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Analysis of a Vendor Managed Consignment Inventory System with Kanban Withdrawals and Payment Delays

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Abstract: Vendor Managed Inventory (VMI) System with Consignment Inventory (CI) policy is a solution for many supply chain leaders in a highly competitive market. In this paper, totally eight different inventory supply chain models are studied. The profit function of supplier and manufacturer in different environments are compared in order to show the profitability of the overall supply chain management system in a manufacturing industry with different time horizons. The inventory systems are applied on a supply chain consisting of a single supplier and a manufacturer. The main focus of this study is to analyze the effect of payment deferral and the time value of money in push and pull (Kanban) manufacturing systems when VMI-CI policy is applied.

Keywords: Vendor Managed Inventory, Consignment Inventory, Payment deferral, Push System, Pull System, EOQ model, time value

1. Introduction

Vendor managed inventory models (VMI) represent a new generation of supply chain inventory policies, which aim to increase total profitability of the supply chain. The combination of VMI with Consignment Inventory (CI) is referred to as the VMI-CI concept and has been defined in several ways by various researchers. According to Copacino (1993), in a VMI model, the supplier is responsible for monitoring and controlling inventory at their own stores, as well as at warehouses of the manufacturers (customers). In this policy, the supplier makes decisions for manufacturer warehouse inventory replenishment also. Therefore, the supplier inventory planning can be merged with manufacturing and transportation processes and this integration can result in decreasing total inventory related costs, and in increasing total supply chain profitability. The Consignment Inventory System (CIS) is a policy in which inventory even at the manufacturer's warehouse is owned by the supplier and is considered as sold only when it is either used by the manufacturer or is sold by the manufacturer to their customers. For MRP planning and other production decisions, CIS policy can assist decision-makers in keeping better track of products and market demand. Also, CIS policy forces suppliers to increase quality thus reducing defects and sale loss (Copacino, 1993). By combining these two policies together (VMI-CI), the supplier is not only the decision-maker of the inventory in the manufacturer store but is also responsible for the manufacturer's inventory cost.

The coordination between supply chain drivers can be defined as the collaboration of supply chain elements, which is classified into two categories: centralized and decentralized (Giannoccaro and Pontrandolfo, 2004). A centralized supply chain model is where the decisions are made for the whole supply chain as a single system rather than for each part individually (Li et al., 2009). Also, a fully coordinated centralized supply chain (also referred to as a cooperative supply chain) always has greater profitability than decentralized supply chain models (Li et al., 2009). In decentralized (non-cooperative) supply chain models, the decision-maker is usually the buyer and the supplier is the follower (Wang et al., 2004). Although decentralized models have less profitability comparatively, the supply chain leaders can increase the amount of total profit of the supply chain by incorporating consignment contracts (Cachon and Fisher, 2000). According to Cottrill (1997), by applying the VMI-CI system to a decentralized supply chain model, total profitability can be increased, along with an improvement in production and market share (Cottrill, 1997).

The type of inventory control contract is crucial in profitability when developing supply chain models. The traditional policy, which is called Inventory Sourcing (IS), is a purchasing contract in which the customer is billed by the vendor after the product has been delivered. The lack of coordination in this method has led supply chain leaders to adopt new models. However, Inventory Sourcing is still useful in some small markets with low demand rates (Gümüş et al., 2008).

While a few CI models have been studied, the novelty of this work is analysis of CI under a Kanban system. The system is also studied for payment delays, which are reduced to make the supplier support the CIs.

The objective of this study is to compare the individual profit functions of both supplier and buyer in different environments and to obtain optimized profitability functions. Moreover, this paper focuses on extending the work done by Dong and Xu (Dong and Xu, 2002) by considering payment delay in Push and Kanban system in two different time horizons. Also, when focusing on total profitability of the supply chain, a methodology is proposed in order to help the supplier to find a proper manufacturer.

2. Literature Review

In the last decade, there has been a considerable amount of research conducted on synchronization of supply chain drivers including; location, transportation, inventory, and information. However, given its great potential, this area is in need of more development-related ideas. Generally, there is no precise definition for a VMI model since these models are discussed in the literature with different definitions. For example, Quick Response (QR), Continuous Replenishment (CR), Rapid Replenishment (RR), and Collaborative Planning, Forecasting, and Replenishment (CPFR) are all VMI models (Holmström et al., 2002).

When a supply chain adopts VMI policy, the buyer lets the supplier deal with market demand directly by controlling the inventory flow of raw materials to delivery and finally, to customers. Hence, the supplier is able to schedule production, transportation, inventory storage, and delivery in an integrated model, which produces more cooperation within supply chain elements and results in higher efficiency and lower total cost of the whole supply chain (Dejonckheere et al., 2003). For instance, a contract that consists of two parameters of revenue sharing plus surplus subsidy has been developed by Gerchak and Wang for a single manufacturer supply chain model with many suppliers (Gerchak and Wang, 2004).

VMI models have been mostly adopted in retail companies. For example, by applying VMI policy, Wal-Mart and J.C. Penny have had great improvements in inventory turnover and sales (Buzzell and Ortmeyer, 1995; Stalk et al., 1992). Moreover, VMI models have also been applied in other industries such as; steel, books and petrochemicals (Disney et al., 2003; Lamb, 1997). Recently, models those combine VMI with CI, have been utilized by online retailer companies such as Amazon, eBay, and Alibaba (Li et al., 2009) Within all these companies from different domains, the fundamental concept of all VMI models are the same, however, there are several different modifications of VMI/CI models in regard to terms and conditions.

Payment deferral (or delay time) in VMI-CI models is one of the primary focuses of this paper. This strategy helps distributors maintain business with retailers (Michaelraj and Shahabudeen, 2009). Consider a situation where a buyer needs to replenish their own inventory. After receiving an order, the distributor will attempt to deliver the goods as late as possible in order to benefit from payment deferral (Huang and Lin, 2005). In addition, there is a distributer concern that the retailer may change the product, upon which the distributor must provide an additional service in order to remove the excess inventory in the retailer's store. VMI can be developed in order to resolve the aforementioned retailer and distributer issues (Huang and Lin, 2005; Michaelraj and Shahabudeen, 2009). Michaelraj and Shahabudeen (2009) explained VMI models from both retailer and distributer viewpoints. From the retailer's point of view, there is no longer a delay in the transfer of goods since the distributor is still the owner of goods. From distributer's point of view, there is no concern of excess inventory removal from the retailer's store considering the inventory system is monitored and controlled by the distributor. They also developed two replenishment models in order to minimize the total balance of payment (BP) while maximizing total sale (Michaelraj and Shahabudeen, 2009).

Models with different inventory policies vary on their VMI contract type and their supply chain structure. Some examples for two-echelon supply chain model with VMI policy are (Christopher, 1998; Disney et al., 2003; Holmström et al., 2002). On the other hand, (Chen et al., 2000; Lee et al., 1997; Sucky, 2009) studied VMI policy on three-level supply chain models with different type of demands (deterministic and stochastic stationary demands). Depending on the type of supplier and buyer VMI contract, the supplier may be responsible for the buyer's store inventory control, a customer distribution center, or a manufacturing location (Shah and Goh, 2006). However, regardless of the type of VMI contract, Piplani and Viswanathan (2003) showed that the total inventory cost of a supply chain decreases after applying a VMI policy in the supply chain. (Piplani and Viswanathan, 2003).

