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Estimation of return stock rate by using wavelet and kernel smoothers

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ABSTRACT

This article aims to estimate the return stock rate of the private banking sector, with two banks, by adopting a partial linear model based on the arbitrage pricing model (APT) theory, using Wavelet and Kernel Smoothers. The results have proved that the wavelet method is the best. Also, the results of the market portfolio impact and inflation rate have proved an adversely effectiveness on the rate of return, and direct impact of the money supply.

Keywords: Partial linear model, Return stock rate model, Wavelet smoother, Local linear smoother

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1. Introduction

Regression function illustrates the relationship between explanatory variables and their effect on the dependent variable. Many articles and researches have been written about the partial linear models (PLM) for the past three decades for one reason that the use of these models being more flexible than standard linear models since it combines non-parametric and parametric compounds together and the other reason because it gives an easier explanation of the effect of each variable compared with the complete non-parametric regression.
Parametric models may contain variables have unknown distribution and do not care about nonlinear effects. Non-parametric models appear curse of dimensionality problem when increased the number of variables that effect on the estimates.
The main objective of the partial linear model is to reduce the assumptions on one or more of the explanatory variables and stay on the linear relationship among the variables.
All methods of estimation for partial linear regression models (*PLM*) are basically relied on non-parametric regression estimation procedures, thus the method of the non-parameter estimation method could be extended naturally to estimate and to assess the partial linear regression models (*PLM*).
Several methods have been proposed in partial linear models (*PLM*), Engle in 1986 and Heckman Rice in 1986 used the Smoothing Spline technique. Speckman in 1988, introduced the idea of the least squares technique. Robinson 1988 constructed estimate the non-parametric component by using least square method with Nadarya-Watson. In 1997, Hamilton, Genentech, and Truong used the Local Linear Smoother based on Speckman method [1].
The partial linear model is defined as [2] [3]:

$$y_i = X_i^T\beta + m(t_i) + \varepsilon_i \quad (1)$$

Where,
y_i refers to the response observation
X_i is fixed known p-dimensional vectors
t_i stand for the values of an extra univariate variable such as the time at which the observation is made
$t_i = \frac{i}{n}, t \in [0,1]$
β is unknown p-dimensional parameter vector
m is unknown smooth function
ε_i are random errors assumed to be *iid* $\varepsilon \sim N(0, \sigma^2)$ distributed.

The main objective is to estimate the vector parameters β that is unknown and the non-parameter function $m(t)$ [4].
The PLM is considered as part of the Semi-parametric Models that are divided into three types as follows [5]:

