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Optimal Supply Chain Network with Multi-Echelon

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Abstract: The study of the effect of redistribution strategy and aggregation, on a multi-echelon supply chain network by managing demand volatility is discussed in this research. For this an operational supply chain design is considered. Multi-echelon network consisting of manufacturing plants, distribution centers, warehouses, and retailers is used to develop the case study. Aggregation strategy was analyzed in the context of single product and multi-product for a multi-period production problem under demand uncertainty. Product sourcing between echelons and distribution strategies are considered for the study. Objective was to use the redistribution strategy to optimize the objective functions for the network. The objective functions include minimization of total cost, minimization of overage and stock-out conditions, and maximization of the customer service level. The total cost function includes product flow, transportation cost and distance cost. The mathematical formulation is carried out in Mixed Integer Linear Programming (MILP) with the help of Generic Algebraic Modeling System (GAMS). Problem formulation considers three type of demand based on volatility and uncertainty cases as high, medium, and low. The research is divided into three main phases to discuss an optimal multi-echelon supply chain network for single product using aggregation strategy.

1. Introduction

Production and inventory management in supply chain management constitute a key scientific area of focus in operation research. There is a great need for efficient organization and utilization of resource/entities in the supply chain network as well as to ensure proper flow of products from manufacturing to retailers. This need has inspired research and development of analytical tools, mathematical models, and computational solutions. However, many aspects of inventory theory and the associated mathematical models need more research investigation. Concepts such as Vendor Managed Inventory (VMI), Industrial Internet of Things (IIoT), and Just-In-Time Distribution (JITD) among others, demonstrate the need for operating the SCN as a unified enterprise, and to design the supply chain network as an integrated network by utilizing the information to effectively analyze the SCN. The trade-offs between service level and resource cost, and describing optimal parameterization, and control of inventory system under different circumstance for effective Supply Chain Networks (SCN) has inspired recent developments in the supply chain research. There are several literatures that currently exist in Supply Chain Design (SCD). Beamon's (1999) study focused on the operations conducted between links of a supply chain with an aim of designing a robust supply chain structure. The aim of the research was to design a novel structure of a supply chain, which maximizes profit, or minimize cost under ideal production-distributions constraints. Mathematical models, found in SCD literature, are classified as either dynamic or static.

In supply chain design, a product might be manufactured at diverse production units, moving along the chain towards the end-user. The network with multi-echelon layout that is spread across diverse sites will ensure that the right product achieves the right customer in the end. For example, based on the organization's size, a clear methodology on detecting necessary distribution center and storages is required. Additionally, organization's customer regions also contribute to the decision-making in terms of network distribution. When the organization's network is set along all operational facilities, the decisions will be made to actualize the tasks aligned with the company's vision and mission – for example, if customer service should be set as the central priority, or risk/cost minimization is more prioritized, or delivery time is more important, and so on. Within supply chain management, a lot of different distribution approaches can be implemented, but their utilization depends on whether the chain is managed through make-to-stock or make-to-order manufacturing strategy.

This approach is applied in the mainstream manufacturing industry in terms of uncertainty of the customer demand, and the production level is prioritized beforehand (before realization of the demand and demand period), whereas there is not much research available. Overall, it is also vital to mention that the producer is likely to control and operate the entities in the network. This business concept contributes to the model building stage. In terms of defining the distribution approach of the network, distribution center and warehouse do not dispatch products to the retailer based on their demand. Retailer's demand refers to the number of products/items requested from the organization. A big portion of literature has investigated the distribution approach, where the quantity of products delivered to the retailer equaled to the retailer's demand. With reference to the literature on supply chain design, the idea of a producer controlling and running own retail units is not addressed properly. For example, Farahani and Elahipanah (2008) tested the application of just-in-time distribution (JITD) model in their supply chain design combined with a MILP model. Nonetheless, they failed to address the make-to-stock manufacturing strategy. Their distribution approach was to ship as much as the retailer had requested. The given model brings essential changes to the distribution concepts, adding to JITD approach, two vital aspects are presented. First, retailers have to guarantee information on quantity of inventory they actually have. Second, customer demand in last time period should be clarified. When a production center becomes aware of the existing on-hand inventory at the retailer, as well as about possible demand for the specific historically-recorded time period, the quantity of items delivered is a computed decision for the company. JITD usage is not always possible, since the retailers are sometimes reluctant to provide data of classified origins. In addition, the inventory quantity and historical information will give a limited scope for improvements. Thus, in the study, the significance is added to the value of concept of a producer managing the decision on the quantity of products transferred to the optimized and computed amounts. Hence, the objective functions of this study are aimed at reducing costs and improve service level in terms of using a MILP model.