There are some issues in case of applicability of VMI models, the first issue is how to fairly share the benefits resulting from a VMI contract between different supply chain elements. A recent study by Karsten and Basten (2013) carried out a cost allocation problem in spare parts inventory model where back ordering is allowed. They discussed four essential assumptions that are necessary for a stable cost allocation methodology. Another issue comes up with applying VMI model in system with constraints. Drawish and Odah (2010) developed a VMI supply chain model with a single vendor and multiple retailers, which considers storage capacity limitation. In their proposed VMI contract between multiple retailers and one vendor, the vendor considers a penalty cost for items exceeding bounds. The developed model can easily describe supply chains with capacity constraints by selecting high penalty cost (Darwish and Odah, 2010). Dealing with NP-hard models is also an issue for VMI models. Using a meta-heuristic method is considered to be one of the best ways to obtain a "good" answer in a reasonable amount of time for mathematical supply chain models. In fact, there are several studies that focus on solving more complex VMI models (Cárdenas-Barrón et al., 2012a; Cárdenas-Barrón et al., 2012b; Leuveano et al., 2012; Pasandideh et al., 2011; Sadeghi et al., 2013).

As previously mentioned, this study expands on the work done by Dong and Xu (2002) who studied the profit function of the supplier and the buyer while before and after signing a VMI contract. The authors concluded that utilizing VMI strategy can result in many advantages for the supply chain and may also eventually decrease supply chain inventory related costs (Dong and Xu, 2002). After adapting VMI policy, the buyer will have an increase in its profit function in both short and long term and will be able to share the profit with the supplier by increasing purchasing quality and price. Overall, after surveying the literature and reviewing Dong and Xu's paper, there is a potential for extending their work. According to their assumptions, the decision maker for the supply chain is the buyer and the supplier should act based on the buyer's decision. An important point however, is that Dong and Xu do not consider the effect of payment deferral and time value of money in their model while in practice, the buyer will usually try to put off the payment for as long as possible in order to take advantage of the time value of money. Another point is that the amount of payment and profit function will be different when the supply chain adopts a pull system in their manufacturing system. Following Dong and Xu's model, this study also considers the previous points in a supply chain in order develop a more practical model and to analyze the effects on the profitability of supply chain elements.

3. Account Payment Time Model

3.1. Supply Chain Description

This work takes inspiration from an actual case study. The manufacturer (buyer) here provides automotive components to one of the major automobile companies in the U.S. This component manufacturer gets their plastic molds from a plastics supplier. For the plastic molds supplier, this buyer is very important one for their business. The buyer recently initiated with the supplier a consignment inventory policy. Both the parties feel that the new policy would have mutual benefits. As a part of this initiative, the buyer is willing to reduce the days for payment, as an incentive. The buyer already has a Kanban system for their manufacturing and is now trying to integrate that with the consignment inventory system**.**

Model Description

The supply chain network modeled in this study consists of one supplier and one buyer (manufacturer). It is assumed that the system deals with a single product with deterministic demand. Also, there is no time lost across the supply chain. Essentially, the demand at the buyer and the demand at the supplier are is assumed to be stable and hence an inventory policy follows EOQ policy. The material sent by the supplier stays at the warehouse of the buyer, until withdrawn for further processing. The buyer follows a Kanban pull system for manufacturing. Both the buyer and the supplier have agreed to have consignment payment, in which the supplier gets paid only on material that is actually withdrawn by the buyer for use in the plant. Until that time, the supplier is responsible for inventory and the related holding costs. The time value of money is assumed to be known. Anything else here? In the following sections, various models under different scenarios are presented. First, a scenario with instantaneous payment is presented, for a consignment inventory system, with a chosen Lot Size, and for a consignment inventory Kanban pull system. Then the same scenarios are considered for a delayed payment system. Both the above cases are for infinite time horizon situations. A finite time horizon model is illustrated for one of the above cases. In the later part of the paper, an optimization model is presented to minimize the total costs of both the supplier and the manufacturer, without and with a Kanban system.

Model Notations and Assumptions

The following are the notations used in the model:

Q	Order quantity delivery by the supplier
q	Quantity pulled by the manufacturer in Kanban system
T	Replenishment cycle time interval for Q
t	Withdrawal time interval of q in Kanban system
n	Number of withdrawals in a cycle
r	Interest rate (time value of money)
C	Cost of one unit for the buyer or selling price of the supplier per unit (W)
S_1	Sum of discounted payments under Instantaneous payment withdrawal for Q
S_2	Sum of discounted payments under Instantaneous payment for q in Kanban system
$\acute{S}_1$	Sum of discounted payments under payment delay for Q
$\acute{S}_2$	Present Worth of the sum of payments under payment delay for q
t_1	Time delay of payment for Q
t_2	Time delay of payment for q in Kanban system
y	Annual demand for the product
π_M	Profit function of manufacturer before VMI
π_S	Profit function of supplier before VMI
$\acute{\pi}_M$	Profit function of manufacturer in Kanban system
$\acute{\pi}_S$	Profit function of supplier in Kanban system
w_c	Contract purchasing price determined by the manufacturer in Kanban system
C(y)	Production and distribution cost for y
P(y)	Sale price (inverse demand function) of the final product
S_S	Supplier's order set up cost/order
S_M	Manufacturer's order set up cost /order
h_S	Supplier's inventory carrying cost/unit
h_M	Manufacturer's inventory carrying cost/unit
πA^c	Profit function of supplier after VMI (fixed time period case A)
$\pi A^c{}_M$	Profit function of manufacturer after VMI (fixed time period case A)
πB^c	Profit function of supplier after VMI in Kanban system (fixed time period case B)
πB^c	Profit function of manufacturer after VMI in Kanban system (fixed time period case B)
$\acute{\acute{S}}_1$	Sum of discounted payment withdrawals for Q after VMI (fixed time period case A)
$\acute{\acute{S}}_2$	Sum of discounted payment withdrawals for q after VMI in Kanban system
y^c	Annual demand after VMI.
n	The number of Kanban pulls in one inventory replenishment cycle
PON	Point of neutrality

The following are the other assumptions for the system:

- n is an integer and n>=1.
- T (the replenishment cycle interval) is constant.

3.2. Infinitive Cycle Period Models

Here, the planning horizon is taken as infinity. The total discounted payments by the manufacturer, the Present Worth, is calculated.

3.2.1. Instantaneous Payment

The first type of models under infinite cycle periods are of instantaneous payments, where it is assumed that the manufacturer pays the supplier as soon as the supply is received or consigned.

