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Energy-Efficient Location-Routing Problem With Time Windows with Dynamic Demand

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Abstract: Sustainability and energy savings have attracted considerable attention in recent years. However, in the traditional location-routing problem (LRP), the objective function has yet to minimize the distance traveled regardless of the amount of energy consumed. Although, distance is one of the major factors determining the energy consumption of a distribution network, it is not the only factor. Therefore, this paper explains the development of a novel formulation of the LRP that considers energy minimization, which is called the energy-efficient location-routing problem (EELRP). The energy consumed by a vehicle to travel between two nodes in a system depends on many forces. Among those, rolling resistance (RR) and aerodynamic drag are considered in this paper to be the major contributing forces. The presented mixed-integer non-linear program (MINLP) finds the best location-allocation routing plan with the objective function of minimizing total costs, including energy, emissions, and depot establishment. The proposed model can also handle the vehicle-selection problem with respect to a vehicles' capacity, source of energy, and aerodynamic characteristics. The formulation proposed can also solve the problems with hard and soft time window constraints. Also, the model is enhanced to handle the EELRP with dynamic customers' demands. Some examples are presented to illustrate the formulations presented in this paper.

1. Introduction

The problem addressed in this paper is the energy-efficient location-routing problem (EELRP). The traditional location-routing problem (LRP) is a combination of the location-allocation problem (LAP) and the vehicle-routing problem (VRP) or energy-efficient vehicle-routing problem (EEVRP). The LAP involves finding a set of distribution centers (DCs) among potential DCs and assigning customers to them. The VRP originated from the traveling salesman problem (TPS) and is defined as the problem of finding a set of routes originating from depots to serve customers. Each customer must be visited only once, and all vehicles should return back to the same depot from which they departed. The demand of customers should not exceed the vehicle capacity. The objective of the VRP is usually minimization of transportation cost based on distance traveled. It has been vastly investigated by researchers (Laporte, 1992; Laporte, et al. 1988; Laporte et al., 2000; Eksioglu et al., 2009).

The increase in fuel costs and other problems caused by fuel consumption, such as emissions, carbon footprint, global warming, etc., increases the importance of an energy-efficient and emission-efficient LRP. Although the number of research and methods developed for the LRP is significant, research is limited relative to the LRP with the objective function of minimizing emissions and energy consumption. Kara et al. (2007) present an energy minimization objective function for the VRP in which all vehicles are identical and have the same capacity, and the problem is separately investigated for symmetric and asymmetric distance matrix. Also, the formulation solves for both the collection problem and the delivery problem. Gusikhin et al. (2010) present a heuristic for solving a mixed-fleet VRP for minimizing fuel consumption and environmental emissions. This heuristic is a variation of the multi-label shortest-path problem, and they did not consider vehicle weight as one of the contributing factors for fuel consumption and emissions. Artmeier et al. (2010) present a generic shortest-path algorithm for battery-powered vehicles in which constraints such as limited cruising range, long recharge times, and energy recovery ability are considered. In that method, graph theory is applied for formulating and solving the problem.

The increase in energy consumption also results in increased emissions. This is another important aspect of energy consumption that should not be ignored. The amount of emissions differs among vehicles based on their sources of energy. Some sources of energy, such as petroleum-based fuel, are the major source of emissions, while other sources, such as electric power, have very few emissions. Hirashima et al. (2002) present a method for calculating the amount of emissions based on a road gradient factor. They compared the results when the objective is distance minimization and when it is emissions minimization. They show that their method provides a better result in terms of emissions.

An EELRP in which each customer`s demand must be satisfied within a time interval is called an EELRP with a time window. The traditional location-routing problem with a time window (LRPTW) is a class of LRP in which each customer must be visited within a specific time window. It has been an interesting subject of research in the last three decades. Many heuristic and meta-heuristic methods have been developed for solving this problem under different conditions and constraints. Desrochers et al. (1987) provide an early survey on the solution methods of the VRP. The solution methods are classified based on mathematical formulations and models. Braysy and Gendreau (2005) present a comprehensive survey on the heuristic and meta-heuristic algorithms developed for solving the vehicle-routing problem with time window (VRPTW). Desrochers et al. (1987) provide an early survey of the VRP solution methods, which are developed based on existing mathematical formulations and models. Kallehauge (2008) reviews the formulation and exact algorithm of the VRPTW, and categorized the formulation and exact methods developed for VRPTW into four major categories: arc formulation, arc-node formulation, spanning tree formulation, and path formulation. In the arc formulation of the VRPTW, each arc of an underlying directed graph is associated with a binary variable. Dantzig et al. (1954), Kallehauge et al. (2007), and Mak and Ernst (2007) present an arc formulation of the VRPTW. In the arc-node formulation of the problem, binary variables are also associated with nodes of the directed graph. This method of formulating the VRPTW can be found in the work of Miller et al. (1960) and Bard et al. (2002). The spanning tree formulation, in brief, is “a method to find lower bounds for the VRPTW, with the help of time- and capacity-constrained shortest spanning trees and Lagrangian relaxation or Dantzig-Wolfe decomposition” (Held and Karp, 1970 and 1971). Several researchers have focused on solutions to the path formulation approach in the last two decades (Chabrier, 2006; Cook and Rich, 1999; Danna and Pape, 2005; Desrochers et al., 1992; Feillet et al., 2004; Fisher et al., 1997; Halse, 1992; Houck, 1978; Kohl and Madsen, 1997; Kolen et al., 1987; Larsen, 1999, 2004). Mirzaei and Krishnan (2011) present a node formulation of the LRP in which each node is represented by a set of binary variables. This formulation provides a generic optimization model, which handles the LRP with time-dependent demand (LRPTD) as well as time windows. In this paper, the formulation presented by the LRP will be extended for application in the EELRP with a time window and with a time-dependent demand.

In the traditional LRP/LRPTW formulation, the result obtained is link-based, i.e., each route is formed by a set of links. The position (order) of customers on each route cannot be determined unless the links are connected in the right order. The interpretation of the sequence of visits in each route is thus obtained after the solution is obtained and hence cannot be used to formulate the problem. On the other hand, in the proposed model, the positions (order) of the customers in each route are presented by a set of binary variables, which can be used for the purpose of formulation and hence is called node formulation. The proposed node-based model can also be used wherever there is any constraint, cost, or risk associated with the sequence of customers in a route. When dealing with EELRP, the weight of the vehicle in each node in the system depends on the sequence of service; hence, node formulation is necessary.

