MODELLING SELF-SIMILAR TRAFFIC OF MULTISERVICE NETWORKS

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Abstract

Simulation modelling is carried out, which allows adequate describing the traffic of multiservice networks with the commutation of packets with the characteristic of burstiness. One of the most effective methods for studying the traffic of telecommunications systems is computer simulation modelling. By using the theory of queuing systems (QS), computer simulation modelling of packet flows (traffic) in modern multi-service networks is performed as a random self-similar process. Distribution laws such as exponential, Poisson and normal-logarithmic distributions, Pareto and Weibull distributions have been considered.

The distribution of time intervals between arrivals of packages and the service duration of service of packages at different system loads has been studied. The research results show that the distribution function of time intervals between packet arrivals and the service duration of packages is in good agreement with the Pareto and Weibull distributions, but in most cases the Pareto distribution prevails.

The queuing systems with the queues M/Pa/1 and Pa/M/1 has been studied, and the fractality of the intervals of requests arriving have been compared by the properties of the estimates of the system load and the service duration. It has been found out that in the system Pa/M/1, with the parameter of the form a>2, the fractality of the intervals of requests arriving does not affect the average waiting time and load factor. However, when $\alpha \leq 2$, as in the M/Pa/1 system, both considered statistical estimates differ.

The application of adequate mathematical models of traffic allows to correctly assess the characteristics of the quality of service (QoS) of the network.

Keywords: simulation modelling, self-similarity of traffic, Hurst exponent, distribution density.

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1. Introduction

Computer simulation modelling is the most effective way to study the processes taking place in telecommunication systems. Until 1980s the main type of services provided to subscribers was telephony. For modelling telephone systems of communication, the simplest flow defined by $P_i(t)$ probability family of receiving i (i=0…∞) calls during the interval of time t was used for describing the input traffic. The probability of receiving i calls for the simplest flow during the interval of time t is defined by the well-known Poisson formula:

$$P_i(t) = \frac{(\lambda t)^i}{i!} e^{-\lambda t},$$

where λ is a parameter of the flow, characterizing the intensity of receiving calls.

Poisson formula describes, with accuracy sufficient for practice, the phone load and, therefore, has been successfully applied in the design and modelling of telephone communication systems. However, with the appearance of personal computers and especially multimedia services, the nature of traffic in telecommunication networks has fundamentally changed. In practice, while analyzing the load in computer networks with packet commutation, it was noted that bursts of packets were present in traffic and long-terms dependences were observed, therefore the traffic can't already described correctly with Poisson formula. During recent years studies related to the analysis of network traffic shows that it has the characteristic of scaling invariance, i.e. has the characteristic of self-similarity [1–4].

2. Characteristics of self-similar traffic

The main distinctive features of self-similar traffic are following [3]:

1. Slow decrease of the dispersion during the increase of observation period.
2. Availability of long-term dependency (aftereffects).
3. The fluctuation nature of the power spectrum.

Statistical characteristics of self-similar traffic (average values, dispersion, spectral density, autocorrelation function etc.) are very different from the exponential (Poisson) regularities.

Continuous stochastic process X(t) is considered as statistically self-similar with the parameter H ($0.5<H\leq 1$), if for any positive number a the processes X(t) and $a^{-H}X(at)$ will have identical distribution. Practically the statistical self-similarity means that the following conditions are met [3, 5–7]:

– average

$$E\left[X(t)\right]=\frac{E\left[X(at)\right]}{a^{H}};$$

– dispersion

$$V_{ar}\left[X(t)\right]=\frac{V_{ar}\left[X(at)\right]}{a^{2H}};$$

– autocorrelation function

$$R(t,\tau)=\frac{R(at,a\tau)}{a^{H}},$$

where H is Hurst exponent, a is positive number.