3.2.1.1 Consignment Inventory - Traditional Model

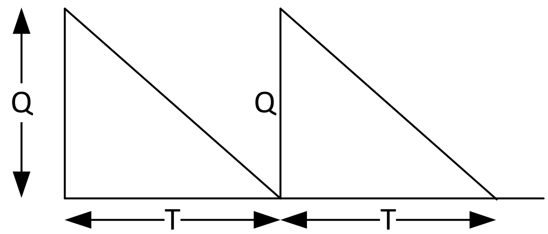

Figure 1: The Instantaneous Payment Supply Chain Inventory Model for Q (The Push system)

This is before adapting VMI-CI policy, and an EOQ model is used in order to minimize the inventory cost. This is a push model, where the supplier tries to sell the product to the manufacturer at every fixed duration of time and receives the payment instantaneously. Figure 1 represents the manufacturer's inventory behavior. As shown in Figure 1, in every cycle, a fixed quantity of Q is supplied to the manufacturer at every T and the payment withdrawal by the supplier is exactly after delivery withdrawals T, 2T, 3T …. The amount Q could be an arbitrarily chosen one, or an EOQ from the perspective of the manufacturer. The Present Worth of the sum of discounted payments is given by:

$$S_1 = QC + QCe^{-rT} + QCe^{-r2T} + QCe^{-r3T} + \cdots \tag{1}$$

$$S_1 = QC\,[1 + e^{-rT} + e^{-r2T} + e^{-r3T} + \cdots] \tag{2}$$

$$S_1 = QC\frac{1}{(1 - e^{-rT})} \tag{3}$$

In this case, the manufacturer does not enjoy any payment deferral. Also, following such a push system may lead to an increase in inventory holding cost for the manufacturer as well as increases in other related costs.

3.2.1.2 Consignment Inventory – Kanban System

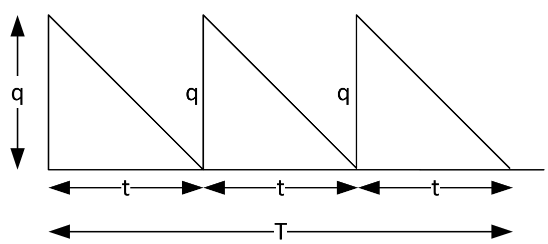

Figure 2: The Instantaneous Payment Supply Chain Inventory Model for q (for n=3)

In this system, the manufacturer withdraws Kanban quantities of q from the inventory consigned by the supplier. The supplier is paid on these quantities. Let t represent the interval time for each withdrawal, and in this case, withdrawal payment. Figure 2 shows the withdrawals in such a Kanban system, for a supplier's supply of Q and cycle time of T, when there are 3 withdrawals per cycle. The discounted present worth of instantaneous payment withdrawal for q, for an infinite horizon is calculated below:

$$S_2 = qC + qCe^{-rt} + qCe^{-r2t} + qCe^{-r3t} + \cdots \tag{4}$$

$$S_2 = qC\,[1 + e^{-rt} + e^{-r2t} + e^{-r3t} + \cdots] \tag{5}$$

$$S_2 = qC\frac{1}{(1 - e^{-rt})} \tag{6}$$

For these two models the consumption rate is equal. The manufacturer should always try to pay the smaller amount of money more frequently. From equations (3) and (6), it can be shown that $S_1 > S_2$ always. Therefore, the amount of saving for the manufacturer is:

$$\begin{aligned} S_1 - S_2 &= QC\frac{1}{(1 - e^{-rT})} - qC\frac{1}{(1 - e^{-rt})} \\ &= \frac{QC}{(1 - e^{-rT})} - \frac{\left(\frac{Q}{n}\right)C}{(1 - e^{-rt})} \\ &= QC\left[\frac{1}{(1 - e^{-rT})} - \frac{t}{n(1 - e^{-rt})}\right] \end{aligned} \tag{7}$$

It can be seen that S_2 decreases as n increases. A sample numerical experiment shows the behavior of this function S_2[1] for C = 10, in Figure 3.

Payment Delay

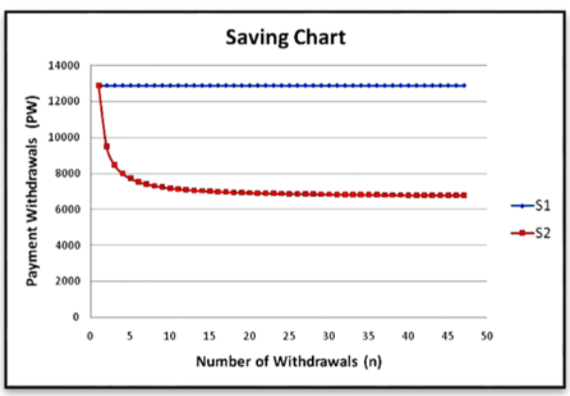

Figure 3: Comparison of S1 and S2 with increasing n

To have a successful supply chain system, a close relationship between supplier and manufacturer is essential. However, in many cases, the payment is not done instantaneously and there are usually some delays between delivering the goods by the supplier and when the payment is paid by the manufacturer. In practice, this is very common in that the manufacturer

[1] The calculation details are in table A.1 in APPENDIX A.

prefers to pay the payment with maximum possible delay. On the other hand, the supplier prefers to be paid as soon as possible. In some cases, the payment delay or the payment schedule is specified in the contracts.

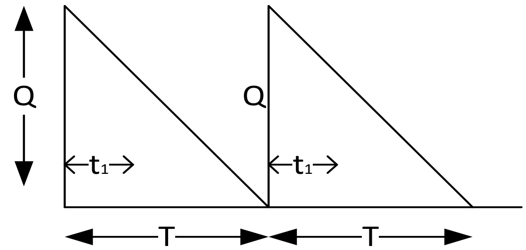

Figure 4: The Payment Delay-Supply Chain Inventory Model for Q

Here, the effects of payment deferral are studied on both push and pull systems. In the push model, where Q is economy order quantity and is received by the manufacturer every T time period, the delay t_1 effects the payment withdrawal calculation. Figure 4 represents the schematic inventory and payment model. Below the sum of payment withdrawals for Q (push model) with delay of t_1 after delivery interval T, 2T, 3T, … for infinite period of time is calculated:

$$S_1' = QCe^{-r\,t_1} + QCe^{-r(T+t_1)} + QCe^{-r(2T+t_1)} + \cdots \quad (8)$$

$$= QCe^{-rt_1}[1 + e^{-rT} + e^{-r2T} + \cdots] \quad (9)$$

$$S_1' = QCe^{-rt_1}\frac{1}{(1-e^{-rT})} = S_1e^{-rt_1} \quad (10)$$

From equation (10) it can be seen that the discounted sum of payments received by the supplier decreases as t_1 increases. Hence in many cases this period of payment delay is agreed upon by both the parties.