As already mentioned, vehicle energy consumption is considered a function of distance traveled (speed), vehicle weight, coefficient of rolling resistance, and aerodynamic characteristics of the vehicle. The node-based property of the model makes it flexible to involve many parameters in the model. With growing attention toward energy consumption and emission, it is essential to approach the LRP as a trade-off between energy and emission cost and profit. Hence, the proposed formulation in this paper approaches the LRP with such trade-offs. The main objective of this paper is summarized as follows:

- If a formulation for EELRP that can solve problems with a symmetric or asymmetric distance matrix.
- Development of a formulation for EELRP that can handle time-window restrictions for serving customers and also dynamic demands.

The objective function presented in this paper is to minimize energy, emissions, and depot-establishment cost, while maximizing profit. The following questions are expected to be answered by solving the model:

- What is the best strategy regarding the location of DCs?
- How are customers allocated to DCs?
- What is the routing plan of DCs to serve customers?
- What vehicle type should be used in each route?

Section 3 of this paper provides a detailed definition of the problem under investigation. A mathematical formulation of the problem and an extension of the problem for solving the EEVRP with a hard time window, a soft time window, and time-dependent demand is presented in section 4. Section 5 provides illustrative examples. Sections 6 and 7 provide a summary and conclusions as well as future work under investigation, respectively.

2. Problem Statement

Notations used to formulate the problem are as follows:

N Total number of customers
M Total number of DCs
K Total number of vehicles
I Set of customers, $I = \{1, 2,\ldots, N\}$
J Set of DCs, $J = \{1,2,\ldots, M\}$
P Set of possible positions that a customer can take in a route, $D = \{1, 2,\ldots, N\}$
V Set of vehicles, $V = \{1,2,\ldots,K\}$
V_v Speed of vehicle v, $\forall v \in V$
w_v Mass of vehicle v when fully loaded(tare mass plus load mass), $\forall v \in V$
λ_v Emission cost for producing 1 NM energy from vehicle v, $\forall v \in V$
γ_v Cost of consuming 1 Newton meter (NM) of energy in vehicle v, $\forall v \in V$
A_v Frontal area of vehicle v, $\forall v \in V$
Cd_v Vehicle v coefficient of drag, $\forall v \in V$
ψ_{gh} Density of air on the road that connect nodes g and h, $\forall g,h \in \{I \cup J\}$
Y_v Capacity of vehicle v, $\forall v \in V$
g/h Index used for all nodes
T_{gh} Travel time between node g and h, $\forall g,h \in \{I \cup L\}$
S_g Service time at node g, $\forall g \in I$
A_{mv} Arrival time at position m of route v, $\forall m \in I, \forall v \in V$
X_{mgv} $\begin{cases} 1 \text{ if node } g \text{ is in position } m \text{ of the vehicle } v; \\ 0 \text{ otherwise} \end{cases}$ $\forall g \in \{I \cup J\}; \forall m \in D; \forall v \in V$
$C^{rr}{}_{ghv}$ Coefficient of rolling resistance between vehicle v tires and the road that connects node g to node h, $\forall v \in V, \forall g,h \in \{I \cup L\}$
L_{mv} $\begin{cases} 1 \text{ if } m \text{ is the last taken position of route } v; \\ 0 \text{ otherwise} \end{cases}$ $\forall m \in D; \forall v \in V$
O_g $\begin{cases} 1 \text{ if there is any vehicle assigned to node } g; \\ 0 \text{ otherwise} \end{cases}$ $\forall g \in J$
z_{vh} $\begin{cases} 1 \text{ If vehicle } v \text{ is assigned to node h}; \\ 0 \text{ otherwise} \end{cases}$ $\forall h \in J; \forall v \in V$
D_{gh} Distance between node g and h, $\forall g,h \in \{I \cup J\}$
F_g Fixed cost for establishing node g, $\forall g \in J$
d_g Demand of node g, $\forall g \in I$
$f_g(t)$ Demand of node g at time t, $\forall g \in I$
S_g Cost of departure from node g, $\forall g \in J$
α_{mv} $\begin{cases} 1 \text{ if earliest arrival at position } m \text{ of route } v \text{ is violated}; \\ 0 \text{ otherwise} \end{cases}$ $\forall m \in D; \forall v \in V$
β_{mv} $\begin{cases} 1 \text{ if latest arrival at position } m \text{ of route } v \text{ is violated}; \\ 0 \text{ otherwise} \end{cases}$ $\forall m \in D; \forall v \in V$
a_g Earliest arrival time at customer g, $\forall g \in I$

b_g	Latest arrival time at customer g, $\forall g \in I$
Δa_g	Maximum deviation permitted from earliest arrival time at customer g, $\forall g \in I$
Δb_g	Maximum deviation permitted from latest arrival time at customer g, $\forall g \in I$
ρ_g	penalty costs associated to lower time limit violation at customer g, $\forall g \in I$
φ_g	Penalty costs associated with upper time limit violation at customer g, $\forall g \in I$
λ	Lost-order cost
γ	Percentage change in unit price of extra product delivered
Ω	Profit obtained from selling a unit of product
u	Unit product mass
g^G	Gravitational acceleration, 9.81 m/s^2
w^1_g	1 if demand has decreased at the delivery time to customer g; 0 otherwise, $\forall g \in I$
w^2_g	1 if demand has increased at the delivery time to customer g; 0 otherwise, $\forall g \in I$

The supply chain network for this problem consists of a set of customers, a set of potential distribution centers, a plant, and a set of available vehicles. The vehicles can consume different types of fuel, such as electricity, gasoline, diesel, coal, etc. Two costs are associated with the consumption of unit energy in a vehicle: energy cost and emissions cost. Other costs such as greenhouse gas emissions, etc., also can be simply added to the model. Vehicles depart DCs fully loaded and travel at a constant speed. There is only a single type of product in this problem. Each customer demand must be at most equal to the vehicle capacity, i.e., $d_g \leq VC$, $\forall g \in I$.