By strategically distributing the amount of product in the network, it helps to reduce the stock out and overage at the retailers. On the strategic distribution, the aggregated amount of the products are located at the distribution center and warehouse while reducing the cost and increasing the service level. The aim of this research is to optimize cost and service level under stochastic and volatile conditions of demand. The model attempts to locate the product at different locations in the network with the best cost and service level "trade- off". The routes are based on the optimization and hence the routes are different for each time period. In section 2 literature review of, the mathematical models for cost reduction and improvement of service level is presented. Case studies with high, medium, and low volatility are used to explain the mathematical models is shown in section 3. Section 4 discusses the Conclusion and future work.

2. Literature Review

In 2001, Tsiakis et al. offered a mixed integer non-linear programming model (MINLP) for supply chain framework of multi-echelon network with uncertainty in demand. The model aimed at covering the entities' location, with addressing related transportation and inventory costs. Pan, F., & Nagi, R., (2013) consider a supply chain network design problem introduced in the agile manufacturing situation at different levels of management and different periods in the context of a great demand of a number of customers. They focus on the problem and generate a solution, considering all options to reduce the total operational costs using a Lagrangian heuristic. Zanjani et al. (2010) created a multi-period, multi-item stochastic project to cope with the production planning with demand uncertainty. In addition, Petridis (2013) presented a MILP model to identify the best network formation. The model's key configuration was added in terms of lead-time by utilizing likelihoods for such operations as overstocking and understocking, accordingly. Baghalian et al. (2013) insisted on the significance of making design for the supply chain network by emphasizing on the service level. Authors addressed the supply chain network representing three echelons based on manufacturers, distribution units, and retail shops. Within their project, Baghalian et al. (2013) paid attention to the random demand with the recognized function on distribution. Authors suggested the retailers request products prior to the start of each time period. Hence, they produced cut-set models (MINLP) with providing a sensitivity evaluation of models developed to identify the best location for entities that are relevant in the strategic perspective for the business.

Sawik, T. (2015), in this paper, a new decision-making problem is discussed. The combinatory stochastic problem represents a stochastic mixed integer program, which includes the organized measured averaging aggregation of the two contradictory objective functions. The incorporated choice of supply portfolio and planning of customer orders with a number of suppliers exposed to independent regional and local destructions are described. The stochastic mixed integer programs used for both equally effective optimization of expected customer service level and cost is discussed.

Sabri and Beamon (2000) categorized supply chain into two domains: strategic and operational. Strategic domain is associated with the supply chain design, since tasks attributed to design (for instance, picking up a location for network facilities) are strategic by nature for any corporation. Operational decisions are those related to production, delivery, and so on. Sabri and Beamon (2000) succeeded in synchronizing both levels, using two sub-models within their solution technique. Authors focused on four echelons, such as suppliers, manufacturing centers, distribution units, and customer regions. Their models became stochastic because of the demand uncertainty as well as lead time. For this reason, authors relied on ε- constraint approach along with MILP objective function. These functions were able to reduce both fixed and variable costs. Shen et al. (2003) produced a cost-based site inventory project, with reference to the above-mentioned model. Over time, Shen and Daskin

(2005) improved it by putting the customer service component in. Their model was targeted to estimate trade-offs relating to cost and service. The weighting technique was utilized along with the heuristic solution method grounded on the generic algorithm. Thus, a non-linear model was able to define the distribution unit location associated with demand nodes to streamline the administration of cost and service. By reviewing this model, it was possible to notice that the traditional location approach with excluded variable (inventory) expenditures rather provided a scope for higher fixed cost, with total cost being underrated. Thus, a business could invest more in fixed capital cost and resultantly have higher total cost. Because of this, the location models provide extensional distribution units than a business actually requires or might effective manage with the funds available. A rationale for adding customer service component to the efforts of Shen and Daskin (2005) is substantially important. Hence, IBM generated their supply chain model by emphasizing on customer service variable while applying the related competitive threshold (Ma & Wilson 2002). Farhani and Elahipanah (2008) optimized the supply chain design by focusing on the service level specifically. Eventually, they utilized just-in-time distribution (JITD) in their supply chain network, using an algorithm to solve the MILP.

3. Problem Definition

The research aims at developing a comprehensive supply chain network with multi-echelon structure depending on number of manufacturing, distribution centers, and warehouse and retailer retailers for the purpose of meeting customer needs. The goal is to optimize the network's total cost, number of echelons and entities within a single echelon to design an optimal supply chain network (Short path) to achieve maximum service level. A mathematical MILP model is provided in the study. In addition to cost cutting and service level improvement, this research also investigates the effect of variability in demand on the service level and cost, which is as follows:

3.1 Mathematical Formulation

Sets and Indices

- T Set of times (periods), t ϵ T
- K Number of echelons k = {1,2, …..K}
- N Set of entities (i=j); 'i' is the entity at current echelon 'k' and j represents entity at echelon 'k+1'. i= {1, 2..(K-1) & j= {2, 3,..,K)