Self-similarity concept is closely linked with the renowned idea of fractals and chaos theory. From a mathematical point of view, a fractal object, first of all, has a fractional dimension, which is defined as

$$d=\frac{\log N}{\log 1/r},$$

where N is number of equal parts into which the object is to be divided, and each piece will be the copy of integer reduced in 1/r times.

The fractal dimension can be considered as a measure of imperfection of the rugged surface of object $d\in[n, n+1]$ in the n-dimensional space, and more imperfect, "uneven" surfaces correspond to higher values of d.

Another parameter that characterizes self-similarity is Hurst exponent $H\in[0; 1]$. There are three different classifications for various Hurst exponents:

– at $0<H<0.5$ – antipersistent time series, i. e. the series at which the so-called "reversion to the mean" takes place: if the system grows in a certain period, then the next period it is necessary to expect a recession. In reality, these processes are very few. Antipersistent time series is called "pink noise";

– at H=0.5 – time series is stochastic. This process is called "white noise". The equality H=0.5 indicates an absence of self-similarity;

– at 0.5<H<1 – persistent time series (these processes are also called "black noise"). Time series is characterized by the effect of long-term memory. If the series starts to grow, it will grow further and if it decreases today, then will also decrease tomorrow. With regard to networks, this means that traffic is a fractal. Closer this parameter to 1, the fractal characteristics becomes more apparent.

3. Simulation modeling: results and discussion

Streams of packets (traffic) in the modern multi-service packet communication networks is random self-similar process and computer simulation modelling is one of the effective methods of modelling such processes. For solving this problem, as a rule, the theory of systems of mass service (SMS) is applied. SMS is a mathematical model designed for the servicing applications incoming at random time intervals, where the duration of servicing is also random. Main place in the general mathematical model of SMS takes the model of incoming stream of applications received by the system for servicing (traffic model). The accuracy of calculation of main characteristics of SMS, which characterizes the operation of the whole system, depends on the correct choice of this model.

It is not quite necessary to use expensive equipment in order to get the overall results for the systems servicing self-similar streams. The different software tools are used to develop simulation models. At the present time for carrying out scientific experiments, it is necessary and sufficient to use the systems of simulation modelling. A powerful tool for carrying out simulation experiments of systems of mass services as models of telecommunication systems is a general-purpose simulation modelling system GPSS World. In this case the study of classical models is only necessary for verifying the adequacy of models built in the system.

In this article more acceptable mathematical models derived from the results of measurements and simulation modelling of parameters of packet communication networks traffic are given prove.

The random process of applications (packets) coming in the system is characterized by a distribution law, establishing the link between the value of the random variable and the probability of occurrence of this value. This stream can be described by a probability distribution function of the time intervals between adjacent applications or probability distribution function of the number of applications for the standard unit of time. The following distribution laws have been considered: the exponential, Poisson and normal-logarithmic distribution, the Pareto distribution and the Weibull distribution.

In the Poisson stream the interval between events is described by an exponential distribution. The probability density of this distribution is as follows:

$$P(x) = \lambda e^{-\lambda t}.$$

One of the main methods of forming self-similar stream is the method originally proposed by Mandelbrot. This method provides for the existence of multiple independent ON/OFF sources. For each source, these periods are strictly alternating.

Duration ON (as well as OFF) periods are also independent and identically distributed, and the distribution of durations of ON periods may differ from the distribution of OFF periods. Each source generates packets only in a position of ON. The resulting value in each period of time is the sum of the values generated by all sources. The emergence of self-similarity is explained by the effect of Noah (Noah effect) in the distribution of durations of ON/OFF periods. The Pareto distribution, which has the following distribution function, can be used to achieve this effect:

$$P(x) = \frac{a}{b}\left(\frac{b}{x}\right)^{a+1}, \quad x > b \text{ and } a > 0. \tag{1}$$

The parameter α is the parameter of form that defines a finiteness or infiniteness of average value and the dispersion for distribution, while b parameter assigns the minimum value of the random variable x. The parameter α assigns the average value and the dispersion as follow:

1) for $0 < a \le 1$ the distribution has the infinite mathematical expectation and dispersion;

2) for $1 < a \le 2$ the distribution has the finite mathematical expectation and the infinite dispersion;

3) for $a > 2$ the distribution has the finite mathematical expectation and dispersion.