The total transportation cost includes the cost of departure from the DCs (S_g, $\forall g \in L$), energy cost, and emissions cost. The energy cost is equal to the total energy consumed by a vehicle times its energy unit cost, γ_v. The amount of energy used (work) to travel between each pair of nodes in the network is presented by equation (1).

$$W = Force * Acceleration * Distance \tag{1}$$

Under a steady state of driving on flat ground at a constant speed, equation (1) can be rewritten as

$$Work = Force * Distance \tag{2}$$

where force is the steady-state force required to overcome friction and aerodynamic drag. The friction force is the rolling resistance, which is calculated using equation (3):

$$Rolling\ Resistance\ (RR) = C^{rr}_{ghv} * Mass * g^G \tag{3}$$

where C^{rr}_{ghv} is the coefficient of rolling resistance and depends on the vehicle tire and road surface. The gravitational acceleration is almost the same and equal to 9.81 m/s^2 for all objects. The aerodynamic drag force in the steady state is given by equation (4):

$$F_A = ½ * Cd_v * A_v * \psi_{gh} * V_v^2 \tag{4}$$

where Cd_v and A_v depend on the characteristics of the vehicle v. Hence, the EELRP is not only sensitive to vehicle capacity but also influenced by characteristics of vehicles with respect to energy consumption. The variable Cd_v describes the smoothness of the vehicle shape. During the vehicle design stage, improving the drag coefficient is a high priority. In addition, the frontal area (A_v) is just as important. The variable ψ_{gh} is the density of air in the road between nodes g and h, and is usually about 1.3 kg/m^3 but can vary with temperature and barometric pressure.

By substituting equations (3) and (4) into equation (2), work can be presented by equation (5):

$$Work = (C^{rr}_{ghv} * Mass * g^G + ½ * Cd_v * A_v * \psi_{gh} * V_v^2) * Distance \tag{5}$$

From equation (5), it is important to calculate the weight of vehicles at each node. The vehicle mass at each node depends on the previous customers' demands on the same route; hence, the vehicle mass after delivering load d_g at position m on route v is calculated by equation (6):

$$\Gamma_{mv} = \left[w_v - \sum_{m'=1}^{m} \sum_{g \in I} u * d_g X_{m'gv} \right] \sum_{g \in I} X_{mgv} \qquad \forall m \in P, v \in V \tag{6}$$

Variables L_{mv} in equation (7) and z_{vh} in equation (8) are defined to simplify the objective function and constraints. A binary variable, LS_{mv}, is used to identify the last customer on a route and is defined by the recursive equations presented in equation (7).

$$L_{mv} = \begin{cases} \sum_{g \in I} X_{mgv} & m = N \\ \prod_{m'=m+1}^{N} (1 - L_{m'v}) \sum_{g \in I} X_{mgv} & \forall m \in \{P / \{N\}\} \end{cases} \qquad v \in V \tag{7}$$

Location 0 of each route is reserved for a DC. However, even if the related binary variable, $X_{0gv}, \forall g \in L, \forall v \in V$ holds a value of 1, this does not mean that node g is selected to be the assigned DC for route v. Node g will not be route v's DC unless there is a link between the depot and a customer in the network. Hence, z_{vh} is introduced to specify whether there is a connection between a depot and a customer in the system. The variable z_{vh} works like a connectivity between DCs and routes, and connects the location decision to the routing decision.

$$z_{vh} = \sum_{g \in I} X_{0hv}.X_{1gv} \qquad \forall h \in J, v \in V \tag{8}$$

3. Mathematical Formulation of the Problem

3.1 Formulation of EELRP

The objective function of the problem is to minimize the total cost of the system while maximizing profit. The mathematical formulation for EELRP is as follows:

Objective Function:

Min

$$E_{mv} = \sum_{v \in V} \sum_{m \in P} \sum_{h \in \{I \cup J\}} \sum_{g \in \{I \cup J\}} D_{gh} \left(X_{m-1gv} X_{mhv} + X_{mgv} X_{0hv} L_{mv} \right) \left(g^G C_{ghv}^{rr} \Gamma_{mv} + \frac{1}{2} c d_v A_v \psi_{gh} V_v^2 \right) (\gamma_v + \lambda_v)$$
$$+ \sum_{g \in J} \sum_{v \in V} z_{vg} S_g + \sum_{g \in J} F_g O_g \tag{9}$$

Subject to:

$$\sum_{v = V} \sum_{m \in P} X_{mgv} = 1 \qquad \forall g \in I \tag{10}$$

$$\sum_{g \in I} X_{mgv} \le 1 \qquad \forall m \in P, v \in V \tag{11}$$

$$\sum_{g \in I} \sum_{m \in P} d_g X_{mgv} \le VC_v \qquad \forall v \in V \tag{12}$$

$$\sum_{v\in V}\sum_{g\in J}\sum_{m\in P} X_{mgv} = 0 \tag{13}$$

$$\sum_{v\in V}\sum_{g\in I} X_{0gv} = 0 \tag{14}$$

$$\sum_{g\in\{I\cup J\}} X_{m-1gv} \geq \sum_{g\in I} X_{mgv} \qquad \forall m \in P, v \in V \tag{15}$$

$$\sum_{g\in J} z_{vg} \leq 1 \qquad \forall v \in V \tag{16}$$

$$O_g \leq \sum_{v\in V} z_{vg} \leq KO_g \qquad \forall g \in J \tag{17}$$

$$1 \leq \sum_{v\in V}\sum_{g\in L} z_{vg} \leq K \tag{18}$$

Equation (9) is the objective function, which minimizes the total cost of the network while maximizing profit. The first term in the objective function calculates energy and emission cost. The second term is the cost of dispatching vehicles from DCs. The third term is the DC's fixed costs in case they are open.

Equations (10) to (18) are constraints for this problem. Constraint (10) ensures that each customer appears in only one route. Constraint (11) guarantees that each position on a route is not taken by more than one customer. Constraint (12) ensures that the total demand of customers assigned to a route is less than the vehicle capacity. It is assumed that position zero of each route is reserved for DCs. This assumption implies that DCs cannot take any other position in routes and also that customers cannot take position 0 of their assigned route. The former is enforced by constraint (13), and the latter is enforced by constraint (14). Constraint (15) ensures that position *m+1* on route v cannot be taken unless position *m* is taken. Constraint (16) guarantees that a route cannot be assigned to more than one DC. Constraint (17) determines if a DC is open or is closed. Constraint (18) keeps the total number of vehicles between one and the number of available vehicles.

If there is any energy restriction for vehicle *v*, constraint (19) can be added to the model. For instance, for an electric vehicle *v* with power restriction, it might be of interest to enforce the vehicle to return to the depot from which it originated before its battery becomes discharged.

$$\sum_{m\in P}\sum_{h\in I}\sum_{g\in\{I\cup J\}} D_{gh} X_{mhv}\left(X_{m-1gv} + X_{0gv}L_{mv}\right)\left(g^G C^{rr}_{ghv}\Gamma_{mv} + \frac{1}{2}cd_v A_v \psi_{gh} V_v^2\right) \leq \pi_v \qquad \forall v \in V \tag{19}$$

where the left side of the inequality is the energy consumed by vehicle *v* to travel its assigned route, and the right side of the inequality is the energy limit of vehicle *v*.

