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THE DELIVERY OF MARKET TIMING SERVICES: NEWSLETTERS VERSUS MARKET TIMING FUNDS

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ABSTRACT

This paper examines the dissemination of market timing information (signals on the overall performance of risky assets relative to the risk free rate). We consider two delivery systems. Under the newsletter delivery system market timing information is disseminated solely through newsletter. Under the fund delivery system, timers set up timing funds in which investors can invest. In the absence of market imperfections we find that both systems produce the same result. With restrictions on borrowing or with other nonlinearities we find the newsletter system to be superior. This is one possible explanation for the plethora of market timing newsletters and the paucity of market timing funds.

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THE DELIVERY OF MARKET TIMING SERVICES: NEWSLETTERS VERSUS MARKET TIMING FUNDS

I. Introduction

Market timing information is information about overall performance of equities relative to risk-free assets. Merton [4] has shown that even such generalized information can lead to astronomical gains. Market timing delivery systems attempt to package and to deliver market timing information. The present paper examines strategies for the delivery of such information.

We envision two delivery systems, one in which market analysts with timing information set up timing funds and one in which market analysts with timing information sell their information in the form of newsletters. (A market analyst with timing information will be referred to as a "timer" henceforth.) We address the issue of the most efficient delivery system. We find that, in the absence of market imperfections, both systems give identical results. If there are restrictions on borrowing (or if there are other nonlinearities) then the newsletter delivery system is superior.

In section II we present the benchmark model which is an extension of a model by Dybvig and Ross [1]. Section III derives the unconstrained optimal use of timing information, and provides a benchmark against which to measure the performance of various information delivery systems. The description of the timing funds and newsletter delivery systems is given in section IV, and

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the evaluation of their performance under ideal conditions is presented in section V. In section VI we assess the effects of borrowing constraints, and in section VII the effect of non-linear response functions on the efficacy of the delivery system. Section VIII concludes.

II. The Model

Investors are partitioned into two classes: passive and active. Passive investors allocate their funds between a money market fund (holding virtually risk-free assets), and an index fund which mimics a broad market-based portfolio of risky assets.

Active investors (or investors who buy into actively managed portfolios) may be engaged in either market timing, or security analysis, or both. Passive investors, as well as active investors who restrict themselves to security analysis, are assumed uninformed about the probability distribution of the index portfolio. Market timers possess information about the market index.

For simplicity, assume that the variance of the index is constant. More specifically, assume that a dollar's worth of the index portfolio will sell in one period for e^x, and that σ^2 is the variance of x. Investors that choose not to invest in market timing information assume that the expected value of x is $r+\pi$, where r is the (known) risk-free rate and π is the market risk premium. The reward to volatility ratio (Sharpe's [5] measure), π/σ, is taken by the uninformed investors to be the market price of risk.

Investors that choose to become informed about the market assume that $x = r + \pi + s + \varepsilon$ where s is the random signal that they imperfectly observe and ε is random noise. (Uninformed investors observe neither s nor ε.) The risk-free rate, r, is assumed to be zero (this is unimportant) and s and ε

are independent normals with zero means and variances of σ_s^2 and σ_ε^2, respectively. Uninformed investors estimate the variance of x to be $\sigma^2 = \sigma_s^2 + \sigma_\varepsilon^2$. The timer invests a unit amount under a constant absolute risk aversion utility function $U(\omega) = -\exp[-A\omega]$ with $A > 0$. We assume that there are no principal-agent problems.

