A stochastic Model for Predicting Irrigation Water Requirements (IWR)

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ABSTRACT

The main objective of this paper is to develop a stochastic time series model with trend, periodic and irregular components using a ten years IWR decade data for three different types of cotton crops cultivated in Gezira Scheme, SUDAN. The model was applied to cotton Brackat and then used to Shmbat & Akala cotton. In the analysis of IWR time series the correlogram technique was used to detect the periodicity which then smoothed by Fourier series method. The series is then tested for stationary and the dependent part of irregular component is found to be well expressed by the first order autoregressive model for all the crops. The developed model superimposes a periodic-deterministic process and an irregular component.

INTRODUCTION

The design and operation of an irrigation project requires detailed information about the irrigation needs of a specific crop with respect to time. Various methods are used to provide this information. Most of the existing methods are either deterministic or probabilistic in nature. While the former do not consider the random effect of the various input parameters, the latter employs the concept of probability to the extent that the time-based characteristics of irrigation needs are ignored. With the ever-increasing demand for water, these methods are no longer sufficient. The irrigation needs of a crop are stochastic in nature because they are affected by the random climatologic parameters (Gupta *et. al*. 1986), i.e. stochastic climate variations are transferred to become stochastic component of irrigation water requirements. Hence, the irrigation needs should be computed considering both the deterministic and stochastic parts of the process. Considering all other factors, soil, topography, quality of water, irrigation methods and practices, etc., known or assumed. The irrigation need is a function of the stochastic variation of the local weather i.e., evapotranspiration rates and precipitation. Accordingly, stochastic analysis of irrigation requirement time series will provide a mathematical model that accounts for the deterministic and stochastic portions and also reflect the decade variations of irrigation needs of a crop.

During the past years many investigators have analyzed the time series of stream flow and rainfall and developed the autoregressive, trigonometric regression and other forms of the stochastic models for data generation. However, the study on the stochastic structure of irrigation requirement time series has not been made (Gupta, *et. al*. 1986).

METHODOLOGY

This model is based upon a simple linear programming approach. Analysis was done to achieve the following objectives:

- To test the stationarity of the IWR time series,
- To identify and remove the trend and periodic components,
- To study the structure of the dependent stochastic component, and
- To recognize the independent stochastic component using diagnostic checking approach.

The general additive model used to describe the time series X_t, irrigation water requirement (IWR) is given as:

$$X_t = T_t + P_t + Z_t \qquad (1)$$

In which

T_t = Trend component at time t = 1, 2… N; P_t = the periodic component, Z_t = the irregular component having a dependent and independent parts and, N = number of data points.

Trend Component (T_t)

T_t was identified by using the seasonal IWR values [S_i, i = 1, 2, 3..... n, n = number of seasons] obtained by the algebraic sum of IWR decade data during each season. For detecting trend, a hypothesis of no-trend was made and the following statistical test was selected to check the hypothesis.

Turning Point Test

In an observed sequence S_i, a turning point R occurs at time i, and its S_i is either greater than S_{i-1} and S_{i+1} or less than the two adjacent values. The expected number of R in a random series has mean and Variance that can be determined as following:

(2) Mean E(R) = 2(n-2)/3

(3) Var V(R) = (16n-29/90)

Consequently $m^{/}$ can be expressed as standard measures, that is:

$$m' = \frac{R - mean}{(Var)^{1/2}} \quad or \quad m' = \frac{R - \mathrm{E}(R)}{\sqrt{V(R)}}$$

Which is treated approximately as a standard normal variant. The value of $m^{/}$ was compared with its value from tables at 5% or 1% level of significance. If the calculated value of $m^{/}$ is within the limits, then the hypothesis of no trend will be accepted, but if present, then it will be removed. After removing the trend, a trend-free series can be obtained. **Periodic Component (Pt)**

The seasonal cycle, (P_t) is modelled by Shetters period gram (Matalas, 1967) which is given as follows.

