),\mu^N(z)\}=\max\{\mu^N(x),\mu^N(z)\}$,

finally ,we can prove that from ($Tb_3$) Defintion3.3

$\lambda^P(z*x)\leq \max\{\lambda^P(z*(z*x)),\lambda^P(z)\}=\max\{\lambda^P(x),\lambda^P(z)\}$. Then $B=(x,\mu^N,\mu^P,\lambda^P)$ is an Tripolar fuzzy ku-sub-algebra of X. The proof is complete.

The following example shows that the converse of Theorem 3.8 may not be true

**Example 3.9** Let X = {0, 1, 2, 3, 4} in which * is given by the Table 1

Define a Tripolar fuzzy set $B=(x,\mu^N,\mu^P,\lambda^P)$ by

Table 4

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| | 0 | 1 | 2 | 3 | 4 |
|---|---|---|---|---|---|
| $\mu^N$ | **-0.8** | **-0.3** | **- 0.3** | **- 0.3** | **-0.3** |
| $\mu^P$ | **0.7** | **0.1** | **0.1** | **0.1** | **0.1** |
| $\lambda^P$ | **0.1** | **0.6** | **0.6** | **0.6** | **0.6** |

Then $\mathrm{B}=(x,\mu^N,\mu^P,\lambda^P)$ is a Tripolar sub -algebra of X ,but it not Tripolar fuzzy ku -sub implicative ideal of KU-algebra X, since For z=0 , x=1 and y=2, we get from L.H.S of $(Tb_2)$ Defintion3.3 : $\mu((1*2)*2)=\mu(1)=0.2$

$R.H.S\ of\ (Tb_2)\ of\ \text{Defintion3.3}:\min\left\{\mu^P(0*((\overbrace{1*2}^{1})*((\overbrace{2*1}^{0})*1),\mu^P(0)\right\}=\mu(0)=0.7$, in this case $\mu^P(x*y^2)\ngeq\ \min\{\mu^P(z*((x*y)*((y*x^2)),\mu^P(z)\}$.

**Lemma 3.10.**

If X is implicative KU-algebra, then every Tripolar fuzzy ideal of X is and Tripolar fuzzy **ku -sub** implicative ideal of KU-algebra X.

Proof. Let $\mathrm{B}=(x,\mu^N,\mu^P,\lambda^P)$ be an Tripolar fuzzy ideal of X, then for all $x,y,z\in X$

(b1) $\mu^P(0)\geq\mu^P(x)$ and $\mu^N(0)\leq\mu^N(x)$,

(b2) $\mu^P(y)\geq\min\{\mu^P(x*y),\mu^P(x)\}$, $\mu^N(y)\leq\max\{\mu^N(x*y),\mu^N(x)\}$ ,

(b3) $\lambda^P(0)\leq\lambda^P(x)$, $\lambda^P(y)\leq\max\{\lambda^P(x*y),\lambda^P(x)\}$ .

Substituting $x*y^2$ for $y$ in $(b_2)$ and $(b_3)$ Definition 3.2(b) **,** we have

$\mu^P(x*y^2)\geq\min\{\mu^P(z*(x*y^2)),\mu^P(z)\}$, $\mu^N(x*y^2)\leq\max\{\mu^N(z*(x*y^2)),\mu^N(z)\}$ and

$\lambda^P(x*y^2)\leq\max\{\lambda^P(z*(x*y^2)),\lambda^P(z)\}$,but KU- algebra is implicative i.e $(x*y^2)=(x*y)*(y*x^2)$, hence

$\mu^P(x*y^2)\geq\min\{\mu^P(z*(x*y^2)),\mu^P(z)\}=\min\{\mu^P(z*(x*y)*(y*x^2)),\mu^P(z)\}$,

$\mu^N(x*y^2)\leq\max\{\mu^N(z*(x*y^2)),\mu^N(z)\}=\max\{\mu^N(z*(x*y)*(y*x^2)),\mu^N(z)\}$ and

$\lambda^P(x*y^2)\leq\max\{\lambda^P(z*(x*y^2)),\lambda^P(z)\}=\max\{\lambda^P(z*(x*y)*(y*x^2)),\lambda^P(z)\}$.

Which shows that $B=(x,\mu^N,\mu^P,\lambda^P)$ is Tripolar fuzzy **ku -sub** implicative ideal of KU-algebra X. The proof is complete.

