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Fact-checking of timetabling principles: a case study on the relationship between planned headways and delays

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Abstract

The tradeoff between reliability and level of service is a central focus for railway operators and infrastructure managers. A well-performing timetable must include an optimal level of buffer time between conflicting train movements, such that a high service delivery and a high service quality are maintained. This focus on buffer time has informed the research within the fields of timetable optimization, capacity utilization and delay propagation modeling. Despite recent and ongoing advancements in these fields, there are still disconnects between the theoretical models and their application in the design, planning and evaluation of railway timetabling. Parameters that are used in timetabling, as well as, as input to the analytical assessment models, are typically derived from practical experience and based on the macroscopic limitations of a system, rather than the microscopic conflicts inherent in its signaling system. The objective of this paper is to support the design of fact-based timetables by introducing a method of applying statistical analysis of the relationship between planned headways and recorded delays to estimate the minimum feasible headway between conflicting train movements in a railway system. This method is applied on the busiest railway line in Denmark and the results from recorded operations are validated through microsimulation.

Keywords

Railway Delays, Headways, Timetables, Data Analysis, Train detection systems

Introduction

The reliability and punctuality of a railway system are of utmost importance to its operators and infrastructure managers, as these factors directly influence the service delivery and service quality of the system. Both performance measures can be improved by decreasing the risk of conflict between trains in the network. One well-established method for reducing the risk of conflict in a timetable is the addition of buffers to the individual timetable components, such as running time and dwell time. Buffer time can also be added between conflicting train movements to ensure that the timetable can be operated, even in the case of moderate disruption; this is referred to as headway buffer.

Headway buffer is defined as the difference between the planned headway time and the minimum headway time, which is a function of the infrastructure, as well as, the features of the trains involved in the interaction (Goverde & Hansen, 2013). The larger the headway buffer between trains, the lower the chance that the delay of one train will propagate to the other trains in the network (Hansen & Pachel, 2014). While buffer time increases the robustness of a system, it also increases the capacity consumption and thus leads to a reduction in the level of service for passengers. This tradeoff between reliability and level of service is a central focus of research within railways, particularly in the fields of timetable optimization (Huisman & Boucherie, 2001; Schittenhelm, 2011; Sels, et al., 2015; Jovanović, Kecman, Bojović, & Mandić, 2017), capacity utilization (Gibson, Cooper, & Ball, 2002; Landex, 2008; Armstrong & Preston, 2017; Jensen, Landex, Nielsen, Kroon, & Schmidt, 2017) and delay propagation modeling (Hofman, Madsen, Groth, Clausen, & Larsen, 2006; Şahin, 2017; Zieger, Weik, & Nießen, 2018; Harrod, Cerreto, & Nielsen, 2019). Many of the models presented or applied in these fields of research emphasize the importance of minimum headway in assessing the performance of a railway timetable and identifying the optimal buffer times that should be used in the planning of these timetables. Although it is included as an input parameter in all the referenced models, the minimum headway was either left as a theoretical concept or was applied as a generalized value without reference to its validation.

In their simulation model for testing timetable robustness and recovery strategies on the DSB S-train, Hofman et al. (2006) applied a general value of 1,5 minutes for the minimum headway between all trains at all locations in the network. However, they admitted that this generalization decreased the precision of the model and that it could be improved by applying actual, verified minimum headways values. Zieger et al. (2018), who used Monte-Carlo simulation to model delay propagation, explained that the minimum headway is dependent on the train type and infrastructure, and asserted that it is the responsibility of the infrastructure manager to identify this parameter to ensure that all timetables are planned with respect to it.

While a realistic estimation of the minimum feasible headway is proven to be essential for the design of robust timetables with adequate buffers to absorb the most common disturbances, it is still common practice in railway planning for practitioners to design planned headways based on experience and rule-of-thumb estimations at an aggregated line level and without consideration of the actual conflicts at the block-section level (Andersson, Peterson, & Törnquist Krasemann, 2011; Palmqvist, Olsson, & Hiselius, 2018). A poor estimation of the minimum headway time leads to infeasible timetables and sequences of trains with a negative headway buffer and thus, an increase in the delay across consecutive trains.

In this paper, the relationship between the planned headways separating conflicting movements and the difference in delay of the second train involved in the conflict is investigated. Historical data recorded by the signaling system components and the automatic train detection system is deployed to estimate the minimum feasible headway between conflicting movements. These values could then be used as input to models or calculation methods that assist in the designing and planning of optimal railway timetables.