It is assumed that the volume of transactions that is associated with investors who engage in market timing is difficult to distinguish from hedging, liquidity trading, and that which is associated with security analysis. We assume that the timers are price takers and that the aggregate timer transaction volume is too small to affect prices.[1]

We assume that timer i does not observe s directly but receives a signal, z_i', that is conditionally normally distributed with mean s and variance σ_i^2. This is summarized by the following notation:

$$z_i' \mid s \sim N(s, \sigma_i^2)$$

<u>Unconditionally</u>, z_i' will be correlated with s and we will denote the correlation coefficient by ρ_i, where

$$\rho_i = \frac{Cov(z_i', s)}{[Var(z_i')\sigma_s^2]^{1/2}}$$

Note also that

$$Cov(z_i', s) = E_s E(z_i' s \mid s) = \sigma_s^2$$

$$Var(z_i') = \sigma_s^2 + \sigma_i^2$$

so

$$\rho_i = \frac{\sigma_s^2}{[(\sigma_s^2 + \sigma_i^2)\sigma_s^2]^{1/2}} = \frac{\sigma_s}{(\sigma_s^2+\sigma_i^2)^{1/2}}$$

or

$$\sigma_i^2 = \sigma_s^2\left(\frac{1-\rho_i^2}{\rho_i^2}\right)$$

Now, noting that the unconditional expectation of z_i' is zero we can use the above relationships to state the conditional and unconditional distributions of z_i' in terms of s, σ_s^2 and ρ_i:

$$z_i'|s \sim N\left(s, \frac{1-\rho_i^2}{\rho_i^2}\sigma_s^2\right)$$

$$z_i' \sim N(0, \sigma_s^2/\rho_i^2)$$

Since we know the distributions of z_i', $z_i'|s$, and s we derive the distribution of $s|z_i'$ by noting

$$f(s|z_i') = \frac{f(z_i'|s)f(s)}{f(z_i')}$$

where f refers to the probability density functions. (See Theil [6].) This yields

$$s|z_i' \sim N(\rho_i^2 z_i, \ (1-\rho_i^2)\sigma_s^2) \ .$$

Although z_i' is not an unbiased estimator of s we can get one by defining

$$z_i = \rho_i^2 z_i'$$

Now note that

$$z_i \sim N(0, \ \rho_i^2 \sigma_s^2)$$

$$z_i|s \sim N(\rho_i^2 s, \ \rho_i^2(1-\rho_i^2)\sigma_s^2)$$

$$s|z_i \sim N(z_i, \ (1-\rho_i^2)\sigma_s^2)$$

In practice, ρ_i^2 can be estimated from past forecasting errors. Recall now that the return on the market can be written

$$x = \pi + s + \varepsilon$$

(since r, the risk-free rate, has been assumed to be zero) and thus the distribution of x conditional on z_i is just

$$x|z_i \sim N(\pi + z_i, \ (1-\rho_i^2)\sigma_s^2 + \sigma_\varepsilon^2)$$

A timer with signal z_i will invest a proportion, γ_i, of the fund in the market and another proportion, $1-\gamma_i$, in the risk-free asset. These proportions depend on the information, z_i, available to the timer so we can write $\gamma_i(z_i)$. The return on the portfolio (recalling that the risk-free rate is assumed to be zero) is just $\gamma_i(z_i)x$. We assume that γ_i is chosen to maximize a negative exponential utility function

$$U = E[-\exp(-A\gamma_i x) \mid z_i]$$

where x is normally distributed conditional on z_i.

With this utility function and with the assumption of normality,

$$\gamma_i(z_i) = \frac{E(x \mid z_i)}{A \operatorname{Var}(x \mid z_i)} \tag{1}$$

(See, for example, Dybvig and Ross [1], Grant [2], Kane and Marks [3], and Tobin [7].) In our case, this yields,

$$\gamma_i(z_i) = \frac{\pi + z_i}{A[(1-\rho_i^2)\sigma_s^2+\sigma_\varepsilon^2]} \tag{2}$$

III. The Optimal Unconstrained Use of Information

We now consider a portfolio manager who receives the n conditionally independent signals. The purpose is to establish an ideal benchmark by which to measure performance under the two information delivery systems. The distribution of the market return given all signals is stated in the following proposition:

Proposition 1: The distribution of x conditional on $z_1, z_2, \ldots, z_n$ is given by

$$(x|z_1, z_2 \ldots z_n) \sim N(\pi + Z,\ (1-R^2)\sigma_s^2 + \sigma_\varepsilon^2)$$

where Z is a sufficient statistic for the set of z_i given by

$$Z = \sum w_i z_i \ ,$$

R^2 is the squared correlation coefficient between Z and s given by

$$R^2 = \sum w_i \rho_i^2,$$

and the w_i are optimal scaling factors for the signals given by:

$$w_i = \frac{\dfrac{1}{1 - \rho_i^2}}{1 + \sum \dfrac{\rho_j^2}{1 - \rho_j^2}}$$

(The proof of proposition is in the appendix.)

Note that the signal scaling factors add up to more than 1.0 as

$$\sum w_i = \frac{n + \sum \dfrac{\rho_j^2}{1 - \rho_j^2}}{1 + \sum \dfrac{\rho_j^2}{1 - \rho_j^2}} > 1 \ .$$

Recall that each raw signal, z_i', was first discounted by ρ_i^2 to account for its noise. The upscaling of the multiple signal reflects the portfolio effect of the independent forecasting errors.

For n forecasters of equal ability (ρ^2) we have the following

$$R^2 = \frac{n\rho^2}{1+(n-1)\rho^2} \quad ; \quad \lim_{n \to \infty} R^2 = 1$$

$$w_i = \frac{1}{1+(n-1)\rho^2} \quad ; \quad \lim_{n \to \infty} w_i = 0$$

$$\sum w_i = \frac{n}{1+(n-1)\rho^2} \quad ; \quad \lim_{n \to \infty} \sum w_i = \frac{1}{\rho^2}$$

With two independent forecasts of equal reliability ($\rho_1=\rho_2=\rho$) we have

$$w_1 = w_2 = \frac{1}{1+\rho^2} = w$$

$$Z = \frac{z_1 + z_2}{1+\rho^2}$$

$$R^2 = \frac{2\rho^2}{1+\rho^2}$$

For example, if $\rho^2 = 1/2$, then $w=2/3$, that is, each signal is weighted by 2/3.

We now can derive the optimal market position of a timer who possesses all signals $z_1, \ldots, z_n$. This position follows directly from Proposition 1 and equation (1).

Proposition 2: The best position in the market for an investor with information $(z_1,\ldots,z_n)$ is

$$\gamma^*(z_1,\ldots,z_n) = \frac{\pi + Z}{A[(1-R^2)\sigma_s^2+\sigma_\varepsilon^2]}$$

which is the multiple signal analog to equation (2).

IV. Delivery Systems

We now consider two delivery systems. Under the first, the newsletter delivery system, timers disseminate market timing information through newsletters. The individual investor collects this information and uses it to make timing decisions. Under the second, fund delivery system, each timer sets up a separate fund and the individual investor diversifies among the many funds. Under both scenarios the individual may borrow or lend at the risk-free rate and also may invest directly in the market.

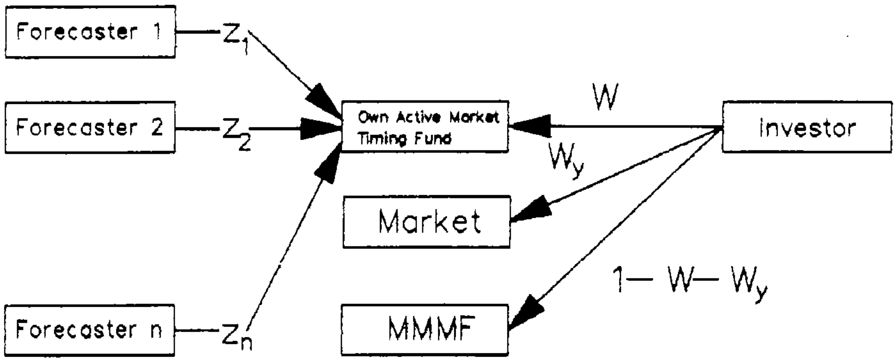
Under the newsletter delivery system we will assume that the investor devotes a proportion W to active timing. A proportion W_y is passively invested in the market and the remainder is passively invested in risk-free assets. (See Figure 1A.)