$$P_t = A_o + \left[\sum_{k=1}^{k/2} A_k \cos(2\pi t / h) + B_k \sin(2\pi kt / h) \right] \qquad (4)$$

In which h= time span of periodicity, k= number of harmonics, 1≤k≤h/2. The Fourier coefficients A_K and B_k of equation (4) are computed by the following:

$$A_k = \frac{2}{N} \sum_{t=1}^{N} \left[P_t Cos(2\pi kt / h) \right] \qquad (5)$$

$$B_k = \frac{2}{N} \sum_{t=1}^{N} \left[P_t Sin(2\pi kt / h) \right] \qquad (6)$$

$$A_o = \frac{2}{N} \sum_{t=1}^{N} P_t \qquad (7)$$

P_t was then computed using equation (4) for particular number of harmonic then, the computed P_t would be removed from original series which leaves only the irregular component Z_t for further analysis.

Irregular Component (Zt)

It was assumed that the value of Z_t at time it would be the combined effect of the weighted sum of the past values so that the dependent part of Z_t can be represented by the p-th order autoregressive process AR (p), and is governed by the following equation:

$$Z_t = \Phi_1 Z_{t-1} + \Phi_2 Z_{t-2} \ldots\ldots\ldots \phi_p Z_{t-p} + a_t \qquad (8)$$

Where a_t, is a white noise process with zero mean and a finite variance σ_a^2?

The AR (ρ) process is characterised by the number (ρ+1) and the parameters $\Phi_1, \Phi_2 ... \Phi_\rho$ And σ_a^2. The fitting procedure of this model involves two stages (Box and Jenkins, 1976):

(1) Selection of the model order, p; and (2) Estimation of autoregressive coefficients$\phi_1, \phi_2 ... \phi_\rho$

For selection of orderρ, the residual variance method was used to obtain the orderρ. The residual variance (Z^2_s) can be computed using the following equation:

$$Z_s^2(\rho) = \frac{1}{N-2\rho-1} Z(\mu, q_1, q_2, ...q_\rho) \qquad (9)$$

In which $Z(\mu,q_1,q_2,\ldots,q_\rho) = (N-p)(c_o- \phi_1 c_1 - \phi_2 c_2 \ldots \phi_\rho c_\rho)$ (residual sum of square); $c_0, c_1 .... c_\rho$.are auto covariance functions at lags ρ, ρ= 0,1,2,3,4,5 respectively . The value of c_ρ for any ρ was computed as;

$$C\rho = E[(Z_t - \mu)(Z_{t+\rho} - \mu)] \qquad (10)$$

Where $\mu = E(Z_t)$. And E (.) denotes the mathematical expectation (mean).

The minimum value of Z_S^2 (ρ) computed by equation (9), gave the appropriate order of the autograssive parameterρ. For representing the dependence structure of IWR time series, the autocorrelation coefficients (r_ρ) can be determined as:

$$r_\rho = \frac{c_\rho}{c_o} \qquad (11)$$

For each value of c_ρ computed by Eq. (10), the partial autoregressive coefficients $\phi_{\rho,\rho}$ expressed as function of $r_\rho$could be computed by using the following regressive formula, (Kottegoda, 1980):

$$\phi_{\rho,\rho} = \left[\frac{r_\rho - \sum_{k=1}^{\rho-1} \{(\phi_{\rho-1,k}) r_{\rho-k}\}}{1 - \sum_{k=1}^{\rho-1} \{(\phi_{\rho-1,\rho-k}) r_k\}} \right] \qquad (12)$$

And

$$\phi_{\rho,k} = \left[\phi_{\rho-1} - \phi_{\rho,\rho}\left(\phi_{\rho-1,\rho-k}\right)\right] \quad (13)$$

In which k=1, 2...ρ

The first three linear autoregressive orders (i.e. ρ =1, ρ =2, and ρ= 3 of equation (8) usually are good approximation for representing the time series. Then they can be used to analyze the Z_t of IWR time series.