It is also possible to enforce the tour duration constraint for all or some of the vehicles. Considering the fact that the arrival time at each customer depends on the arrival times at previous customers on the same route, the arrival time at position *m* on route *v* is calculated using equation (20):

$$A_{mv} = \left[\sum_{m'=1}^{m}\sum_{h\in I}\sum_{g\in\{I\cup J\}} X_{m'-1gv} X_{m'hv}\left(S_g + T_{gh}\right)\right]\sum_{g\in I} X_{mgv} \qquad \forall m \in P, v \in V \tag{20}$$

where T_{gh} is the travel time between nodes g and h $\forall g,h \in \{I \cup J\}$, and S_g represents the service time at node g, $\forall g \in I$. By using the definition of L_{mv} presented in equation (7), the tour completion constraint will be

$$\sum_{m\in P} L_{mv}\left(A_{mv} + \sum_{h\in I}\sum_{g\in L} T_{gh} X_{0gv} X_{mhv} \right) \le \chi_v \qquad v \in V \tag{21}$$

where, the first term on the left side of the inequality is the time elapsed to meet the last customer on route v, and the second term on the left side of the inequality is the travel time from the last customer on route v to the originated DC. The summation of these two terms is the total tour duration imposed to be less than the tour duration, χ_v.

Extension of the model to a multi-product case is straightforward. The following section presents an example to illustrate this problem.

3.2 Formulation of EELRP with Time Windows

An EELRP in which each customer`s demand must be satisfied within a time interval is called an EELRP with a time window. The time window can be hard or soft. In a formulation with a “hard time window,” each customer has an associated time window during which the demand must be met. The vehicle cannot deliver products to the customer before the start of the time window or after the time window has elapsed, i.e., late or early arrival at a customer is not acceptable. In a VRPTW formulation with a “soft time window,” the customer can be served before and after the preferred time window, i.e., early or late arrival at a customer is acceptable up to a predefined limit. However, there is usually a penalty cost associated with the violation of the time window, which results in late or early service to the customer. In this section, the formulation proposed in section 3.1 is modified to tackle the LRP with hard and soft time windows.

3.2.1 Formulation of EELRP with Hard Time Windows

In the LRP with a hard window constraint, a customer’s demand does not dynamically change with time and is either equal to the initial demand or zero, depending on the arrival time. When the time window is hard, no violation from the time intervals is acceptable. Therefore, the time window can be easily represented in the model with a constraint that enforces the arrival time within the related time intervals.

$$\sum_{g\in I} a_g X_{mgv} \le A_{mv} \le \sum_{g\in I} b_g X_{mgv} \qquad \forall m \in P, v \in V \tag{22}$$

Constraint (22) must be added to the formulation presented in Section 3.1 to enforce the arrival time at customer to be within the related time interval.

3.2.2 Formulation of EELRP with Soft Time Windows

In addition to solving the EELRP with a hard time window, the model can handle EELRP with soft time window constraints, which allows the time interval violation for serving customers with related penalty costs. The modification will be adding a constraint to enforce the arrival time:

$$\sum_{g\in I} a_g X_{mgv} - \left(\sum_{g\in I} \Delta a_g X_{mgv}\right)\alpha_{mv} \le AT_{mv} \le \sum_{g\in I} b_g X_{mgv} + \left(\sum_{g\in I} \Delta b_g X_{mgv}\right)\beta_{mv} \qquad \forall m \in P, v \in V \tag{23}$$

Constraint (23) is used to enforce the arrival time at each spot of a route to be within the related time interval with associated allowed deviations.

The objective function also needs to be modified by adding a term related to the penalty costs associated with the time window violation. This term is presented in equation (24).

$$\sum_{g\in I}\sum_{v\in V}\sum_{m\in I} X_{mgv}\left(\rho_g \alpha_{mv} + \varphi_g \beta_{mv}\right) \tag{24}$$

3.2.3 Formulation of EELRP with Time-Dependent Customer Demand

The proposed model can also solve the EELRP with a time-dependent demand. In the EELRP with a time window, a customer's demand does not dynamically change with time and is either equal to the initial demand or zero, depending on the arrival time. The node-based property of the proposed EELRP enables it to solve the problem when customers' demands are any arbitrary function of time (Mirzaei and Krishnan, 2011). The objective function of the problem is to minimize the total cost of the system, including energy and emission cost, while maximizing profit. Each customer has an initial demand, d_g, which will dynamically change with time after initiating, i.e., $d'_g=f(d_g,\tau_g), \forall g \in I$ The special case of this problem is the EELRP with a time window, in which the demand function is defined by equation (25) (Mirzaei and Krishnan, 2011):

$$f_g(\tau_g) = \begin{cases} 0 & if \quad \tau_g \le a_g \\ d_g & if \quad a_g \le \tau_g \le b_g \\ 0 & if \quad \tau_g \ge b_g \end{cases} \qquad \forall g \in I \tag{25}$$

where a_g is the earliest arrival time at node g, and b_g is the latest arrival time at node *g*. Based on the basic assumptions of vehicle-routing problems, $d'_g \le VC, \forall g \in I$. If instead of the function presented in equation (25) any other function is defined for a customer's demand, the problem is no longer an LRP with a time window but rather an LRP with a time-dependent demand.

To formulate the problem for the EELRP with the time-dependent demand, it is necessary to calculate arrival times at each customer *g* using equation (26):

$$\tau_g = \sum_{v\in V}\sum_{m\in N} X_{mgv} AT_{mv}, \forall g \in I \tag{26}$$

For customers with a decreasing demand function, there is a "lost-order cost," which is the cost resulting from not meeting a customer demand completely or partially, i.e., $(D_g - f_g(\tau_g))\lambda$, where λ is the lost-order cost per unit of product. The profit is the product of the total quantity delivered to the customer, the profit per unit of product, and the customer-fulfillment level, i.e., $D'\Omega B$ in which B is the customer fulfillment level defined by equation (27):

$$E_g = \frac{f_g(\tau_g)}{d_g} \qquad \forall g \in I \tag{27}$$

The value of E_g is dynamic and depends on the time of delivery. The value of E_g for customers with a monotonously increasing demand function is greater than one, and the value of E_g value for customers with a monotonously decreasing demand function is less than one. The unit product price for the additional number of products delivered to customers with increasing demand can be different from the initial price. For example, the price can be cheaper due to a quantity discount. The constant γ represents the percentage of decrease or increase in price.