**Theorem 3.11.** Let $B=(x,\mu^N,\mu^P,\lambda^P)$ be a Tripolar fuzzy set in X satisfying the condition $(Tb_2)$ and $(Tb_3)$ Defintion3.3 , then $B=(x,\mu^N,\mu^P,\lambda^P)$ satisfies the following inequalities:

$(Tb_2')$ $\mu^P(x*y^2)\geq\mu^P((x*y)*(y*x^2))$, $\mu^N(x*y^2)\leq\mu^N((x*y)*(y*x^2))$ and

$(Tb_3')$ $\lambda^P(x*y^2)\leq\lambda^P(z*(x*y)*(y*x^2))$.

Proof. Let $B=(x,\mu^N,\mu^P,\lambda^P)$ satisfying $(Tb_2)$ and $(Tb_3)$ of Defintion3.3. Take z = 0 in $(Tb_2)$, $(Tb_3)$ and using ( $ku_3$ ),then we get

$$\mu^P(x*y^2)\geq\min\{\mu^P(0*((x*y)*((y*x^2)),\mu^P(0)\}=\mu^P((x*y)*((y*x^2))\quad ,$$
$$\mu^N(x*y^2)\leq\max\{\mu^N(0*((x*y)*((y*x^2)),\mu^N(0)\}=\mu^N((x*y)*((y*x^2))\text{ --}(Tb_2')$$

$$\lambda^P(x*y^2)\leq\max\{\lambda^P(0*((x*y)*((y*x^2)),\lambda^P(0)\}=\lambda^P((x*y)*((y*x^2))\text{ --}(Tb_3')$$

The proof is complete.

We now give a condition for a Tripolar fuzzy ideal to be a Tripolar fuzzy ku-sub-implicative ideal.

**Theorem 3.12**. Every Tripolar fuzzy ideal $B=(x,\mu^N,\mu^P,\lambda^P)$ of X satisfying the condition $(Tb_2')$ and $(Tb_3')$ is a Tripolar fuzzy ku-sub-implicative ideal of X .

Proof. Let $B=(x,\mu^N,\mu^P,\lambda^P)$ be Tripolar fuzzy ideal of X satisfying the condition $(Tb_1')$ and $(Tb_2')$ , then we get $\mu^P(x*y^2)\geq\mu^P(((x*y)*((y*x^2))$ , hence

$\mu^P(x*y^2)\geq\min\{\mu^P(z*((x*y)*((y*x^2)),\mu^P(z)\}$ (by Definition3.2.(b) - $(b_2)$ Tripolar fuzzy ideal ) , hence

$$\mu^P(x*y^2)\geq\overbrace{\mu^P(((x*y)*((y*x^2))\geq\min\{\mu^P(z*((x*y)*((y*x^2)),\mu^P(z)\}}^{by\ Definition..b},$$

$\mu^N(x*y^2)\leq\{\mu^N(((x*y)*((y*x^2))\}$ , hence

$\mu^N(x*y^2)\leq\max\{\mu^N(z*((x*y)*((y*x^2)),\mu^N(z)\}$ by Definition3.2.(b) Tribipolar fuzzy ideal ,

hence $\mu^N(x*y^2)\leq\mu^N(((x*y)*((y*x^2))\leq\overbrace{\max\{\mu^N(z*((x*y)*((y*x^2)),\mu^N(z)\}}^{by\ Definition..b}$, and

$\lambda^P(x*y^2)\leq\{\lambda^P(((x*y)*((y*x^2))\}$ , hence

$\lambda^P(x*y^2)\le\max\{\lambda^P(z*((x*y)*((y*x^2)),\lambda^P(z)\}$ (by Definition 3.2.(b) Tribipolar fuzzy ideal ) ,

hence $\lambda^P(x*y^2)\le \overbrace{\lambda^P(((x*y)*((y*x^2))\le\max\{\lambda^P(z*((x*y)*((y*x^2))}^{by\ Definition..b_3},\lambda^P(z)\}$,

which proves the conditions $(Tb_2)$ and $(Tb_3)$. The proof is complete.

**Proposition 3.13.** Let $\mathrm{B}=(x,\mu^N,\mu^P,\lambda^P)$ be a Tripolar fuzzy ku-sub-implicative ideal of $X$. If the inequality $x*y\le z$ holds in X, then $\mu^N(x)\le\max\{\mu^N(y),\mu^N(z)\}$, $\mu^P(x)\ge\min\{\mu^P(y),\mu^P(z)\}$ and $\lambda^P(x)\le\max\{\lambda^P(y),\lambda^P(z)\}$ for all $x,y,z\in X$.

Proof. Let $\mathrm{B}=(x,\mu^N,\mu^P,\lambda^P)$ be a Tripolar fuzzy ku-sub-implicative - ideal of KU - algebra X and let x, y, z ∈ X be such that y*x ≤ z, then z * (y*x)= 0 or y * (z*x)= 0 i.e $z*x\le y$ we get $\mu(z*x)\ge\mu(y)$ ..... (a) .