Parameters

- C_{ijt} Number of available products that can be supplied from 'i' to 'j' in time period t ϵ T, (i, j) ϵ N
- D_{jt} Demand for product at entity j for time period t ϵ T, where j=K
- T_{ij} Unit transportation cost of the product for unit distance to (i, j) ϵ N
- E_{ij} Distance between entities (i, j) ϵ N
- I_{jt} Inventory of product at entities j for time period t ϵ T, (i, j) ϵ N
- H_{ij} Unit inventory holding cost at entities j, (i, j) ϵ N
- O_{jkt} Overage at entity j for time period t ϵ T, ∀ k=2 to K
- M_{jkt} Stock-out at entities j for time period t ϵ T, ∀ k=2 to K
- A_{jk} Storage Capacity of entities j, ∀ k=2 to K

Decision Variables

- V_{ijt} Quantity of units shipped from i to j in time period t, t ϵ T
- S_{jt} Quantity of products stored at entity j in time period t, t ϵ T ∀ k=2…K
- G_{ij} 1 if entity i supply to entity j (i, j) ϵ N, 0 otherwise

3.2 Objective Function

$$Min\ Z1\left\{\sum_{k=1}^{K}\sum_{i=j=1}^{N}\sum_{t=1}^{T}\left(V_{ijt}^{k+n}.\ T_{ij}^{k+n}.E_{ij}^{k+n}+\left(H_{ij}^{k+n}.I_{jkt}^{k+n}\right)\right)\right\} \quad (1)$$

$$Min\ Z2\sum_{k=1}^{K}\sum_{i=j=1}^{N}\sum_{t=1}^{T}\left\{M_{jkt}^{k+n}+O_{jkt}^{k+n}\right\} \quad (2)$$

$$Max\ Z3 \sum_{k=1}^{K} \sum_{i=j=1}^{N} \sum_{t=1}^{T} \quad I_{jt}^{k+n} / D_{jt}^{k+n} \tag{3}$$

The objective function Equation 1 minimizes the total cost, in which the first term determines the total shipping cost and the second term represents the inventory holding cost. The second objective function (Equation 2) minimizes the stock-out (M_{jkt}^{k+n}) and overage (O_{jkt}^{k+n}) cost. The third objective function (Equation 3) maximizes the service level by the ratio of existing inventory (I_{jt}^{k+n}) to demand (D_{jt}^{k+n}), which indicates the service level.

Subject to Constraints

$$\sum_{k=1}^{K} \sum_{i=j=1}^{N} V_{ijt} \quad \leq \ C_{ijt} \qquad \forall\ (i,\ j) \epsilon\ N\ ,\ t \in T \tag{4}$$

$$\sum_{K=1}^{K} \sum_{i=j=1}^{N} I_{ijt} \quad \leq \ A_{jt} \qquad \forall\ (i,\ j)\ \epsilon\ N\ ,\ t \in T \tag{5}$$

$$\sum_{k=1}^{K} \sum_{i=j=1}^{N} V_{ijt} \ \leq \ V_{i(j+1)t} \ + I_{i(j+1)t-1} \quad \forall\ (i,\ j)\ \epsilon\ N\ ,\ t \in T \tag{6}$$

$$\sum_{k=1}^{K} \sum_{i=j=1}^{N} V_{ijt} \quad \leq A_{jt} - O_{jkt} \quad \forall\ (i,\ j)\ \epsilon\ N\ ,\ t \in T \tag{7}$$

$$\sum_{k=1}^{K} \sum_{i=j=1}^{N} I_{ijt-1} + V_{ijt} \leq V_{i(j+1)t} + M_{jkt-1} + I_{ijt} - M_{jkt}\ \forall\ (i,\ j) \epsilon\ N,\ t \in T \tag{8}$$

$$V_{ijt},\ O_{jkt},\ I_{ijt}\ \geq 0 \qquad \forall\ (i,\ j) \epsilon\ N, (i,\ j) \epsilon\ A,\ t \in T \tag{9}$$

$$G_{ij} \in \{0,1\} \qquad \forall\ (i,\ j) \epsilon\ N, (i,\ j) \epsilon\ A,\ t \in T \tag{10}$$

The manufacturing capacity constraints are given by Equation 4. Equation 5 is used to ensure that the quantity of products stored is not greater than the storage capacity of the entity. Equation 6 ensures that the number of products shipped out of an entity is lesser than or equal to the number of products shipped to that entity. Equation 7 ensures that the number of products arriving at any entity is less than or equal to the storage capacity minus the overage at the entity for any time-period. Equation 8 is used to ensure that the inventory from (t-1) and the quantity of products shipped is lesser than or equal to the number of products shipped out of an entity and the stock out from (t-1) in the end of each time period. Equation 9 ensures non-negativity for the number of units of product shipped in each time-period, overage and inventory. Equation 10 is a binary constraint that indicates whether products are shipped from entity i in echelon k to entity j in echelon (k+1).