The objective function for the TDLRP with a time-dependent demand is given by

Objective Function:
Min

$$E_{mv} = \sum_{v\in V}\sum_{m\in P}\sum_{h\in I}\sum_{g\in\{I\cup J\}} D_{gh}X_{mhv}\left(X_{m-1gv}+X_{0gv}L_{mv}\right)\left(g^G C^{rr}_{ghv}\Gamma_{mv}+\frac{1}{2}cd_v A_v \psi_{gh} V_v^2\right)(\gamma_v+\lambda_v) \tag{28}$$

$$\sum_{g\in J}\sum_{v\in V} z_{vg}S_g + \sum_{g\in J} F_g O_g + \sum_{g\in I}\left(w^1_g * \lambda * (d_g - f_g(\tau_g)) \pm w^2_g * \Omega * \gamma * (f_g(\tau_g) - d_g) - \Omega E_g f_g(\tau_g)\right)$$

In equation (28), the first term determines the energy and emission cost. The second term is the cost of dispatching vehicles from DCs. The third term is the DCs' fixed establishment cost. The last term includes the lost-order cost, cost/profit of additional items requested by customers, and sales profit, respectively.

Two modifications are required for the set of constraints presented in equations (10) to (18) in order to handle the TDLRP with a time-dependent demand, as opposed to the formulation in section 3.1. First, it is necessary to change the vehicle-capacity constraint of equation (12) by equation (29) to consider the demand variability.

$$\sum_{g \in I} \sum_{m \in D} f_g(\tau_g) * X_{mgv} \leq Y_v \qquad \forall v \in V \tag{29}$$

Second, the set of constraints presented in equation (30) should be added to the set of constraints:

$$\begin{aligned} & w_g^1 \left(d_g - f_g(\tau_g) \right) \geq 0 && \forall g \in I \\ & w_g^2 \left(d_g - f_g(\tau_g) \right) < 0 && \forall g \in I \\ & w_g^1 + w_g^2 = 1 && \forall g \in I \end{aligned} \tag{30}$$

This set of constraints is used to determine whether a customer's demand at the time of delivery is higher or lower compared to the initial demand.

3.3 Illustrative Examples

3.3.1 Example of EELRP

Figure 1 shows a three-layer network problem used to illustrate the proposed mathematical model. The problem consists of one plant, four customers, and two DCs. Nodes 1, 2, 3, and 4 represent the customers, and nodes 5 and 6 represent potential DCs. Coordinates of the nodes and their associated demand are presented in Table 1. No demand is associated with depots.

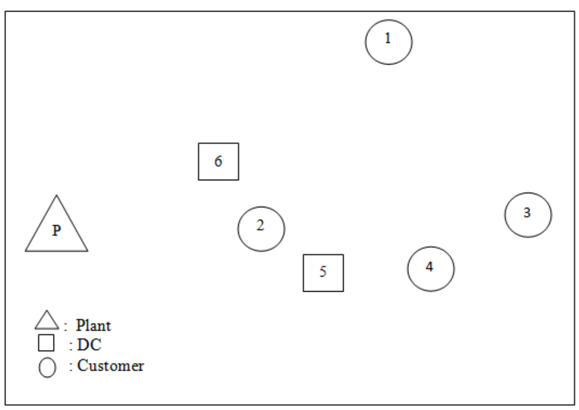

Figure 1. Three-layer network problem with one plant (p), four customers (nodes 1–4), and two dcs (nodes 5–6)

Table 1. Node coordinates and associated demands

Node	Coordinate	Demand
1	(80,45)	100
2	(70,15)	25
3	(100,20)	65
4	(90,10)	60
5	(75,10)	-
6	(60,25)	-

Four vehicles are available. Information regarding the vehicle types, capacity, initial mass, drag coefficient, frontal area, cost of emissions, and energy cost are provided in Table 2. The coefficient of the rolling resistance for each of the four vehicles on each road is shown in Table 3, Table 4, and Table 5. Air density is considered to be the same and equal to 1.3 kg/m³ for all roads. All vehicles travel at a constant speed of 40 kilometer per hour. The travel time between nodes is then calculated accordingly in Table 6. Distances are calculated as Euclidean and presented in Table 7.

Table 2. Vehicle Information

Vehicle Number	Type	*VC* (Unit of Product)	Initial Mass (kg) (Tare Mass + Load Mass)	Drag Coefficient	Frontal Area (m^2)	Emissions Cost ($)	Energy Cost ($)
1	Gasoline	100	6000 = 4000+20×100	0.75	4.0	0.1×10^{-4}	0.2×10^{-4}
2	Electric	70	4200 = 2800+20×70	0.6	3.5	0	0.2×10^{-4}
3	Hybrid	70	4200 = 2800+20×70	0.6	3.5	0.5×10^{-4}	2.7×10^{-5}
4	Diesel	115	9600 = 7100+20×125	0.89	4.5	0.2×10^{-4}	0.1×10^{-4}

Table 3. Coefficient of rolling resistance for vehicle 1

Node	1	2	3	4	5	6
1	-	0.012	0.013	0.010	0.012	0.013
2	0.012	-	0.013	0.013	0.010	0.015
3	0.013	0.013	-	0.015	0.030	0.025
4	0.010	0.013	0.015	-	0.030	0.025
5	0.012	0.010	0.030	0.030	-	0.030
6	0.013	0.015	0.025	0.025	0.030	-

Table 4. Coefficient of rolling resistance for vehicles 2 and 3

Node	1	2	3	4	5	6
1	-	0.010	0.025	0.015	0.017	0.018
2	0.010	-	0.010	0.015	0.080	0.010
3	0.025	0.010	-	0.012	0.010	0.035
4	0.015	0.015	0.012	-	0.020	0.015
5	0.017	0.080	0.010	0.020	-	0.010
6	0.018	0.010	0.035	0.015	0.010	-

Table 5. Coefficient of rolling resistance for vehicle 4

Node	**1**	**2**	**3**	**4**	**5**	**6**
1	-	0.011	0.022	0.016	0.014	0.010
2	0.011	-	0.011	0.010	0.012	0.010
3	0.022	0.011	-	0.013	0.022	0.030
4	0.016	0.010	0.013	-	0.020	0.019
5	0.014	0.012	0.022	0.020	-	0.011
6	0.010	0.010	0.030	0.019	0.011	-

Table 6. Transportation time between nodes (hours)

Node	**1**	**2**	**3**	**4**	**5**	**6**
1	0.00	0.53	0.53	0.61	0.59	0.47
2	0.53	0.00	0.51	0.34	0.12	0.24
3	0.53	0.51	0.00	0.24	0.03	0.67
4	0.61	0.34	0.24	0.00	0.25	0.56
5	0.59	0.12	0.45	0.25	0.00	0.35
6	0.47	0.24	0.67	0.56	0.35	0.00

Table 7. Distance between nodes (km)

Node	**1**	**2**	**3**	**4**	**5**	**6**
1	0	31.6	32	36.4	35.4	28.3
2	31.6	0	30.4	20.6	7.1	14.1
3	32	30.4	0	14.1	2	40.3
4	36.4	20.6	14.1	0	15	33.5
5	35.4	7.1	26.9	15	0	21.2
6	28.3	14.1	40.3	33.5	21.2	0

The cost of vehicle departure from node 5 is \$45 and from node 6 is \$50. The transportation cost (energy and emission cost) from the plant to the DCs is fixed in the planning horizon of the problem and is equal to \$250 for node 5 and \$200 for node 6. In addition, the fixed cost for establishing DC 1 is \$40 and for DC 2 is \$35. The profit obtained from selling each unit of product is \$100. This problem is a single product with unit product mass of 20 kg.