In order to separate the dependent portion of Z_t from the independent one, the following expression can be used.

$$a_t = \left[Z_t - \sum_{k=1}^{\rho} \phi_{\rho,k} Z_{t-k}\right] \quad (14)$$

The a_t series is called the residual series in the subsequent discussion.

Diagnostic Checking

Diagnostic checking is used for statistically verifying the adequacy of the formulated model. Diagnostic was done to confirm the randomness of the residuals, which is the condition for accepting the formulated autoregressive model. For this paper, the residual series was examined for any lack of randomness. Autocorrelation coefficients of residual series for lag1 (L_1=50) were computed and were drawn against L_1 with 95% confidence limits. If the obtained correlogram fits within the limits, then it can be proved that residuals are normally distributed with zero mean and var = (1/L).

Model Efficiency

Nash and Sutcliffe (1970) can describe the overall model fit using model efficiency criterion such as that provided. The form of this criterion is:

$$R^2 = \frac{F_O - F}{F_O} \quad (15)$$

Where:

$$F_O = \sum_{t=1}^{N}\left(X_t - \overline{X_t}\right)^2 ; F = \sum_{t=1}^{N}\left(X_t - \hat{X}_t\right)^2 \quad (16)$$

In which R^2 is model efficiency, F_O is an initial variance, F is residual variance, and N is number Of data points and X_t is observed series, and $\hat{x}$ is the predicted value.

Model Application, Cotton (Barakat)

The methodology described in the above part was used in investigating the structure of time series of irrigation water requirements (IWR) for three different types of crops in Gezira Scheme. Computing the crop Evapotranspiration and effective rainfall obtained the IWR values during growing period (1980-1989) for three different cotton crops (Akala; Barakat and Shambat). The growing period of each crop was divided into 20 sub-periods, each of one-decade duration.The IWR time series is composed of its decade values of ten cropping seasons in the present analysis, the efficiency of the irrigation system has not been considered, hence the data of IWR has to be adjusted in order to obtain the gross IWR of the crop. Table (1) shows a few statistical characteristics of the decade IWR that indicates that the coefficient of variation range from 0.0173 to 0.1424, which signifies the importance of temporal variability of IWR values. The values of serial correlation coefficient (r_p) in table (1) are also significantly different from zero, which shows that IWR is mutually dependent. Thus, the IWR time series can be modelled on stochastic Theory. Fig. (1) Shows the mean values of the decade IWR of a cotton crop over a ten periods under analysis.

Table (1): Statistical Characteristics of Season IWR series of Cotton (Barakat)

Season No. No.	Total (mm)	Avg. (mm)	Std.	C_p	r_p
1	231.2	23.12	3.293023	0.142432	0.9103
2	234.99	23.499	2.069785	0.08808	0.7525
3	233.5	23.35	2.096691	0.089794	0.5234
4	234.18	23.418	1.766055	0.075414	0.2443
5	285.5	28.55	3.632189	0.127222	-0.0486
6	360.75	36.075	3.719848	0.103114	-0.3212
7	445.84	44.584	3.995487	0.089617	-0.5522
8	538.55	53.855	4.592417	0.085274	-0.7319
9	659.21	65.921	3.331778	0.050542	-0.8294
10	709.5	70.950	1.228341	0.017313	-0.8568
11	729.32	72.932	1.289787	0.017685	-0.8195
12	715.25	71.525	1.514238	0.021171	-0.7184
13	703.59	70.359	2.559507	0.036378	-0.5375
14	628.24	62.824	2.838302	0.045179	-0.3130
15	590.73	59.073	3.657085	0.061908	-0.0453
16	523.16	52.316	2.104457	0.040226	0.2357
17	494.34	49.434	2.740400	0.055524	0.5072
18	443.48	44.348	1.709300	0.038543	0.7276
19	442.15	44.215	2.660774	0.060178	0.8864
20	415.76	41.576	2.230093	0.053639	0.9610