Next, the payment for q (pull system) with a certain amount of delay (t_2) after withdrawal intervals t, 2t, 3t, etc., for an infinite period of time is considered. In this situation, the Kanban and payment deferral policies are integrated. Therefore, the manufacturer pays based on market demand and also enjoys the deferral of the payment. Figure 5 depicts a schematic view of the model. If Q is kept constant, then when increasing the number of withdrawals per cycle, the quantity of q decreases[2]. The discounted sum will be given by

$$S_2' = qCe^{-rt_2}\frac{1}{(1-e^{-rt})} = S_2e^{-rt_2}$$

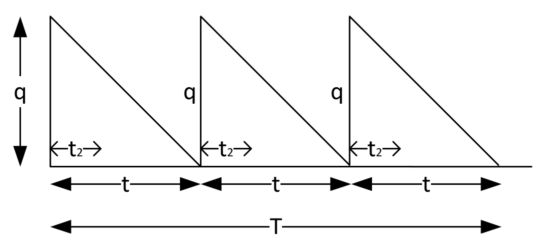

Figure 5: The Payment Delay-Supply Chain Inventory Model for q (n=3)

2 The calculation details are in table A.2 in APPENDIX A.

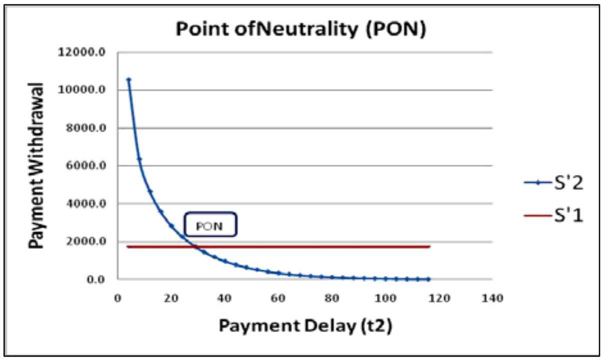

Figure 6: Point of Neutrality (PON) when $\acute{S}_1 = \acute{S}_2$, for a given t_1

The behavior of the sum of discounted payments for q for increasing payment delays is shown in Figure 6. It is the largest for $t_2 = 0$ and decreases exponentially as t_2 increases. In comparing to a similar sum for the case of payments for a supplier's lot of Q, this behavior can sometimes be classified in three parts. Initially, the amount of payment withdrawal for Q (push system) for a given payment delay t_1 may be lower than the amount of payment withdrawal for q (pull system). By increasing the amount of delay (t_2), the payment withdrawal for q decreases. There is a point in which these two models have an equal amount of payment withdrawal. The corresponding delay of this point is called Point of Neutrality (PON). The Point of Neutrality is where both of the financial forces are equal. Hence, if the manufacturer has to make a payment withdrawal low, then there must be an increase of payment delay time above the PON value. Similarly, if the manufacturer works in the range of time delay before the PON, then it will help the supplier with its profit functions. But it is always recommended for both the manufacturer and supplier to be at the PON so that both entities can remain at the same profit level as before. Also, the supplier can equally benefit by still working with the manufacturer. Knowing the assumptions: $S_1' = S_2'$ and $\frac{Q}{q} = \frac{T}{t} = n \rightarrow T = nt$, the value of point of neutrality is calculated[3]:

$$PON: t^*_2 = \ln\left[\frac{k}{n}\right]/r + t_{1,} \text{ where,} \tag{11}$$

$$k = \left[\frac{(1-e^{-rnt})}{(1-e^{-rt})}\right] \tag{12}$$

Moreover, according to the last equation, two useful statements can be derived[4]:

Lemma I: The time delay of payment for t_2 (q) is less than time delay of payment for t_1 (Q).
Since $S_1 > S_2$, for the same demand rate and interest rate, comparison of the payment delays will yield that t_1 will be greater than t_2, if the discounted total payments in both the cases of lots of Q and Kanban lots of q are equal. That is, if $S_1' = S_2'$, then $t_2 < t_1$
Lemma II: If the payment delay is kept same, then switching to pull system reduces the payment withdrawal.
In order to apply Kanban system, it is essential for supply-chain leaders to be able to understand and analyze the behavior of the payment withdrawal functions in different situations. As an illustration, the behavior of payment withdrawal functions is compared for two different situations. Figure 7 represents the behavior of payment withdrawals for both push and pull models. When delay times for both systems are equal, the withdrawal quantity (q) amount is constant and the amount of order (Q) increases[5]. Hence, the manufacturer can take advantage by increasing the number of withdrawals per cycle. Figure 8 is also showing the behavior of payment withdrawals for both models in which the withdrawal quantity (q) and t_1 are held constant and Q and t_2 are increasing[6].

[3] See APPENDIX B. for proof.
[4] See APPENDIX C. for proof.
[5] For details, see APPENDIX A. Table A.3
[6] For details, see Table A.4 in APPENDIX A.

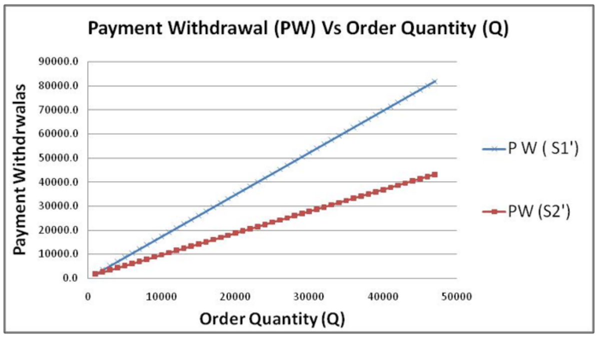

Figure 7: Comparison of $\acute{S}_1$ *and* $\acute{S}_2$ with increasing (Q) when $t_1 = t_2$.

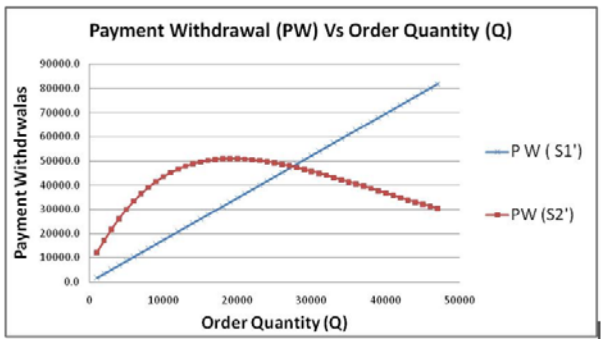

Figure 8: Comparison of $\acute{S}_1$ *and* $\acute{S}_2$ with increasing (Q) and t_2 , while t_1 is constant.