The mathematical formulation was solved using LINGO optimizer software, and results of the EELRP are shown in Table 8. Figure 2 shows the network configuration solution for the EELRP. Customers 1 and 4 are assigned to vehicles 1 and 3, respectively, and customers 2 and 3 are assigned to vehicle 4. In this case, both DCs are selected to be open. Thus, customer 1 will be served through DC 1, and the other customers will be served through DC 2. The negative value of the objective function shows the network profit, which is \$13,942.

Table 8. Results of EELRP

Objective Value	$-13941.9
Computation Time	00:04:08
Binary Variables with Value of 1	X_{111}, X_{124}, X_{143}, X_{234}, X_{052}, X_{053}, X_{054}, X_{061}, X_{062}, O_5, O_6, z_{16}, z_{35}, z_{45}, LS_{11}, LS_{13}, LS_{24}

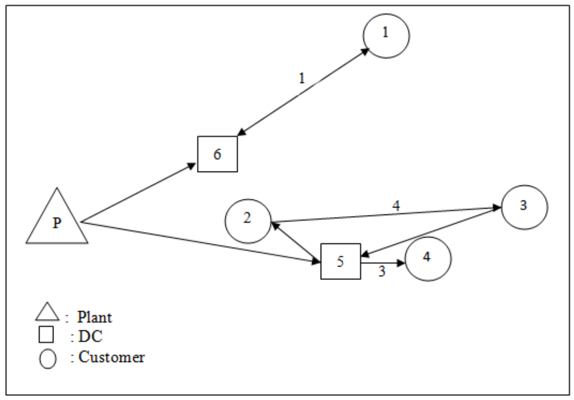

Figure 2. Network configuration solution for EELRP

The same problem is solved by traditional formulation of the LRP (Mirzaei and Krishnan, 2011). Results of the LRP are presented in Table 9. In this case, the transportation cost is assumed to be $1/km. Figure 3 shows the network configuration solution for the LRP in which the objective function is the trade-off between distance and profit. By comparing the distance and energy consumption between the two results, presented in Table 10, the distance traveled is increased by 7.9%. However, the energy and emission cost has decreased more than 36%.

Table 9. Results of LRP

Objective Value	$-24458.9
Computation Time	00:0:53
Binary Variables with Value of 1	X_{111}, X_{122}, X_{144}, X_{234}, X_{051}, X_{052}, X_{054}, O_5, z_{15}, z_{25}, z_{45}, LS_{11}, LS_{12}, LS_{24}

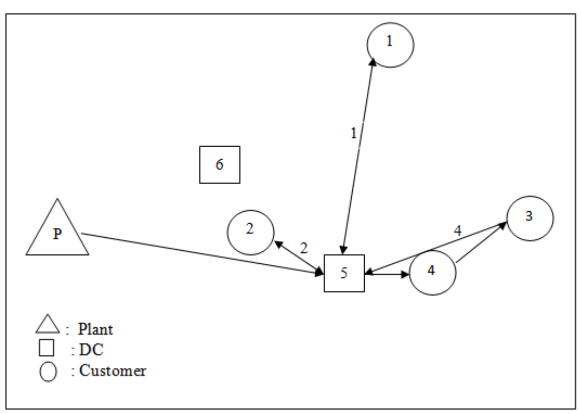

Figure 3. Network Configuration Solution for LRP

Table 10. Comparison of EELRP and LRP results

Formulation	Distance Traveled (KM)	Energy and Emission Cost ($)
EELRP	126.10	1553.32
LRP	116.10	2429.37

3.3.2 Example of EELRP with Hard Time Windows

This section illustrates the EELRP with a hard time window. The example is similar to the one presented in section 3.4.1, except that all customers have a time window assigned to them, as shown in Table 11. The time interval implies that each customer's demand is d_g if it is served within the specified time window; otherwise, it is zero.

Table 11. Time interval assigned to each customer for EELRP with hard time windows

Customer Number	1	2	3	4
Time Interval (Hours)	[35,40]	[50,58]	[20,30]	[14,16]

The mathematical formulation was solved using LINGO optimizer software, and results for the EELRP with a hard time window are presented in Table 12. Figure 4 shows the network configuration solution for the EELRP with hard time window. Customers 1 and 4 are assigned to vehicle 1 and 3, respectively, and customers 3 and 2 are assigned to vehicle. In this case, only depot 1 is selected to serve the customers. The negative value of objective function shows a network profit of $7,037.6.

Table 12. Results of EELRP with hard time windows

Objective Value	$-7037.6
Computation Time	00:00:23
Binary Variables with Value of 1	X_{111}, X_{134}, X_{143}, X_{224}, X_{051}, X_{053}, X_{054}, O_5, z_{15}, z_{35}, z_{45}, LS_{11}, LS_{13}, LS_{24}

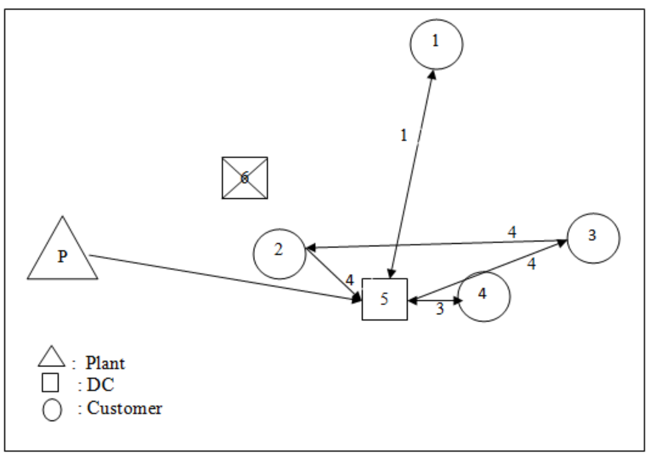

Figure 4. Network configuration solution for EELRP with hard time windows

By solving the same problem with a traditional LRP with a hard time window, the same network configuration is obtained, except that vehicle 2, instead of vehicle 3, is assigned to serve customer 4. Results for this configuration are shown in Table 13 and Figure 5. This happens because the traditional formulation of LRP does not distinguish between the vehicles in terms of energy and emissions, only with respect to their capacities. Thus, there is no difference between vehicles 2 or 3 when using the traditional LRP because they have the same capacity.