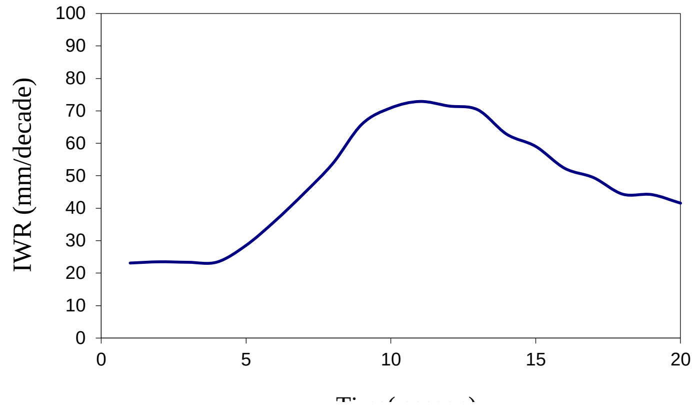

Fig (1): Mean Decade Values of IWR for Cotton (Bracket)

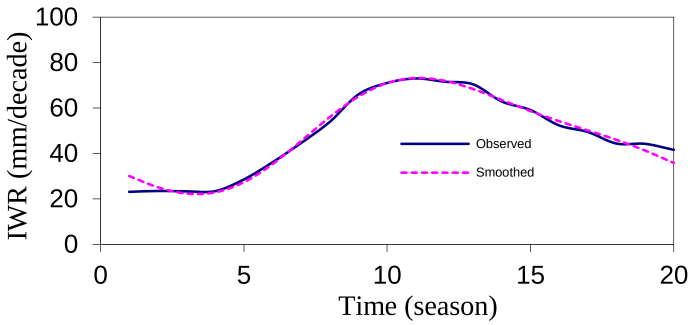

Fig (2): IWR Estimated from 10-Season Means & Obtained by harmonic Fitted Point.

Removal of Components of IWR time Series

Trend Components

Before we actually estimate the trend, the first thing to find is whether or not any trend is present at all. To check this, tests for randomness were performed on the series using Turing point test as follows: m' = (R-mean)/ standard deviation. The seasonal values, S_i for ten seasons, are 937.3, 984.4, 951.041, 979.4, 951.97, 978.33, 949.86, 980.76, 950.5 and 955.68mm. The computed m', for the turning point test is 1.38, which is within the limits of ±1.960 at 5% level of significance. It reveals the absence of T_t in the IWR time series, hence X_t is treated as the trend free series.

Periodic Component

Presence of P_t in IWR series was confirmed by the oscillating shape of the correlogram in Fig (3) which has peaks at lags equal 20 and other multiples of it, indicating the time span of periodicity as 20 decades.

Table (2): Smoothing via Constrained Fourier Series of Cotton Crop (Brakat) data

Number of harmonic	Fourier coefficients		Amplitude	Variance accounted by the j^{th} harmonics
	A(j)	B(j)		
1	-17.59	-15.67	23.56	93.23
2	5.21	-1.29	5.37	4.84

Table (2) reflects the percentage of variance accounted for by a certain number of harmonics as 93.23% which recommends the selection of the first harmonic only. For representing the P_t of the IWR time series, Fourier coefficients A_k and B_k are given in Table (3) were found to be -17.59 and -15.67 respectively, therefore P_t can be expressed as:

P_t was computed using Eq. (17) for all values of t (t_{max}=200). The P_t thus obtained was removed

$$P_t = 48.09 - 17.59\cos\left(\frac{2\pi t}{h}\right) - 15.67\sin\left(\frac{2\pi t}{h}\right) \qquad (17)$$

from the original time series in order to get a new stationary series Z_t.