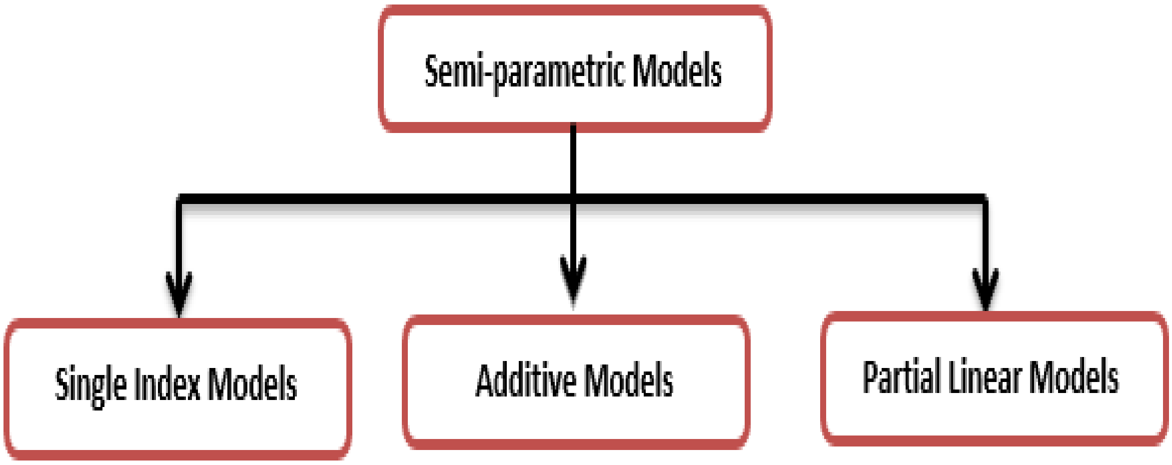

Figure 1. The types of semi-parametric models

2. Wavelet smoother

Wavelets are relatively modern mathematical tool, and considered as an extension of the ideas of the Fourier theory in 1800. Wavelet analysis is used to represent, analyze, process and synthesize the various data which studied in mathematics, science, engineering, economics and social studies (radar signal, sound, image and others).
Haar has been the first who discovered the wavelet in 1910 and he described some of its characteristics that are useful in the construction of public wavelet systems. The main idea of wavelet transform is to get rid of the problem of loss of data within the frequency and time fields in Fourier transform.
The continuous wavelet transformation has been discovered by Alex Grossman and Jean Morlet in 1984. Then, the orthogonal wavelet transformation had been discovered by Lemarie and Meyer in 1986, then in 1988 Daubechies founded the orthogonal bases made of wavelets compactly supported. In 1989, Mallat designed algorithm of the fast wavelet transform, whereas Victor Wickerhauser had introduced wavelet packs in 1991 and applied them to data compression. In 1994, Donoho and Johnstone had introduced a threshold that minimizes noises of the signal or image and others [6, 7].
It is worthy to be noting that wavelet transform helps us to know the frequency and time information at the same time, where wavelet transformations rely on small waves, called waves. The general method in the wavelet transform lies in creating an approximate compound using scaling function (low frequency filter) and wavelets functions (high frequency filter), which produces a window of variable dimensions to obtain high accuracy in time and frequency [6] [7].

$$\hat{m}(a,b) = \frac{1}{\sqrt{a}}\int_{-\infty}^{\infty} m(t)\ \psi(\frac{t-b}{a})dt \qquad (2)$$

Where, a: scaling coefficient
b: Translation coefficient
t: Time
In 1989, Mallat [8], introduced an algorithm for Fast wavelet transform based on low and high filters in the process of transform, the sample size should be 2^j because when the wavelet transform of filters, 2 filters becomes 4 filters and 4 filters become 8 filters and so on . Figure 2 shows the quick wavelength transform according to the cascade algorithm which states that we should start by calculating the scaling function $\emptyset_0(t)$ until we get the scaling function $\emptyset_n(t)$ as in the following formula:

$$\emptyset_{n+1}(t) = \sqrt{2}\sum_{k=0}^{2N-1} h_k \emptyset_n(2t-k) \qquad (3)$$

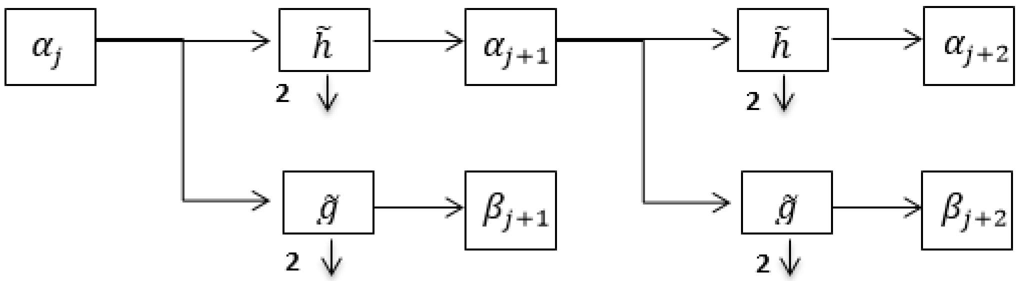

Figure 2. The fast wavelet transform for Mallat using the filters

To find the inverse of wavelet transform, we reverse the process above.

Wavelet Shrinkage and Denoising is a way to reduce the noise of a signal, image, and so on. Donoho and Johnstone in 1994, proposed a non-linear wavelet estimator for non-parametric function based on the reconstruction of the wavelet coefficients and retention of the coefficients of scaling. The wavelet is reduced by the threshold employed to transform the low-frequency signal to zero and the high-frequency signal keeps it or gets it close to zero [9].

Donoho and Johnstone provided two types of signal denoising that are as follows:

1- Hard threshold function: this function transform the values (wavelet coefficients vector) that are smaller than the threshold value to zero and values greater than the threshold remain the same.

2- Soft threshold function: transform the smallest values than the threshold value to zero and shrinks larger values than threshold value.

There are several ways to find the threshold value, one of these is the Universal threshold method proposed by Donoho and Johnstone in 1994:

$$\lambda_{UV} = \sigma \sqrt{2 \ \log(n)} \tag{4}$$

$\hat{\sigma}$ represents the standard deviation of the noise level and is given by

$$\hat{\sigma} = {MAD\ (y^{1,d})}/{0.6745} \tag{5}$$

The other method is Cross Validation (CV) that proposed by "Nason" in 1996. This method is used on a wide range and based on dividing it into two equal groups of sample size. The first group contains even numbers and the second group contains odd numbers so that the odd ordered data is used. To predict the ordered even data and vice versa, leading to the "Leave-out-half" strategy and how to calculate the value of the parameter λ, so we have the vector $y_i = (y_1, \dots\dots, y_n)'$ with a sample size $n = 2^j$ [10].