Under the fund delivery system investors allocate a proportion W_i to fund i, W_x is passively invested in the market and the remainder is passively invested in risk-free assets. (See Figure 1B.)

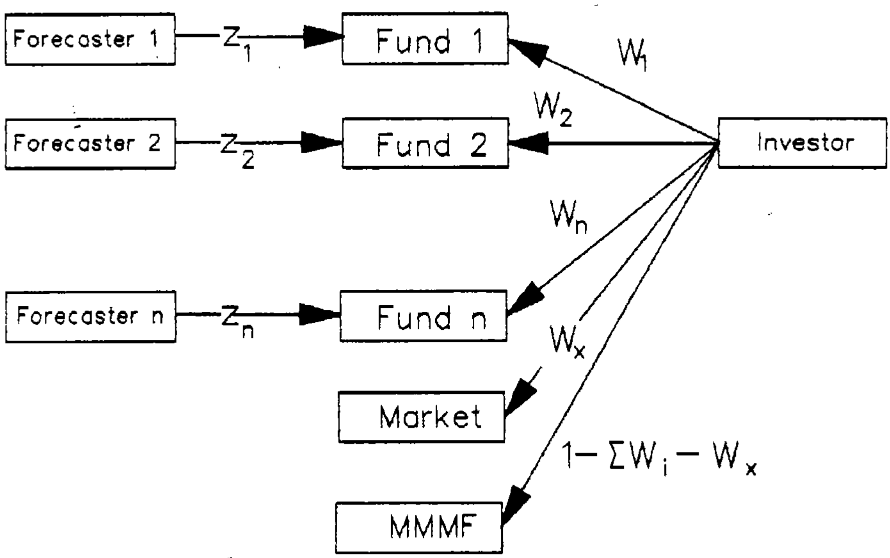
Figure 1 Goes Here

Figure 1

A. Newsletter Delivery System



B. Fund Delivery System



File Name: SPECIAL

Under both systems, a set of rational expectations signals, z_i, are generated. The i-th signal is obtained by the i-th timer. Under the newsletter system these are passed on to investors through newsletters. Under the fund system the timers do not sell the information but rather set up timing funds.

The notation for each system is summarized below:

IV.1 Newsletter Delivery System

z_i' = raw signal received by i-th timer and reported in i-th newsletter

z_i = signal adjusted for accuracy ($z_i = \rho_i^2 z_i'$)

The investor collects all of the newsletters and sets up his own timing fund to which a proportion of the investor's assets is devoted. In order to facilitate the comparison of the newsletter delivery system to the timing fund system, we (artificially) split the risky investment under the newsletter system into two parts: that which is passively invested in the market, W_y, and that which is invested in the active timing fund on the basis of the obtained signals, W. The summary of the notation is given below:

W = proportion of investor's assets invested directly in own timing fund

W_y = proportion of investor's assets invested directly in market (not subject to timing)

$\gamma(z_1,\ldots,z_n)$ = proportion of own timing fund's assets invested in market

$\gamma_N(z_1,\ldots,z_n)$ = proportion of investor's assets invested directly or indirectly in market. Again, the investor will have some assets invested directly in the market and some assets invested in the investor's own timing fund which are then invested by the fund in the market:

$$\gamma_N(z_1,\ldots,z_n) = W\gamma(z_1,\ldots,z_n) + W_y$$

IV.2 Fund Delivery System

z_i' = signal received by each timer

z_i = signal adjusted for accuracy ($z_i = \rho_i^2 z_i'$)

Each timer sets up a fund:

W_i = proportion of investor's assets invested directly in fund i

W_x = proportion of investor's assets invested directly in market

$\gamma_i(z_i)$ = proportion of the i-th fund's assets invested in market

$\gamma_T(z_1,\ldots,z_n)$ = proportion of the total investor's assets invested directly or indirectly in market. That is, the investor will have some assets invested directly in the market and some assets invested in many timing funds, part of which are then invested by the funds in the market:

$$\gamma_T(z_1,\ldots,z_n) = \sum W_i\gamma_i(z_i) + W_x$$

V. Evaluating Performance Without Constraints

We will first consider the outcomes under the two systems given the above assumptions. In a later section, we will relax the assumption of a linear reaction function (negative exponential utility). The following proposition states that the optimal use of information (given in Proposition 2) can be achieved under the newsletter system:

Proposition 3: Under the newsletter delivery system the investor's overall position in the market is equal to the optimal position, that is,

$$\gamma_N(z_1,\ldots z_n) = \gamma^*(z_1,\ldots,z_n) = \frac{\pi + Z}{A[1-R^2)\sigma_s^2+\sigma_\varepsilon^2}$$

if

$$\gamma(z_1,\ldots,z_n) = a + bZ$$

and if the investor invests W in the investor's own timing fund and W_y directly in the market where

$$W = \frac{1}{b[A(1-R^2)\sigma_s^2+\sigma_\varepsilon^2]}$$

$$W_y = \frac{b\pi - a}{b[A(1-R^2)\sigma_s^2+\sigma_\varepsilon^2]}$$

Proof:

$$\gamma_N(z_1,\ldots,z_n) = W_\gamma(z_1,\ldots,z_n) + W_y = \frac{a + bZ + b\pi - a}{b[A(1-R^2)\sigma_s^2+\sigma_\varepsilon^2]} = \gamma^*(z_1,\ldots,z_n) \quad \text{Q.E.D.}$$

This result is powerful in the sense that a single fund collecting all newsletters could serve the interests of many investors even if each investor had a different amount of risk aversion, A. Note that W and W_y can be chosen without regard to the signals themselves.

The following proposition states that the optimal use of information (given in Proposition 2) can also be achieved under the fund delivery system.

Proposition 4: Under the fund delivery system the investor's overall position in the market is equal to the optimal position, that is, $\gamma_T(z_1,\ldots,z_n) = \gamma^*(z_1,\ldots,z_n)$ if

$$\gamma_i(z_i) = a_i + b_i z_i \ ,$$

and if the investor invests proportions W_i in each fund and W_x directly in the market where,

$$W_i = \frac{w_i}{b_i A[(1-R^2)\sigma_s^2+\sigma_\varepsilon^2]}$$

$$W_x = \frac{\pi - \sum(a_i/b_i)w_i}{A[(1-R^2)\sigma_s^2+\sigma_\varepsilon^2]}$$

where w_i and R depend only on $\rho_1,\ldots,\rho_n$.

Proof:

$$\gamma_T(z_1,\ldots,z_n) = W_x + \sum W_i\gamma_i(z_i) = \gamma^*(z_1,\ldots,z_N). \quad \text{Q.E.D.}$$

As in the newsletter case, investors with different amounts of risk aversion A could be served by the same set of timing funds.

VI. Market Imperfections

In the model above, information disseminated through timing funds has as much value as information disseminated through newsletters. This result, however, depends on two key assumptions: 1) there are no restrictions on borrowing and short positions in the market, and 2) the optimal amount invested in the market is linear in the signal. If we relax either of these assumptions, information disseminated through newsletters is more valuable than information disseminated through timing funds.

VI.1 Restrictions on Borrowing

We will now introduce borrowing restrictions both on funds and on investors. We will assume that an investor can borrow a proportion of his or her net assets equal to B_I. Thus the investor can invest $1 + B_I$ dollars for every dollar of net assets. Likewise, funds can borrow a proportion of net assets equal to B_F and thus can invest $1 + B_F$ dollars in the market for every dollar of net assets. Of course, neither the investor nor the fund has to borrow the full amount available.