3.3. Case Studies

The model is explained using case studies. These case studies are for single product, four time-periods, and a four echelon system (Figure 1). This case study provides further explanation and understanding to the mathematical model. In this case study the system assumes a make-to- stock approach. In this research, the stock-out (M_{jkt}) is computed as a back-order since all back-orders are assumed to be filled in subsequent time-periods. Demand (D_{jt}) at the retailers is stochastic. In this study, the supply chain design covers the quantity of products shipped from one echelon to another. GAMS (BARON) solver is used to solve the problem. The objective function reduces cost and enhances service level, while considering demand volatility. Demand varies from retailer to retailer as well as from one time period to the next. The three case studies are used to demonstrate the impact of volatility on service level and cost.

3.3.1 Case Study 1 (High Volatility)

This case study consists of one manufacturing plant, two distribution centers, three warehouses and four retailers as mentioned before (Figure 3.1). Volatility in the demand can be defined as ratio of the mean demand to the standard deviation of the demand for each retailer. Based on the Empirical Rule for the normal distribution, about 68% of the data lie within ± 1 standard deviation of the mean, 95% of the data lie within ± 2 standard deviation of the mean, and 99.7% of the data lie within ± 3 standard deviation of the mean. In this research ± 1 standard deviation of the mean retailer demand (3000) represent low volatility whereas ± 2 and ± 3 standard deviation of the mean retailer demand (3000) represent medium and high volatility respectively.

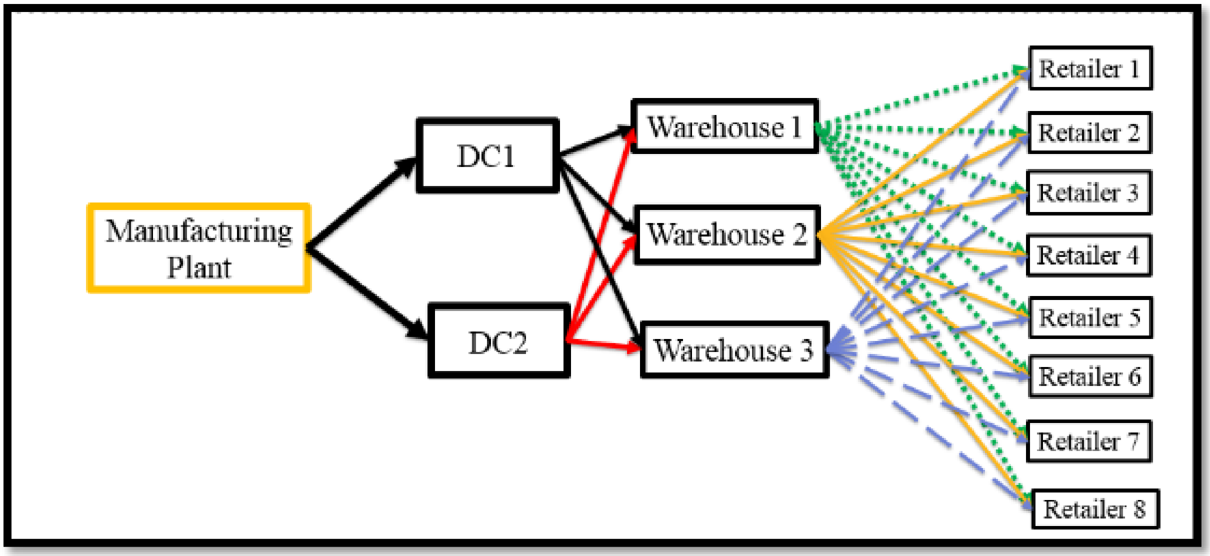

Figure 1. Supply Chain Network

Table 1 gives the holding cost and storage capacity at each retailer. Table 2 provides the transportation cost per unit between entities. Table 3 represent the distance between entities. Table 4 provides the customer demand. The mean of the demand and the standard deviation for the demand are calculated. The volatility for all retailers are above 99.7%.

Table 1. Retailer holding cost and storage capacity

Retailer Location	Holding Cost $	Entity Capacity
11	0	-
21	11	20000
22	15	20000
31	20	10000
32	18	10000
33	20	10000
41	42	3000
42	44	3000
43	46	3000
44	48	3000
45	50	3000
46	51	3000
47	53	3000
48	54	3000

Table 2. Transportation cost between entity

Echelons		1	2		3			4							
Entities location		11	21	22	31	32	33	41	42	43	44	45	46	47	48
1	11	0	2	3	0	0	0	0	0	0	0	0	0	0	0
2	21	0	0	0	4	6	9	0	0	0	0	0	0	0	0
	22	0	0	0	8	5	3	0	0	0	0	0	0	0	0
3	31	0	0	0	0	0	0	12	15	18	26	25	30	38	45
	32	0	0	0	0	0	0	30	25	15	17	19	22	28	33
	33	0	0	0	0	0	0	47	44	40	31	23	23	13	10