All the odd values of the vector y_i are to be deleted, so that the sample size becomes $n = 2^{j-1}$ and then this reconfigured data is used to create an estimator of the $\hat{g}_\lambda^E$ function for even numbers by a practical threshold either a threshold hard or soft as in the following formula [11]**:**

$$\bar{g}_{\lambda,j}^E = \frac{1}{2}\left(\hat{g}_{\lambda,j+1}^E + \hat{g}_{\lambda,j}^E\right), \quad j = 1, \dots\dots\dots, {n}/{2} \tag{6}$$

So, $\hat{g}_{\lambda,{n}/{2}+1}^E = \hat{g}_{\lambda,1}^E$ because the function g is assumed to be periodic. The function $\hat{g}_\lambda^O$ for the reconfigured individual numbers is estimated to become:

$$\bar{g}_{\lambda,j}^O = \frac{1}{2}\left(\hat{g}_{\lambda,j+1}^O + \hat{g}_{\lambda,j}^O\right), \quad j = 1, \dots\dots\dots, {n}/{2} \tag{7}$$

The final estimator $\widehat{M}(\lambda)$ for this method is then obtained so that the parameter $\lambda_{\min}$ is the only minimizer of this estimator, as that shown in the following formats:

$$\widehat{M}(\lambda) = \sum_{j=1}^{n/2}\left[(\bar{g}_{\lambda,j}^E - y_{2j+1})^2 + (\bar{g}_{\lambda,j}^O - y_{2j})^2\right] \tag{8}$$

$$\lambda_{min} = \arg\min_{\lambda \geq 0} \widehat{M}(\lambda) \tag{9}$$

The final parameter of the "cross-validation" method is then obtained as in the formula below:

$$\lambda^{CV} = \left(1 - \frac{\log 2}{\log n}\right)^{-1/2} \lambda_{min} \tag{10}$$

The partial linear model estimation is an extension for the non-parametric estimation process, so the non-parametric estimation process will be illustrated. The non-parametric regression model is defined as follows:

$$y_i = m(t_i) + \varepsilon_i \tag{11}$$

Where, $m(t_i)$ is an unknown function, ε_i usually assumes (*i.i.d*) and is normally distributed *N* (0,1). We can write the model in terms of matrices as it becomes as follows:

$$\underline{Y} = \underline{m} + \underline{\varepsilon} \qquad (12)$$

Equation (12) is multiplied by the discrete wavelet transform matrix W to become the model as follows:

$$WY = Wm + W\varepsilon \qquad (13)$$

Where, $\theta = Wm$, $w = Wy$, thus the equation (12) could be rewritten in another form:

$$Y = W^{-1}\theta + \varepsilon \qquad (14)$$

The estimator $\hat{\theta}$ can be find by using the soft threshold:

$$\hat{\theta}_s = sgn(w) \circ (|w| - \lambda e)_+ = \arg\min_{\theta_i} \frac{1}{2}(w_i - \theta_i)^2 + \lambda|\theta_i| \qquad (15)$$

$$i = 1, \dots \dots, n$$

Where, λ indicates threshold value, and (∘) means multiplying the components between two vectors.

The non-parametric estimator $\hat{m}$ or $S(Y)$ of the response variable (Y) can then be obtained by multiplying the soft threshold estimator $\hat{\theta}_s$ by the inverse of the wavelet transform matrix W.

$$\hat{m}(Y) = S(Y) = W^{-1}\hat{\theta}_s \qquad (16)$$

In 2004, Xiao-Wen Changa & Leming Qu [12] proposed a Backfitting method for estimating a partial linear model using wavelet smoother. This method is based on a descent iterative algorithm. This algorithm is more complex than the normal Backfitting algorithm, this method stated to estimate the partial linear model by minimizing the square of L2 Norm of the residual vector at parts of the norm L1 for non-parametric wavelet coefficients concerning with the non-parametric compound. This method is used to avoid non-parametric regression problems.

In 2003, Qu [13], proposed another method for estimating the partial linear model using the threshold function, a method similar to the Speckman method. The Wavelet Smoother can be applied to this method as in the Kernel Smoother and Spline Smoother.