3.3. Fixed Cycle Period Time (One Year)

This section studies the effect of payment delay in push and Kanban systems in short term. The basic elements are the same as the infinite cycle model. Like the previous section, two models are surveyed. The first model considers a one-year cycle period for the push system ($mT = 1$). The amount of payment withdrawal for Q in this model (S_1'') is calculated as:

$$S_1'' = QCe^{-rt_1} + QCe^{-r(T+t_1)} + QCe^{-r(2T+t_1)} + \ldots\ldots QCe^{-r((m-1)T+t_1)} \quad (13)$$

$$S_1'' = QC[e^{-rt_1} + e^{-r(T+t_1)} + e^{-r(2T+t_1)} + \ldots\ldots e^{-r((m-1)T+t_1)}] \quad (14)$$

$$S_1'' = QCe^{-rt_1} + QCe^{-r(T+t_1)} + QCe^{-r(2T+t_1)} + \ldots\ldots QCe^{-r((m-1)T+t_1)} \quad (13)$$

$$S_1'' = QC[e^{-rt_1} + e^{-r(T+t_1)} + e^{-r(2T+t_1)} + \ldots\ldots e^{-r((m-1)T+t_1)}] \quad (14)$$

$$S_1'' = \frac{QCe^{-rt_1}\left(1 - e^{-mrT}\right)}{1 - e^{-rT}} \quad (15)$$

Another model (pull system), where the delay in payment withdrawal is represented by t_2 and the amount of delivery is q after intervals $t, 2t, 3t, \ldots, mnt$; the payment withdrawal (S_2'') is obtained by the following equations ($mnt = 1$):

$$S_2'' = qCe^{-rt_2} + qCe^{-r(t+t_2)} + qCe^{-r(2t+t_2)} + \cdots \ldots \ldots \ldots qCe^{-r((mn-1)t+t_2)} \quad (16)$$

$$S_2'' = qC[e^{-rt_2} + e^{-r(t+t_2)} + e^{-r(2t+t_2)} + \cdots \ldots \ldots \ldots e^{-r((mn-1)t+t_2)}] \quad (17)$$

$$S_2'' = \frac{qCe^{-rt_2}(1 - e^{-mnrt})}{1 - e^{-rt}} \tag{18}$$

When these two models are compared, as in the previous section for infinite model, delays are kept equal and constant as well as the withdrawal quantity (q). Using these assumptions and by increasing the number of withdrawals per cycle (n), Figure 9 is generated[7]. As shown, while $t_1 = t_2$, it can be proven (same as previous section) that $S_1'' > S_2''$. PON is also calculated in the same way and has the same results as the previous section. Thus, for both infinite and short-term models, there is no change in the value of delay for which the payment withdrawals are found to be equal. Also, the same result for Lemma II can be obtained. Therefore, for the manufacturer, it is always beneficial to adopt Kanban system while the payment delays are equal.

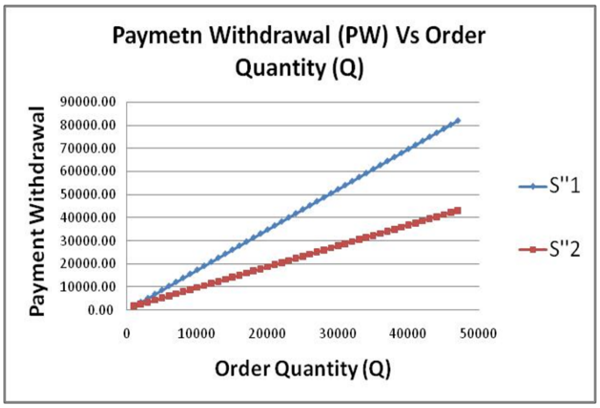

Figure 9: Comparison of $\acute{S}_1$ *and* $\acute{S}_2$ with increasing (Q) when $t_1 = t_2$.

4. Optimization Model

The main reason for adopting VMI-CI is to systematically increase the profit functions of the total supply chain. Dong and Xu suggested a methodology to increase the amount of profit function of both players (supplier and buyer). As explained previously in VMI-CI policy, the inventory of the manufacturer is owned and controlled by the supplier. Further expanding their work, this section analyzes the amount of yearly profitability by incorporating payment deferral and time value of money in the supply chain VMI model for both push and Kanban systems. Notations and assumptions are the same as the previous section.

4.1. Model Structure

4.1.1. Before VMI

This section briefly describes the profit function of the players before applying VMI policy when the manufacturer and the supplier are owners of their inventory. As a result, for the manufacturer, the best case is that they follow EOQ inventory rules in which the economy order quantity is obtained as follows:

$$Q_M = EOQ = \left(\frac{2 * \text{Annual Demand} * \text{Manufacturer's set up Cost}}{\text{Manufacturer's Inventory Holding Cost/Unit}}\right)^{\frac{1}{2}}$$

$$Q_M = EOQ = \left(\frac{2S_M y}{h_M}\right)^{\frac{1}{2}} \tag{19}$$

[7] Table A.5 contains the calculation details, see the APPENDIX A.

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To achieve the profit function of the manufacturer, the total amount of annual costs (in the case of inventory) should be subtracted from the annual purchasing income. Total supplier's cost is obtained by the following equation:

$$\begin{pmatrix}\text{Annual Production}\\ \text{and distribution Cost}\end{pmatrix} + \begin{pmatrix}\text{Supplier's Order set up}\\ \text{Cost /order}\end{pmatrix} * \frac{\text{Annual Demand}}{\text{Q}_\text{M}} + \begin{pmatrix}\text{Supplier's Inventory}\\ \text{Holding Cost /unit}\end{pmatrix} * \frac{\text{Q}_\text{M}}{2}$$

$$\text{Total Supplier's Cost} = c(y) + \left(\frac{S_S\, y}{Q_M} + \frac{h_S Q_M}{2}\right) \tag{20}$$

As stated before, in this system the supplier should follow the economic order quantity of the manufacturer. The annual profit function of the supplier:

$$\begin{pmatrix}\text{TOTAL SUPPLIER'S}\\ \text{PROFIT FUNCTION}\end{pmatrix} = \begin{pmatrix}\text{Annual Contract}\\ \text{Purchasing Price}\end{pmatrix} - \begin{pmatrix}\text{Annual Production}\\ \text{and distribution Cost}\end{pmatrix} - \begin{pmatrix}\text{Supplier's}\\ \text{Inv Holding and Order set up Cost}\end{pmatrix}$$

$$\pi_S = w\,y - c(y) - \left(\frac{S_S\, y}{Q_M} + \frac{h_S Q_M}{2}\right) \tag{21}$$

With replacing Q_M, with equation (19), the profit function of the supplier would be as the below:

$$\pi_S = w\,y - c(y) - \left(\frac{h_M S_M y}{2}\right)^{\frac{1}{2}} \left(\frac{S_S}{S_M} + \frac{h_S}{h_M}\right) \tag{22}$$

After identifying the profit function of the supplier, the same process takes place for calculating profit function of the company.

$$\text{Total Buyer's Cost} = \begin{pmatrix}\text{Contract Purchasing}\\ \frac{\text{Price}}{\text{unit}}\end{pmatrix} * \text{Annual Demand} +$$

$$\begin{pmatrix}\text{Manufacturer's Order set up}\\ \text{Cost /order}\end{pmatrix} * \frac{\text{Annual Demand}}{\text{Q}_\text{M}} + \begin{pmatrix}\text{Manufacturer's Inv Holding}\\ \text{Cost /unit}\end{pmatrix} * \frac{\text{Q}_\text{M}}{2}$$

$$\text{Total Buyer's Cost} = w\,y + \left(\frac{S_M\, y}{Q_M} + \frac{h_M Q_M}{2}\right) \tag{23}$$

$$\begin{pmatrix}\text{TOTAL MANUFACTURER'S}\\ \text{PROFIT FUNCTION}\end{pmatrix} = (\text{Annual Sale Price}) - \begin{pmatrix}\text{Annual Contract}\\ \text{Purchasing Cost}\end{pmatrix} - \begin{pmatrix}\text{Manufacturer's Inv Holding}\\ \text{and Order set up Cost}\end{pmatrix}$$

$$\pi_M = P(y)y - w\,\mathbf{y} - (\mathbf{2}h_M S_M y)^{\frac{1}{2}} \tag{24}$$

4.1.2. After VMI-CI (Dong and Xu Model)

As previously discussed, in Consignment Inventory management, the supplier not only decides when to supply the product but is also responsible for determining the amount of order quantity for the manufacturer.