Echelons		1	2		3			4							
Entities location		11	21	22	31	32	33	41	42	43	44	45	46	47	48
4	41	0	0	0	0	0	0	0	0	0	0	0	0	0	0
	41	0	0	0	0	0	0	0	0	0	0	0	0	0	0
	43	0	0	0	0	0	0	0	0	0	0	0	0	0	0
	44	0	0	0	0	0	0	0	0	0	0	0	0	0	0
	45	0	0	0	0	0	0	0	0	0	0	0	0	0	0
	46	0	0	0	0	0	0	0	0	0	0	0	0	0	0
	47	0	0	0	0	0	0	0	0	0	0	0	0	0	0
	48	0	0	0	0	0	0	0	0	0	0	0	0	0	0

Table 3. Distance between entity

Echelons		1	2		3			4							
Entities location		11	21	22	31	32	33	41	42	43	44	45	46	47	48
1	11	0	300	320	0	0	0	0	0	0	0	0	0	0	0
2	21	0	0	0	160	170	200	0	0	0	0	0	0	0	0
	22	0	0	0	220	190	170	0	0	0	0	0	0	0	0
3	31	0	0	0	0	0	0	50	90	110	460	490	520	550	720
	32	0	0	0	0	0	0	220	200	120	90	65	110	250	680
	33	0	0	0	0	0	0	600	550	500	480	230	180	100	80
4	41	0	0	0	0	0	0	0	0	0	0	0	0	0	0
	41	0	0	0	0	0	0	0	0	0	0	0	0	0	0
	43	0	0	0	0	0	0	0	0	0	0	0	0	0	0
	44	0	0	0	0	0	0	0	0	0	0	0	0	0	0
	45	0	0	0	0	0	0	0	0	0	0	0	0	0	0
	46	0	0	0	0	0	0	0	0	0	0	0	0	0	0
	47	0	0	0	0	0	0	0	0	0	0	0	0	0	0
	48	0	0	0	0	0	0	0	0	0	0	0	0	0	0

Table 4. Customer demand at the retailers with high volatility

Retailer Location	Time Period 1	Time Period 2	Time Period 3	Time Period 4	Mean	Standard Deviation	Volatility
41	2940	3097	3018	2889	2986	91.049	32.795
42	3145	3006	2928	3149	3057	108.705	28.122
43	2887	3140	2865	2909	2950.25	127.769	23.090
44	2980	3094	3043	2971	3022	57.706	52.369
45	3095	3123	3149	3137	3126	23.238	134.522
46	2936	2920	3025	3094	2993.75	81.242	36.850
47	3021	3000	2865	3102	2997	98.377	30.464
48	2973	3114	2909	3060	3014	90.962	33.135

3.3.1.1 Simulation Analysis

Using the data for the case study, optimization provides the optimal amount of products shipped from one echelon to the next for each time-period. Table 5 show the quantity of products shipped from warehouse to retailers for each time-period. Table 6 show the quantity of products shipped from distribution center to warehouses for each time-period. Table 7 show the quantity of products shipped from manufacturing to distribution center for each time-period.

Table 5. Amount of products transported from warehouse to retailer

		Time Period 1	Time Period 2	Time Period 3	Time Period 4
31	41	3000	2940	3097	3018
	42	3000	3145	3006	2928
	43	3000	2887	3140	2865
32	43	0	0	0	0
	44	3000	2980	3094	3043
	45	3000	3095	3123	3149
	46	3000	2936	2920	3025
33	46	0	0	0	0
	47	3000	3021	3000	2865
	48	3000	2973	3114	2909

Table 6. Amount of products transported from distribution center to warehouse

		Time Period 1	Time Period 2	Time Period 3	Time Period 4
21	31	9000	8972	9243	8811
	32	6000	6075	6217	6192
	33	0	0	0	0
22	31	0	0	0	0
	32	0	0	0	0
	33	9000	9953	9040	7770

Table 7. Amount of products transported from manufacture to distribution center

		Time Period 1	Time Period 2	Time Period 3	Time Period 4
11	21	15500	14547	15460	16730
	22	9000	9953	9040	7770

Table 8 and 9 show the optimized quantity of products stored at distribution center and warehouse for each time-period. In addition, the solver determined the best location to aggregate the products with the minimum volatility and cost in order to obtain high service level. Table 10 and Table 11 show the stock out and the overage respectively.