In 1988, Speckman [14], proposed an algorithm for estimating a partial linear model using partial residual analyzes. Under appropriate assumptions, the convergent bias and variance are obtained. Thus the β estimation by partial residual method shows better bias and no loss of convergence in contrast as well.

Using the partial residual method of the partial linear model in equation (1), we get both the estimator β for the parametric part and the estimator $\hat{m}$ for the non-parametric part as follows:

$$\varepsilon'\varepsilon = (Y - X\beta - Wm)'(Y - X\beta - Wm)$$

By taking the derivative of both the parametric and non-parametric part and equating it to zero, we get the estimators as follows:

$$\hat{\beta} = \left(X'(I-K)'(I-K)X\right)^{-1}X'(I-K)'(I-K)Y$$

$$K = W\left(W'W\right)^{-1}W'$$

Suppose that:

$$\tilde{Y} = (I-K)Y = Y - KY$$

$$\tilde{X} = (I-K)X = X - KX$$

Therefore, the parametric estimator $\hat{\beta}_{LS}$ and non-parameter estimator $\hat{m}(t)$ are as follows:

$$\therefore \hat{\beta}_{LS} = \left(\tilde{X}'\tilde{X}\right)^{-1}\tilde{X}'\tilde{Y} \qquad (17)$$

$$\therefore \hat{m}(t) = K(Y - X\hat{\beta}) \qquad (18)$$

Where, $S(Y)$ and $S(X)$ are represent the nonlinear wavelet shrinkage as previously defined in equation (16). $S(Y)$ And $S(X)$ are used to obtain the parametric estimator $\hat{\beta}_W$ and the non-parametric estimator $\hat{m}_W(t)$ as below:

$$\hat{m}(Y) = S(Y) = W^{-1}\hat{\theta}_s$$

$$\underline{X} = \underline{m} + \underline{\varepsilon} \qquad (19)$$

$$WX = Wm + W\varepsilon \qquad (20)$$

As long as $\alpha = Wm$ and $A = WX,$ thus the equation (19) can be re-written in another form:

$$X = W^{-1}\alpha + \varepsilon \qquad (21)$$

The estimator α^is found by using a soft threshold:

$$\hat{\alpha}_s = sgn(A) \circ (|A| - \lambda e)_+ \qquad (22)$$

Then $S(X)$ can be obtained by multiplying $\hat{\alpha}_s$ by the inverse of the wavelet transform matrix reverse.

$$\hat{m}(X) = S(X) = W^{-1}\hat{\alpha}_s \qquad (23)$$

So, the wavelet smoother could be applied to get:

$$\tilde{Y}_W = (I - S)Y = Y - S(Y)$$

$$\tilde{X}_W = (I - S)X = X - S(X)$$

Then, the estimators of the parametric part could be found according to the following formula:

$$\therefore \hat{\beta}_W = \left(\tilde{X}'_W \tilde{X}_W\right)^{-1} \tilde{X}'_W \tilde{Y}_W \quad (24)$$

To estimate the non-parametric portion of the partial linear model (*PLM*), be:

$$Z = Y - X\hat{\beta}_W \quad (25)$$

A new vector Z, is produced and the nonlinear wavelet smoother is shrunk by the soft threshold as previously defined on the vector (Z) to become as follows:

$$\underline{Z} = \underline{m} + \underline{\varepsilon} \quad (26)$$

$$WZ = Wm + W\varepsilon \quad (27)$$

Since $D = WZ$ and $\gamma = Wm$, the equation (26) can be re-written in another form:

$$Z = W^{-1}\gamma + \varepsilon \quad (28)$$

$$\hat{\gamma}_s = sgn(D) \circ (|D| - \lambda e)_+ \quad (29)$$

The final estimation of the non-parametric part is then obtained as follows:

$$\hat{m}_W(t) = S(Z) = S\left(\mathrm{Y} - \mathrm{X}\hat{\beta}_W\right) = W^{-1}\hat{\gamma}_s \quad (30)$$

3. Kernel smoother

The mathematical ideas and skills have been developed in the analysis of non-parametric regression estimation to semi-parametric in the past three decades and that Kernel Smoother is a wider smoothness and most famous. As we mentioned earlier, the kernel regression estimators are endowed with the advantage of intuitive and mathematical simplicity and are used with a fixed and random designs. Within each of these non-parametric estimators, there are also many different methods. Examples of these methods are Nadarya Watson, Nadarya & Watson, Kernel local polynomial estimator, Muller, Clevel, Stone, Kernel alternative estimators and Pristley Chao.

There are several methods for Kernel smoother and the most important of these methods is the method of local linear smoother [5].