Table 13. Results of LRP with hard time windows

Objective Value	$-24409.8
Computation Time	00:00:02
Binary Variables with Value of 1	X_{111}, X_{134}, X_{142}, X_{224}, X_{051}, X_{052}, X_{053}, X_{054}, X_{063}, O_5, z_{15}, z_{25}, z_{45}, LS_{11}, LS_{12}, LS_{24}

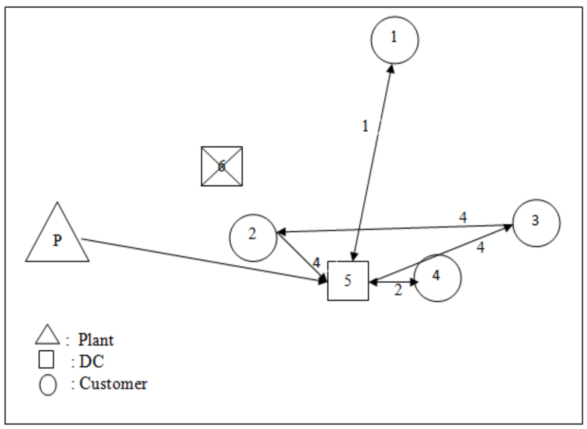

Figure 5. Network configuration solution for LRP with hard time windows

Table 14 shows the comparison between results obtained from the LRP and the EELRP with hard time windows. From this table it can be concluded that the EELRP formulation results in more than 18% savings in energy and emission costs while the distance traveled remains the same.

Table 14. Comparison of results of EELRP and LRP with hard time windows

Formulation	Distance Traveled (KM)	Energy and Emission Cost ($)
EELRP	165.2	$1717.15
LRP	165.2	$2099.25

In the next section, the formulation and an example of EELRP with a soft time window is presented.

3.3.3 Example of EELRP with Soft Time Windows

The example in this section is similar to the one presented in section 3.4.2 except for time intervals. In this example, violation from the time intervals is allowed up to a specified limit (Table 15). A penalty cost, associated with violating a lower or upper limit of a time interval, is also provided in this table.

Table 15. Time interval, violation, and penalty cost assigned to each customer for EELRP with soft time windows

Customer Number	1	2	3	4
Time Interval (Hours)	[36,40]	[50,55]	[25,30]	[14,14.5]
Lower Limit Violation Allowed (Hours)	1.5	0.5	2	1
Upper Limit Violation Allowed (Hours)	0.5	2.3	0.5	1
Penalty Cost ρ_q ($)	20	10	50	30
Penalty Cost φ_q ($)	50	70	20	10

The mathematical formulation was solved using LINGO optimizer software, and results for the EELRP with a soft time window are shown in Table 16. Figure 6 shows the network configuration solution. Customers 1 and 4 are assigned to vehicle 1 and 2, respectively, and customers 3 and 2 are assigned to vehicle 4. In this case, only one depot (node 5) is selected to serve the customers. The negative value of the objective function shows the network profit of $6,938.

Table 16. Results of EELRP with soft time windows

Objective Value	$-6937.65
Computation Time	00:1:38
Binary Variables with Value of 1	X_{111}, X_{134}, X_{143}, X_{224}, X_{051}, X_{053}, X_{054}, O_5, z_{15}, z_{35}, z_{45}, LS_{11}, LS_{13}, LS_{24}, α_{11}, β_{13}, β_{24}

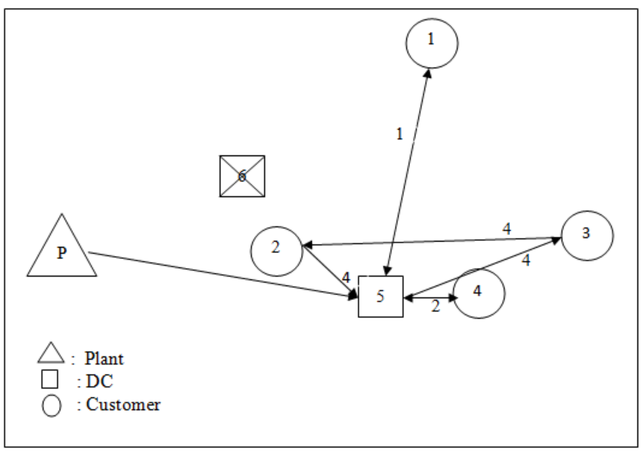

Figure 6. Network configuration solution for EELRP with soft time windows

By solving the same problem with the traditional LRP with soft time windows, the same network configuration is obtained except for a difference in vehicle assignments. Results are shown in Table 17 and Figure 7. As previously mentioned, this happens because the traditional formulation of the LRP does not differentiate among vehicles in terms of energy and emissions, only with respect to their capacities.

Table 17. Results of LRP with soft time windows

Objective Value	$-24309.8
Computation Time	00:5:06
Binary Variables with Value of 1	X_{114}, X_{131}, X_{142}, X_{221}, X_{051}, X_{052}, X_{054}, O_5, z_{15}, z_{25}, z_{45}, LS_{12}, LS_{14}, LS_{21}, α_{14}, α_{31}, α_{34}, β_{12}, β_{13}, β_{21}, β_{31}, β_{34}

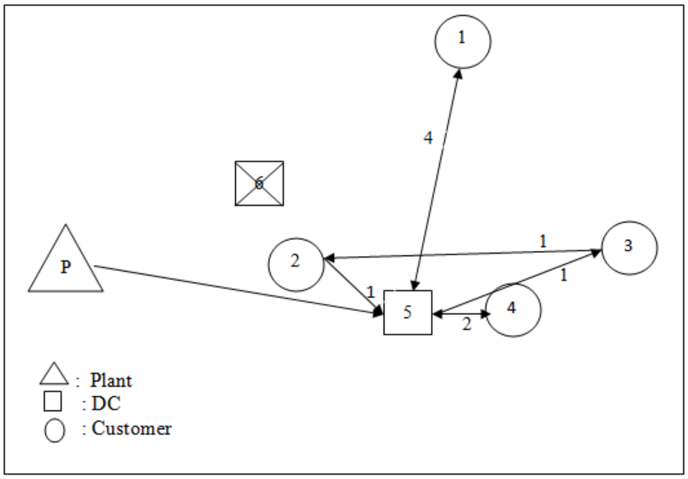

Figure 7. Network configuration solution for LRP with soft time windows

By comparing the results, as shown in Table 18, it can be concluded that the proposed formulation will result in more than 26% savings in energy and emission cost, while the distance traveled remains the same as in the traditional formulation.

