Determination of Optimal Investment Strategies For A Defined Contribution (Dc) Pension Fund With Multiple Contributors, Proportional Administrative Costs And Taxation

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Abstract

This study centres on determining the optimal investment strategies for defined contribution (DC) pension fund with multiple contributors, administration cost and taxation on the invested fund. We assume that a certain proportion of the member's contributions as administrative cost which is remitted to the pension fund manager also following the Nigerian Pension Reform Act of 2004 the invested fund is subjected to tax. We obtained an optimized equation using Hamilton Jacobi equation, then solve the equation using Legendre transformation method to obtained explicit solutions of the optimal investment strategy for CARA utility function. We observed that the tax has a direct effect on the investment strategies.

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Introduction

Defined contribution pension according to Antolin et.al. (2010) is very crucial in retirement income system in a lot of countries and there is a growing trend to automatically involve all workers in it. In as much the DC scheme is relatively new compared to the defined benefit (DB) pension scheme, it forms a determining factor of the old age income adequacy for future retirees. In a defined contribution pension fund, contributions are fixed in advance. DC pension fundhas received more attention in the literature; see, Haberman and Vigna (2002), Gerrard etal. (2004)and Deelstra etal. (2004). Over the years sizeable literatures on optimal investment strategies for DC pension funds have been recorded some of which include Gao (2008) who studied an asset allocation problem under a stochastic interest rate. Boulier et al (2001) studied optimal investment for DC with stochastic interest rate and Battocchio and Menoncin (2004)] where the interest rate was Vasicek model, Chubing and Ximing (2013), Deelstra et al (2003) and Gao (2008), studied the affine interest rate which include the Cox- Ingeroll- Ross (CIR) model and Vasicek model. Recently, more attention has been given to constant elasticity of Variance (CEV) model in DC pension fund investment strategies. As Geometric Brownian motion (GBM) can be considered as a special case of the (CEV) model, such work extended the research of Xiao et al (2007) where they applied (CEV) model to derive dual solution of a CRRA utility function via Legendre transform, also Gao (2009) extended the work of Xiao et al (2007) by obtaining solutions for investor with CRRA and CARA utility function. Blake et al.(2012) investigate an asset allocation problem under a loss-averse preference..

Recently Dawei and Jingyi (2014) extended the work in Gao (2009) by modelling pension fund with multiple contributors, they went on to find the explicit solution for CRRA and CARA using power transformation method. Osu et al (2017) studied optimal investment strategies in DC pension fund with multiple contributions using Legendre transformation method to obtain the explicit solution for CRRA and CARA. Akpanibah and Samaila (2017) studied stochastic strategies of optimal investment for DC pension fund with multiple contributors where they considered the rate of contribution to be stochastic. Nkeki (2014),studied a mean-variance portfolio selection problem with stochastic salary, proportional administrative costs and taxation in the accumulation phase of a defined contribution (DC) pension scheme

In this paper, we extend the work of Dawei and Jingyi (2014)and investigate optimal investment strategies for DC pension fund with multiple contributors, administration cost and taxation on the invested fund. We took into consideration administrative cost where certain proportion of the members fund is paid to the pension fund manager also from the Nigerian Pension Reform Act of 2004 where the invested fund is subjected to tax. We solve the optimal investment problem using Legendre transformation method to obtain the optimal investment strategies for CARA utility function.

2. Preliminaries

Starting with a complete and frictionless financial market which is continuously open over a fixed time interval $0 \leq t \leq T$, where$T$is the retirement time of a given shareholder.

Let the market be of a risk free asset (cash) and a risky asset (stock). Suppose (Ω, F, P) is a complete probability space such thatΩ is a real space and$P$ a probability measure, $\{W_o(t), W^\circ(t): \mathrm{t} \geq 0\}$ is a standard two dimensional motion such that they orthogonal to each other.$F$is the filtration and denotes the information generated by the Brownian motion $\{W_o(t), W^\circ(t)\}$.