3.1. Local linear kernel smoother

In 1977, Stone suggested a local Linear Smoother method for estimating the non-parametric model using the weighted function W. The results were compared with other methods based on the Kernel smoother estimation. The results showed, by using the local linear smoother (LLS), that convergent bias and variance conditions are not affected at the boundary. Cleveland in 1979, proposed a method called robust locally weighted regression on observations (*X*, *Y*) and estimated points used as a smoother estimator. In this method numbers of points are used where the window size of the points (*t*) varies from one to another.

When finding these estimators using the weighted regression will reduce the effect of outlier values.

In 1993 , Jianqing Fan used MiniMax to prove that the estimators achieve the best probability of stability and convergence rates and has the characteristics of testing important samples that are adapted to the fixed and random designs , the local linear smoother (LLS) depends on the following minimizer problem [15][16]:

$$\min_{(a,b)} \sum_{j=1}^{n} (Y_j - \alpha - \beta(t_j - T))^2 K(\frac{t_j - T}{h}) \quad (31)$$

Where the two estimators $\hat{a} = \hat{a}(\mathrm{t})$, $\hat{\beta} = \hat{\beta}(t)$ are consistent estimator for the non-parametric function $m(t)$ and for the first derivative of the non- parametric function $m'(t)$ [17] [18].

Therefore, using the Speckman method to estimate the partial linear regression model, the estimation of the parameter β for the parametric part and the non- parametric estimation $\hat{m}(t)$ are obtained according to the following formulas [19][20]:

$$\hat{\beta}_{Speck} = (X^T(I - S)^T(I - S)X)^{-1}X^T(I - S)^T(I - S)Y \quad (32)$$

Where, $\tilde{X}^T = X^T(I - S)^T$, $\tilde{X} = (I - S)X$,$\tilde{Y} = (I - S)Y$

$$\hat{\beta}_{Speck} = \left(\tilde{X}^T \tilde{X}\right)^{-1} \tilde{X}^T \tilde{Y} \quad (33)$$

$$\hat{m}_{Speck} = S\left(Y - X^T \hat{\beta}_{Speck}\right) \quad (34)$$

Since I is a one-dimensional matrix of $n \times n$ the S matrix of the local linear smoother used in finding the final estimates of the partial linear model (*PLM*) can be obtained as follows:

$$S = \left(V_{t_1}^T, \ldots\ldots\ldots, V_{t_n}^T\right)^T \quad (35)$$

V_t is defined as follows:

$$V_t = e_t^T (Q_t^T W_t Q_t)^T Q_t^T W_t \quad (36)$$

$$Q_t = \begin{bmatrix} 1 & (T_1 - t) \\ \vdots & \vdots \\ 1 & (T_n - t) \end{bmatrix}$$

$$W_t = diag\{K_h(T_1 - t), \quad \ldots\ldots \quad , K_h(T_n - t)\}$$

$$K_h(u) = \frac{1}{h} K\left(\frac{u}{h}\right) \quad , \quad e_t^T = (1, 0, \ldots., 0)$$

W_t represents matrix of weights and e_t^T is a vector whose first value is equal to one and the rest are zero.

Wand and Jones explained the case of a fixed design, that the non-parametric variable $t_i = \frac{i}{n}$ follows the following assumptions [21]:

1. The function $m^{''}$ is continuous (has a second derivative) for the period [0,1]
2. *K* is symmetric around zero and supported for the period [-1,1]
3. The Bandwidth is a sequence verifying $n \to \infty \quad nh \to \infty \quad h \to 0$
4. The point t at which the estimation is taking place satisfies $h < t < 1 - h, \ for\ all\ n \geq n_0$ where n_0 is fixed

Where the fourth condition implies that *t* is more than the band width away from the boundary for each n large enough. The simplified way to represent the serial of weights $w_{ni}(t)_{i=0}^n$ is to describe the weighted function w_{ni} by the *K*(u) Kernel function and the scaling parameter that modifies the size and shape of the relative weights on *t* [16].

The Gaussian & Uniform Kernel functions will be chosen, it should be non-negative, symmetric around zero, continuous and have a second derivative [22].

Table 1. Kernel functions

Kernel	Explicit form
Gaussian kernel	$, u \in [-\infty, \infty]$)*K(u)* = $\frac{1}{\sqrt{2\pi}}$ *exp(*$-u^2$
Uniform kernel	*K(u)* = $\frac{1}{2}$, $u \in [-1,1]$

The choice of the smoother parameter or bandwidth is considered most important than the choice of Kernel functions.