We will show that with borrowing restrictions, the newsletter delivery system is superior. To this end we will show that i) under the newsletter delivery system one can replicate the best outcome achievable under the fund delivery system, ii) under the fund delivery system one cannot replicate the best newsletter. Taken together these two imply that the newsletter system must be strictly superior.

Proposition 5: With borrowing restrictions the newsletter delivery system outcome can replicate the best fund delivery system.

Proof: The best timer outcome is found by choosing $\gamma_i(z_i)$, W_i and W_x to maximize investor utility subject to borrowing constraints on both the investor and the fund:

$$\gamma_T(z_1,\ldots,z_n) = \sum W_i \gamma_i(z_i) + W_x$$

subject to:

$$W_x + \sum W_i \leq 1 + B_I$$

$$\gamma_i(z_i) \leq 1 + B_F \qquad \text{for all:}$$

The newsletter outcome is defined by

$$\gamma_N(z_1,\ldots,z_n) = W\gamma(z_1,\ldots,z_n) + W_y$$

subject to:

$$W + W_y \leq 1 + B_I$$

$$\gamma(z_1,\ldots,z_n) \leq 1 + B_F$$

Now if we set the investor's newsletter timing fund strategy as:

$$\gamma(z_1,\ldots z_n) = \frac{\sum W_i \gamma_i(z_i) + W_x}{W}$$

$$W = 1 + B_I$$

$$W_y = 0$$

then $\gamma_N(z_1,\ldots,z_n) = \gamma_T(z_1,\ldots,z_n)$ without violating borrowing constraints since

$$W + W_y = 1 + B_I + 0 \leq 1 + B_I$$

and

$$\gamma(z_1,\ldots,z_n) = \frac{\sum W_i\gamma_i(z_i) + W_X}{W} \leq \frac{\sum W_i(1+B_F) + W_X}{(1+B_I)} \leq 1 + B_F \qquad \text{Q.E.D}$$

Thus, with borrowing restrictions the newsletter system can replicate any fund delivery system outcome and, in particular, can replicate the optimal fund delivery system. The reverse is not true as the following proposition indicates

<u>Proposition 6</u>: With borrowing restrictions the fund delivery system cannot replicate the best newsletter outcome.

<u>Proof</u>: Assume that the fund delivery system can replicate the optimal newsletter outcome. That is, suppose there exists $\gamma_i(z_i)$, W_i, and W_X such that

$$\sum W_i\gamma_i(z_i) + W_X = \gamma_N^*(z_1,\ldots,z_n)$$

where γ_N^* is the optimal newsletter outcome with borrowing constraints. We know that the distribution of x depends only on $Z = \sum w_i z_i$ and not on the individual z_i. Thus, we can write $\gamma_N^*(z_1,\ldots,z_n) = f(Z)$.
Substituting yields:

$$\sum W_i\gamma_i(z_i) + W_X = f(Z)$$

Taking the total differential with respect to the z_i yields

$$\sum W_i \gamma_i'(z_i)dz_i = f'(Z)\sum w_i dz_i$$

which implies

$$W_i \gamma_i'(z_i) = f'(Z)w_i \qquad \text{for all } i.$$

From this it is clear that $\gamma_i'(z_i)$ does not depend on z_i but only on Z and thus $\gamma_i(z_i)$ is linear. (That is, by holding Z constant and varying z_i we can see that $\gamma_i'(Z_i)$ is constant for all values of z_i.) Also $f'(Z)$ does not depend on Z but only on z_i. Therefore, it too must be linear. (That is, by holding z_i constant and varying Z we can see that $f^1(Z)$ is constant for all Z.) Thus the timer outcome can replicate the optimal newsletter outcome only if $\gamma_i(z_i)$ are linear in z_i and $f(Z)$ is linear in Z. With borrowing restrictions neither the $\gamma_i(z_i)$ nor $f(Z)$ are linear.

