Nonlinear Forced Vibration Of Piezoelectric And Electrostatically Actuated Nano/Micro Piezoelectric

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Abstract

In this study, the nonlinear vibration analysis of nano/micro electromechanical (NEMS/MEMS) piezoelectric beam exposed to simultaneous electrostatic and piezoelectric actuation. NEMS/MEMS beam actuate with combined DC and AC electrostatic actuation on the through two upper and lower electrodes. An axial force proportional to the applied DC voltage is produced by piezoelectric layers present via a DC electric voltage applied in the direction of the height of the piezoelectric layers. The governing differential equation of the motion is derived using Hamiltonian principle based on the Eulere-Bernoilli hypothesis and then this partial differential equation (PDE) problem is simplified into an ordinary differential equation (ODE) problem by using the Galerkin approach. Hamiltonian approach has been used to solve the problem and introduce a design strategy. Phase plane diagram of piezoelectric and electrostatically actuated beam has plotted to show the stability of presented nonlinear system and natural frequencies are calculated to use for resonator design. The result compare with the numerical results (fourth-order Runge-Kutta method), and approximate is more acceptable and results show that one could obtain a predesign strategy by prediction of effects of mechanical properties and electrical coefficients on the stability and forced vibration of common electrostatically actuated micro beam.

Keywords: NEMS/MEMS, Piezoelectric and Electrostatically actuated, Hamiltonian approach, Nonlinear Force Vibrations, Euler-Bernoulli theory, Galerkin method.

1. Introduction

Today, Nano/Micro electromechanical system (NEMS/MEMS) devices are in the heart of new technologies which are widely used in aerospace, automotive, biotechnology, instrumentation, robotics or manufacturing. The application of Nano/micro electromechanical system (NEMS/MEMS) devices especially the electrically actuated MEMS devices which require few mechanical components and small voltage levels for actuation is continuously growing. Analysis, modeling and experimental results related to the nonlinear behavior of NEMs / MEMs systems have numerously been reported in recent years that in references [1–14] consider such as them.

Azizi et al. considered Mass detection based on pure parametric excitation of a micro beam actuated by piezoelectric layers [1]. They showed that the micro beam was excited nearby the periodic solutions outside the boundaries of the region of parametric resonance. Also application of piezoelectric actuation to regularize the chaotic response of an electrostatically actuated micro-beam investigated by Azizi et al. [2]. Maani Miandoab et al. reported study of nonlinear dynamics and chaos in MEMS/NEMS resonators [3]. The presented results revealed that chaotic motion occurs when the system steady state response intersects with the homoclinic orbit. It is interesting to note that for a resonator with high damping ratio, the chaotic motion occurs where the corresponding maximum velocity approaches to the homoclinic orbit velocity; on the other hand, a resonator with low damping ratio becomes chaotic where its vibration amplitude approaches the homoclinic orbit amplitude. Mahmoodi et al. investigated non-linear vibrations and frequency response analysis of piezoelectrically driven microcantilevers [4]. They concluded that in microscale beams, a small change in amplitude of excitation could increase the amplitude of vibration considerably. Yiming Fu et al. studied the application of the energy balance method to a nonlinear oscillator arising in the microelectromechanical system

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(MEMS) [5]. Mashinchi et al. analytical solution for nonlinear vibration of Micro-Electro-mechanical system (MEMS) by analytical approach [6]. Study on the use of perturbation technique for analyzing the nonlinear forced response of piezoelectric microcantilevers by M. Zamanian et al. [7].In this paper, a comparison was made between direct and indirect perturbation approaches for solving the non-linear vibration equations of a piezoelectrically actuated cantilever microbeam. Ghanbari et al. investigated squeeze film damping in a micro-beam resonator based on micro-polar theory [8]. Sadeghzadeh et al. proposed Higher Order Hamiltonian Approach to the Nonlinear Vibration of Micro Electro Mechanical Systems [9]. Rhoads et al. studied the dynamic response of a class of electrostatically driven MEM oscillators [10]; cubic type of nonlinearity due to the nonlinear spring and time-varying linear and nonlinear stiffness due to electrostatic actuation were included in their formulation. Shabani et al investigated the development of superharmonics and chaotic response in an electrostatically actuated torsional micro-mirror near pull-in condition [11]. They reported DC and AC symmetry breaking in their model, which led to chaotic response by increasing the amplitude of the harmonic excitation. DC and AC symmetry breaking in NEMS/MEM devices was previously reported by De and Aluru [12]. Haghighi and Markazi [13] proposed a MEM SDOF system with electrostatic actuation on both sides of the proof mass. Using Melnikov's theorem they investigated the chaotic response of the system in terms of the governing parameters. They proposed a robust adaptive fuzzy control algorithm to regularize the chaotic response of the system. The model studied in the present study is a clamped–clamped micro-beam, sandwiched with two piezoelectric layers through the length of the micro-beam. Piezoelectrically sandwiched micro-beams were first proposed by Rezazadeh et al. [14] to control the static pull-in instability of a MEM device, and later on similar models were studied. Nonlinearity in NEMS/MEMS may cause some difficulties in computations.

Recently, some approximate methods are considered to be the powerful methods capable of handling strongly nonlinear behaviors, especially in NEMS/MEMS systems and can converge to an accurate periodic solution for smooth nonlinear systems that can be showed in works of Hashemi kachapi et al. in references [15-24].

In current study, the methodology of the Hamiltonian for solving an ordinary differential equation with strong power nonlinearity is presented. Numerical comparisons (forth order range- kutta method) and results were carried out to confirm the rightness and accuracy of the applied method. Deriving the dimensionless equation of motion and separation with assumed mode method, first approximation of Hamiltonian of system has proposed and then, natural frequency calculated for each case. Finally, we show the effects of various parameters on the frequency of piezoelectric and electrostatically actuated nano/micro beam, concluding Hamiltonian approach is completely efficient and agreeable. Other similar work, the effects of voltage V_{AC} which leads to forced harmonic vibration is not consideration and is usually considered to be zero. The main feature of the present work to consider the voltage V_{AC} on the natural frequency and dynamic response.

2. Mathematical Model

As illustrated in Fig. 1, the studied model is an isotropic clamped–clamped nano-micro beam of length l, width a, thickness h, density ρ, and Young's modulus E. The nano-micro beam is sandwiched with two piezoelectric layers throughout the length of the microbeam. The piezoelectric layers are of thickness h_p and density ρ_p. The Young's modulus of the piezoelectric layers is denoted by E_p , and the equivalent piezoelectric coefficient is denoted by $\bar{e}_{31}$. Two electrodes are placed underneath and on top of the micro-beam. Initial gaps between the micro-beam and the electrodes are both g_0 and the applied electrostatic voltages by the upper and lower electrodes are denoted by V_u and V_l , respectively.