Irregular Component

Z_t was analysed by fitting the auto-regressive or Morkov schemes to the series as outlined in equation (8)

The order of the model was determined by comparing the order of the model and its R^2 as shown in Table (5), also the correlogrom of residual series in Fig (4) with 95% tolerance limit indicates that the model is within order 1.The empirical values of ϕ_1 is 0.911 .Therefore Z_t. may be expressed as:

The AR (1) is stationary because the parameter ϕ_1 satisfies the condition $-1< \phi <1$. Then the

$$Z_t = 0.911 Z_{t-1} + a_t \qquad (18)$$

developed model describes the periodic irregular behaviour of the original series and it is a trend free series.

The IWR Model

The developed model is a super position of a harmonic-deterministic process and first order Markorv Model and it can be denoted as follows:

$$X_t = 48.09 - 17.59\cos\left(\frac{2\pi t}{h}\right) - 15.67\sin\left(\frac{2\pi t}{h}\right) + 0.911X_{t-1} + a_t \qquad (19)$$

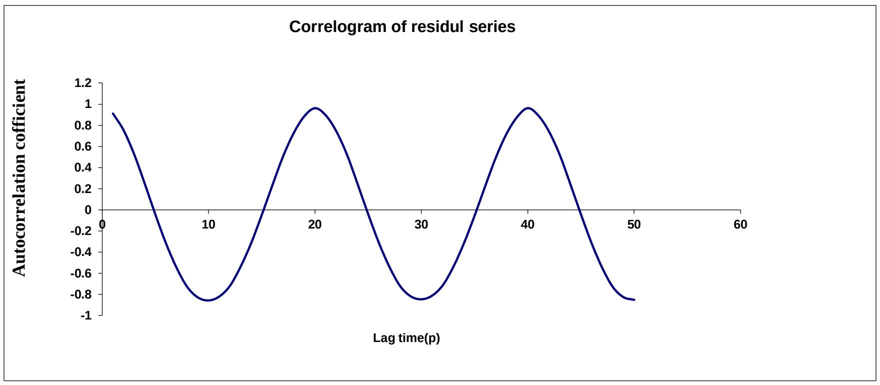

Fig (3): Correlogram of season IWR for Cotton (Braket)

The three terms constitute the deterministic part of the IWR time series. The first term is a constant, indicating the arithmetic mean of IWR. The second and third terms are the harmonic portion of the The fourth term represents the dependent stochastic deterministic part, and are functions of time. component of the model. The last term is a random independent part of the stochastic component with a zero mean. The formulated model was subjected to various checks to test its adequacy for representing the time dependent structure of the IWR. The correlogram in Fig. (4) Shows that almost all the auto - correlation coefficient has a mean value of 0.00126, nearly zero, and the variance of 0.021, which is approximately (1/50 = 0.02).This leads to the conclusion that the residuals are independent and normally distributed.

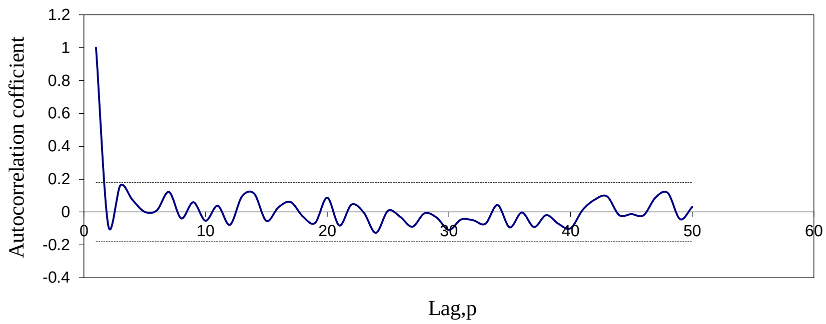

Fig. (4): Correlogram of Residual Series with 95% Tolerance limits

The IWR, generated by the model, were plotted with corresponding observed values in Fig (5) which indicates closeness of the values, and thereby reflects the appropriateness of the formulated IWR model. Therefore, the model may be employed to generate decade IWR values of cotton crops for use in planning and designing irrigation projects. (3):