The small bandwidth is connected with areas of intensive data, while the large bandwidth is desired in areas with sparse data. The value of the bandwidth can be found using the Cross Validation (CV) method [23].

4. Application

In 1976, Steven Ross introduced the arbitrage pricing theory (APT) to explain the return on investment in securities, it is an alternative theory of capital asset pricing model (CAPM). This theory stated that there is one beta coefficient that is the Return Rate of the market portfolio effecting return stocks Rate, whereas the theory of arbitrage pricing refers to the return Rate of the market portfolio is not the only factor. The model of the arbitrage pricing theory It examines the relationship between systemic market risk and return Rate. This model indicates that the return of securities depends on a variety of factors and these factors affect the return and the market in general [24].

What distinguishes the pricing theory of arbitrage (APT) from the theory of capital asset pricing model (CAPM) is its comprehensiveness of all risks. The APT model assumes that financial markets are competitive and can reflect the returns of assets as a linear function of the risk factors group k [25].

The APT model is used to calculate the return Rate on appropriate investments with the usual stocks is hugely used [26].

Therefore, the partial linear regression model will be constructed in light of the arbitrage pricing theory (APT) as follows:

$$\mathrm{R_i} = \beta_1 R_m + \beta_2\, PI + \beta_3 M_2 + m(t) + \varepsilon \quad (37)$$

Where,
R_i: Return stock Rate for each organization or company
R_m: Return Rate of Market Portfolio
PI: Inflation Rate.
M_2: Broad presentation of the monetary
t : Time
ε: The non – systematic risk for each organization or company.
The data were collected from the Central Bureau of Statistics and Stocks and securities with a sample size equal to (128) months. These data include:
1. Return Rate for the two Banks (the Iraqi Investment Bank and the Iraqi Credit Bank).
2. Market portfolio.
3. Inflation rate of consumer prices based on 2007 as the bases year.
4. Money supply in its broad sense.
4. Time of 128 months.

5. Results

In order to estimate the partial linear model (PLM) and to obtain the results according to the algorithms mentioned in the theoretical aspect, we must first extract preliminary estimates for the parametric linear model (PLM) and according to the method of least squares normal (OLS) as in the following formula:

$$\beta = (X'X)^{-1}X'Y$$

5.1. Iraqi investment bank

The return stock rate will be estimate for Iraqi Investment Bank. Where the estimates are found for the parametric part and then the non-parametric part of the partial linear model (PLM) as shown in the table below:

Table 2. Parametric estimation values β using OLS, wavelet smoother and local linear smoother method

Parameters / Method	β_1	β_2	β_3
OLS	- 0.1111	- 0.3212	0.2507
Wavelet (C.V)	- 0.6223	- 1.5890	- 0.0849
Wavelet (UN.)	- 0.6343	- 1.2697	1.4220
LLS (Gaussian)	- 0.0972	- 0.9926	0.2799
LLS (Uniform)	- 0.0885	- 1.2933	- 0.9325

The Mean Squared Error is then calculated for the methods used in estimating the partial linear model (PLM) of the return stock rate data (banking sector / Iraqi investment bank), as in the table below:

Table 3. The values of mean squared error (MSE) of the return stock rate of the investment bank

Estimation Method	Wavelet (C.V)	Wavelet (UN.)	LLS (Gaussian)	LLS (Uniform)
MSE	1.055876	0.9759171	1.452446	6.354019

Figure 3 explains the real and the estimated data for the observation (Y) return stock rate of investment bank based on different kernel functions.