Table 18. Comparison of results of EELRP and LRP with soft time windows

Formulation	Distance Traveled (KM)	Energy and Emission Cost ($)
EELRP	165.2	1717.15
LRP	165.2	2320.61

The following section presents a generic formulation of EELRP that can handle the dynamicity of customers' demands.

3.3.4 Example of EELRP with Time-Dependent Demand

The example investigated in this section is similar to the example presented in section 3.4.1 However, in this case, it is assumed that each customer's demand varies with time after initiation according to a function provided in Table 19. The mathematical formulation was solved using LINGO optimizer software, and results of the EELRP with a time-dependent demand are shown in Table 20. Figure 8 shows the network configuration solution. Customers 1 and 4 are assigned to vehicle 4 and 3 respectively, and customers 2 and 3 are assigned to vehicle 1. In this case, both depots are selected to serve the customers. The negative value of the objective function shows the network profit of $11,791.

Table 19. Customers' demand information for EELRP with time-dependent demand

Node	Coordinate	Initial Demand	Demand Function
1	(80,45)	100	$100\text{-}0.2\tau_1$
2	(70,15)	25	$25\text{-}0.2\tau_1$
3	(100,20)	65	$65\text{-}0.3\tau_1$
4	(90,10)	60	$60\text{-}0.5\tau_1$
5	(75,10)	-	-
6	(60,25)	-	-

Table 20. Results of EELRP with time-dependent demand

Objective Value	$-11791.0
Computation Time	00:04:14
Binary Variables with Value of 1	X_{114}, X_{121}, X_{143}, X_{231}, X_{051}, X_{053}, X_{064}, O_5, O_6, z_{15}, z_{35}, z_{46}, LS_{13}, LS_{14}, LS_{21}, Γ_2, Γ_4

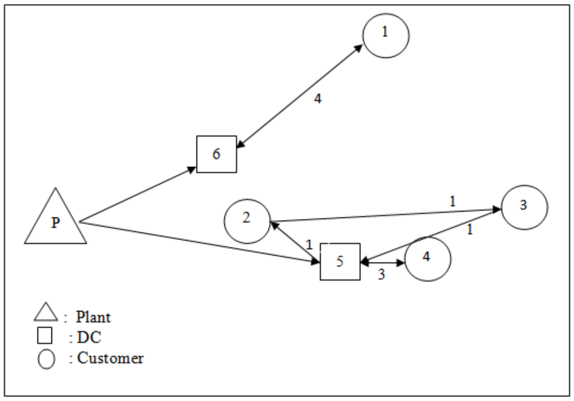

Figure 8. Network configuration solution for EELRP with time-dependent demand

The same problem is solved using the traditional formulation of the LRP. Results are presented in Table 21, and Figure 9 shows the network configuration solution.

Table 21. Results of LRP with time-dependent demand

Objective Value	$-24469.8
Computation Time	00:01:15
Binary Variables with Value of 1	X_{114}, X_{121}, X_{142}, X_{231}, X_{051}, X_{052}, X_{054}, X_{063}, O_5, z_{15}, z_{25}, z_{45}, LS_{12}, LS_{14}, LS_{21}, Γ_2, Γ_4

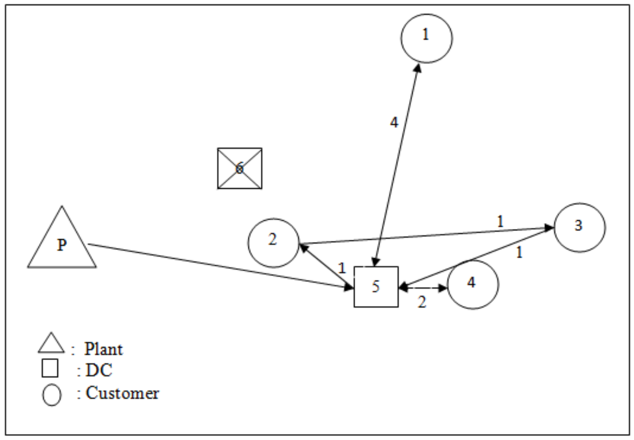

Figure 9. Network configuration solution for LRP with time-dependent demand

The distance traveled along with energy and emission cost obtained from the EELRP and LRP with a time-dependent demand are compared in Table 22. It can be concluded that distance traveled to serve the customers from depots is reduced more than 10% (traveled distance does not include the distance from the plant to depots, because it is already considered a fixed cost of establishing the depots). In addition, the energy and emission cost has been reduced by more than 37% in the routes obtained by the EELRP with the time-dependent demand.

Table 22. Comparison of results of EELRP and LRP with time-dependent demand

Formulation	**Distance Traveled in Tours Serving Customer (KM)**	**Energy and Emission Cost ($)**
EELRP with Time-Dependent Demand	126.10	1447.01
LRP with Time-Dependent Demand	140.30	2306.18

4. Conclusions

In this paper, the traditional LRP is approached with a novel perspective that considers energy and emission cost of a logistics problem. In the presented mathematical formulation, the objective function is the trade-off between energy and emission cost and profit. In addition, the model deals with the problem of vehicle selection when the available vehicles have different sources of energy, different aerodynamic structures, and consequently different costs of fuel and emission, while the traditional LRP is sensitive only to vehicle capacity. The node-based property of the presented model enhances the flexibility of the model. To illustrate the flexibility of the formulation, other possible scenarios of the EELRP, such as the EELRP with a soft or hard time window and the EELRP with a time-dependent demand are formulated. The mathematical formulations are illustrated with examples. The comparison of results between the EELRP and LRP shows that the model provides more economical solutions in terms of energy and emission cost.

Since the problem is NP-hard, for networks with a high number of nodes, heuristic or meta-heuristic algorithms for solving the problem are required. A closer look at constraints (13) and (14) reveals that they can be decoupled from the route/vehicle, and hence, applying column generation or Benders decomposition for finding the exact solution of larger-size problems may be possible. Adopting heuristic methods to tackle large-size problems is currently under investigation. The node-based formulation is also critical for configuring supply networks under risk associated with the sequence of customer visits. This aspect of the problem also is currently under investigation.

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