Table 8. Amount of products stored at Distribution center

Retailer location	Time Period 1	Time Period 2	Time Period 3	Time Period 4
21	0	0	254	673
22	0	0	322	478

Table 9. Amount of products stored at Warehouse

Retailer location	Time Period 1	Time Period 2	Time Period 3	Time Period 4
31	500	300	229	0
32	0	586	345	0
33	0	137	455	0

Table 10. Stock-out

Retailer location	Time Period 1	Time Period 2	Time Period 3	Time Period 4
41	0	157	0	0
42	145	0	0	221
43	0	253	0	44
44	0	114	0	0
45	95	28	26	0
46	0	0	105	69
47	21	0	0	237
48	0	141	0	151

Table 11. Overage

Retailer location	Time Period 1	Time Period 2	Time Period 3	Time Period 4
41	60	0	79	129
42	0	139	78	0
43	113	0	275	0
44	20	0	51	72
45	0	0	0	12
46	64	16	0	0
47	0	21	135	0
48	27	0	205	0

Table 12 shows the values obtained for the three objective functions: total cost (Z1), cost of the overage and stock-out (Z2) and the service level (Z3) for the first scenario high volatility.

Table 12. Objective function Values for the high volatility case

Z1(total cost)	29714779
Z2(overage and stock-out)	3303
Z3(service level)	0.96

3.3.2 Case Study 2 (Medium Volatility)

The network design in this case and input data is the same as the previous case study. However, the customer demand volatility is medium based on the categorization. Tables 14, 15 and 16 shows the amount of products transported from warehouse to retailer, distribution center to warehouse and manufacture to distribution center, respectively.

Table 13. Customer demand at the retailers with medium volatility

Retailer Location	Time Period 1	Time Period 2	Time Period 3	Time Period 4	Mean	Standard Deviation
41	3018	2986	3029	3026	3014.75	19.72097
42	2929	2950	3077	2930	2971.5	70.99531
43	3005	3086	2960	2990	3010.25	53.85397
44	3021	3067	3037	2946	3017.75	51.49353
45	3035	3021	3077	3021	3038.5	26.50157
46	3049	2981	3043	3018	3022.75	30.90173
47	3004	3030	3039	2999	3018	19.51068
48	3045	3021	2940	2925	2982.75	59.16291

Table 14. Amount of products transported from warehouse to retailer

		Time Period 1	Time Period 2	Time Period 3	Time Period 4
31	41	3000	3018	2986	3029
	42	3000	2929	2950	3077
	43	3000	3005	3086	2960
32	43	0	0	0	0
	44	3000	3021	3067	3037
	45	3000	3035	3021	3077
	46	3000	3049	2981	3043
33	46	0	0	0	0
	47	3000	3004	3030	3039
	48	3000	3045	3021	2940

Table 15. Amount of products transported from distribution center to warehouse

		Time Period 1	Time Period 2	Time Period 3	Time Period 4
21	31	9000	8952	9022	9066
	32	6000	6056	6088	6114
	33	0	0	0	0
22	31	0	0	0	0
	32	0	0	0	0
	33	9000	9992	9390	7770

Table 16. Amount of products transported from manufacture to distribution center

11		Time Period 1	Time Period 2	Time Period 3	Time Period 4
	21	15500	14508	15110	16730
	22	9000	9992	9390	7770

Table 17 and 18 shows the optimized amount of products stored at distribution center and warehouse for each time period. The solver found the best location to store the aggregate quantity of products with the minimum volatility and cost in the distribution center and warehouse, on the other hand reach high service level. Table 19 and Table 20 shows the stock out and overage respectively. The case study was studied to verify the model which is solved by using GAMS (BARON Solver). Table 21 shows the optimal objectives (total cost Z1, the overage and stock-out Z2, and the customer service level Z3) for the second case with medium demand volatility.

Table 17. Amount of products stored at distribution center

Retailer location	Time Period 1	Time Period 2	Time Period 3	Time Period 4
21	0	0	300	550
22	0	0	334	366

Table 18. Amount of products stored at Warehouse

Retailer location	Time Period 1	Time Period 2	Time Period 3	Time Period 4
31	400	397	417	0
32	0	298	319	0
33	0	299	516	0

Table 19. Stock outs

Retailer location	Time Period 1	Time Period 2	Time Period 3	Time Period 4
41	18	0	43	0
42	0	21	127	0
43	5	81	0	30
44	21	46	0	0
45	35	0	56	0
46	49	0	62	0
47	4	26	9	0
48	45	0	0	0

Table 20. Overage

Retailer location	Time Period 1	Time Period 2	Time Period 3	Time Period 4
41	0	32	0	3
42	71	0	0	147
43	0	0	126	0
44	0	0	30	91
45	0	14	0	56
46	0	68	0	25
47	0	0	0	40
48	0	24	81	15

Table 21. Objective

Z1(total cost)	25765550
Z2(overage and stock-out)	1501
Z3(service level)	0.99

3.3.3 Case Study 3 (Low Volatility)

The third case study shows low volatility in the customer demand. The data for the demand is shown in Table 22. Tables 23, 24 and 25 shows the amount of products transported from warehouse to retailer, distribution center to warehouse and manufacture to distribution center, respectively. The case study model is solved using GAMS (BARON Solver).