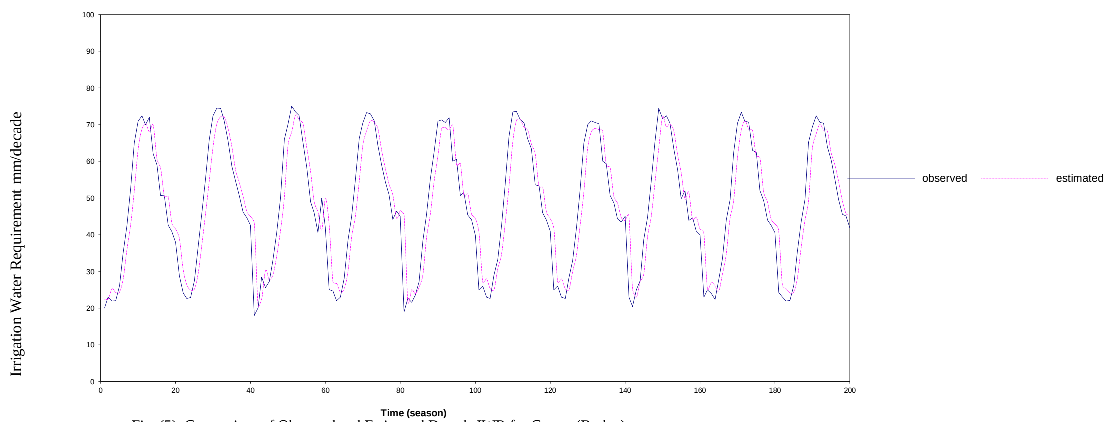

Fig. (5): Comparison of Observed and Estimated Decade IWR for Cotton (Brakat)

Smoothing via Constrained Fourier series for Cotton (Shambat)

Number of harmonic	Fourier coefficients		Amplitude	Variance accounted by the jth harmonics
	A(j)	B(j)		
1	-2.46	-1.49	2.88	16.95
2	-4.18	1.31	4.38	39.23

Table (4): Smoothing via Constrained Fourier series for Cotton (Akala)

Number of harmonic	Fourier coefficients		Amplitude	Variance accounted by the jth harmonics
	A(j)	B(j)		
1	-8.8	-0.51	9	94
2	0.09	-0.7	0.75	0.59

Table (5): Comparing Model Parameters and Efficiency

Auto Model order p	Estimated Parameters				Model Efficiency R^2 %	Types of Crops
	ϕ_1	ϕ_2	ϕ_3	ϕ_4		
1 2	0.9105 1.34	 -0.468			84.20 65.97	Cotton Shambat
1 2 3 4	0.7116 0.843 0.84 0.84	 -0.184 -0.166 -0.18	 -0.022 0.038	 -0.071	61.01 59.92 59.93 59.95	Cotton Shambat
1 2 3 4	0.81 0.97 0.944 0.922	 -0.199 -0.071 -0.083	 -0.132 0.027	 -0.168	72.81 70.85 71.46 71.89	Cotton Akala

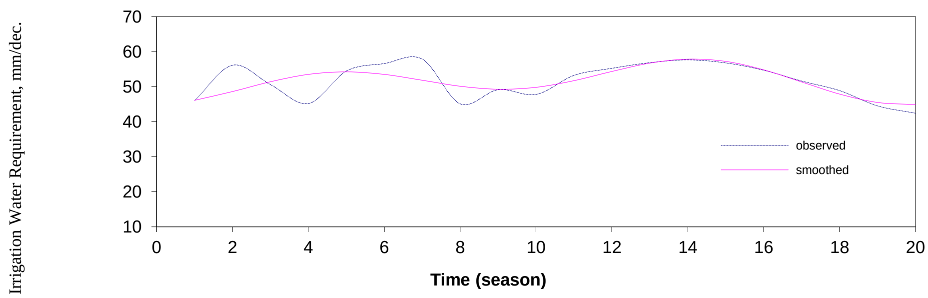

Fig. (6): IWR Estimated from 10 season decade means and obtained by harmonic fitted points for Cotton (Shambat)