There is a connection between Hurst exponent and the parameter a:

$$H = \frac{(3-a)}{2}.$$

The parameter a is called the fractal indicator of time series.

During the practical generating the random variable of the time interval between events according to the Pareto distribution (1) it is necessary to make transition from an equal distribution by the inverse function method:

$$Z_i = \frac{b}{\sqrt[a]{U_i}},$$

where Z_i – i interval between events, U – random number, equally distributed at the interval [0, 1].

The Weibull distribution, which is used for modelling the self-similar traffic, has the parameter a (it can vary from 0 to 1) and b. Its density function is shown below:

$$P(x) = a * b * x^{(a-1)} \cdot e^{-b * x^a}. \quad (2)$$

There is the following connection between Hurst exponent and the parameter *a*.

$$H = \frac{2-a}{2}.$$

During the practical generating the random variable of the time interval between events according to the Weibull distribution (2) it is necessary to make transition from an equal distribution by the inverse function method:

$$Z_i = \left(\frac{-1}{b}\ln U_i\right)^{\frac{1}{a}}.$$

Mathematically to achieve the Noah effect one can also use the log-normal distribution, which is also often referred to as heavy-tailed distributions. At the log-normal distribution not the variable itself, but its logarithmic value is subject to the normal law, i. e. in the dependence Z=log(X), Z – normally distributed random variable, and X – a random variable distributed according to the log-normal law, which has the following form:

$$P(x) = \frac{1}{x\sqrt{2\pi\sigma}} e^{\frac{-(\log(x)-m)^2}{2\sigma^2}}, \quad x > 0. \quad (3)$$

Here, σ – the mean square deviation of the random variable Z, and m – mathematical expectation. These parameters can be determined on the basis of experimental data using the following formulas:

$$m = \frac{1}{n}\sum_{i=1}^{n}\log(x_i); \quad \sigma = \sqrt{\frac{1}{n}\sum_{i=1}^{n}(\log(x_i) - m)^2}.$$

During the practical generating the random variable of the time interval between events according to the Weibull distribution (3) it is necessary to make transition from an equal distribution by the inverse function method:

$$Z_i = e^{U_i}.$$

In studies on the distribution of the number of applications during the time interval and on the distribution of the time interval between applications and so on, correspondence between the observed and theoretical values of the variables are checked by the relevant graphs, or by eye. In fact, one needs to perform a quantitative evaluation of statistical hypothesis testing. To do this, there are certain criteria, among which a special place belongs to the criteria x^2 – of Pearson and Kolmogorov [8–10].

The adequacy of the experimental and theoretical distributions can be judged by Pearson's criterion of consent, which value is calculated according to the following expression:

$$x^2 = \sum_{i=1}^{j} \left(\frac{n_i - m_i}{m_i} \right)^2.$$

Here n_i – experimental data, m_i – theoretical data, calculated according to a specific distribution function. Based on the obtained x^2 and degrees of freedom $k = j - 1 - l$, where j – the intervals number of breakdown, and l – the quantity of distribution parameters, one can find the probability that when the received level of significance is 5 %, the experimental data are in agreement with theoretical one.

Kolmogorov's criterion is based on a comparison of the integral curves of distributions:

$$\lambda = \frac{\max|f - F|}{\sqrt{N}},$$

where f – the accumulated experimental frequency of distribution, F – the accumulated theoretical frequency of distribution calculated according to a specific distribution function.

Based on the statistical data on number and size of transmitted packets, on the time intervals between the packets during the established connection (communication session), on the data on durations of the established connections, etc., one can create a mathematical model of the real traffic [3].