Let y = x in $(Tb_2)$ and $(Tb_3)$ Definition3.3, we get

By $(Tb_2)$: $\mu^P(x*y^2)\ge\min\{\mu^P(z*((x*y)*((y*x^2)),\mu^P(z)\}$, Let y = x in

$$\mu^P(x*x^2)\ge\min\left\{\mu^P(z*((\overbrace{x*x}^{0})*((\overbrace{x*x^2}^{x})),\mu^P(z)\right\}=\min\{\mu^P(z*x),\mu^P(z)\},\ \text{i.e}$$

$$\mu^P(x)\ge\min\{\mu^P(z*x),\mu^P(z)\}\ge\min\{\mu^P(y),\mu^P(z)\}\ \text{( by (a)) ,}$$

$$\mu^N(x*y^2)\le\max\{\mu^N(z*((x*y)*((y*x^2)),\mu^N(z)\},$$

$$\mu^N(x*x^2)\le\max\left\{\mu^N(z*((\overbrace{x*x}^{0})*((\overbrace{x*x^2}^{x})),\mu^N(z)\right\}=$$

$$=\min\{\mu^N(z*x),\mu^N(z)\},\text{i.e}$$

$$\mu^N(x)\le\max\{\mu^N(z*x),\mu^N(z)\}\le\max\{\mu^P(y),\mu^P(z)\}\text{and}$$

By Defintion3.3 $(Tb_3)$: $\lambda^P(x*y^2)\le\max\{\lambda^P(z*((x*y)*((y*x^2)),\lambda^P(z)\}$,

$$\lambda^P(x*x^2)\le\max\left\{\lambda^P(z*((\overbrace{x*x}^{0})*((\overbrace{x*x^2}^{x})),\lambda^P(z)\right\}$$

$$=\max\{\lambda^P(z*x),\lambda^P(z)\},\text{i.e }\ \lambda^P(x)\le\max\{\lambda^P(z*x),\lambda^P(z)\}\le\max\{\mu^P(y),\mu^P(z)\}\ .$$

The proof is complete.

**Definition3.14** Let $\mathrm{B}=(x,\mu^N,\mu^P,\lambda^P)$ be a Tripolar fuzzy set and $t,u\in[0,1]$, $s\in[-1,0]$.

The set $\mathrm{B}^{t,u,s} = \{x \in X : \mu^P(x) \geq t, \lambda^P(x) \leq u, \mu^N(x) \leq s\}$, is called (t ,u , s)- Tri cut of $\mathrm{B} = (x, \mu^N, \mu^P, \lambda^P)$.

**Theorem 3.15.** A Tri-polars fuzzy set $\mathrm{B} = (x, \mu^N, \mu^P, \lambda^P)$ is Tripolar fuzzy ku-sub-implicative - ideal of KU - algebra X if and only if $\mathrm{B}^{t,u,s} = \{x \in X : \mu^P(x) \geq t, \lambda^P(x) \leq u, \mu^N(x) \leq s\} \neq \Phi$

is a sub-implicative ideal of X.

**Proof:** Suppose that $\mathrm{B} = (x, \mu^N, \mu^P, \lambda^P)$ Tripolar fuzzy ku-sub-implicative of KU - algebra and $\mathrm{B}^{t,u,s} = \{x \in X : \mu^P(x) \geq t, \lambda^P(x) \leq u, \mu^N(x) \leq s\} \neq \Phi$ for any $t, u \in [0,1]$, $s \in [-1,0]$. there exists $x \in \mathrm{B}^{t,u,s}$ so that $\mu^P(x) \geq t, \lambda^P(x) \leq u, \mu^N(x) \leq s$. It follows from Definition3.3 $(Tb_1)$ that $\mu^P(0) \geq \mu^P(x) \geq t, \lambda^P(0) \leq \lambda^P(x) \leq u, \mu^N(0) \leq \mu^N(x) \leq s$ so that $0 \in \mathrm{B}^{t,u,s}$. Let $x, y, z \in X$ be such that $z*((x*y)*((y*x^2) \in \mathrm{B}^{t,u,s}$ and $z \in \mathrm{B}^{t,u,s}$. Using Definition3.3 $(Tb_2),(Tb_3)$, we know that

$$\mu^p(x*y^2) \geq \min\{\mu^p(z*((x*y)*((y*x^2)), \mu^p(z)\} = \min\{t,t\} = t \quad ,$$
$$\mu^N(x*y^2) \leq \max\{\mu^N(z*((x*y)*((y*x^2)), \mu^N(z)\} = \max\{s,s\} = s \text{ and}$$

$$\lambda^p(x*y^2) \leq \max\{\lambda^p(z*((x*y)*((y*x^2)), \lambda^p(z)\} \max\{u,u\} = u$$

,thus $x*y^2 \in \mathrm{B}^{t,u,s}$. Hence $\mathrm{B}^{t,u,s}$ is a sub-implicative ideal of X.