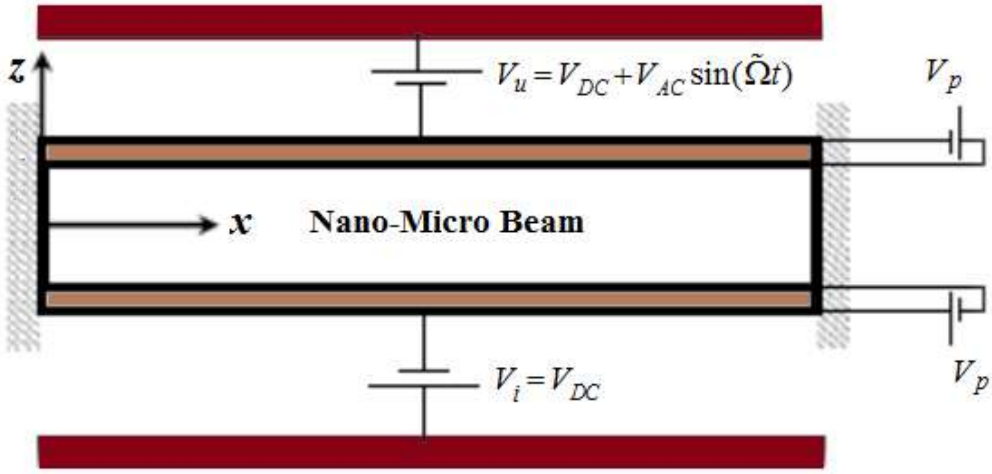

Fig. 1. Schematics of the clamped–clamped piezoelectrically sandwiched nano-micro beam and the electrodes

The applied voltage through the upper electrode is supposed to be a combination of a DC voltage V_{DC} and an AC voltage with amplitude V_{AC} and frequency $\tilde{\Omega}$; the voltage applied through the lower electrode is a pure DC voltage, the same as the DC component of the upper electrode. The coordinate system, as illustrated in Figure 1 is attached to the midplane of the very left end of the micro-beam, where x and z are respectively the horizontal and vertical coordinates. The deflection of the micro-beam along the z axis is denoted by $w(x,t)$. When a clamped–clamped beam undergoes bending, the extended length of the beam (l') becomes larger than its initial length l, leading to the introduction of an axial force as follows [25]:

$$F_a = \frac{Eah + 2E_p ah_p}{l}(l' - l) \approx \frac{Eah + 2E_p ah_p}{2l}\int_0^l \left(\frac{\partial w}{\partial x}\right)^2 dx \tag{1}$$

Here l' is estimated based on the integration of the arc length ds as [26]:

$$l' = \int_0^l ds \approx \int_0^l \sqrt{1 + \left(\frac{\partial w}{\partial x}\right)^2} dx = l + \frac{l}{2}\int_0^l \left(\frac{\partial w}{\partial x}\right)^2 dx \tag{2}$$

The governing equation of the transverse motion can be obtained by the minimization of the Hamiltonian using variational principle. The total potential strain energy of the nano-micro beam includes the bending and axial strain energies (U_b, U_a) and the electrical energy U_e as [27]:

$$U(t) = U_b + U_a + U_e = \frac{EI}{2}\int_0^l \left(\frac{\partial^2 w}{\partial x^2}\right)^2 dx + \frac{E_p hah_p(\frac{h}{2} + h_p)}{2}\int_0^l \left(\frac{\partial^2 w}{\partial x^2}\right)^2 dx + a\bar{e}_{31}V_p\int_0^l \left(\frac{\partial w}{\partial x}\right)^2 dx$$
$$+ \frac{Eha + 2E_p ah_p}{8l}\left(\int_0^l \left(\frac{\partial w}{\partial x}\right)^2 dx\right)^2 + \frac{\varepsilon_0 aV_u^2}{2}\int_0^l \frac{dx}{(g_0 - w)} + \frac{\varepsilon_0 aV_l^2}{2}\int_0^l \frac{dx}{(g_0 + w)} \tag{3}$$

where I and V_p denote respectively the moment of inertia of the cross-section about the horizontal axis passing through the center of the surface for the cross-section of the micro-beam, and the applied voltage to the piezoelectric layers. In Eq. (3) the first two terms are the strain energies due to the bending of the microbeam, the third term is the strain energy due to the axial force of the piezoelectric layers, the fourth term is the strain energy due to the stretching of the midplane and the last two terms indicate the electrical potential energy stored between the micro-beam and the two substrates, underneath and above; ε_0 is the dielectric constant of the gap medium.

The kinetic energy of the micro-beam is represented as [27]:

$$T=\frac{\rho ah}{2}\int_0^l\left(\frac{\partial w}{\partial t}\right)^2dx+\rho_p ah_p\int_0^l\left(\frac{\partial w}{\partial t}\right)^2dx=\left(\frac{\rho ah}{2}+\rho_p ah_p\right)\int_0^l\left(\frac{\partial w}{\partial t}\right)^2dx \tag{4}$$

The Hamiltonian is represented in the following form:

$$H=T-U \tag{5}$$

Substituting Eqs. (3) and (4) into Eq. (5), the Hamiltonian reduces to

$$H=\left(\frac{\rho ah}{2}+\rho_p ah_p\right)\int_0^l\left(\frac{\partial w}{\partial t}\right)^2dx-\frac{EI}{2}\int_0^l\left(\frac{\partial^2 w}{\partial x^2}\right)^2dx-\frac{E_p hah_p(\frac{h}{2}+h_p)}{2}\int_0^l\left(\frac{\partial^2 w}{\partial x^2}\right)^2dx-a\bar{e}_{31}V_p\int_0^l\left(\frac{\partial w}{\partial x}\right)^2dx$$
$$-\frac{Eha+2E_p ah_p}{8l}\left(\int_0^l\left(\frac{\partial w}{\partial x}\right)^2dx_a\right)^2-\frac{\varepsilon_0 aV_u^2}{2}\int_0^l\frac{dx}{(g_0-w)}-\frac{\varepsilon_0 aV_l^2}{2}\int_0^l\frac{dx}{(g_0+w)} \tag{6}$$

Based on the fact that the variation of the integral of the Hamiltonian over the time period [0, t] vanishes, namely, $\delta\int_0^l(T-U)dt=0$, the governing equation of motion and the corresponding boundary conditions are obtained as

$$(EI)_{eq}\frac{\partial^4 w(x,t)}{\partial x^4}+(\rho A)_{eq}\frac{\partial^2 w(x,t)}{\partial t^2}-\left(F_p+\frac{(EA)_{eq}}{2l}\int_0^l\left(\frac{\partial w(x,t)}{\partial x}\right)^2dx\right)\times\frac{\partial^2 w(x,t)}{\partial x^2}$$
$$=\frac{\varepsilon_0 a(V_{DC}+V_{AC}\sin(\tilde{\Omega}t))^2}{2(g_0-w(x,t))^2}-\frac{\varepsilon_0 aV_{DC}^2}{2(g_0+w(x,t))^2} \tag{7}$$

subject to the following boundary conditions for climbed- climbed:

$$w(0,t)=w(l,t)=0,\qquad \frac{\partial w(0,t)}{\partial x}=\frac{\partial w(l,t)}{\partial x}=0 \tag{8}$$

where:

$$(EI)_{eq}=EI+E_p hah_p(\frac{h}{2}+h_p),\qquad F_p=2a\bar{e}_{31}V_p$$
$$(\rho A)_{eq}=\rho ah+2\rho_p ah_p,\qquad (EA)_{eq}=Eah+2E_p ah_p \tag{9}$$

The integral term in Eq. (7) represents the midplane stretching of the micro-beam due to the immovable edges. Nonlinearities in resonant microsystems generally arise from three sources: (i) large (finite) structural deformations, (ii) displacement dependent excitations (stiffness parametric excitation), and (iii) tip/sample interaction potentials (e.g. electrostatic interactions, and the Lennard–Jones potential).