Literature survey

Headway times, and particularly minimum headway times, serve as input parameters to the models of delay generation and propagation found in the literature. However, there is a smaller set of research studies that have used empirical data to focus specifically on the relationship between realized delay and planned headway.

The relationship between delay and headway was studied by Landex (2008) by identifying a delay propagation factor as a function of capacity consumption and an initial delay value, given in terms of the minimum headway. The author asserted that the planned headway, along with the minimum headway and the initial delay, could be used to estimate the realized secondary delay but did not explore this assertion further. Haith et al. (2014) validated this assertion and concluded that planned headway values increase the precision of finding and assessing the reactionary delays in a system in comparison to using a compression method to assess capacity usage and the corresponding realized delay.

Hansen (2004) modelled the stochastic nature of realized block occupation by analyzing the distributions of the realized time registrations of trains in relation to their planned values. The author then asserted that these findings could be used to determine the optimal planned headway since it assured that there was an

acceptable probability that conflicts would be avoided. This analysis focused on the planned headway at the line level, rather than at the detailed block section level.

Daamen et al. (2009) developed a conflict identification tool that uses detailed historical operations data, including signal aspect data, as input to the model. Goverde & Meng (2011) extended the usability of this tool by introducing a statistical analysis tool that automatically identifies secondary delays based on the identification of route conflict chains. The focus of this research was to provide a method for identifying the signals in the system with the greatest number of conflicts or largest changes in delay in order to identify systemic bottlenecks.

Richter (2012) had a similar research goal and used an aggregated dataset of detailed signal aspect records to study the source of train delays on both the train level and the signal level. The authors investigated the difference in delay between consecutive trains, but only connected this to the planned headway through visual inspection. A similar method was applied by van Oort et al. (2015), who assessed the service quality on a bus line through visual comparison of the realized headways and realized delays at each stopping location on the line. A value for the minimum headway could have been estimated through this visualization technique, but it is not sufficient for clarifying its direct relationship to delay, nor does it include the relationship between the planned headway and the realized delay.

Corman & Kecman (2018) assessed the relationship between the planned headway between two consecutive trains and the change in delay of the second train, in the case that at least one of the trains was a freight train. They used visual inspection to assert that, in general, large changes in delay correspond to shorter planned headway times. The authors also took this investigation one step further and used regression analysis to conclude that the change in delay for this subset of trains could not be explained statistically by the planned headway.

Minimum headway and its direct relationship to delay was investigated by Yabuki et al. (2015) in their assessment of the effectiveness of a delay reduction measure applied on a metro line. This delay reduction measure involved upgrading the signaling system to enable a decrease in the minimum headway on the line, and therefore, an increase in the buffer time when the planned headway is unchanged. The authors analyzed empirical data by association rules and concluded that reducing the minimum headway was successful in reducing the level of delays in the network. However, they did not extend their research to include the derivation of the minimum feasible headway time inherent in the system.

There is agreement throughout the literature on the importance of understanding the relationships between minimum feasible headway, planned headway and realized delay. There is also a clear need for the derivation of accurate values of minimum headway to be used as input for models of timetable optimization, capacity utilization and delay propagation. This research focuses on the relationship between planned headway and realized secondary delays; it expands the usefulness of this relationship by identifying a method for applying statistical analysis to derive the minimum feasible headway inherent in a railway system. In addition to the derivation of the minimum headway from standardly accessible historical operations data, the second major contribution of this work is the focus on specific conflicting movements, rather than on conflicting train paths at the line level.

Identification of the minimum feasible headway

Headway times in railway planning describe the time separation between conflicting train movements at a specified location. The planned headways can be considered as the summation of two main components. The first is the minimum feasible headway, which describes the technical time necessary for the itinerary reset after a train passes and for the transfer of movement authority to the second train. The second part is commonly referred to as headway buffer, and it is used to reduce the interferences between train movements in case of small disturbances (Hansen, 2004). This relationship is described in (1), with h_i being the planned headway between trains i and $i - 1$, $h_{i_{min}}$ being the minimum feasible headway, and b_i being the headway buffer.