The preliminary analysis of simulation modelling results showed that the traffic has self-similar property, and the Hurst parameter is not lower than 0.8. These data testify that the multi-service traffic is characterized by a strong irregularity of the intensity of receiving applications and packets. The applications and packets are not smoothly spread across the various time intervals, and are grouped into "bursts" in certain intervals, and are completely absent or are very few in other time intervals [3]. Because of this, in the burst traffic at a relatively small average value of the packet arrival intensity (traffic intensity) there is a sufficient quantity of relatively large emissions.

To study the distribution of time intervals between arrivals of packets and the length of packet service at different loads of the system and at different values of Hurst parameter, the statistical data are agreed by the above-mentioned distribution laws. The research results showed that the distribution function of time intervals between arrivals of packets and the length of packet service are well agreed with the Pareto distribution with parameters a=0.316, $b = 3.02610^{-4}$ (**Fig. 1**) and the Weibull distribution with parameters a=0.836, $b = 1.28510^{-5}$ (**Fig. 2**).

Testing of the statistical hypotheses is made by the Pearson criterion of consent. In the **Table 1**, the results of corresponding the experimental data to the Pareto distributions and the Weibull distributions with degrees of freedom k=12 are shown, under the different loads of system and the different values of Hurst parameter. As this **Table 1** shows, the Pareto distribution (with a higher probability) and the Weibull distribution adequately describe the experimental data, but the Pareto distribution prevails in most cases.

Let's consider the Pareto distribution properties at $a \le 1$ or $a \le 2$, which are most relevant to the current researches of the network traffic. One can derive the following formula from (6) for determining the m-th order of the start time:

$$M\left(x^m\right)=\int_0^\infty x^m P(x)dx=\begin{cases}\dfrac{ab^m}{a-m}, & a>m;\\ \infty,\ a\le m, & (m=1,2,...).\end{cases} \tag{4}$$

Let's generate using a random number generator in an amount of $N=10^8$ independent values of the random variable $xP(b,a)$ for b=1 a=1.1 and calculate the mathematical expectation and the mean square deviation using the formula (4):

$$M(x)=11 \text{ and } \sigma=\sqrt{M\left(x^2\right)}=\infty.$$

However, processing of the statistical data generated by a random number generators, showed that $\hat{M}(x)\approx 7.5$ and $\hat{\sigma}\approx 380,$ which differ from the true values 11 and ∞.

In comparison with the "classical cases", we note that hundred times shorter sample of 10^6 values of exponential random variable gives estimates for its mean and dispersion of more accurate approximations. For $a \le 1$ the difference of the sample from the Pareto distribution with the "classic" samples markedly enhanced.

In designing, simulation modelling channels of data transmission networks and servicing the fractal network, the problems caused by its "non-classical" nature are complicated. Let's imagine the channel of data transmission network in the form of the system of mass service (SMS) channel G/G/1 [9, 10]. Using the notation Pa for the Pareto distribution, let's define the type of SMS of interest to us in the form of G/Pa/1. For analytical assessment of the problems arising from its study we will consider the system M/Pa/1. Let's consider the queue M/Pa/1, the distribution moments of which can be calculated using the known analytical expressions [9]. To calculate the average waiting time in the queue, one can use the Pollaczek-Khinchine formula:

$$W=\frac{\lambda^2 M\left(x^2\right)/2}{1-\rho}, \tag{5}$$

where λ – the intensity of the input (exponential) applications stream, $\rho=\lambda M(x)$ – the load factor.