Conversely, suppose that $\mathrm{B}^{t,u,s} \neq \Phi$ is a sub-implicative ideal of X ,

for every $t, u \in [0,1]$, $s \in [-1,0]$ and any $x \in X$, let $\mu^P(x) = t, \lambda^P(x) = u, \mu^N(x) = s$. Then $x \in \mathrm{B}^{t,u,s}$. Since $0 \in \mathrm{B}^{t,u,s}$, it follows that $\mu^P(0) \geq \mu^P(x) = t, \lambda^P(0) \leq \lambda^P(x) = u, \mu^N(0) \leq \mu^N(x) = s$ for all $x \in X$. Now, we need to show that $\mu^P(x), \lambda^P(x), \mu^N(x)$ satisfies Definition3.3 $(Tb_2),(Tb_3)$. If not, then there exist $a, b, c \in X$ such that

$$\mu^P(a*b^2) \leq \min\{\mu^P(c*((a*b)*((b*a^2)), \mu^P(c)\}$$

Taking $t_0 = \frac{1}{2}[(\mu^P(a*b^2) + \min\{\mu^P(c*((a*b)*((b*a^2)), \mu(c)\}]$ then we have

$$\mu^P(a*b^2) < t_0 < \min\{\mu(c*((a*b)*((b*a^2)), \mu^P(c)\},$$

then there exist $a, b, c \in X$ such that

$$\mu^N(a*b^2) \geq \max\{\mu^N(c*((a*b)*((b*a^2)), \mu^N(c)\}$$

. Taking $\varsigma_0 = \frac{1}{2}[(\mu^N(a*b^2) + \max\{\mu^N(c*((a*b)*((b*a^2)), \mu^N(c)\}]$ then we have

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$\mu^N(a*b^2) > \varsigma_0 > \max\{\mu^N(c*((a*b)*((b*a^2)),\mu^N(c)\}$ and

$\lambda^P(a*b^2) \geq \max\{\lambda^P(c*((a*b)*((b*a^2)),\lambda^P(c)\}$

. Taking $\tau_0 = \frac{1}{2}[(\lambda^P(a*b^2) + \max\{\lambda^P(c*((a*b)*((b*a^2)),\lambda^P(c)\}]$ then we have

$\lambda^P(a*b^2) > \varsigma_0 > \max\{\lambda^P(c*((a*b)*((b*a^2)),\lambda^P(c)\}$

Hence in all cases $c*((a*b)*(b*a^2)) \in \mathrm{B}^{t,u,s}$ and $c \in \mathrm{B}^{t,u,s}$, but $a*b^2 \notin \mathrm{B}^{t,u,s}$ which means that $\mathrm{B}^{t,u,s}$ is not a sub-implicative ideal of X, this is contradiction. Therefore $\mathrm{B} = (x, \mu^N, \mu^P, \lambda^P)$ is Tripolar fuzzy sub-implicative ideal of X . The proof is complete.

**4-**Image (Pre-image) Tripolar fuzzy sub-implicative - ideals

**efinition 4.1** Let $(X,*,0)$ and $(Y,*',0')$ be KU-algebras. A mapping $f: X \to Y$ is said to be a homomorphism if $f(x*y) = f(x)*' f(y)$ for all $x, y \in X$. Note that if $f: X \to Y$ is a homomorphism of KU-algebras, then $f(0) = 0'$.

Let $f: X \to Y$ be a homomorphism of KU-algebras for any Tri -polar fuzzy set $\mathrm{B} = (x, \mu^N, \mu^P, \lambda^P)$ in Y, we define new bipolar fuzzy set $\mathrm{B}_f = (x, {\mu_f}^N, {\mu_f}^P, {\lambda_f}^P)$ in X by $\mu_f^N(x) := \mu^N(f(x))$, $\mu_f^P(x) := \mu^P(f(x))$ and $\lambda_f^P(x) := \lambda^P(f(x))$ for all $x \in X$.