According to Eq. (7), two types of nonlinearities exist in this model. The nonlinearity of the dynamics of the structure adds interesting behavior to the response of the system. For convenience the following non-dimensional parameters (with over-hats) are introduced:

$$W=\frac{w}{g_0},\quad \xi=\frac{x}{l},\quad \tau=\frac{t}{\tilde{t}},\quad \Omega=\tilde{\Omega}\tilde{t} \tag{10}$$

where $\tilde{t}$ is a timescale defined as follows:

$$\tilde{t}=\sqrt{\frac{(\rho A)_{eq}}{(EI)_{eq}}} \tag{11}$$

Substituting Eq. (10) into Eq. (7) and dropping the hats and assuming the amplitude of the AC voltage to be much less than the DC voltage, the equation of the motion in the non-dimensional form is obtained:

$$\frac{\partial^4 W(\xi,\tau)}{\partial\xi^4}+\frac{\partial^2 W(\xi,\tau)}{\partial\tau^2}-\left(\alpha_1+\alpha_2\int_0^l\left(\frac{\partial W(\xi,\tau)}{\partial\xi}\right)^2 d\xi\right)\frac{\partial^2 W(\xi,\tau)}{\partial\xi^2} \tag{12}$$
$$=\alpha_3 V_{DC}^2\left(\frac{1}{(1-W(\xi,\tau))^2}-\frac{1}{(1+W(\xi,\tau))^2}\right)+\frac{2\alpha_3 V_{DC}V_{AC}\sin(\Omega\tau)}{(1-W(\xi,\tau))^2}$$

Where

$$\alpha_1=\frac{F_p l^2}{(EI)_{eq}},\qquad \alpha_2=\frac{(EA)_{eq}g_0^2}{2(EI)_{eq}},\qquad \alpha_3=\frac{\varepsilon_0 a l^4}{2g_0^3(EI)_{eq}} \tag{13}$$

To approximate the homoclinic trajectory of Eq. (12) with the homoclinic orbit of the well-known Duffing equation, Galerkin method is used to discretize Eq. (13); therefore the approximate solution is supposed to be in the form

$$W(\xi,\tau)=\sum_{i=1}^{n}\phi_i(\xi)u_i(\tau) \tag{14}$$

The deflection $W(\xi,\tau)$ in Eq. (14) is expressed as a sum of spatial shapes that, a priori, satisfy the imposed kinematic boundary conditions and n is the number of degrees of freedom, $\phi_i(\xi)$ is the i th eigenfunction of the beam and $u_i(\tau)$ is the i th time-dependent deflection parameter of the beam. Based on a single degree-of- freedom model of the beams (n=1), Eq. (14) can be solved with appropriate accuracy.

For example, $\phi(\xi)$ can be assumed as

$$\phi(\xi)=16\xi^2(1-\xi)^2 \tag{15}$$

or

$$\phi(\xi)=\left(\cosh\left(\frac{\alpha\xi}{l}\right)-\cos\left(\frac{\alpha\xi}{l}\right)\right)-\left(\frac{\cosh\alpha-\cos\alpha}{\sinh\alpha-\sin\alpha}\right)\left(\sinh\left(\frac{\alpha\xi}{l}\right)-\sin\left(\frac{\alpha\xi}{l}\right)\right),\quad \alpha=4.730040745 \tag{16}$$

Eqs. (15) or (16) are the first eigenfunction of a double-clamped beam that satisfy all the kinematic boundary conditions.

In order to avoid division by zero in the electrostatic force term, we multiply Eq. (5) by $(1-W^2(\xi,\tau))^2$, as a result;

$$\begin{aligned}T(W(\xi,\tau)) &= (1-W^2(\xi,\tau))^2(\frac{\partial^4 W(\xi,\tau)}{\partial\xi^4}+(1-W^2(\xi,\tau))^2\frac{\partial^2 W(\xi,\tau)}{\partial\tau^2}\\ &-(1-W^2(\xi,\tau))^2\left(\alpha_1+\alpha_2\int_0^l\left(\frac{\partial W(\xi,\tau)}{\partial\xi}\right)^2 d\xi\right)\frac{\partial^2 W(\xi,\tau)}{\partial\xi^2}\\ &-4W(\xi,\tau)\alpha_3 V_{DC}^2-2(1+W(\xi,\tau))^2\alpha_3 V_{DC}V_{AC}\sin(\Omega\tau)=0\end{aligned} \tag{17}$$

The one-parameter Galerkin method can be computed by:

$$\int_0^1 \phi(\xi)T\left(W(\xi,\tau)\right)d\xi=0 \tag{18}$$

After substituting for $T\left(W(\xi,\tau)\right)$ from Eq. (17) into Eq. (18), multiplying by $\phi(\xi)$, and integrating the outcome from 0 to 1, we obtain

$$\int_0^1\left(\begin{aligned}&\left(\ddot{u}(\tau)\right)\left(\phi(\xi)^2-2\phi(\xi)^4u(\tau)^2+\phi^6u(\tau)^3\right)-\left(2\alpha_3V_{DC}V_{AC}\phi\right)\sin(\Omega\tau)\\ &+\left(\left(\phi\phi''''-\alpha_1\phi\phi''-4\alpha_3V^2{}_{DC}\phi^2\right)-\left(4\alpha_3V_{DC}V_{AC}\phi^2\right)\sin(\Omega\tau)\right)u(\tau)\\ &\left(-2\alpha_3V_{DC}V_{AC}\phi^3\right)\sin(\Omega\tau)u(\tau)^2+\left(-2\phi^3\phi''''+2\alpha_1\phi^3\phi''-\alpha_2\phi\phi''\int_0^1(\phi')^2d\xi\right)u(\tau)^3\\ &\left(\phi^5\phi''''-\alpha_1\phi^5\phi''+2\alpha_2\phi^3\phi''\int_0^1(\phi')^2d\xi\right)u(\tau)^5+\left(-\alpha_2\phi^5\phi''\int_0^1(\phi')^2d\xi\right)u(\tau)^7\end{aligned}\right)d\xi=0 \tag{19}$$

That the nonlinear ordinary differential equation is;

$$\begin{aligned}&\ddot{u}(\tau)\left(\kappa_1+\kappa_2u(\tau)^2+\kappa_3u(\tau)^4\right)+\kappa_4\sin(\Omega\tau)+\left(\kappa_5+\kappa_6\sin(\Omega\tau)\right)u(\tau)\\ &+\kappa_7\sin(\Omega\tau)u(\tau)^2+\kappa_8u(\tau)^3+\kappa_9u(\tau)^5+\kappa_{10}u(\tau)^7=0\end{aligned} \tag{20}$$

or

$$\ddot{u}(\tau)\left(\kappa_1+\kappa_2 u(\tau)^2+\kappa_3 u(\tau)^4\right)+\kappa_5 u(\tau)+\kappa_8 u(\tau)^3+\kappa_9 u(\tau)^5+\kappa_{10} u(\tau)^7 \qquad (21)$$
$$=\left(\kappa_4+\kappa_6 u(\tau)+\kappa_7 u(\tau)^2\right)\sin(\Omega\tau)$$

Where

$$\kappa_1=\int_0^1\left(\phi^2\right)d\xi,\ \ \kappa_2=\int_0^1\left(-2\phi^4\right)d\xi,\ \ \kappa_3=\int_0^1\left(\phi^6\right)d\xi,\ \ \kappa_4=\int_0^1\left(-2\alpha_3 V_{DC}V_{AC}\phi\right)d\xi, \qquad (22)$$
$$\kappa_5=\int_0^1\left(\phi\phi''''-\alpha_1\phi\phi''-4\alpha_3 V^2{}_{DC}\phi^2\right)d\xi,\ \ \kappa_6=\int_0^1\left(-4\alpha_3 V_{DC}V_{AC}\phi^2\right)d\xi,$$
$$\kappa_7=\int_0^1\left(-2\alpha_3 V_{DC}V_{AC}\phi^3\right)d\xi,\ \ \kappa_8=\int_0^1\left(-2\phi^3\phi''''+2\alpha_1\phi^3\phi''-\alpha_2\phi\phi''\int_0^1(\phi')^2 d\xi\right)d\xi,$$
$$\kappa_9=\int_0^1\left(\phi^5\phi''''-\alpha_1\phi^5\phi''+2\alpha_2\phi^3\phi''\int_0^1(\phi')^2 d\xi\right)d\xi,\ \ \kappa_{10}=\int_0^1\left(-\alpha_2\phi^5\phi''\int_0^1(\phi')^2 d\xi\right)d\xi$$

here a overdot $(.)$ denotes differentiation with respect to the time variable τ, while a prime $(')$ indicates the partial differentiation with respect to the coordinate variable ξ. Eq. (21) is a nonlinear ordinary differential equation which can be solved by analytical approaches especially Hamiltonian method. In the next section, the Hamiltonian approach is utilized to study this nonlinear system.