Combining propositions 5 and 6 we get:

Proposition 7: With borrowing restrictions, the newsletter delivery system is superior to the timer delivery system.

The reason for this result is that under the newsletter delivery system extreme signals are netted out so that the constraints are binding much less of the time than under the timer delivery system.

VII. Nonlinear Response Functions

In the above we have assumed that the unconstrained optimal market position was linear in Z. (This came from normality and the negative exponential utility function.) Other utility functions will not have this property. For example, the unconstrained optimum for quadratic utility is:

$$f(z_1,\ldots,z_n) = \frac{\pi + Z}{Q[(\pi+Z)^2+(1-R^2)\sigma_s^2+\sigma_\varepsilon^2]}$$

where Q is a measure of risk aversion. Consider the following propositions for arbitrary reaction functions.

Proposition 8. In the absence of constraints, the newsletter delivery system outcome can replicate any arbitrary reaction function $f(z_1,\ldots,z_n)$.

Proof: Simply let $\gamma_N(z_1,\ldots,z_n) = f(z_1,\ldots,z_n)$

This is not the case for the fund delivery system:

Proposition 9. In the absence of constraints, the fund delivery system can replicate an arbitrary reaction function $f(z_1,\ldots,z_n)$ if and only if $f(z_1,\ldots,z_n)$ is linear (affine) in the z_i.

Proof: See proof of proposition 6.

As a result we get the following proposition:

Proposition 10. In the absence of constraints and with a nonlinear optimal response function, the newsletter delivery system outcome is superior to the fund delivery system outcome.

In actual practice, there are borrowing restrictions and other nonlinearities. We thus hypothesize that in the real world the newsletter delivery system leads to superior outcomes. This may explain the plethora of market timing newsletters and the paucity of pure timing funds.

VIII. Conclusions

We investigated two market timing information delivery systems, one in which timers set up timing funds and one in which timers sell their information through newsletters. In the absence of market imperfections we found that both systems produce the same result. With restrictions on borrowing, or with nonlinear response functions, we found the newsletter delivery system to be superior. This is one possible explanation for the plethora of market timing newsletters and the paucity of market timing funds.

APPENDIX

Proof of Proposition 1

Proposition 1: The distribution of x conditional on $z_1, z_2, \ldots z_N$ is given by

$$(x \mid z_1, \ldots z_N) \sim N(\pi + Z,\ (1-R^2)\sigma_s^2 + \sigma_s^2)$$

where

$$Z = \sum w_i z_i$$

$$R^2 = \sum w_i \rho_i^2$$

$$w_i = \frac{\frac{1}{1-\rho_i^2}}{1 + \sum \frac{\rho_j^2}{1-\rho_j^2}}$$

Proof: Recall that

$$z_i = \rho_i^2 z_i'$$

$$z_i' \mid s \sim N(s,\ \sigma_s^2/k_i)$$

where

$$\text{where } k_i = \frac{\rho_i^2}{1 - \rho_i^2}$$

Now, using Bayes formula, we have

$$f(s|z_i',\ldots,z_n') = \frac{f(z_i',\ldots,z_n'|s)f(s)}{f(z_i',\ldots,z_n')}$$

where f refers to the probability density function of the corresponding random variable. Since we want to derive the distribution of s given the z_i we can write, for simplicity,

$$f(s|z_1',\ldots,z_n') \propto f(z_1',\ldots,z_n'|s)f(s)$$

where α means proportional to. Because the z_1' are conditionally independent normals and because s is unconditionally normal, we can write

$$f(s|z_1,\ldots,z_n) \;\alpha\; f_1(z_1'|s)f(z_2'|s)\ldots f(z_n'|s)f(s)$$

$$\propto \exp\left[\left(\sum \frac{(z_i'-s)^2}{2\sigma_s^2/k_i}\right) + \left(\frac{-1}{2}\frac{s^2}{\sigma_s^2}\right)\right]$$