$$h_i = h_{i_{min}} + b_i. \quad (1)$$

When the planned headway between conflicting movements of two trains is equal to the minimum feasible headway, any delay of the first train will be transferred and result in a delay of the second train at least

equal to the delay of the first. This delay can only be recovered if there is a buffer in the planned headway between the trains. In this case, the delay of the second train is greater than or equal to the delay of the first train minus the planned headway buffer. The headway buffer represents, thus, the upper limit in the delay recovery between consecutive trains at a specified location. This relationship is explained by the equations below:

$$d_i \geq d_{i-1} - b_i \quad (2)$$

$$\Delta d_i := d_i - d_{i-1} \geq h_{i_{min}} - h_i, \quad (3)$$

where d_i is the deviation from the plan measured for train i at a timing point, and Δd_i is the difference in the deviation from the plan measured between consecutive trains at a timing point. Note that the relations are valid both for positive and negative deviations from the schedule, respectively delays and earliness, as the minimum headway between conflicting movements is independent from the timetable, hence the definition of d_i as a deviation rather than a delay. From (1), the minimum feasible headway corresponds to a value of planned headway that contains no buffer and therefore allows for no recovery between consecutive trains. It is important to note, that when the development of the deviation between consecutive timing points is considered, the deviation from the plan for a single train is also dependent on the running time supplement included in its schedule. However, the focus of this paper is the difference in deviation between two consecutive train movements at a single timing point. Therefore, the deviations measured for a pair of consecutive train movements are independent of the running time supplements planned for each train around the timing point.

Difference in deviation from previous train vs. Planned Headway

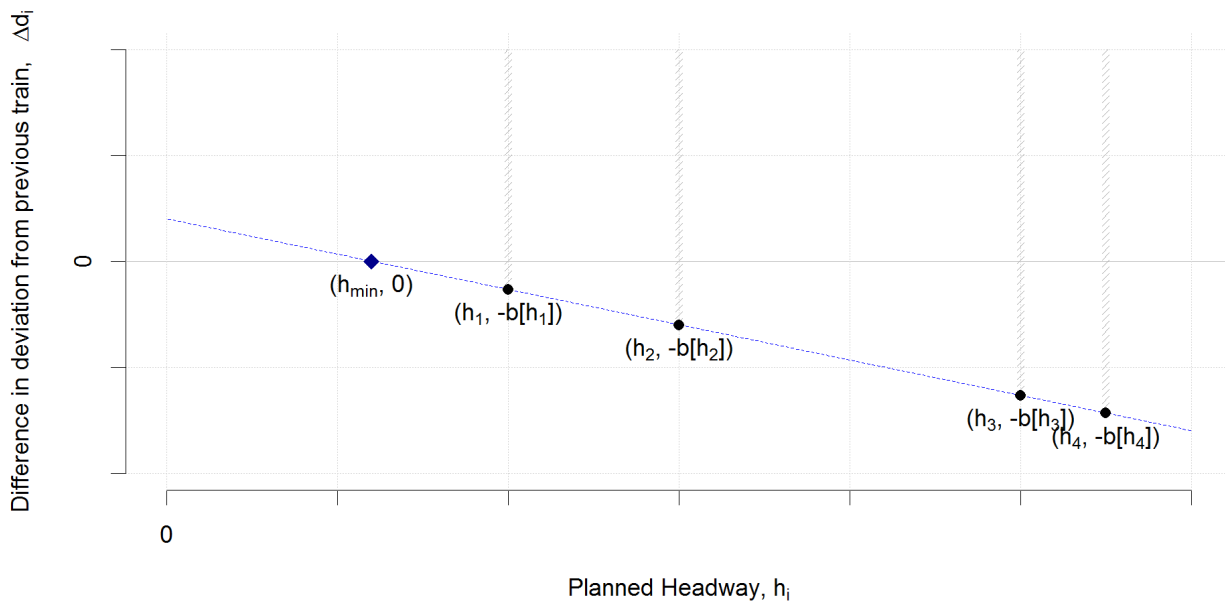


Figure 1: Relationship between planned headway and difference in deviation between trains with discrete values on planned headways.

Railway schedules are often characterized by few discrete values of planned headway, due to the rounding to entire minutes in the public timetables (Hansen, 2004). The continuous domain of (1) becomes thus discrete, and the distributions of realized differences in delays can be analyzed as conditional to the individual values of planned headway. The minimum feasible differences in deviations from the schedule still lie on the straight line defined in (1), as depicted in Figure 1.

In this paper, the relationship between the planned headways and the difference in deviation between consecutive trains is investigated through historical data recorded by the signaling system and the automatic train detection system. The timestamps of all the trains operated at one location are compared to the schedule to identify the deviations. The time differences between the scheduled times of

consecutive trains represent the planned headway. The difference in deviation between consecutive trains is then compared to the respective planned headway. For a given value of planned headway, the minimum difference in deviation recorded between trains identifies an upper boundary to the buffer as it expresses the maximum recorded recovery between consecutive trains (cf. (2), (3)). The regression of the minimum differences in deviation against the planned headways returns the linear relationship between the headway buffer and the planned headway. The minimum headway between conflicting movements can be calculated, then, as the value of planned headway that gives zero buffer.