3. Solution Procedure

For the application of Hamiltonian approach, consider a general nonlinear oscillator in the form:

$$\ddot{u}(\tau)+f\left(u(\tau)\right)=0, \qquad u(0)=A,\ \dot{u}(0)=0 \qquad (23)$$

Where u and τ are generalized dimensionless displacement and dimensionless time and A is amplitude of oscillator. Based on the variational principle, by implementing the semi-inverse method and He's method [15-18], variation parameter could be written as;

$$J(u)=\int_0^t\left(-\frac{1}{2}\dot{u}^2+F(u)\right)dt \qquad (24)$$

Where $T=2\pi/\omega$ is period of the oscillator and $\frac{\partial F}{\partial u}=f(u)$. Thus, Hamiltonian of presented problem could be expressed as;

$$H=\frac{1}{2}\dot{u}^2+F(u)=F(A) \qquad (25)$$

Then defining a new function as;

$$R(t)=\frac{1}{2}\dot{u}^2+F(u)-F(A) \tag{26}$$

From Eq. (26) we can obtain approximate frequency–amplitude relationship of a nonlinear oscillator. For current special problem, we have following Hamiltonian equation:

$$\begin{aligned} H=&\frac{1}{2}\dot{u}^2\left(\kappa_1+\kappa_2u^2+\kappa_3u^4\right)+\kappa_4\sin(\Omega\tau)u+\left(\kappa_5+\kappa_6\sin(\Omega\tau)\right)\frac{u^2}{2}\\ &+\kappa_7\sin(\Omega\tau)\frac{u^3}{3}+\kappa_8\frac{u^4}{4}+\kappa_9\frac{u^6}{6}+\kappa_{10}\frac{u^8}{8}=\kappa_4\sin(\Omega\tau)A+\left(\kappa_5+\kappa_6\sin(\Omega\tau)\right)\frac{A^2}{2}\\ &+\kappa_7\sin(\Omega\tau)\frac{A^3}{3}+\kappa_8\frac{A^4}{4}+\kappa_9\frac{A^6}{6}+\kappa_{10}\frac{A^8}{8} \end{aligned} \tag{27}$$

or

$$\begin{aligned} R=&\frac{1}{2}\dot{u}^2\left(\kappa_1+\kappa_2u^2+\kappa_3u^4\right)+\kappa_4\sin(\Omega\tau)(u-A)+\left(\kappa_5+\kappa_6\sin(\Omega\tau)\right)\left(\frac{u^2}{2}-\frac{A^2}{2}\right)\\ &+\kappa_7\sin(\Omega\tau)\left(\frac{u^3}{3}-\frac{A^3}{3}\right)+\kappa_8\left(\frac{u^4}{4}-\frac{A^4}{4}\right)+\kappa_9\left(\frac{u^6}{6}-\frac{A^6}{6}\right)+\kappa_{10}\left(\frac{u^8}{8}-\frac{A^8}{8}\right) \end{aligned} \tag{28}$$

For satisfying of the initial conditions, its initial approximate guess can be expressed as

$$u=A\cos\omega\tau \tag{29}$$

Substituting Eq. (29) into Eq. (28), the frequency–amplitude relationship can be obtained from;

$$\left\langle \begin{aligned} &R=\frac{1}{2}A^2\omega^2\sin^2\omega\tau\left(\kappa_1+\kappa_2A^2\cos^2\omega\tau+\kappa_3A^4\cos\omega\tau^4\right)\\ &+\kappa_4A\sin\Omega\tau\left(\cos\omega\tau-1\right)+\frac{A^2}{2}\left(\kappa_5+\kappa_6\sin\Omega\tau\right)\left(\cos^2\omega\tau-1\right)\\ &+\frac{A^3}{3}\kappa_7\sin\Omega\tau\left(\cos^3\omega\tau-1\right)+\frac{A^4}{4}\kappa_8\left(\cos^4\omega\tau-1\right)+\frac{A^6}{6}\kappa_9\left(\cos^6\omega\tau-1\right)\\ &+\frac{A^8}{8}\kappa_{10}\left(\cos^8\omega\tau-1\right)=0 \end{aligned}\right\rangle \tag{30}$$

Therefore, equation (30) could be solved, and the natural frequency could be obtained as;

(31)

For the trial function Eq. (15) and use of the relations (22) values of κ_1 - κ_{10} are obtained as follows.

$$\begin{Bmatrix} \kappa_1 = 40.64, \quad \kappa_2 = -59.91, \quad \kappa_3 = 24.82, \quad \kappa_4 = -106.7\alpha_3 V_{DC} V_{AC}, \\ \kappa_5 = -162.54\alpha_3 V_{DC}^2 - 487.62 + 487.62\alpha_1, \quad \kappa_6 = -162.54\alpha_3 V_{DC} V_{AC}, \\ \kappa_7 = -68.2\alpha_3 V_{DC} V_{AC}, \quad \kappa_8 = 872.94 + 2377.72\alpha_2, \quad \kappa_9 = 376.01\alpha_1, \\ \kappa_{10} = 1833.52\alpha_2 \end{Bmatrix} \tag{32}$$

By choosing any arbitrary point-like $\omega\tau = \pi / 4$, and setting $R\left(\tau = \frac{\pi}{4\omega}\right) = 0$, also Setting values $\kappa_i \ (i = 1..10)$ in Eq. (31),the natural frequency could be obtained as following:

$$\omega = \frac{0.23570\left(\begin{array}{l} -3A\left(0.062042A^4 - 0.29954A^2 + 0.40635\right) \\ \left(\begin{array}{l} 7.498\alpha_3 V_{DC} V_{AC} \sin\Omega\tau + 3.5269A^2 V_{DC} V_{AC} \sin\Omega\tau \\ -51.568A^7\alpha_2 - 6A(-1.6254\alpha_3 V_{DC}^2 - 4.8762 + 4.8762\alpha_1) \\ -4.5A^3(8.7294 + 23.777\alpha_2) + 9.7524A\alpha_3 V_{DC} V_{AC} \sin\Omega\tau \\ -13.16A^5\alpha_1 \end{array}\right) \end{array}\right)^{1/2}}{A\left(0.40635 - 0.29954A^2 + 0.062042A^4\right)} \tag{33}$$

As a result, the natural frequency is dependent on the parameters of $A, \alpha_1, \alpha_2, \alpha_3, V_{DC}, V_{AC}$ and Ω.

Substituting Eq. (33) into Eq. (29) yields:

$$u(\tau) = A\cos(\frac{0.23570\left(\begin{array}{l} -3A\left(0.062042A^4 - 0.29954A^2 + 0.40635\right) \\ \left(\begin{array}{l} 7.498\alpha_3 V_{DC} V_{AC} \sin\Omega\tau + 3.5269A^2 V_{DC} V_{AC} \sin\Omega\tau \\ -51.568A^7\alpha_2 - 6A(-1.6254\alpha_3 V_{DC}^2 - 4.8762 + 4.8762\alpha_1) \\ -4.5A^3(8.7294 + 23.777\alpha_2) + 9.7524A\alpha_3 V_{DC} V_{AC} \sin\Omega\tau \\ -13.16A^5\alpha_1 \end{array}\right) \end{array}\right)^{1/2}}{A\left(0.40635 - 0.29954A^2 + 0.062042A^4\right)}\tau) \quad ()$$

Where shows dynamic response of piezoelectric and electrostatically actuated nano/micro beam and shows such approach is much simpler and has been widely used.