$$\propto \exp \frac{-1}{2\sigma_s^2}\left[\left(\sum k_i(z_i'-s)^2\right) + s^2\right]$$

$$\propto \exp \frac{-1}{2\sigma_s^2}\left[s^2(1+\sum k_i) - 2s(\sum z_i'k_i) + (U)\right]$$

$$\propto \exp \frac{-(1+\sum k_i)}{2\sigma_s^2}[(s-\frac{\sum z_i' k_i}{1+\sum k_i})^2 + (V)]$$

$$\propto \exp \frac{-1}{2\sigma_s^2(1+\sum k_i)^{-1}}[(s-\frac{\sum z_i' k_i}{1+\sum k_i})^2]$$

where (U) and (V) are terms that do not include s. This is enough to conclude that

$$s|z_1',\ldots,z_n' \sim N(\frac{\sum z_i' k_i}{1+\sum k_i},\ \sigma_s^2(1+\sum k_i)^{-1})$$

Now noting that $z_i = \rho_i^2 z_i'$ and $k_i = \rho_i^2/(1-\rho_i^2)$, and defining $(1-R^2) = (1+\sum k_i)^{-1}$ and, finally, recalling that $x = \pi + s + \varepsilon$ we get the result to be proved. Note that $R^2 = (\sum k_i)(1+\sum k_i)^{-1}$ is the (squared) correlation between Z and s. This can be seen as follows:

$$[Corr(Z,s)]^2 = \frac{[Cov(Z,s)]^2}{Var(Z)Var(s)}$$

where $Z = \sum w_i z_i$. Thus

$$Cov(Z,s) = \sum w_i \rho_i^2 \sigma_s^2$$

$$Var(Z) = E(\sum w_i z_i)^2$$

$$= \sum w_i^2 \rho_i^2 \sigma_s^2 + \sum \sum w_i w_j Cov(z_i, z_j)$$

$$\mathrm{Cov}(z_i, z_j) = E_s E[(z_i - \rho_i^2 s)(z_j - \rho_j^2 s) + \rho_i^2 z_j s + \rho_j^2 z_i s - s^2 | s]$$

$$= E_s[\rho_i^2 \rho_j^2 s^2]$$

$$= \rho_i^2 \rho_j^2 \sigma_s^2$$

$$\mathrm{Var}(s) = \sigma_s^2$$

Thus

$$[\mathrm{Corr}(Z, s)]^2 = \frac{(\sum w_i \rho_i^2)^2}{\sum w_i^2 \rho_i^2 + \sum\sum_{i \neq j} w_i w_j \rho_i^2 \rho_j^2}$$

$$= \frac{(\sum w_i \rho_i^2)^2}{(\sum w_i \rho_i^2)^2 + \sum w_i^2 \rho_i^2 (1 - \rho_i^2)}$$

$$= \frac{(\sum k_i)^2}{(\sum k_i)^2 + \sum (k_i)}$$

$$= \frac{\sum k_i}{\sum k_i + 1}$$

Q.E.D.

Footnote

1. We believe that these assumptions are fairly realistic given the large amount of stock selection, hedging, liquidity, trading, etc. and the low abilities of (and thus, the low correlation between) timers. The assumption that timers are price takers avoids problems associated with strategic behavior and the assumption that aggregate timer transaction volume is small obviates the need for a general equilibrium model. An alternative model might allow nontimers to distinguish timing volume from nontiming volume and thus to engage in game behvior. Principle results from the (considerably more complicated) model that accounts for the gaming activity will not be different if one assumes, as we do, that the desired transaction volume that would result from information developed by a professional timer is the same, whether the timer manages the accounts of the clients personally, or transmits the information to clients by a newsletter.

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