**Theorem 4.2** Let $f: X \to Y$ be a homomorphism of KU-algebras. If $\mathrm{B} = (x, \mu^N, \mu^P, \lambda^P)$ is Tri -polar fuzzy ku - sub-implicative ideal of Y, then $\mathrm{B}_f = (x, {\mu_f}^N, {\mu_f}^P, {\lambda_f}^P)$ is Tripolar fuzzy ku -sub-implicative - ideal of X.

**Proof.** $\mu_f^N(x) := \mu^N(f(x)) \geq \mu^N(0) = \mu^N(f(0)) = \mu_f^N(0)$,
$\mu_f^P(x) := \mu^P(f(x)) \leq \mu^P(0) = \mu^P(f(0)) = \mu_f^P(0)$, and

$\lambda_f^P(x) := \lambda^P(f(x)) \geq \lambda^P(0) = \lambda^P(f(0)) = \lambda_f^P(0)$ for all $x, y \in X$.

Now

$$\mu_f^N(x*y^2)) = \mu^N(f(x*y^2)) \leq \max\{\mu^N(f(z*((x*y)*((y*x^2)))),\mu^N(f(z))\},$$
$$\mu_f{}^N(x*y^2)) = \mu^N(f(x*y^2))) \leq \max\{\mu^N(f(z*((x*y)*((y*x^2)))),\mu^N(f(z))\}$$
$$\mu_f{}^N(x*y^2)) \leq \max\{\mu^N(f((z*((x*y)*((y*x^2)))),\mu^N(f(z))\} =$$
$$\max\{\mu^N(((f(z)*((f(x)*f(y)*((f(y)*f(x^2)))),\mu^N(f(z))\} =$$
$$\max\{\mu^N(((f(z*((x*y)*(y*x^2)))),\mu^N(f(z))\}$$
$$\{\mu_f^P(z*(x*y)*(y*x^2)))) \wedge \mu_f^P(z))\} \quad ,$$

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$$\mu_f^P(x*y^2)) = \mu^P(f(x*y^2)) \geq \min\{\mu^P(f(z*((x*y)*((y*x^2)))), \mu^P(f(z))\},$$
$$\mu^P(x*y^2)) = \mu^P(f(x*y^2))) \geq \min\{\mu^P(f(z*((x*y)*((y*x^2)), \mu^P(f(z))\}$$
$$\mu^P(x*y^2)) \geq \min\{\mu^P(f((z*((x*y)*((y*x^2)), \mu^P(f(z))\} =$$
$$\min\{\mu^P(((f(z)*((f(x)*f(y)*((f(y)*f(x^2))))), \mu^P(f(z))\} =$$
$$\min\{\mu^P(((f(z*((x*y)*(y*x^2)))), \mu^P(f(z))\} =$$
$$= \min\{\mu_f^P(z*(x*y)*(y*x^2)))), \mu_f^P(z))\} \qquad and$$

$$\lambda_f^P(x*y^2)) = \lambda^P(f(x*y^2)) \leq \max\{\lambda^P(f(z*((x*y)*((y*x^2)))), \lambda^P(f(z))\},$$
$$\lambda_f{}^P(x*y^2)) = \lambda^P(f(x*y^2))) \leq \max\{\lambda^P(f(z*((x*y)*((y*x^2)))), \lambda^P(f(z))\}$$
$$\lambda_f{}^P(x*y^2)) \leq \max\{\lambda^P(f((z*((x*y)*((y*x^2)))), \lambda^P(f(z))\} =$$
$$\max\{\lambda^P(((f(z)*((f(x)*f(y)*((f(y)*f(x^2)))), \lambda^P(f(z))\} =$$
$$\max\{\lambda^P(((f(z*((x*y)*(y*x^2)))), \lambda^P(f(z))\}$$
$$\max\{\lambda_f{}^P(z*(x*y)*(y*x^2)))), \lambda_f{}^P(z))\} \ .$$

Hence $\mathrm{B}_f = (x, \mu_f{}^N, \mu_f{}^P, \lambda_f{}^P)$ is Tripolar fuzzy ku- sub-implicative ideal of X.

The proof is complete.

**Theorem 4.3** Let $f: X \to Y$ be an epimorphism of KU-algebras .If $\mathrm{B}_f = (x, \mu_f{}^N, \mu_f{}^P, \lambda_f{}^P)$ is Tripolar fuzzy ku- sub-implicative ideal of X, then $\mathrm{B} = (x, \mu^N, \mu^P, \lambda^P)$ is Tripolar fuzzy ku- sub-implicative ideal of Y.