4. Results and discussion

The geometrical and mechanical properties of the case study are represented in Table 1.

Table 1. Geometrical and material properties of the nano/micro and piezoelectric layers

Geometrical and material property	Micro beam	Piezoelectric layers
Length (L)	600 μm	600 μm
Width (a)	30 μm	30 μm
Height (h)	3 μm	0.01 μm

Initial gap (w_0)	$2\ \mu m$	-
Young`s modulus (E)	$169.61\,GPa$	$76.6\,GPa$
Density (ρ)	$2331\ kg/m^3$	$7500\ kg/m^3$
Piezoelectric constant ($\bar{e}_{31}$)	-	-9.29
Permittivity constant (ε_0)	$8.85\times10^{-12}\ F/m$	-
Mass (ng)	41.958	2.7

The comparison between Hamiltonian approach and fourth-order Runge-Kutta method is plotted in Figs. (2-4) that natural frequency of piezoelectric and electrostatically actuated nano/micro beam is considered.

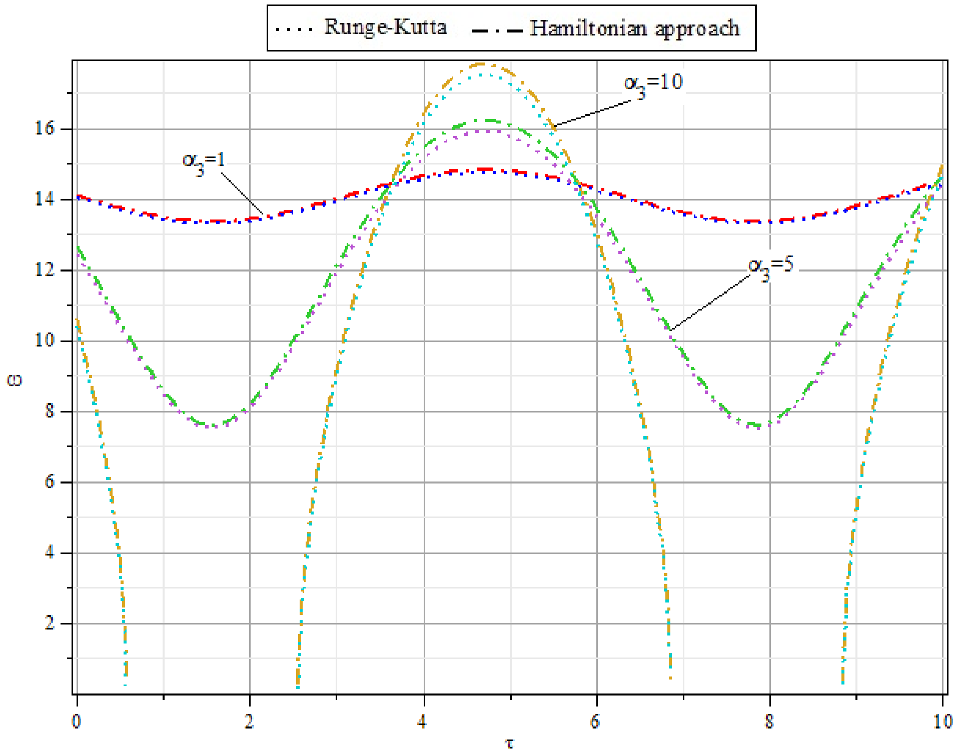

Fig.2. Effect of α_3 parameter on natural frequency piezoelectric and electrostatically actuated nano/micro beam with $A=1,\alpha_1=1,\alpha_2=1,\Omega=1,V_{DC}=1,V_{AC}=1$

Figure 2 depicts the effect of parameter α_3 on natural frequency of nano/micro beam with parameters $A=1,\alpha_1=1,\alpha_2=1,\Omega=1,V_{DC}=1,V_{AC}=1$ and various values for α_3. It can be observed that the frequency increases with increasing α_3. Obtained results by the first-order Hamiltonian are close to the higher-order numerical solution, especially for low amplitudes.

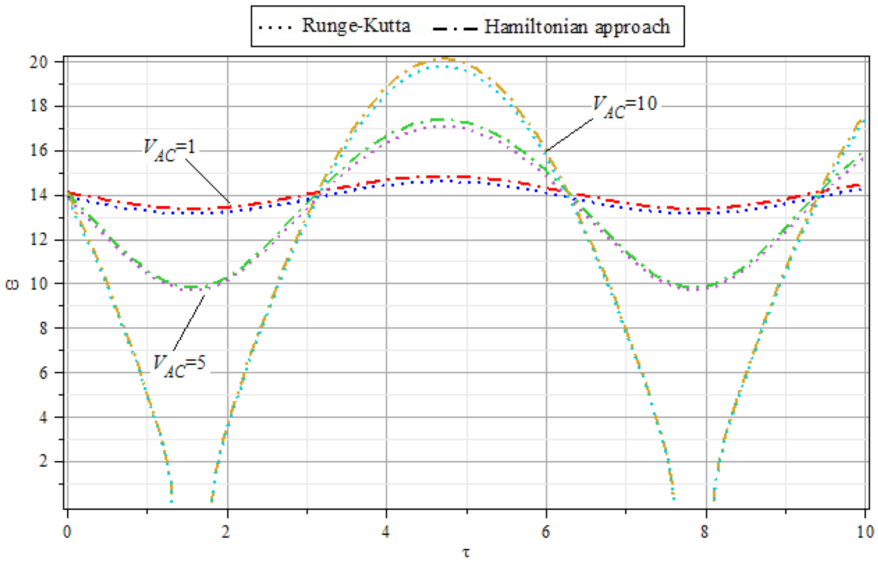

Fig.3. Effect of V_{AC} parameter on natural frequency piezoelectric and electrostatically actuated nano/micro beam with $A=1,\alpha_1=1,\alpha_2=1,\alpha_3=1,\Omega=1,V_{DC}=1$

In Figure 3 shows the effect of applied voltage V_{AC} on natural frequency of nano/micro beam with parameters $A=1,\alpha_1=1,\alpha_2=1,\alpha_3=1,\Omega=1,V_{DC}=1$ and various values for V_{AC}. In comparison with previous figure, the frequency increases with increasing V_{AC} but the maximum value of natural frequency is higher.

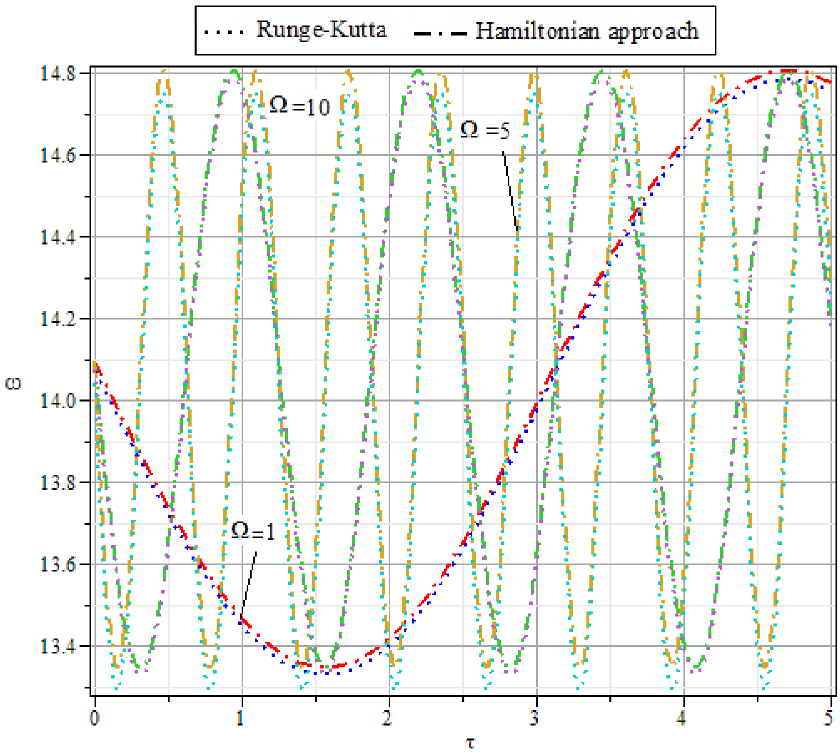

Fig.4. Effect of Ω parameter on natural frequency piezoelectric and electrostatically actuated nano/micro beam with $A=1,\alpha_1=1,\alpha_2=1,\alpha_3=1,V_{DC}=1,V_{AC}=1$.