Let's assume the interval τ of applications arrival has the mathematical expectation

$$M(\tau)=\lambda^{-1}=22$$

time units and $x\in P(1,1.1).$ Since

$$M(x)=ab/(a-1)=11,$$

then

$$\rho=M(x)=11/22=0.5,$$

and since a=1.1<2, then $M\left(x^2\right)=\infty$ and according to (5) $W=\infty.$ These are the exact values of the characteristics ρ and W of the considered SMS. And they themselves are paradoxical: the channel is idle half, and the queues are endless in average. In simulation modelling this system M/Pa/1 to GPSS World, performing experiments lasting 10^4, 10^5 and 10^6 etc. time units we get the "strange" sequence of estimates. After passing through the SMS of tens of millions of applications the estimate for ρ converges approximately 0.36 (but not to the true value of 0.5), and the estimate for W is stabilized at about the end value 250.

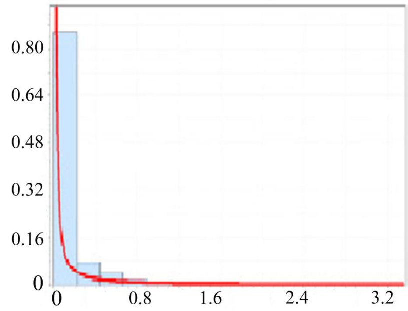

Fig. 1. Bar chart of measurements of packets arrival duration

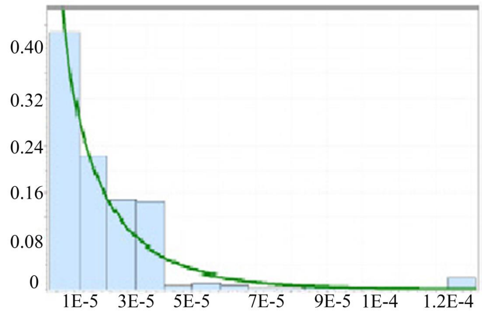

Fig. 2. Bar chart of measurements of packets service duration

Table 1

Results of corresponding experimental data to the Pareto distributions and the Weibull distributions

System load	Values of Pearson criterion							
	The Pareto distributions							
	H=0,55	H=0,6	H=0,65	H=0,7	H=0,75	H=0,8	H=0,85	H=0,9
0,1	6,324	6,414	7,326	8,325	8,386	7,328	8,553	8,333
0,2	7,500	8,670	8,598	9,501	9,511	0,504	0,509	0,543
0,3	5,611	5,754	9,611	8,698	8,664	9,620	9,659	9,900
0,4	6,673	6,779	8,673	9,683	9,731	9,829	9,027	8,454
0,5	7,703	9,732	8,783	8,861	8,982	9,184	9,514	8,024
0,6	6,847	5,925	6,031	9,182	9,393	9,719	9,188	9,000
0,7	6,144	7,281	8,467	8,705	9,019	9,466	9,180	9,540
0,8	7,714	7,928	9,184	8,513	8,929	8,539	9,603	8,588
0,9	9,787	9,081	8,435	7,888	6,511	8,381	7,391	9,700
The Weibull distributions								
0,1	7,026	5,398	6,321	8,325	6,329	5,337	7,349	5,368
0,2	5,536	5,542	5,547	6,557	7,569	8,591	5,628	6,711
0,3	6,074	5,756	7,774	6,795	9,825	6,872	7,694	5,142
0,4	5,972	5,995	7,022	7,064	6,118	7,199	8,353	7,611
0,5	5,237	5,271	6,321	7,379	7,469	7,570	7,782	7,281
0,6	5,557	5,605	5,978	7,765	6,883	7,105	6,444	5,261
0,7	7,957	6,035	6,136	6,269	7,461	7,789	8,352	6,584
0,8	8,516	9,636	5,781	8,996	5,291	5,759	5,678	7,882
0,9	7,465	6,634	7,899	5,231	5,691	5,491	6,449	5,832

Exit from the critical range of a, $a \le 2$ does not save the statistical characteristics of the system from oddities. Let's replace in the model, for example, the value of 1.1 of the parameter a to 2.1. Now $M(x) = 1.90909$ and one must also replace the mean value of 22 with the value $3.81818 = 2M(x)$, in order to maintain the same load $\rho = 0.5$. Performing simulation modelling, let's obtain the estimate for ρ, equal to 0.500 (true), but the estimate for W converges slowly to about 0.72, while the true meaning of W, according to (5), is equal to 1.19. The true estimate for W can be achieved only at $a > 3$.