**Proof.** For any $a \in Y$, there exists $x \in X$ such that $f(x) = a$ .Then

$$\mu^N(a) = \mu^N(f(x)) = \mu_f{}^N(a) \geq \mu_f{}^N(0) = \mu^N(f(0)) = \mu^N(0).$$

$$\mu^P(a) = \mu^P(f(x)) = \mu_f{}^P(a) \leq \mu_A^f(0) = \mu^P(f(0)) = \mu^P(0),$$

$$\lambda_f{}^P(a) = \lambda^P(f(x)) = \mu_f{}^N(a) \geq \mu_f{}^N(0) = \mu^N(f(0)) = \mu^N(0).$$

Let $a, b, c \in Y$. Then $f(x) = a,\ f(y) = b,\ f(z) = c$, for some $x, y, z \in X$. It follows that

$$\mu^N(c*a^2) = \mu^N(f(z*x^2)) = \mu_f^N(z*x^2) \leq \max\{\mu_f^N((z*((x*y)*(y*x^2)), \mu_f^N(z)\}$$
$$= \max\{\mu^N(f((z*((x*y)*(y*x^2)), \mu^N(f(z))\} =$$
$$= \max\{\mu^N(f(z)*((f(x)*f(y))*((f(y)*f(x)*f(x)), \mu^N(f(z))\}$$

$$= \max\{\mu^N(c*((a*b)*(b*a^2))), \mu^N(c)\},$$

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$\mu^P(c*a^2)=\mu^P(f(z*x^2))=\mu_f^P(z*x^2)\geq\min\{\mu_f^P((z*((x*y)*(y*x^2)),\mu_f^P(z)\}$
$=\min\{\mu^P(f((z*((x*y)*(y*x^2)),\mu^P(f(z))\}$
$=\min\{\mu^P(f(z)*((f(x)*f(y))*((f(y)*f(x)*f(x)),\mu^P(f(z))\}$

$=\min\{\mu^P(c*((a*b)*(b*a^2))),\mu^P(c)\}$, and

$\lambda^P(c*a^2)=\lambda^P(f(z*x^2))=\lambda_f^P(z*x^2)\leq\max\{\lambda_f^P((z*((x*y)*(y*x^2)),\lambda_f^P(z)\}$
$=\max\{\lambda^P(f((z*((x*y)*(y*x^2)),\lambda^P(f(z))\}=$
$=\max\{\lambda^P(f(z)*((f(x)*f(y))*((f(y)*f(x)*f(x)),\lambda^P(f(z))\}$

$=\max\{\lambda^P(c*((a*b)*(b*a^2))),\lambda^P(c)\}$. The proof is complete.

**5.** Product of Tripolar ku- sub-implicative ideals

**Definition 5.1** Let $\mu$ and $\lambda$ be two fuzzy sets in the set X. the product $\lambda\times\mu: X\times X\to[0,1]$ is defined by $(\lambda\times\mu)(x,y)=\min\{\lambda(x),\mu(y)\}$, for all $x,y\in X$.

**Definition 5.2** Let $A=(X,\mu_A^N,\mu_A^P,\lambda_A^P)$ and $B=(X,\mu_B^N,\mu_B^P,\lambda_B^P)$ are two Tripolar fuzzy set of X, the Cartesian product $A\times B=(X\times X,\mu_A^N\times\mu_B^N,\mu_A^P\times\mu_B^P,\lambda_A^P\times\lambda_B^P)$ is defined by

$(\mu_A^N\times\mu_B^N)(x,y)=\max\{\mu_A^N(x),\lambda_B^N(y)\}$, $(\mu_A^P\times\mu_B^P)(x,y)=\min\{\mu_A^P(x),\lambda_B^P(y)\}$ and

$(\lambda_A^P\times\lambda_B^P)(x,y)=\max\{\lambda_A^P(x),\lambda_B^P(y)\}$, where $\mu_A^N\times\mu_B^N: X\times X\to[-1,0]$
, $\mu_A^P\times\mu_B^P: X\times X\to[0,1]$, and $\lambda_A^P\times\lambda_B^P: X\times X\to[0,1]$, for all $x,y\in X$.

**Remark 5.3** Let X and Y be KU-algebras, we define* on X ×Y by:

For every $(x,y),(u,v)\in X\times Y$, $(x,y)*(u,v)=(x*u,y*v)$. Clearly $(X\times Y;*,(0,0))$ is

KU-algebra.

**Proposition 5.4** Let $A=(X,\mu_A^N,\mu_A^P,\lambda_A^P)$ and $B=(X,\mu_B^N,\mu_B^P,\lambda_B^P)$ are two Tripolar fuzzy ku- sub-implicative ideals of X, then $A\times B$ is Tripolar KU- sub-implicative - ideal of $X\times X$.