By adopting VMI-CI in the supply chain, the manufacturer is no longer responsible for controlling its own inventory. Instead, the supplier is the owner and controller of the entire inventory (in its own store and/or at the manufacturer store). Based on this modification, the profit functions of the supplier and the manufacturer are re-calculated:

$$Q_S = EOQ = \left[\left(\frac{2(S_S + S_M)y}{(h_S + h_M)}\right)\right]^{\frac{1}{2}} \tag{25}$$

$$\pi^c{}_s = w_c y - c(y) - \left[\frac{y(S_S + S_M)}{Q_S} + \frac{Q_S(h_S + h_M)}{2}\right] \tag{26}$$

$$\pi^c{}_s = w_c y - c(y) - [2(S_S + S_M)(h_S + h_M)y\ ]^{1/2} \tag{27}$$

And the manufacturer’s profit function with eliminating inventory costs is:

$$\pi^c{}_M = P(y)y - w_c y \tag{28}$$

Regarding the above formula for traditional and VMI models, it is understood that after VMI, the profit function of the supplier decreased, and the profit function of the manufacturer increased.

4.1.3. Payment delay– withdrawal for Q

The new VMI-CI models, which consider the effect of time deferral in payment withdrawal are discussed in this section. For the push model, it is proposed that the manufacturer accepts to pay the entire amount of Q (economy order quantity) with a time delay of t_1. The amount of payment (S_1'') for this model is as follows:

$$S_1'' = \frac{Q\ Ce^{-rt_1}\left(1 - e^{-mrT}\right)}{1 - e^{-rT}} \tag{29}$$

Subsequently, the profit function of the supplier and manufacturer is obtained:

$$\pi A^c{}_s = S_1''\ - c(y) - [2(S_S + S_M)((h_S + h_M)y\ ]^{1/2} \tag{30}$$

$$\pi A^c{}_M = P(y)y - S_1'' \tag{31}$$

Comparing the profit functions of VMI-CI models with and without payment deferral, the equation (30) shows that the profit function of the supplier decreased after including payment delay in the contract and equation (31) represents that the manufacturer’s profit increases with considering payment deferral. Equation (32) and (32) comparing the profit functions of supplier and manufacturers in different inventory models:

$$\pi A^c{}_M > \pi^c{}_M > \pi_M \tag{32}$$

$$\pi_S > \pi^c{}_s > \pi A^c{}_s \tag{33}$$

4.1.4. After VMI with payment delay – withdrawal for q

As mentioned earlier, the manufacturer will adopt a VMI-CI policy in order to be released from inventory costs and instead focus on other issues such as market and quality. In a VMI-CI contract for a Kanban system, the manufacturer only pays the supplier for the amount of quantity, which is pulled by the market (q) after every t (withdrawal time interval) time period. The payment withdrawal with considering t_2 as the allowed delay is:

$$S_2'' = \frac{qCe^{-rt_2}(1-e^{-mnrt})}{1-e^{-rt}} \tag{34}$$

With respect to this equation, the profit functions of supplier and manufacturer are calculated as:

$$\pi B^c{}_s = S_2'' - c(y) - \left[2(S_S+S_M)(h_S+h_M)y\ \right]^{\frac{1}{2}} \tag{35}$$

$$\pi B^c{}_M = P(y)y - S_2'' \tag{36}$$

By comparing the profit of these two profit functions with the previous ones, we have the following results:

$$\pi B^c{}_M > \pi A^c{}_M > \pi^c{}_M \tag{37}$$

$$\pi^c{}_s > \pi A^c{}_s > \pi B^c{}_s \tag{38}$$

By applying VMI-CI policy with payment delay in a Kanban system, the profit function of the manufacture increases the most. However, the benefit is not for only manufacturer. After the manufacturer's profits increase, the manufacturer may try to reduce the final market price of the product. As a result of this price reduction, the market demand (y) may increase. With a need for more materials, the manufacturer will order higher quantities from the supplier thus leading to an increase in the supplier's demand rate. This new demand will therefore help the supplier increase its profit function by making use of the so-called 'low margin high demand' policy.

4.2. Maximizing the Profit Function Of Supplier At New Annual Demand y^c

From the supplier's point of view, they may want to maximize their profit function by making a contract with a manufacturer who has a certain amount of annual market demand. The amount of desirable annual demand is obtained when the first derivative of the profit function is equal to zero. Also, in order to make sure that the profit function of the supplier concave down; the second order derivative is calculated. APPENDIX D. shows the calculation of the first and second order derivatives of the supplier's profit function.

$$y^c = \frac{[(S_S+S_M)(h_S+h_M)]}{2[(S_2'')' - c'(y)]^2} \tag{39}$$

Hence, at this demand point, the supplier can have the most amount of profit function. Thus, the maximized profit functions for supplier are:

$$\pi B^c{}_s = S_2'' - c(y^c) - [2(S_S+S_M)((h_S+h_M)y^c\,]^{1/2} \tag{40}$$

$$\pi B^c{}_M = P(\,y^c)\,y^c - S_2''\,y^c \tag{41}$$

5. Summary and Conclusions

This study first described Vendor Managed Inventory models and then illustrated how they are becoming important in current supply chain strategies. The aim of this study is to develop a two-echelon supply chain model with a single supplier and a single manufacturer in which both push and Kanban systems are considered separately. Essentially, the manufacturer usually tends to have a delay in payment withdrawal while the supplier always wants the payment as soon as possible. By incorporating this fact into the model, the influence of payment deferral and time value of money for Push and Kanban manufacturing systems in both infinite and short term (one year) time horizons are analyzed. Although signing a VMI-CI contract has considerable benefits for a supply chain in the long term, the amount of supplier's profitability is reduced. Therefore, a methodology proposed to help supplier to maximize its profit function. In reality, however, the effect of payment deferral is always a big issue in supply chain contracts. Studying and analyzing this issue is one of the future directions in this area for other supply chain models. This study has some limitations. For instance, the demand can be considered as deterministic, however, this condition cannot be always true in the real world. Using a stochastic demand variable in assumptions can help develop a more realistic inventory model.

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Appendix A

Table A.1.: Payment Withdrawals for Instantaneous Payment, Infinite Cycle models at Same Order Quantity.