Let $S_0(t)$ denote the price of the risk free asset, it model is given as

$$\frac{dS_0(t)}{S_0(t)} = rdt. \tag{1}$$

Let $S(t)$ denote the risky asset and its dynamics is given based on its stochastic nature and the price process described by the CEV model in Gao (2009) as

$$\frac{dS_t(t)}{S_t(t)} = \varphi dt + \gamma dW_o. \tag{2}$$

Where φ is an expected instantaneous rate of return of the risky asset and satisfies the general condition $\varphi > r_0$. $\gamma$is the instantaneous volatility,

In a pension fund system with multiple contributors, it is expected that payment are remitted to contributors who have retired from service and the payment continues till the death of a specific contributor after which payment is stopped for that particular contributor. As stated by Dawei and Jingyi (2014) that the payment is a stochastic process and assume the Brownian motion with drift as follows

$$dC(t) = udt - vdW^{\circ}(t), \quad (3)$$

where$u$ and v are positive constants and denote the amount given to the retired contributors and that which which is due death contributors which are out of the system..

Consider that in DC plans the contributions provided by the contributors are fixed and then without loss of generality, we assume that the number of contributors is constant and so is the contribution rate c.Let$0 < \vartheta < 1$be the proportion of contribution to be paid as administrative costs to the pension fund managers. Then, the accumulated contribution by the members at time t is $(1-\vartheta)c(t)$.If there is no investment, the dynamics of the surplus is given by

$$dR(t) = ((1-\vartheta)c(t) - u)dt + vdW^{\circ}(t) \quad (4)$$

$$dR(t) = (\beta c - u)dt + vdW^{\circ}(t). \quad (5)$$

$$\beta = 1 - \vartheta \quad (6)$$

Materials and Methods

Hamilton-Jacobi-Bellman (HJB) equation

Let π be the strategy and we define the utility attained by the members from a given state x at time t as

$$J_{\tau}(t,r,x) = E_{\tau}\left[U\big(X(T)\big) \mid r(t) = r, X(t) = x\right], \quad (7)$$

Where t is the time, r is the short interest rate and $x$is the wealth. The main aim of this section is to find the optimal value function and optimal strategy given as

$$J(t,r,x) = \sup_{\pi} J_{\pi}(t,r,x) \text{ and } \quad \pi^{*}, \quad (8)$$

respectively such that

$$J_{\pi^{*}}(t,r,x) = J(t,r,x). \quad (9)$$

3.2. Legendre Transformation

Here we state the basic theorem on Legendre transform and dual theory; this transformhelps to transform a non linear partial differential equationto a linear partial differential equation.

Theorem 1: Let $f: R^{n} \to R$ be a convex function for $z > 0$, define the Legendre transform

$$L(z) = \max_{x}\{f(x) - zx\}, \quad (10)$$

Where $L(z)$ is the Legendre dual of $f(x)$. Jonsson and Sircar (2002)

Since $f(x)$ is convex, from theorem 3.1 we defined the Legendre transform

$$\hat{J}(t,r,z) = \sup\{J(t,r,x) - zx \mid 0 < x < \infty\} \; 0 < t < T. \quad (11)$$

where$\hat{J}$is the dual of J and $z > 0$ is the dual variable of x.

The value of x where this optimum is attained is denoted by $g(t,r,z)$, so that

$$g(t,r,z) = \inf\left\{x \mid J(t,r,x) \geq zx + \hat{J}(t,r,z)\right\} \; 0 < t < T. \quad (12)$$

The function g and $\hat{J}$are closely related and can be refers to as the dual of J. These functions are related as follows

$$\hat{J}(t,r,z) = J(t,r,g) - zg. \quad (13)$$

Where

$$g(t,r,z) = x, J_{x} = z, \; g = -\hat{J}_{z.} \quad (14)$$

At terminal time, we denote

$$\widehat{U}(z) = \sup\{ U(x) - zx \mid 0 < x < \infty \},$$

and

$$J(z) = \sup\{x \mid U(x) \geq zx + \widehat{U}(z)\}$$

As a result

$$J(z) = (U')^{-1}(z), \tag{15}$$

where$J$ is the inverse of the marginal utility U and note that $J(T, r, x) = U(x)$.