The analysis of historical records can be disaggregated by different factors with a potential influence on the minimum feasible headway. Examples are the train length and dynamic performance, the train category, and the speed profile of the conflicting itineraries. In the following section, the method described above finds application on a Danish case.

A Danish case: the West Line

The *Vestbane* (West Line) is a primarily double tracked railway in the Copenhagen region. This is the the busiest railway line in the Danish railway network of Banedanmark, and it is operated by a manifold traffic: regional, intercity, and international passenger trains, as well as domestic and international freight trains. The passenger service is typically operated from the central station in Copenhagen (KH) to Høje Tåstrup (HTÅ) and beyond, whereas the typical route for freight trains originates from Malmø (Sweden) through the Øresund bridge and reaches the Vestbane at the junction in Hvidovre. Figure 2 depicts the line scheme with the train detection points. Only the westbound tracks are reported as the analysis only includes trains in this direction.

At Copenhagen central station, four platform tracks are connected to the Vestbane, but these tracks all share the same timing point, located just beyond the junction. On the contrary, the two westbound tracks in Høje Taastrup are provided with individual timing points, as the line continues as four-tracked up to Roskilde.

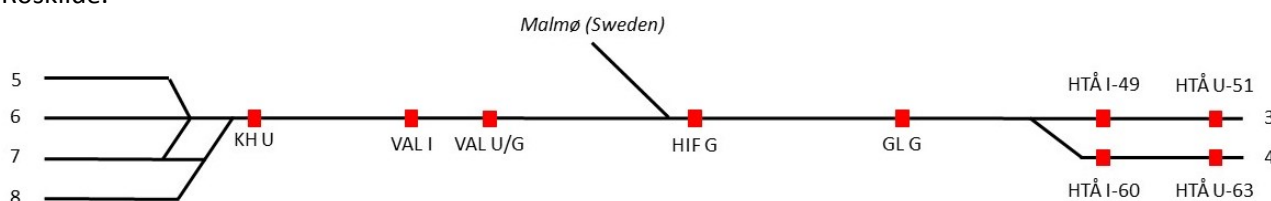


Figure 2: The Vestbane line scheme. Westbound track. The timing point locations are reported in red.

Table 1: Station codes and names on the Vestbane.

Station code	Station name	Distance from KH [km]	Type
KH	Copenhagen central station	0,0	Passenger Station
VAL	Valby	3,9	Halt
HIF	Hvidovre Fjern	7,3	Junction
GL	Glostrup	11,2	Technical station
HTÅ	Høje Taastrup	19,5	Passenger Station

In the resulting charts, the stations are identified by a code specified by the infrastructure manager. The station codes and names are reported in Table 1.

The set of timestamps included in the analysis state the scheduled and realized times of the trains at every timing point on the Vestbane during the period from August to December 2018, as this is the most recent long period without major modifications to the timetable. The daily timeframe of the records spans from 5AM to 8PM to exclude the influences of track possession for routine works and the consequent traffic modifications. A total of 118.965 records were collected and analyzed between Copenhagen central station and Høje Taastrup. The records include information about the operations and the timing points, such as the station name, track section ID, train ID, train category, scheduled time, and recorded deviation. The data is generated by Banedanmark's automatic train detection system, which uses the sensors from the

interlockings and the signaling system components. Typically, the track circuit boundaries do not correspond exactly to the platforms and an offset is generated between the time recorded by the automatic system and the actual time a train arrives at or departs from the platform. A correction factor was calculated by Banedanmark using statistical analyses of GPS positions of train trajectories in collaboration with the main rail operator, DSB (Richter, Landex, & Andersen, 2013). The recorded timestamps are, therefore, an approximation of the real platform times.

The timestamps are divided into three types, which describe the associated types of movement. "I" records indicate the arrival times at the stations (*Indkørsel*, Entrance), whereas "U" records indicate the departure times (*Udkørsel*, Exit). "G" records indicate the pass-through time in case of non-stopping trains (*Gennemkørsel*, pass-through) and are measured at the same locations as the "U" records.

The planned headways and differences in deviation across consecutive trains were calculated from the timestamps by means of the free software R 3.5.3 by the R Foundation for Statistical Computing. For every timing point, the conflicting movements of interest were identified in terms of track ID and type of records (I, U, or G).