Order Quantity	Quantity Pulled by the Buyer	Replenishment Cycle Time for Q	Withdrawal Time Interval of q	Number of Withdrawals in a Cycle	Interest Rate	Instantaneous Payment Withdrawal for Q	Instantaneous Payment Withdrawal for q
Q	q	T	t	n	r %	S1	S2
1000	1000.00	30	30.00	1	0.05	12872.16917	12872.16917
1000	500.00	30	15.00	2	0.05	12872.16917	9476.275672
1000	333.33	30	10.00	3	0.05	12872.16917	8471.646942
1000	250.00	30	7.50	4	0.05	12872.16917	7994.609172
1000	200.00	30	6.00	5	0.05	12872.16917	7716.591827
1000	166.67	30	5.00	6	0.05	12872.16917	7534.686107
1000	142.86	30	4.29	7	0.05	12872.16917	7406.443083
1000	125.00	30	3.75	8	0.05	12872.16917	7311.186482
1000	111.11	30	3.33	9	0.05	12872.16917	7237.647181
1000	100.00	30	3.00	10	0.05	12872.16917	7179.161982
1000	90.91	30	2.73	11	0.05	12872.16917	7131.5395
1000	83.33	30	2.50	12	0.05	12872.16917	7092.011629

Table A.2.: Payment Withdrawals for Instantaneous Payment, Infinite Cycle models at Same Order Quantity.

Order Quantity	Quantity Pulled by the Buyer	Replenishment Cycle Time for Q	Withdrawal Time Interval of q	Number of Withdrawals in a Cycle	Interest Rate	Instantaneous Payment Withdrawal for Q	Instantaneous Payment Withdrawal for q
Q	q	T	t	n	r %	S1	S2
1000	1000.00	30	30.00	1	0.05	12872.16917	12872.16917
2000	1000.00	30	15.00	2	0.05	25744.33834	18952.55134
3000	1000.00	30	10.00	3	0.05	38616.5075	25414.94083
4000	1000.00	30	7.50	4	0.05	51488.67667	31978.43669
5000	1000.00	30	6.00	5	0.05	64360.84584	38582.95914
6000	1000.00	30	5.00	6	0.05	77233.01501	45208.11664
7000	1000.00	30	4.29	7	0.05	90105.18418	51845.10158
8000	1000.00	30	3.75	8	0.05	102977.3533	58489.49186
9000	1000.00	30	3.33	9	0.05	115849.5225	65138.82463
10000	1000.00	30	3.00	10	0.05	128721.6917	71791.61982
11000	1000.00	30	2.73	11	0.05	141593.8608	78446.93449
12000	1000.00	30	2.50	12	0.05	154466.03	85104.13955

Table A.3.: Payment Withdrawals S_2' and S_1' with increasing n and t_2 at constant Order Quantity

Order Quantity	Quantity Pulled by the Buyer	Replenishment Cycle Time for Q	Withdrawal Time Interval of q	Number of Withdrawals in a Cycle	Interest Rate	Time delay of Payment for Q	Payment Withdrawals for Q with Delay	Payment Withdrawals for q with Delay	Time delay of Payment for q
Q	q	T	t	n	r %	t1	S'1	S'2	t2
1000	1000.00	30	30.00	1	0.05	40	1742.1	12244.4	1
1000	500.00	30	15.00	2	0.05	40	1742.1	8574.5	2
1000	333.33	30	10.00	3	0.05	40	1742.1	7291.6	3
1000	250.00	30	7.50	4	0.05	40	1742.1	6545.4	4
1000	200.00	30	6.00	5	0.05	40	1742.1	6009.7	5
1000	166.67	30	5.00	6	0.05	40	1742.1	5581.8	6
1000	142.86	30	4.29	7	0.05	40	1742.1	5219.2	7
1000	125.00	30	3.75	8	0.05	40	1742.1	4900.8	8
1000	111.11	30	3.33	9	0.05	40	1742.1	4614.9	9
1000	100.00	30	3.00	10	0.05	40	1742.1	4354.4	10
1000	90.91	30	2.73	11	0.05	40	1742.1	4114.5	11
1000	83.33	30	2.50	12	0.05	40	1742.1	3892.2	12

Table A.4.: Payment withdrawal S_2' and S_1' with constant order quantity and t_1, and at t_2 increasing

Order Quantity	Quantity Pulled by the Buyer	Replenishment Cycle Time for Q	Withdrawal Time Interval of q	Number of Withdrawals in a Cycle	Interest Rate	Time delay of Payment for Q	Payment Withdrawals for Q with Delay	Payment Withdrawals for q with Delay	Time delay of Payment for q
Q	q	T	t	n	r %	t1	S'1	S'2	t2
1000	1000.00	30	30.00	1	0.05	40	1742.1	12244.4	1
2000	1000.00	30	15.00	2	0.05	40	3484.1	17149.0	2
3000	1000.00	30	10.00	3	0.05	40	5226.2	21874.8	3
4000	1000.00	30	7.50	4	0.05	40	6968.2	26181.7	4
5000	1000.00	30	6.00	5	0.05	40	8710.3	30048.4	5
6000	1000.00	30	5.00	6	0.05	40	10452.4	33491.0	6
7000	1000.00	30	4.29	7	0.05	40	12194.4	36534.6	7
8000	1000.00	30	3.75	8	0.05	40	13936.5	39206.7	8
9000	1000.00	30	3.33	9	0.05	40	15678.5	41534.3	9
10000	1000.00	30	3.00	10	0.05	40	17420.6	43543.8	10
11000	1000.00	30	2.73	11	0.05	40	19162.6	45259.9	11
12000	1000.00	30	2.50	12	0.05	40	20904.7	46706.1	12

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Table A.5.: Payment withdrawal S_2' and S_1' with increasing order quantity and $t_1=t_2$.

Economic Order Quantity	Quantity pulled by Buyer	No of Withdrawals in a Cycle	Cycle Time for (q)	PW for (Q) with Delay (Cycle Period of 1Year) $S_1'' = \frac{QCe^{-rt_1}(1-e^{-mrT})}{1-e^{-rT}}$	PW for (q) with Delay (Cycle Period of 1 Year) $S_2'' = \frac{qCe^{-rt2}(1-e^{-mnrt})}{1-e^{-rt}}$	Rate of Interest	Time Delay t1 for Q	Time Delay t1 for q	Cycle Time
Q	q=Q/n	n	t= T/n	$S_1''(t_1)$	$S_2''(t_2)$	r	t1	t2	T
1000	1000	1	30.00	10078.80	10078.80	0.05	40	40	30
2000	1000	2	15.00	18360.99	13517.05	0.05	40	40	30
3000	1000	3	10.00	26347.59	17340.31	0.05	40	40	30
4000	1000	4	7.50	34181.18	21229.15	0.05	40	40	30
5000	1000	5	6.00	41916.99	25128.34	0.05	40	40	30
6000	1000	6	5.00	49583.34	29023.46	0.05	40	40	30
7000	1000	7	4.29	57197.07	32910.29	0.05	40	40	30
8000	1000	8	3.75	64769.12	36787.83	0.05	40	40	30
9000	1000	9	3.33	74855.35	42088.99	0.05	40	40	30
10000	1000	10	3.00	85379.48	47618.48	0.05	40	40	30
11000	1000	11	2.73	96340.46	53375.29	0.05	40	40	30
12000	1000	12	2.50	107737.32	59358.63	0.05	40	40	30