At terminal time T, we can define

$$g(T, r, z) = \inf_{x>0}\{ x \mid U(x) \geq zx + \hat{J}(t, r, z) \} \, and \, \hat{J}(t, r, z) = \sup_{x>0}\{ U(x) - zx\}$$

so that

$$g(T, r, z) = (U')^{-1}(z). \tag{16}$$

Wealth Formulation with tax on invested fund

Let $X(t)$ denote the wealth of pension fund at $t \in [0, T]$, let π denote the proportion of the pension fund invested in the risky asset $S_t$and $1 - \pi$, the proportion invested in risk free asset. Since the surplus is the contribution of the members, it is tax exempted as stated in Nigerian PensionReform Act, 2004,we only subject the invested fund to tax. Let αbe the rate at which the invested fund is being tax, which implies $(1 - \alpha)$ of the invested fund will be tax free. Hence the dynamics of the pension wealth is given by

$$dX(t) = (1 - \alpha)\left(\pi X(t)\frac{dS_t(t)}{S_t(t)} + (1 - \pi)X(t)\frac{dS_0(t)}{S_0(t)}\right) + dR(t) \tag{17}$$

Substituting (1), (2) and (5) into (17) we have

$$dX(t) = \left[\left(\pi X(t)(\varphi - r)\right)(1 - \alpha) + r(1 - \alpha)X(t) + \beta c - u\right]dt + X(t)(1 - \alpha)\pi\gamma dW_0(t) + vdW^{\circ}(t) \tag{18}$$

The Hamilton-Jacobi-Bellman (HJB) equation associated with (18) is

$$J_t + \omega s J_s + (r(1 - \alpha)x + (\beta c - u))J_x + \frac{1}{2}\gamma^2 s^2 J_{ss} + \frac{1}{2}v^2 J_{xx} + \sup\left\{\frac{1}{2}\pi^2(1 - \alpha)^2 x^2 \gamma^2 J_{xx} + \pi x(1 - \alpha)(\varphi - r)J_x + \pi x(1 - \alpha)\gamma^2 s J_{xs}\right\} = 0. \tag{19}$$

Differentiating equation (19) with respect to π, we obtain the first order maximizing condition as

$$\pi x^2(1 - \alpha)^2\gamma^2 J_{xx} + x(1 - \alpha)\gamma^2 s J_{xs} + x(1 - \alpha)(\varphi - r)J_x = 0 \tag{20}$$

Solving equation (20) for τ we have

$$\pi^* = -\frac{[(\varphi - r)J_x + \gamma^2 s J_{xs}]}{x(1 - \alpha)\gamma^2 J_{xx}}. \tag{21}$$

Substituting (21) into (19), we have

$$J_t + \omega s J_s + (r(1 - \alpha)x + (\beta c - u))J_x + \frac{1}{2}\gamma^2 s^2 J_{ss} + \frac{1}{2}v^2 J_{xx} + \sup\left\{\frac{1}{2}\left(-\frac{[(\varphi - r)J_x + \gamma^2 s J_{xs}]}{x(1 - \alpha)\gamma^2 J_{xx}}\right)^2 (1 - \alpha)^2 x^2 \gamma^2 J_{xx} + \left(-\frac{[(\varphi - r)J_x + \gamma^2 s J_{xs}]}{x(1 - \alpha)\gamma^2 J_{xx}}\right)x(1 - \alpha)(\varphi - r)J_x + \left(-\frac{[(\varphi - r)J_x + \gamma^2 s J_{xs}]}{x(1 - \alpha)\gamma^2 J_{xx}}\right)x(1 - \alpha)\gamma^2 s J_{xs}\right\} = 0 \tag{22}$$

So that

$$J_t + \omega s J_s + (r(1-\alpha)x + (\beta c - u))J_x + \frac{1}{2}\gamma^2 s^2 \left[J_{ss} - \frac{J_{xs}{}^2}{J_{xx}}\right] + \frac{1}{2}v^2 J_{xx} - \frac{(\varphi - r)^2}{2\gamma^2}\frac{J_x{}^2}{J_{xx}} - (\varphi - r)s\frac{J_x J_{xs}}{J_{xx}} = 0. \quad (23)$$

Differentiating (13) with respect to $t, s, and x$ we have the following partial derivatives

$$J_t = \hat{J}_t,\ J_s = \hat{J}_s, J_x = z, J_{sx} = \frac{-\hat{J}_{sz}}{\hat{J}_{zz}}, J_{xx} = \frac{-1}{\hat{J}_{zz}},\ J_{sx} = \hat{J}_{ss} - \frac{\hat{J}_{sz}{}^2}{\hat{J}_{zz}}. \quad (24)$$