**Proof.** For all $x,y\in X$, we have

$$\mu_A^N\times\mu_B^N(0,0)=\max\{\mu_A^N(0),\mu_B^N(0)\}\leq\max\{\mu_A^N(x),\mu_B^N(y)\}\}=\mu_A^N\times\mu_B^N(x,y)$$

$$\mu_A^P\times\mu_B^P(0,0)=\min\{\mu_A^P(0),\mu_B^P(0)\}\geq\min\{\mu_A^P(x),\mu_B^P(y)\}\}=\mu_A^P\times\mu_B^P(x,y)\text{, and}$$

$$\lambda_A^P\times\lambda_B^P(0,0)=\max\{\lambda_A^P(0),\lambda_B^P(0)\}\leq\max\{\lambda_A^P(x),\lambda_B^P(y)\}\}=\lambda_A^P\times\lambda_B^P(x,y)$$

Now let $(x_1,x_2),(y_1,y_2),(z_1,z_2)\in X\times X$, then

$$\max\{(\mu_A^N \times \mu_B^N)\{((z_1,z_2)*[(x_1,x_2)*((y_1,y_2)*((y_1,y_2)*(x_1,x_2)^2))],(\mu_A^N \times \mu_B^N)(z_1,z_2)\} =$$

$$\max\left\{(\mu_A^N \times \mu_B^N)((z_1*((x_1*y_1)*((y_1*x_1{}^2)),(z_2*((x_2*y_2)*((y_2*x_2{}^2))),(\mu_A^N \times \mu_B^N)(z_1,z_2)\right\}=$$

$$= \max\{\{\mu_A^N(z_1*((x_1*y_1)*((y_1*x_1{}^2))),\mu_B^N((z_2*((x_2*y_2)*((y_2*x_2{}^2))\},\{\mu_A^N(z_1),\mu_B^N(z_2)\}\} =$$

$$\max\{\max\{\ \mu_A^N(z_1*((x_1*y_1)*((y_1*x_1{}^2))),\ \mu_A^N(z_1)\},\max\{\ \mu_B^N((z_2*((x_2*y_2)*((y_2*x_2{}^2)),\ \mu_B^N(z_2)\}$$
$$\le \max\{\ \mu_A^N(x_1*y_1^2)\ ,\ \mu_B^N(x_2*y_2^2)\ \} = (\mu_A^N \times \mu_B^N)(x_1*y_1^2,x_2*y_2^2),$$

$$\min\{(\mu_A^P \times \mu_B^P)\{((z_1,z_2)*[(x_1,x_2)*((y_1,y_2)*((y_1,y_2)*(x_1,x_2)^2))],(\mu_A^P \times \mu_B^P)(z_1,z_2)\} =$$

$$\min\left\{(\mu_A^P \times \mu_B^P)((z_1*((x_1*y_1)*((y_1*x_1{}^2)),(z_2*((x_2*y_2)*((y_2*x_2{}^2))),(\mu_A^P \times \mu_B^P)(z_1,z_2)\right\}=$$

$$\min\{\{\mu_A^P(z_1*((x_1*y_1)*((y_1*x_1{}^2))),\mu_B^P((z_2*((x_2*y_2)*((y_2*x_2{}^2))\},\min\{\mu_A^P(z_1),\mu_B^P(z_2)\}\} =$$

$$\min\{\min\{\mu_A^P(z_1*((x_1*y_1)*((y_1*x_1{}^2))),\mu_A^P(z_1)\},$$
$$\min\{\mu_B^P((z_2*((x_2*y_2)*((y_2*x_2{}^2)),\mu_B^P(z_2)\} \le$$

$$\min\{\mu_A^P(x_1*y_1^2),\mu_B^P(x_2*y_2^2)\} = (\mu_A^P \times \mu_B^P)(x_1*y_1^2,x_2*y_2^2)$$

Finally

$$\max\{(\lambda_A^P \times \lambda_B^P)\{((z_1,z_2)*[(x_1,x_2)*((y_1,y_2)*((y_1,y_2)*(x_1,x_2)^2))],(\lambda_A^P \times \lambda_B^P)(z_1,z_2)\} =$$

$$\max\left\{(\lambda_A^P \times \lambda_B^P)((z_1*((x_1*y_1)*((y_1*x_1{}^2)),(z_2*((x_2*y_2)*((y_2*x_2{}^2))),(\lambda_A^P \times \lambda_B^P)(z_1,z_2)\right\}=$$

$$= \max\{\{\lambda_A^P(z_1*((x_1*y_1)*((y_1*x_1{}^2))),\lambda_B^P((z_2*((x_2*y_2)*((y_2*x_2{}^2))\},\{\lambda_A^P(z_1),\lambda_B^P(z_2)\}\} =$$