Figure 4 shows the effect of forced harmonic frequency Ω on natural frequency. This figure shows that the natural frequency is the same for all values of Ω and only the period increased by increasing the amount of Ω.

Dynamic response of piezoelectric and electrostatically actuated nano/micro beam are depicted in Figures (5-11).

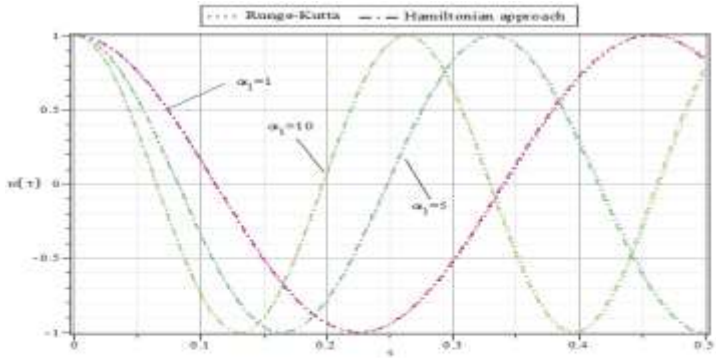

Fig.5. Comparison of dynamic response obtained with Hamiltonian approach and forth order rang-kutta method for different values of α_1 parameter and $A=1, \alpha_2=1, \alpha_3=1, \Omega=1, V_{DC}=1, V_{AC}=1$

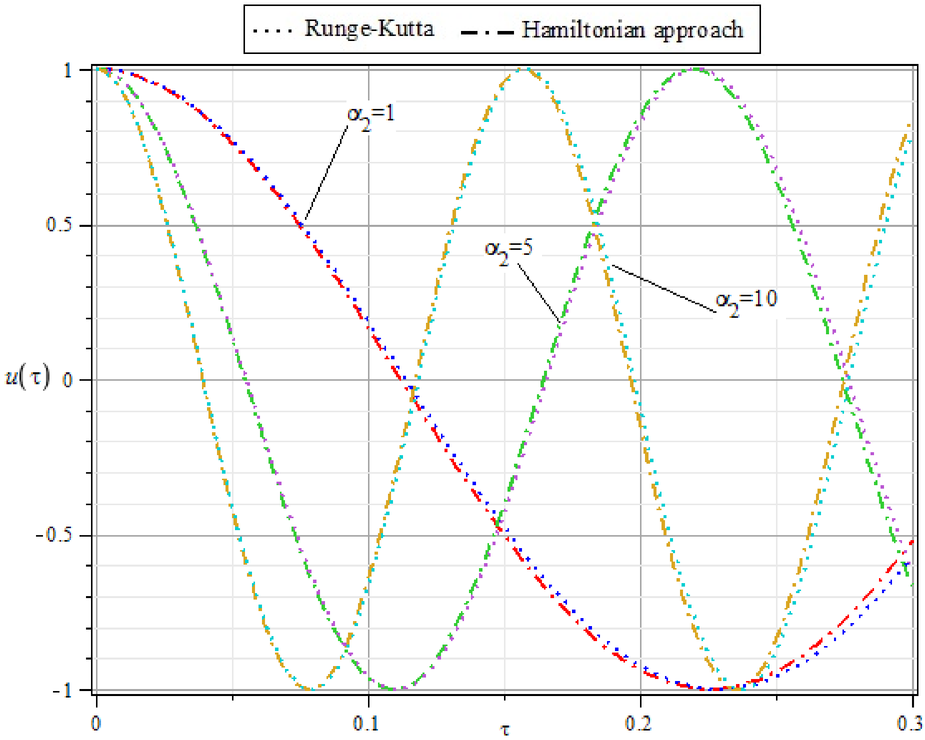

Fig.6. Comparison of dynamic response for different values of α_2 parameter and $A=1, \alpha_1=1, \alpha_3=1, \Omega=1, V_{DC}=1, V_{AC}=1$

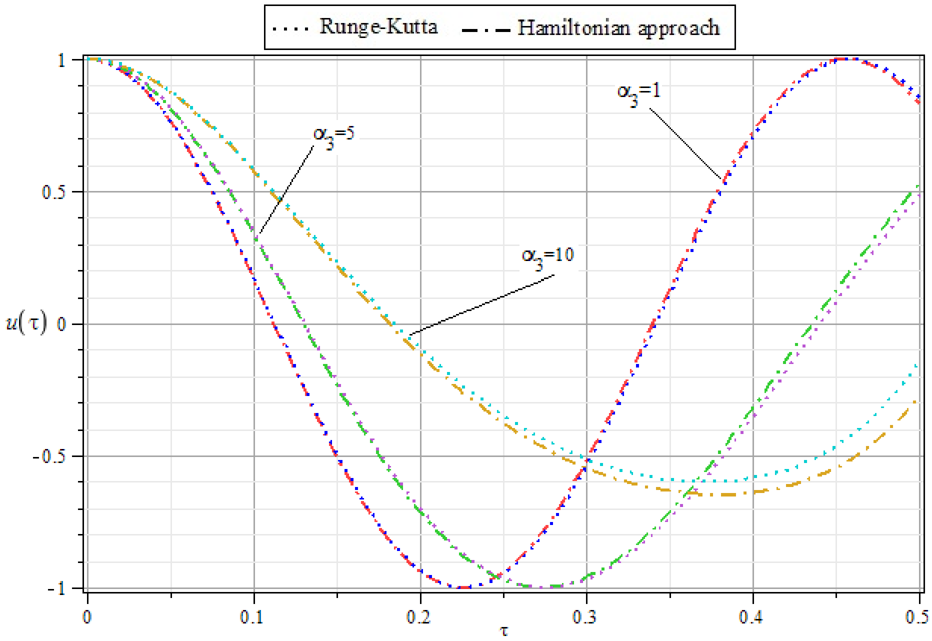

Fig.7. Comparison of dynamic response for different values of α_3 parameter and $A=1, \alpha_1=1, \alpha_2=1, \Omega=1, V_{DC}=1, V_{AC}=1$