Substituting (24) into (23), we have

$$\hat{J}_t + \omega s\hat{J}_s + (r(1-\alpha)x + (\beta c - u))z + \frac{1}{2}\gamma^2 s^2 \hat{J}_{ss} - \frac{1}{2}v^2\frac{1}{\hat{J}_{zz}} - \frac{z^2(\varphi - r)^2}{2\gamma^2}\hat{J}_{zz}{}^2 - (\varphi - r)sz\hat{J}_{zz} = 0 \quad (25)$$

and

$$\pi^* = -\frac{[(\varphi - r)z\hat{J}_{zz} - \gamma^2 s\hat{J}_{sz}]}{x(1-\alpha)\gamma^2}. \quad (26)$$

Differentiating (25) and (26) with respect to$z$ and using $x = g = -\hat{J}_z$, we have

$$g_t + rsg_s - r(1-\alpha)g - (\beta c - u) + \frac{1}{2}\gamma^2 s^2 g_{ss} + \left(\frac{(\varphi - r)^2}{\gamma^2} - r\right)zg_z + \frac{1}{2}v^2\frac{g_{zz}}{g_z{}^2} + \frac{z^2(\varphi - r)^2 g_{zz}}{2\gamma^2} - (\varphi - r)szg_{sz} = 0 \quad (27)$$

and

$$\pi^* = -\frac{[(\varphi - r)zg_z - \gamma^2 sg_s]}{g(1-\alpha)\gamma^2} \quad (28)$$

Assume that the fund manager takes an exponential utility

$$U(x) = -\frac{1}{b}e^{-bx}, \quad b > 0. \quad (29)$$

The absolute risk aversion of a decision maker with the utility described in (29) is constant and is a CARA utility

Since $g(T, s, z) = (U')^{-1}(z)$ with the CARA utility function we obtain

$$g(T, s, z) = -\frac{1}{b}lnz. \quad (30)$$

Hence we developed a solution to (27) as follows

$$g(t, s, z) = -\frac{1}{b}[q(t)(lnz + a(t, s))] + e(t), \quad (31)$$

With boundary conditions $q(T) = 1,\ e(T) = 0, a(T, s) = 0$

$$g_t = -\frac{1}{b}[q'(t)(lnz + a(t, s)) + qa_t] + e'(t),$$

$$g_s = -\frac{1}{b}qa_s,\ g_z = -\frac{q}{bz},\ g_{zz} = \frac{q}{bz^2}, g_{ss} = -\frac{1}{b}qa_{ss},\ g_{sz} = 0. \quad (32)$$

Substituting (32) into (27), we have

$$[q'(t) - r(1-\alpha)q(t)]lnz + [-e'(t) + r(1-\alpha)e(t) + (\beta c - u)]b + [a_t + rsa_s + \frac{1}{2}\gamma^2 s^2 a_{ss} + \frac{(\varphi - r)^2}{2\gamma^2} - r(1-\alpha)a + \frac{q'}{q}a - r(1-\alpha) - \frac{1}{2}v^2]q = 0.$$

Such that

$$q'(t) - r(1-\alpha)q(t) = 0 \quad (33)$$

and

$$a_t + rsa_s + \frac{1}{2}\gamma^2 s^2 a_{ss} + \frac{(\varphi - r)^2}{2\gamma^2} - r(1-\alpha) - \frac{1}{2}v^2 = 0. \quad (34)$$

So that

$$-e'(t) + r(1-\alpha)e(t) + (\beta c - u) = 0 \quad (35)$$

Solving (33) and (35) we obtain

$$q(t) = e^{-r(1-\alpha)(t-T)} \tag{36}$$

and

$$e(t) = -\frac{(\beta c-u)}{r(1-\alpha)}(1 - e^{-r(1-\alpha)(t-T)}). \tag{37}$$

We next formulate a solution for (32) in the following form

$a(t,s) = Y(t) + Z(t)\gamma^2,\ Y(T) = 0, Z(T) = 0, where\ \gamma^2 = k^2 s^{2\delta}$, and δ is the elasticity parameter and satisfies the general condition $\delta < 0$.