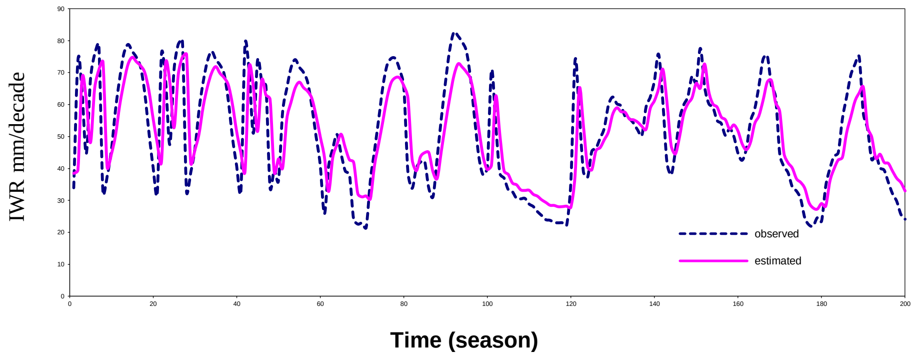

Fig. (8): Comparison of Observed and Estimated 1WR for Cotton Crop (Shambat) for 10 Seasons

Model Application to Shambat & Akala

The cotton crop (Brakat) was taken as an example for the modelling. The study demonstrated that the formulation of such a model is feasible. Also mathematical expressions were developed for two other cotton crops, Shambat and Akala. It was found that decade time series of irrigation requirement was trend-free because the computed turning point test (m') for Shamabat & Akala were found to be 1.167 and 1.65 respectively. These were within the limits of 1.96 at 5% level of significance, and it was periodic stochastic in nature. Hence the developed model superimposed a periodic – deterministic process and irregular component. The deterministic portion of the irrigation requirement series was analyzed using correlogram and Fourier series. Tables (3) & (4) show the periodic component of the IWR represented by the first harmonic for the Cotton Shambat and second harmonic for Cotton Akala respectively. Table (5) shows that the time dependence of the stochastic portion may be approximated by the first order auto- regressive model for both crops with constant auto-regressive coefficients of 0.7116 for Shambat and 0.81 for Akala.

The IWR generated by the formulated model, were plotted with corresponding observed values and were shown in fig. (8) and fig. (7) For Shambat and Akala Cotton with 60% and 73% efficiencies respectively.

CONCLUSION AND RECOMMENDATIONS

The study demonstrated that the formulation of such a model is feasible. Also mathematical expressions were developed for two other cotton crops, Shambat and Akala. It was found that decade time series of irrigation requirement was trend-free because the computed turning point test (m') for Shamabat & Akala were found to be 1.167 and 1.65 respectively. The developed periodic-stochastic model then used for representing the time based

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الملخص

ان الهدف الأساسي لهذه الورقة هو تطوير نموذج رياضي يساعد في ايجاد التنبوءات بكمية المياه المتوقعة إستخدامها لاغراض الري وهذا النموذج يعتمد أساساً على طريقة السلاسل الزمنية بمكونات ثلاثة هي الانحدار والدورية والعشوائية .

تم تطبيق هذا النموذج على محاصيل القطن والتي تزرع بالسودان (بركات شمبات وأكالا) حيث استخدمت عدة تقنيات لاجراء المحاكاة الرياضية في السلاسل الزمنية الأمر الذي ساعدنا في الكشف عن وجود بعض الظواهر من عدمها (الانحدار) حتى تمكنا من ايجاد علاقة رياضية مبنية على الدورية والعشوائية.