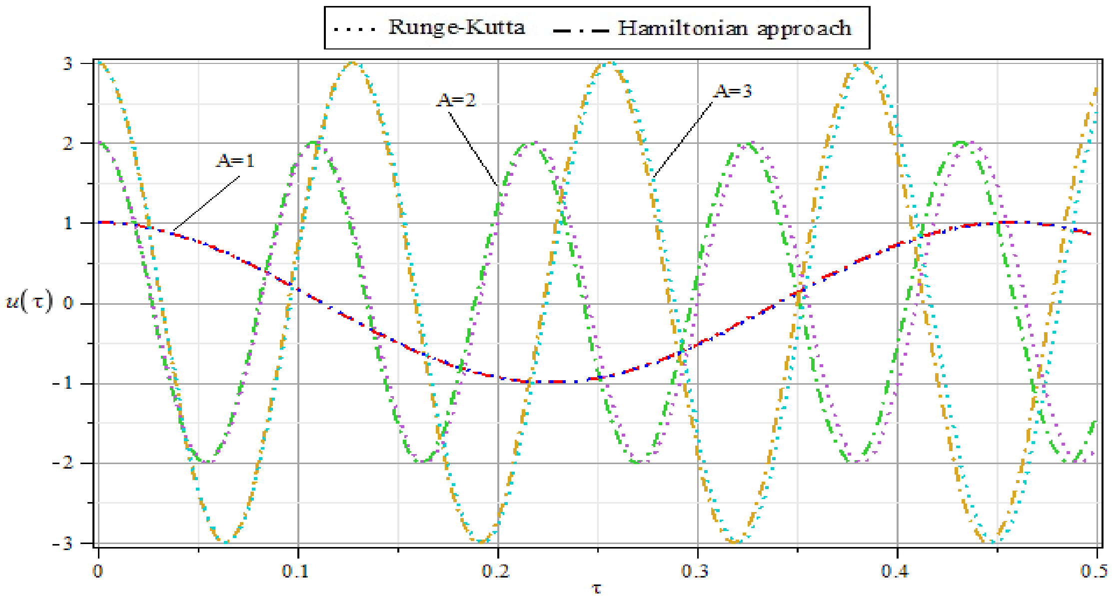

Fig.8. Comparison of dynamic response for different values of A parameter and $\alpha_1=1, \alpha_2=1, \alpha_3=1, \Omega=1, V_{DC}=1, V_{AC}=1$

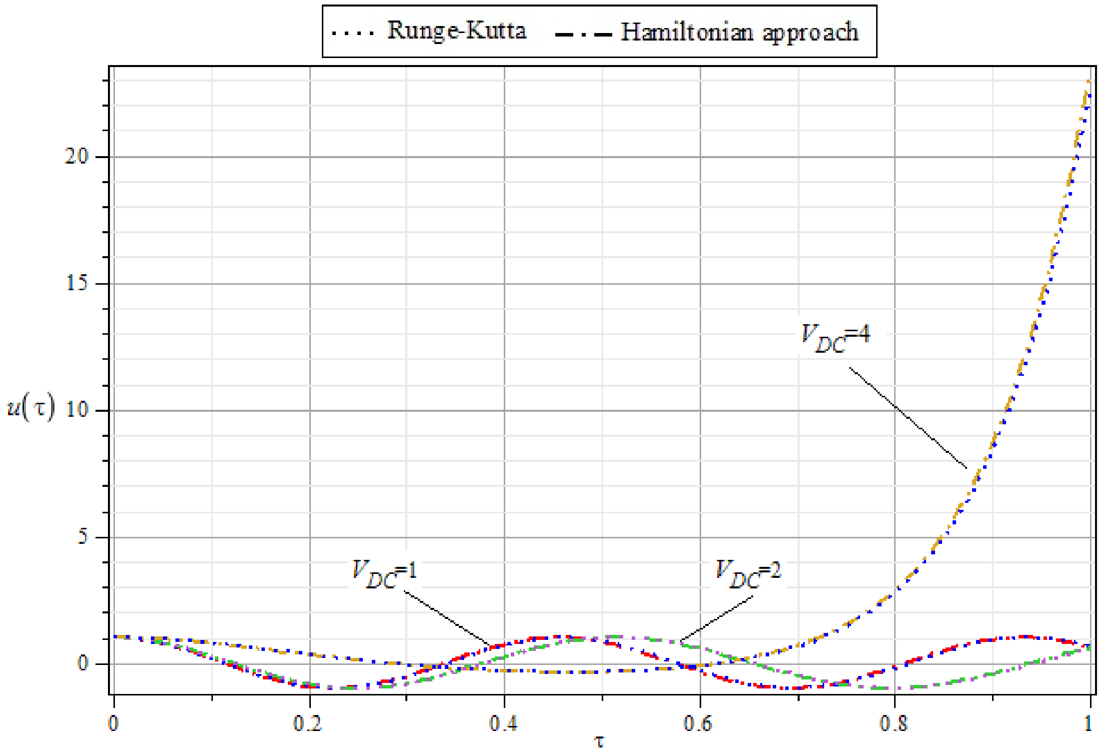

Fig.9. Comparison of dynamic response for different values of V_{DC} parameter and $A=1, \alpha_1=1, \alpha_2=1, \alpha_3=1, \Omega=1, V_{AC}=1$

As shown in Figures 9 and 10, it is obvious that increasing amounts of sauce plays a large role in increasing the amount of output.

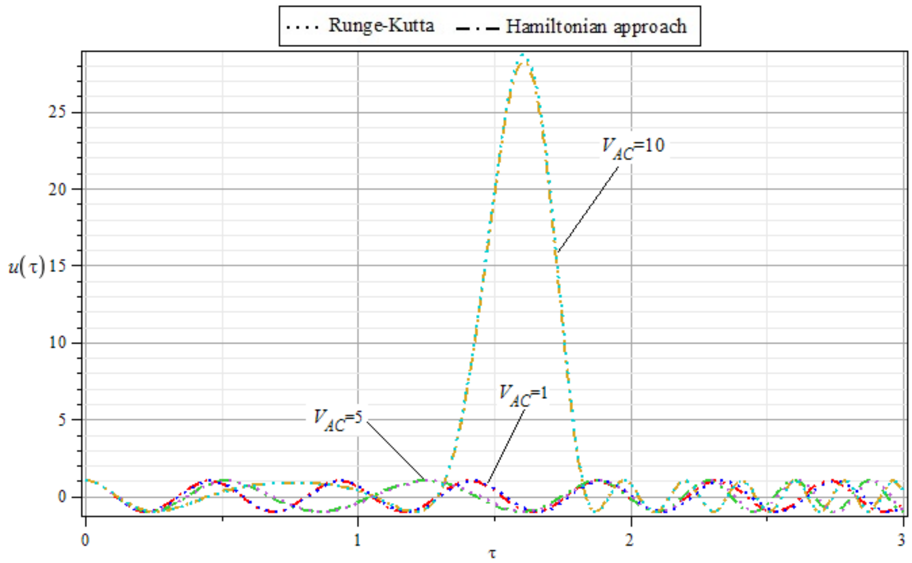

Fig.10. Comparison of dynamic response for different values of V_{AC} parameter and $A=1, \alpha_1=1, \alpha_2=1, \alpha_3=1, \Omega=1, V_{DC}=1$