And now let's consider the system Pa/M/1, where with the infinite dispersion of the duration of τ arrival intervals the average length of the queue W is finite (if $\rho < 1$).

The comparison of the results of simulation modelling the system Pa/M/1 and exact solutions (in order to determine the exact solutions of ρ and W it is necessary to solve, by the numerical method, the equation obtained from the Laplace transform of the Pareto integral density) is shown in the **Table 2**.

Table 2

Comparison of estimates $\rho_з$ and $W_з$ with exact values ρ and W

ρ	a=2.1, M(τ)=1.90909			a=1.1, M(τ)=11		
	$\rho_з$	W	$W_з$	$\rho_з$	W	$W_з$
0.1	0.100	0.0001	0.0001	0.136	0.035	0.047
0.2	0.200	0.0067	0.007	0.275	0.369	0.505
0.3	0.300	0.0364	0.036	0.402	2.037	2.733
0.4	0.400	0.1106	0.111	0.532	10.75	14.409
0.5	0.500	0.2635	0.264	0.673	49.81	98.139
0.6	0.600	0.5640	0.564	0.831	251.2	859.316

As it is seen from the **Table 2**, in the system Pa/M/1 at a>2 the fractality of intervals of applications arrivals does not have an effect on the properties of estimates ρ_{im} and W_{im} for ρ and W – they converge to the exact values. However, at $a \le 2$ as in the system M/Pa/1 both considered statistical estimates diverge.

Let's briefly discuss the results and preconditions of the research. So, by using the theory of queuing systems (QS), computer simulation modelling of packet flows (traffic) in modern multi-service networks was performed as a random self-similar process. It is shown, that a quantitative estimation of the degree of self-similarity of traffic flow is the Hurst parameter, which has a value of not lower than 0.8. This testifies that the multi-service traffic is characterized by a strong irregularity in the intensity of incoming requests and packages. The research results showed that the distribution function of the time intervals between the arrivals of packages and the service duration of the packages are in good agreement with the Pareto and Weibull distributions. The numerical characteristic of the distribution, which can serve as its measure of uncertainty, is the entropy of distribution law. Knowing the entropy, it is possible to calculate the characteristics of the service quality of a queuing system with a queue for the case of servicing traffic that has a self-similarity effect. To calculate the average waiting time in the queue, one can use the Pollaczek-Khinchin formula. Having determined the average number of requests in the system, it is possible to calculate the remaining QoS characteristics (Q, W and T) from known ratios. Therefore, it becomes possible to calculate the QoS characteristics in the QS model with self-similar traffic for any distribution law of the service duration. The obtained results are the development of research in the field of fractal traffic and can be successfully used to solve practical problems of designing telecommunication systems in the fractal traffic conditions.

The fractal QS are practically not amenable to purely analytical research methods. Therefore, for studying the fractal traffic it is necessary to use analytical and simulation methods together.

It is worth mentioning that in simulation modeling, estimates of the mathematical expectation of fractal random variables may converge to true averages for too long (millions, billions of years or more). If the random variable has infinite dispersion, then it is extremely difficult to estimate the mathematical expectation of this random variable by means of simulation modeling. This problem is a matter of the future and requires its own scientific and technical solution.

4. Conclusions

The systems of mass service with the queues M/Pa/1 and Pa/M/1 have been studied and the fractalities of intervals of applications arrivals on the properties of estimates of system loads and service duration have been compared. Simulation modelling results have shown that a more appropriate model of streams in the multi-service networks with the packet commutations are the probability functions of the Pareto and the Weibull distributions. The use of adequate mathematical models of traffic allows to assess correctly the characteristics of service quality (QoS) of the network.

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