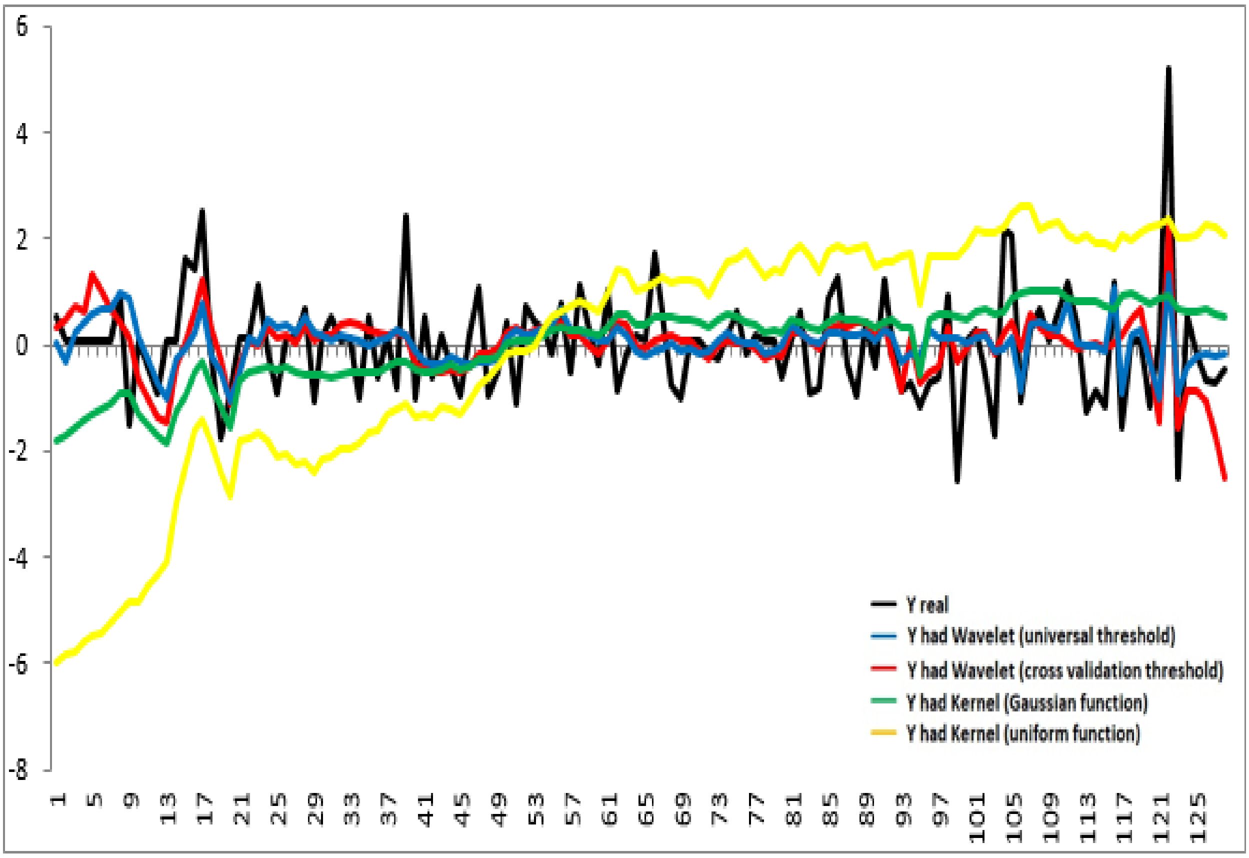

Figure 3. The real and the estimated data for the observation (Y) return stock rate of investment bank

5.2. Iraqi credit bank

The return stock rate will be estimated for the Iraqi Investment Bank. Estimates are found for the parametric part and then the non-parametric part of the partial linear model (PLM) as shown in the table below:

Table 4. Parametric estimation values β using
OLS, wavelet method and local linear smoother method

Method \ Parameters	β_1	β_2	β_3
OLS	- 0.0460	- 0.2707	0.1821
Wavelet (C.V)	- 0.6199	- 2.2064	0.3093
Wavelet (UN.)	- 0.6009	- 1.1585	0.0939
LLS (Gaussian)	- 0.0378	- 0.7288	- 0.1404
LLS (Uniform)	- 0.0213	- 0.7589	- 0.7990

The mean squared error is then calculated for the methods used in estimating the partial linear model (PLM) of the return stock rate data (Iraqi Credit Bank), as in the table below:

Table 5. The values of mean squared error (MSE)
of the return stock rate of the credit bank

Estimation Method	Wavelet (C.V)	Wavelet (UN.)	LLS (Gaussian)	LLS (Uniform)