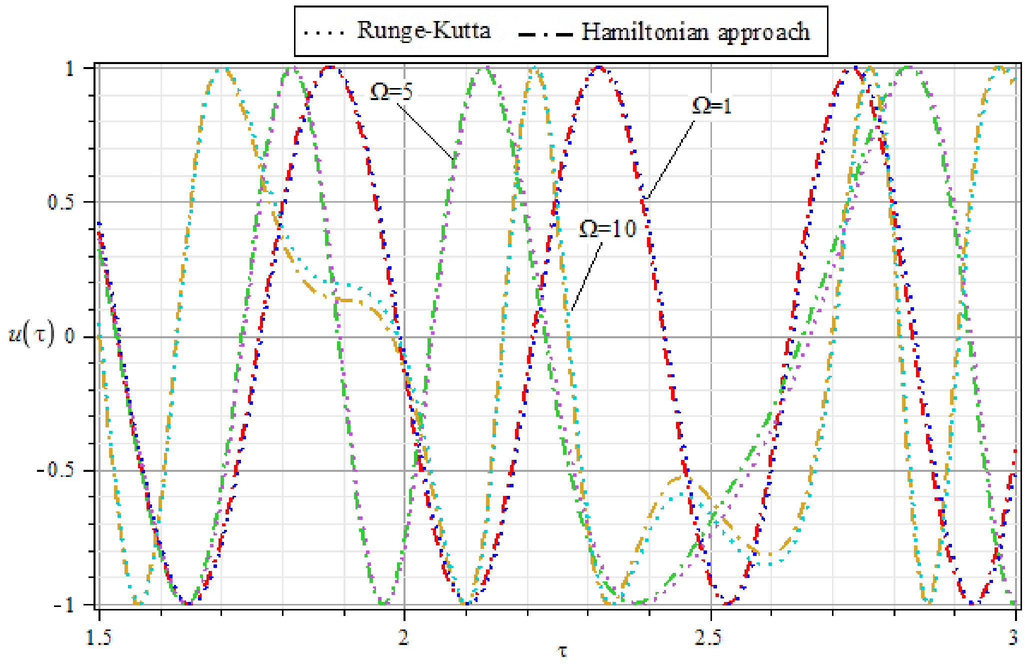

Fig.11. Comparison of dynamic response for different values of Ω parameter and $A=1, \alpha_1=1, \alpha_2=1, \alpha_3=1, V_{DC}=1, V_{AC}=1$

Figures (12-14) shows the effect of parameters α_2, V_{DC} and V_{AC} on dynamic response $W(\xi,\tau)$,respectively.

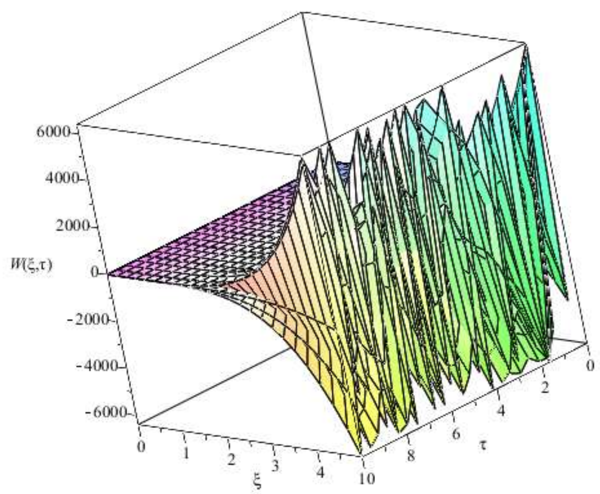

Fig.12. Effect of parameters α_2 on dynamic response $W(\xi,\tau)$.

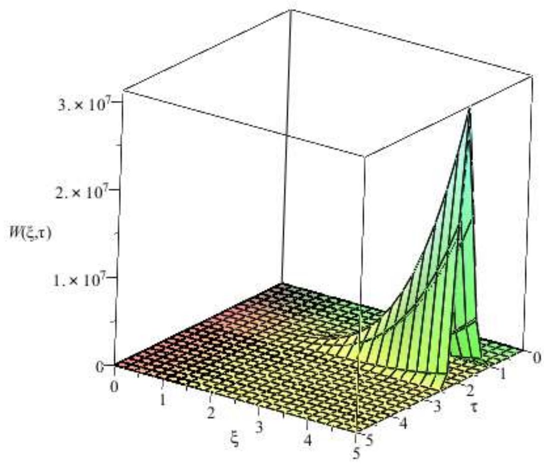

Fig.13. Effect of parameters V_{DC} on dynamic response $W(\xi,\tau)$.

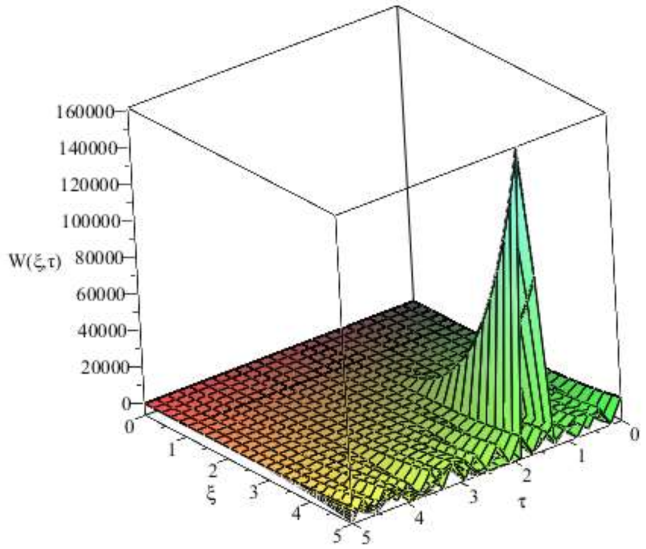

Fig.14. Effect of parameters V_{AC} on dynamic response $W(\xi,\tau)$.

Figure (15) shows the effect of excitation frequency Ω on natural frequency ω in piezoelectric and electrostatically actuated nano/micro beam.

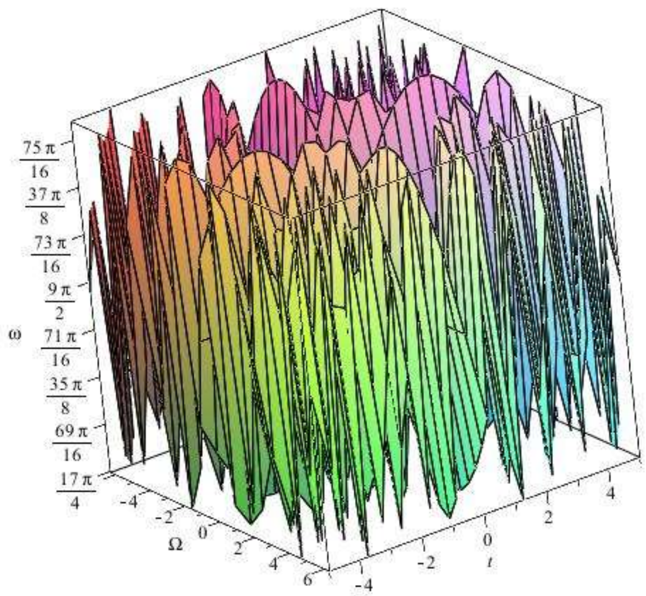

Fig.15. Effect of excitation frequency Ω on natural frequency ω in piezoelectric and electrostatically actuated nano/micro beam with $A=1,\alpha_1=1,\alpha_2=1,\alpha_3=1,V_{DC}=1,V_{AC}=1$

Several simulations and plots could be introduced to consider the fundamental design requirements before any manufacturing process. Based on the presented system, the proposed nonlinear model based on Hamiltonian approach is completely efficient and acceptable to find the effects of parameters on the natural frequency and dynamic response piezoelectric and electrostatically actuated nano/micro beam.

Conclusions

This paper studied the effect of various parameters on the natural frequency and dynamic response of piezoelectric and electrostatically actuated nano/micro beam, applying the Hamiltonian approach and forth orders rang-kutta approach. Other similar work, the effects of voltage V_{AC} which leads to forced harmonic vibration is not consideration and is usually considered to be zero. The main feature of the present work is to consider the voltage V_{AC} on the natural frequency and dynamic response. Due to the nonlinear manner of NEMS/MEMS resonators on actuator design paradigms, this would be used in practical work for more efficient and low cost experiments. The effects of nonlinear parameters on the natural frequency and dynamic response were also illustrated in several figures. Utilized approximate solution converged to the numerical solution and obviously demonstrated a good level of accuracy.

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Conflicts of Interest

The authors report no conflict of interest.

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