Table 22. Customer demand at the retailers with Low volatility

Retailer Location	Time Period 1	Time Period 2	Time Period 3	Time Period 4	M	SD
41	2997	3045	2984	3006	3008	26.26785
42	2986	2983	3001	3012	2995.5	13.52775
43	3029	3038	2994	3048	3027.25	23.48581
44	2969	2992	2969	3007	2984.25	18.64359
45	2987	2959	2966	2978	2972.5	12.4499
46	3027	3024	3023	2971	3011.25	26.88711
47	3025	3017	3034	2973	3012.25	27.07243
48	2972	3015	3041	2984	3003	31.14482

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Table 23. Amount of products transported from warehouse to retailer

		Time Period 1	Time Period 2	Time Period 3	Time Period 4
31	41	3000	2997	3045	2984
	42	3000	2986	2983	3001
	43	3000	3029	3038	2994
32	43	0	0	0	0
	44	3000	2969	2992	2969
	45	3000	2987	2959	2966
	46	3000	3027	3024	3023
	46	0	0	0	0
	47	3000	3025	3017	3034
	48	3000	2972	3015	3041

Table 24. Amount of products transported from distribution center to warehouse

		Time Period 1	Time Period 2	Time Period 3	Time Period 4
21	31	9500	8512	9066	8979
	32	6000	5988	5951	5935
	33	0	0	0	0
22	31	0	0	0	0
	32	0	0	0	0
	33	9000	10000	9483	7663

Table 25. Amount of products transported from manufacture to distribution center

11		Time Period 1	Time Period 2	Time Period 3	Time Period 4
	21	15500	14468	15017	16837
	22	9000	10032	9483	7663

Table 26 and 27 shown the optimized amount of products stored at distribution center and warehouse for each period. It also shows that the solver chose the best location to aggregate the products which is the second and third echelons (distribution center and warehouse) with the minimum volatility and cost, and to achieve high service level at retailer echelon. Table 28 and Table 29 shows the stock out and the overage respectively. The case study was studied to verify the model which is solved by using GAMS (BARON Solver). Table 30 shows the optimal objectives (total cost Z1, the overage and stock-out Z2, and the customer service level Z3) for the second case with low volatility.

Table 26. Amount of products stored at Distribution center

Retailer location	Time Period 1	Time Period 2	Time Period 3	Time Period 4
21	0	0	186	494
22	0	0	320	923

Table 27. Amount of products stored at Warehouse

Retailer location	Time Period 1	Time Period 2	Time Period 3	Time Period 4
31	0	236	400	0
32	200	158	378	0
33	300	614	657	0

Table 28. Stock outs

Retailer location	Time Period 1	Time Period 2	Time Period 3	Time Period 4
41	0	48	0	22
42	0	0	18	11
43	29	9	0	54
44	0	23	0	38
45	0	0	7	12
46	27	0	0	0
47	25	0	17	0
48	0	43	26	0

Table 29. Overage

Retailer location	Time Period 1	Time Period 2	Time Period 3	Time Period 4
41	3	0	61	0
42	14	3	0	0
43	0	0	44	0
44	31	0	23	0
45	13	2	0	0
46	0	3	1	52
47	0	8	0	61
48	28	0	0	57

Table 30. Objective

Z1(total cost)	22669525
Z2(overage and stock-out)	813
Z3(service level)	1.0

3.4 Discussion and Analysis

The routes in our network are different for each time period in each case study. The following analysis shows the relationship between the total cost Z1 and type of demand volatility to improve the objective function value as shown in Figure 2. The optimal total cost for each type of uncertainty and volatilely demand function improved. On the other hand, Figure 3 shows the relationship between the demand fluctuation and service level, the optimal value of the service level for each type of uncertainty and volatilely demand function improved.