$$a_t = Y' + Z'\frac{k^2}{\gamma^2}, a_s = -\frac{2\delta Y}{s}\frac{k^2}{\gamma^2}, a_{ss} = \frac{2\delta(2\delta+1)Z}{s^2}\frac{k^2}{\gamma^2}. \tag{38}$$

Substituting (38) into (34) we have

$$(Y' + 2\delta(2\delta+1)k^2 Z - r(1-\alpha) - \tfrac{1}{2}v^2)\gamma^2 + k^2\left[Z' - 2r\delta Z + \frac{(\varphi-r)^2}{2k^2}\right] = 0, \tag{39}$$

so that

$$Y' + 2\delta(2\delta+1)k^2 Z - r(1-\alpha) - \tfrac{1}{2}v^2 = 0, \tag{40}$$

and

$$Z' - 2r\delta Z + \frac{(\varphi-r)^2}{2k^2} = 0. \tag{41}$$

Solving (41) with the given condition gives;

$$Z(t) = \frac{(\varphi-r)^2}{4k^2 r\delta}\left[1 - e^{2r\delta(t-T)}\right]. \tag{42}$$

Next substituting (42) into (40) and solving (40) with the given condition we have

$$Y(t) = \left[\frac{(2\delta+1)(\varphi-r)^2}{4r} - r(1-\alpha) - \tfrac{1}{2}v^2\right](T-t) - \left[\frac{(2\delta+1)(\varphi-r)^2}{8r^2\delta}\left(1 - e^{2r\delta(t-T)}\right)\right]. \tag{43}$$

$$a(t,s) = \left[\frac{(2\delta+1)(\varphi-r)^2}{4r} - r(1-\alpha) - \tfrac{1}{2}v^2\right](T-t) - \left[\frac{(2\delta+1)(\varphi-r)^2}{8r^2\delta}\left(1 - e^{2r\delta(t-T)}\right)\right] + \left[\frac{(\varphi-r)^2}{4\gamma^2 r\delta}\left(1 - e^{2r\delta(t-T)}\right)\right]. \tag{44}$$

$$g(t,s,z) = -\frac{1}{b}e^{r(1-\alpha)(t-T)}\left\{lnz + \left[\frac{(2\delta+1)(\varphi-r)^2}{4r} - r(1-\alpha) - \tfrac{1}{2}v^2\right](T-t) - \left[\frac{(2\delta+1)(\varphi-r)^2}{8r^2\delta}\left(1 - e^{2r\delta(t-T)}\right)\right] + \left[\frac{(\varphi-r)^2}{4\gamma^2 r\delta}\left(1 - e^{2r\delta(t-T)}\right)\right]\right\} - \frac{(\beta c-u)}{r(1-\alpha)}(1 - e^{-r(1-\alpha)(t-T)}). \tag{45}$$

If $g_z = -\frac{1}{bz}e^{r(1-\alpha)(t-T)}$. and $g_s = \frac{1}{b}e^{r(1-\alpha)(t-T)}\frac{(\varphi-r)^2}{2s\gamma^2 r}\left(1 - e^{2r\delta(t-T)}\right)]$,

then the optimal investment strategy is given as

$$\pi^* = \frac{1}{b}\frac{(\varphi-r)}{\gamma^2 g(1-\alpha)}e^{r(1-\alpha)(t-T)}\left[1 + \frac{(\varphi-r)}{2r}\left(1 - e^{2r\delta(t-T)}\right)\right]. \tag{46}$$

Conclusion

This study centres on determining the optimal investment strategies for DC pension fund with multiple contributors with administration cost and taxation on the invested fund. We took into consideration administrative cost where certain proportion of the members fund is paid to the pension fund manager. Also following the Nigerian Pension Reform Act of 2004 the invested fund is subjected to tax. We obtained an optimized equation using Hamilton Jacobi equation then solve the equation using Legendre transformation method to obtained explicit solutions of the optimal investment strategy for CARA utility function. We observed that the tax has a direct effect on the investment strategy and this provides the pension managers insight on how to invest to maximize profit.

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