$$\max\{\max\{\ \lambda_A^P(z_1*((x_1*y_1)*((y_1*x_1{}^2))),\lambda_A^P(z_1)\},\max\{\lambda_B^P((z_2*((x_2*y_2)*((y_2*x_2{}^2)),\lambda_B^P(z_2)\} \le$$

$$\max\{\ \lambda_A^P(x_1*y_1^2)\ ,\ \lambda_B^P(x_2*y_2^2)\ \} \wedge \sigma = (\lambda_A^P \times \lambda_B^P)(x_1*y_{11}^2,x_2*y_2^2)\ ,$$

then $A \times B$ is Tripolar ku- sub-implicative -ideal of $X \times X$. The proof is complete..

**Conclusion.**

In this paper, we introduced the notion of a tripolar fuzzy set Tripolar as a generalization of a fuzzy set, bipolar fuzzy set and an intuitionistic fuzzy set. We have studied the Tripolar fuzzy ku- sub-implicative -ideal in KU-algebras. Also we discussed few results of Tripolar fuzzy ku- sub-implicative -ideal in KU-algebras under homomorphism, the image and the pre- image of Tripolar fuzzy of ku- sub-implicative under homomorphism of

KU-algebras are defined. How the image and the pre-image of Tripolar fuzzy of ku- sub-implicative under homomorphism of KU-algebras become Tripolar fuzzy of ku- sub-implicative are studied. Moreover, the product of Tripolar fuzzy of ku- sub-implicative to product Tripolar fuzzy of ku- sub-implicative is established. Furthermore. The main purpose of our future work is to investigate the foldedness of other types of fuzzy ideals with special properties such as a Tripolar (interval value) fuzzy n-fold of ideals in some algebras.

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**Conflicts of Interest**

State any potential conflicts of interest here or “The authors declare no conflict of interest”

**References**

1. Atanassov K., intuitionistic fuzzy sets, Fuzzy Sets and Systems, 20(1)(1986),87- 96

2. Atanassov K., New operations defined over the intuitionistic fuzzy sets, fuzzy sets and systems, 61 (2) 1994, 137-142.

3. Lee K. M, Bipolar fuzzy subalgebras and bipolar fuzzy ideals of BCK/BCI-algebras, Bull.Malays. Math. Sci. Soc. 32 3 (2009) 361{373.

4. Lee K. M, Bipolar-valued fuzzy sets and their operations, Proc. Int. Conf. Intelligent Tech-nologies Bangkok, Thailand (2000) 307{312.

5. Lee K. M., Comparison of interval-valued fuzzy sets, intuitionistic fuzzy sets, and bipolar-valued fuzzy sets, Int. J. Fuzzy Log. Intell. Syst. 14 2 (2004) 125{129.

6. Lee K. J., Jun Y. B., Bipolar fuzzy a-ideals of BCI-algebras, Commun. Korean Math. Soc.26 4 (2011) 531{542.

7. Mostafa S. M., Omar R. A. K, Abd El- Baseer, O. W. Sub implicative ideals of KU-Algebras, International Journal of Modern Science and Technology Vol. 2, No. 5, 2017. Page 223-227.

8. Mostafa S. M., Abd-Elnaby M. A. and Yousef M. M. M., Fuzzy ideals of KU-Algebras, Int. Math. Forum, 63 (6) (2011) 3139-3149

9. Mostafa S.M. , Kareem F. “Bipolar fuzzy n-fold KU-ideal of KU-algebras”Mathematica Aeterna, Vol. 4, 2014, no. 6, 633 – 650

10. 3- Mostafa S. M., Baseer O. W. A.. Fuzzy Sub Implicative deals of KU-Algebras J. new theory Year: **2018**, Number: **23**, Pages: **1-12**

11. Muhiuddin G.,Bipolar fuzzy KU-subalgebras and bipolarfuzzy KU-ideals in KU-algebras .Annals of Fuzzy Mathematics and Informatics Volume 8, No. 3, (September 2014), pp. 409{418

12. Prabpayak C. and Leerawat U., "On ideals and congruence in KU-algebras" scientia Magna Journal, Vol. 5(2009), No.1, 54-57.

13. Prabpayak C. and Leerawat U. , On isomorphisms of KU-algebras , Scientia magna journal , 5(2009) , no .3 , 25-31

14. Yaqoob N., Mostafa S.M. and Ansari M.A., On cubic KU-ideals of KU-algebras, ISRN Algebra, (2013) Article ID935905, 10 pages.

15. Zadeh L. A., Fuzzy sets, Information and Control 8 (1965) 33