MSE	1.257762	1.14669	1.789398	3.430014

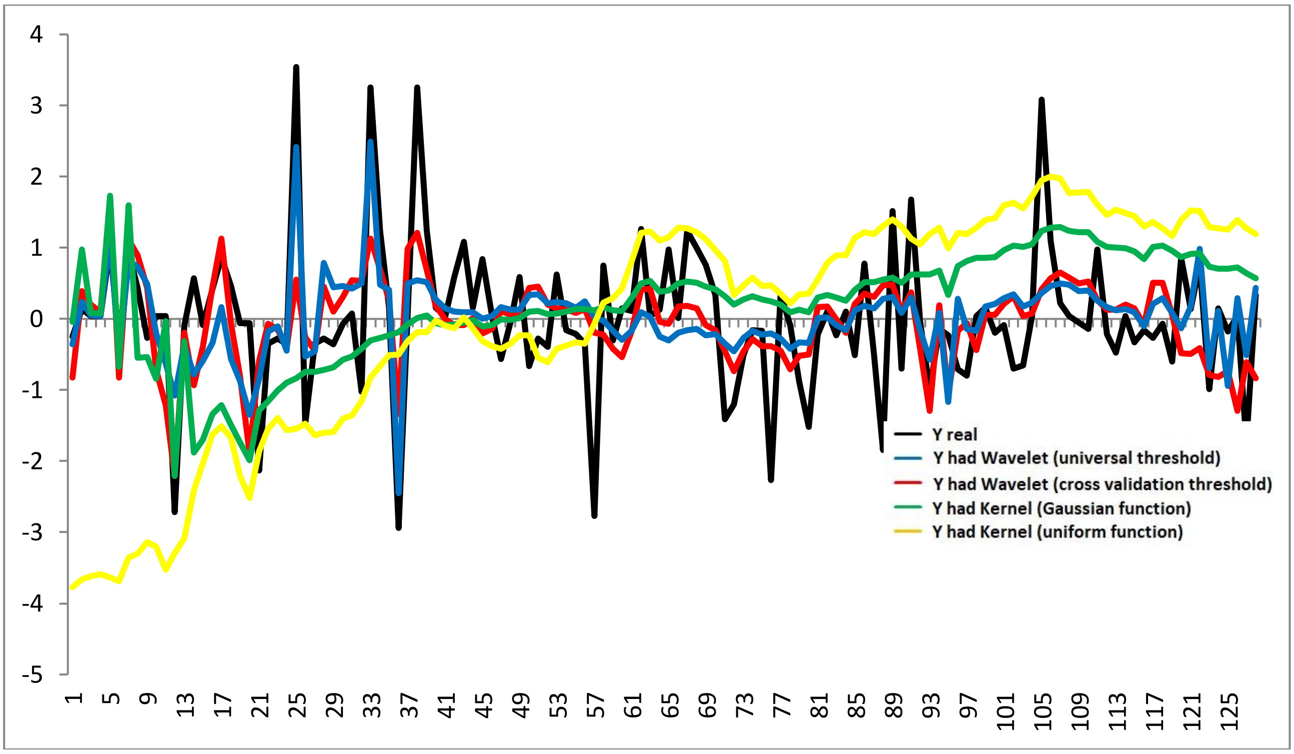

Figure 4. The real and the estimated data for the observation (Y) return stock rate of credit bank

Where,
Wavelet (C.V): Wavelet Smoother when using Cross validation threshold method
Wavelet (UN.): Wavelet Smoother when using Universal threshold method
LLS (Gaussian): Local Linear Smoother when choosing Gaussian Kernel function
LLS (Uniform): Local Linear Smoother when choosing Uniform Kernel function

Based on the results obtained in the process of estimating the partial linear model (*PLM*) using wavelet smoother and kernel smoother, we conclude the following:

1. The above results indicate that the Wavelet Estimation method for partial linear model (*PLM*) of return stock rate data (investment and credit bankers) is better than the local linear Smoother method.
2. The *(MSE*) values using wavelet smoother are smaller than MSE values when using local linear smoother.
3. The universal threshold method is slightly smaller than the Cross-Validation method.
4. The lowest estimator performance is Uniform function in local linear smoother.
5. The values of the parametric estimates β of the PLM are small due to the fact that the monthly percentage change in return stock rate is very low.
6. Noting that the Market Portfolio and inflation rate have a negative impact and money supply positively affects the average return stock of the banks sector.

6. Conclusions

The partial linear regression model based on Arbitrage Pricing Model (APT) is distinguished from the parametric and non-parametric models by obtaining the best unbiased estimates of the parametric and non-parametric data and solving the problem of data variability in light of economic and financial phenomena. It is based on accurate and clear investment decisions as well as short and long term planning.
Using the best estimation method (Wavelet universal), the final form of the partial linear model (PLM) of the return stock rate (Investment Bank) will be as follows:

$$\hat{R}_{\mathrm{IIB}} = -0.6343\ \mathrm{R_m} - 1.2697\ \mathrm{PI} + 1.4220\ \mathrm{M_2} + \hat{\mathrm{m}}(\mathrm{t})$$

And by using the best estimation method (Wavelet universal), the final form of the partial linear model (PLM) of the average return stock (Credit Bank) will be as follows:

$$\hat{R}_{i_{ICB}} = -0.6009\ R_m - 1.1585\ PI + 0.0939\ M_2 + \hat{\mathrm{m}}(\mathrm{t})$$

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