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Appendix B

Calculating the Point of Neutrality (PON):

$$S_1{}' = S_2{}' \rightarrow \frac{QCe^{-r\,t_1}}{(1-e^{-rT})} = \frac{qCe^{-r\,t_2}}{(1-e^{-rt})} \rightarrow \frac{Qe^{-rt_1}}{(1-e^{-rT})} = \frac{Qe^{-r\,t_2}}{n(1-e^{-rt})} \rightarrow \frac{n\,e^{-r\,t_1}}{(1-e^{-rnt})} = \frac{e^{-r\,t_2}}{(1-e^{-rt})}$$

$$\rightarrow \frac{ne^{-r\,t_1}e^{r\,t_2}}{(1-e^{-rnt})} = \frac{1}{(1-e^{-rt})} \rightarrow ne^{r(t_2-t_1)} = \frac{(1-e^{-rnt})}{(1-e^{-rt})} \rightarrow \ln[ne^{r(t_2-t_1)}] = \ln[\frac{(1-e^{-rnt})}{(1-e^{-rt})}]$$

$$\rightarrow \ln[n] + \ln[e^{r(t_2-t_1)}] = \ln[\frac{(1-e^{-rnt})}{(1-e^{-rt})}] \rightarrow \ln[n] + r(t_2-t_1)\,] = \ln[\frac{(1-e^{-rnt})}{(1-e^{-rt})}]$$

$$\rightarrow rt_2 - r\,t_1 + \ln[n] = \ln\left[\frac{(1-e^{-rnt})}{(1-e^{-rt})}\right]$$

$$suppose: \quad \left[\frac{(1-e^{-rnt})}{(1-e^{-rt})}\right] = k$$

$$r\,t_2 = \ln\left[\frac{(1-e^{-rnt})}{(1-e^{-rt})}\right] - \ln[n] + r\,t_1 \rightarrow r\,t_2 = \ln[k] - \ln[n] + r\,t_1 \rightarrow rt_2 = \ln[k/n] + r\,t_1$$

$$\rightarrow t_2 = \ln\frac{\left[\frac{k}{n}\right]}{r} + t_1 \rightarrow \text{PON} \quad \text{or} \quad t^*{}_2 = \ln\left[\frac{k}{n}\right]/r + t_1$$

Appendix C

Proof of Lemma I:

$$\text{We Know k } = \left[\frac{(1-e^{-rnt})}{(1-e^{-rt})}\right] \quad \text{Eq. (C.1)}$$

$$\ln[k] = \ln[(1-e^{-rnt})] - \ln[(1-e^{-rt})] \quad \text{Eq. (C.2)}$$

$$\ln[k] - \ln[n] = \ln[(1-e^{-rnt})] - \ln[(1-e^{-rt})] - \ln[n]] \quad \text{Eq. (C.3)}$$

$$\ln\left[\frac{k}{n}\right] = \ln[(1-e^{-rnt})] - \ln[(1-e^{-rt})] - \ln[n] = \ln[(1-e^{-rnt})] - \ln[(1-e^{-rt})n] \quad \text{Eq. (C.4)}$$

$$\ln\left[\frac{k}{n}\right] = \ln\left[\left(1-e^{-\frac{rnT}{n}}\right)\right] - \ln\left[\left(1-e^{-\frac{rT}{n}}\right)n\right] = \ln\left[\left(1-e^{-rT}\right)\right] - \ln\left[\left(1-e^{-\frac{rT}{n}}\right)n\right] \quad \text{Eq. (C.5)}$$

$$\left(1-e^{-\frac{rT}{n}}\right)n > 1 \; or \; \ln\left[\left(1-e^{-\frac{rT}{n}}\right)n\right] = +\text{ve} \quad \text{Eq. (C.6)}$$

Hence,

$$\ln\left[\frac{k}{n}\right] = -\text{ve or } \ln\frac{\left[\frac{k}{n}\right]}{r} < 0$$

$we\ know\ , t_2 = \ln\frac{\left[\frac{k}{n}\right]}{r} + t_1$ Hence , $t_2 < t_1$

Note: When n=1 $\ln\left[\frac{k}{n}\right] = 0,\ t_2 = t_1$

Proof of Lemma II:

Let $S_1'(\tau)$ and $S_2'(\tau)$ be the payment withdrawals and function of time (τ). Here τ is payment delay, t_1 and t_2 for $S_1'(\tau)$ and $S_2'(\tau)$ respectively. Let t_2 be the time at which both $S_1'(\tau)$ and $S_2'(\tau)$ are equal for a given timet1 i.e. at PON, we have $S_1'(t_1) = S_2'(t_2)$. Now, since we have known:

$S_2'(\tau) = qCe^{-rt2}\frac{1}{(1-e^{-rt})}$ And also we know $t_1 > t_2$, hence $S_2'(t_1) < S_2'(t_2)$ but $S_1'(t_1) = S_2'(t_2)$. Therefore $S_2'(t_1) < S_1'(t_1)$.

Appendix D

Calculating first and second order derivatives of supplier's profit function:

$$\frac{\partial \pi B^{c}{}_{s}}{\partial y} = 0 \qquad \text{Eq. (D.1)}$$

$$\frac{\partial \pi B^{c}{}_{s}}{\partial y} = \frac{\partial}{\partial y}[S_2{}'' - c(y) - [2(S_S + S_M)(h_S + h_M)y]^{1/2}] = 0 \qquad \text{Eq. (D.2)}$$

$$\frac{\partial \pi B^{c}{}_{s}}{\partial y} = (S_2{}'')' - c'(y) - \left[\frac{(S_S + S_M)(h_S + h_M)}{2y}\right]^{\frac{1}{2}} = 0 \qquad \text{Eq. (D.3)}$$

$$(S_2{}'')' - c'(y) = \left[\frac{(S_S + S_M)(h_S + h_M)}{2y}\right]^{\frac{1}{2}} \qquad \text{Eq. (D.4)}$$

$$(2y)^{1/2} = \frac{[(S_S + S_M)(h_S + h_M)]^{\frac{1}{2}}}{[(S_2{}'')' - c'(y)]} \qquad \text{Eq. (D.5)}$$

$$y^{c} = \frac{[(S_S + S_M)(h_S + h_M)]}{2[(S_2{}'')' - c'(y)]^2} \qquad \text{Eq. (D.6)}$$

Second order derivative:

$$\frac{\partial 2\pi B^{c}{}_{s}}{\partial y2} = \frac{\partial 2}{\partial y2}(S_2{}'')' - c'(y) - \left[\frac{(S_S + S_M)(h_S + h_M)}{2y}\right]^{\frac{1}{2}} \qquad \text{Eq. (D.7)}$$

$$\frac{\partial 2\pi B^{c}{}_{s}}{\partial y2} = 0 - c''(y) - \left[\frac{(S_S + S_M)(h_S + h_M)}{8y^3}\right]^{\frac{1}{2}} \qquad \text{Eq. (D.8)}$$

$$\frac{\partial 2\pi B^{c}{}_{s}}{\partial y2} < 0 \qquad \text{Eq. (D.9)}$$