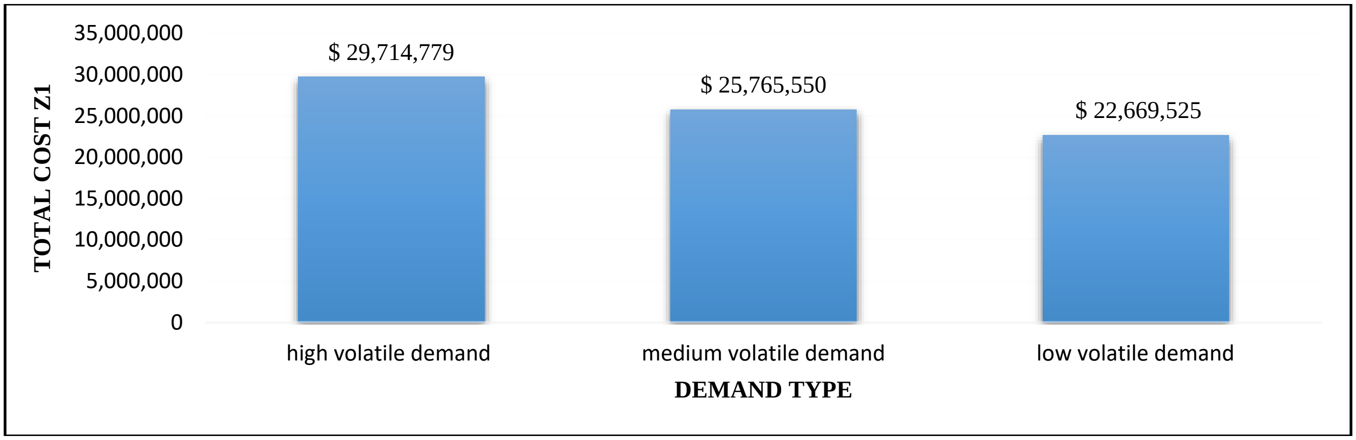

Figure 2. Relationship between the total cost Z1

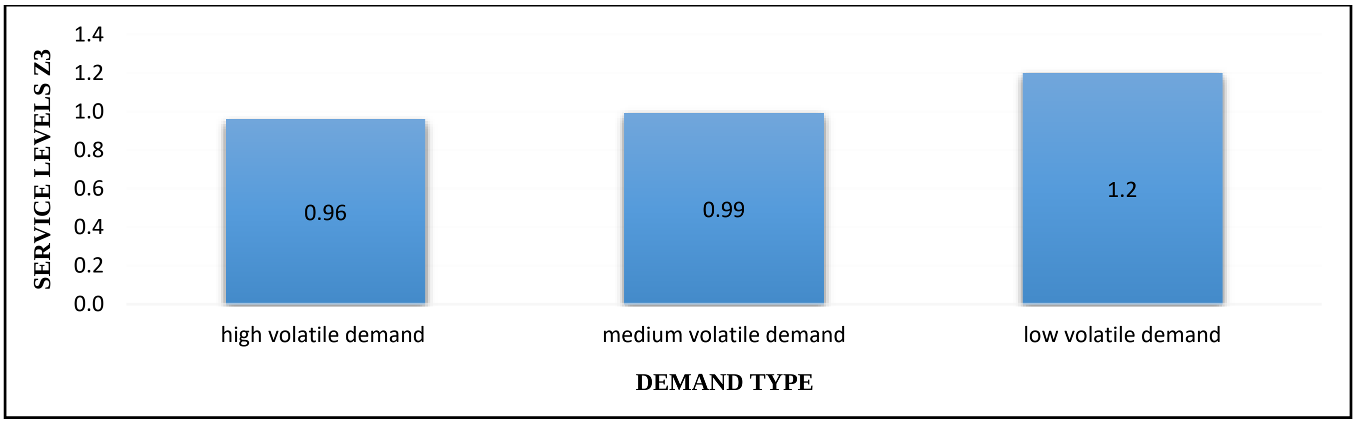

Figure 3. Relationship between the service levels

Tables 31 and 32 compare the products sorted at each distribution center and warehouse for each time period resulting from the aggregation. Moreover, showing the strategic distribution that are follow in this research which is locating the aggregated amount of products at warehouses to help the firm to reduce the cost and increasing the service levels. The other benefit of this strategy the stock-out and the overage at the customer zone been reducing as well as per of our third objective function.

Table 31. Summary of the products sorted at distribution center result of aggregation

Retailer location	Low				Medium				High			
	Time Period 1	Time Period 2	Time Period 3	Time Period 4	Time Period 1	Time Period 2	Time Period 3	Time Period 4	Time Period 1	Time Period 2	Time Period 3	Time Period 4
21	0	0	186	494	0	0	300	550	0	0	254	673
22	0	0	320	923	0	0	334	366	0	0	322	478

Table 32. Summary of the products sorted at warehouse result of aggregation

Retailer location	Low				Medium				High			
	Time Period 1	Time Period 2	Time Period 3	Time Period 4	Time Period 1	Time Period 2	Time Period 3	Time Period 4	Time Period 1	Time Period 2	Time Period 3	Time Period 4
31	0	236	400	0	400	397	417	0	500	300	229	0
32	200	158	378	0	0	298	319	0	0	586	345	0
33	300	614	657	0	0	299	516	0	0	137	455	0

4. Conclusion

This paper provides a network to work on modern supply chain principle such as reaching the consumer directly in short DTC. As well in our network the routes are different for every time period in each case study. The case study with different demand uncertainty were examined in this research to achieve and develop the best design for multi-echelon supply chain network system. The proposed design optimizes total cost and service level when the demand is volatile and the concept of trade-offs of total cost and aggregation is been used. This research is comprises of three objective functions. The first objective aims to minimize the total cost of the products that strategically distributed in the network. The second objective aims to maximize the service level at the costumer's zone which is the last echelons in our network. The third objective aims to minimize the stock-out and overage at the last echelon. The mathematical formulation introduced in this research is Mixed Integer Programming (MILP) with the help of Generic Algebraic Modeling System (GAMS) BARON solver. This model can notify the decision maker with comprehensive view about the plan of increasing the service level with the minimum cost.

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