The relationship between the planned headway and the realized difference in deviation was explored on a subset of the records, which only included passenger trains operated in the scheduled order. Freight and empty trains were excluded as there are fewer timestamps for these trains and they are characterized by larger variations in the recorded deviations (Corman & Kecman, 2018). The dataset was further filtered according to the sequences of trains, as the comparison between planned headway and realized difference in deviation, in fact, is only valid if the realized sequence of trains corresponds to the plan.

From (2), the minimum recorded difference in deviation constitutes an upper boundary for the actual headway buffer and does not necessarily correspond to its magnitude. For this reason, only a subset of the recorded minimum differences in deviation as a function of the planned headway can be considered in the regression to the headway buffer. As a starting point, the selection of the valid points is based on the number of observations recorded for each value of planned headway. The underlying assumption is that, for a large enough sample of observations of train sequences planned with a given headway, there finds at least one case of full recovery. In such cases, the full buffer contributed in the reduction of delay propagation and the delay of the second train of the pair was reduced by exactly an amount corresponding to the headway buffer. In this study, the selection of the valid points was based on the number of observations for every value of planned headway as a percentage of the total number of observations in the complete dataset. The percentage was defined for individual headway studies.

Results

Two representative graphs are reported in this article, as a result of the analysis of the Danish case. Figure 3 and Figure 4 show the relationship between recorded differences in deviation and planned headways. The minimum feasible headways were calculated for the main conflicting movements on the line and compared to the minimum feasible headway times measured through microsimulation. The results are reported in Table 2.

The simulation tests were operated in the commercial software RailSys 10.3.322, by Rail Management Consultants GmbH.

Difference in deviation from previous train vs. Planned headway: KH - U U - 261 261

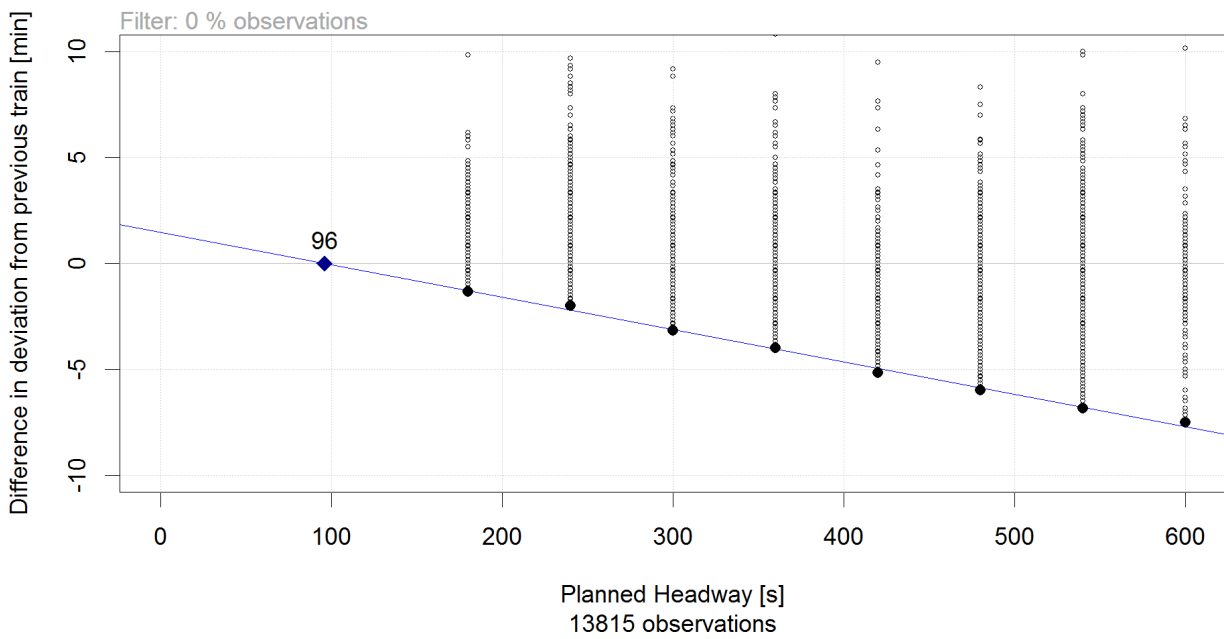


Figure 3: Difference in deviation in relation to the planned headway for departures from Copenhagen central station. The bold dots are the minimum differences in deviation recorded for given planned headways. The blue line is the regression line of the headway buffer as a function of the planned headway. The diamond is the calculated minimum feasible headway.

Difference in deviation from previous train vs. Planned headway: HTÅ - U I - 63 60

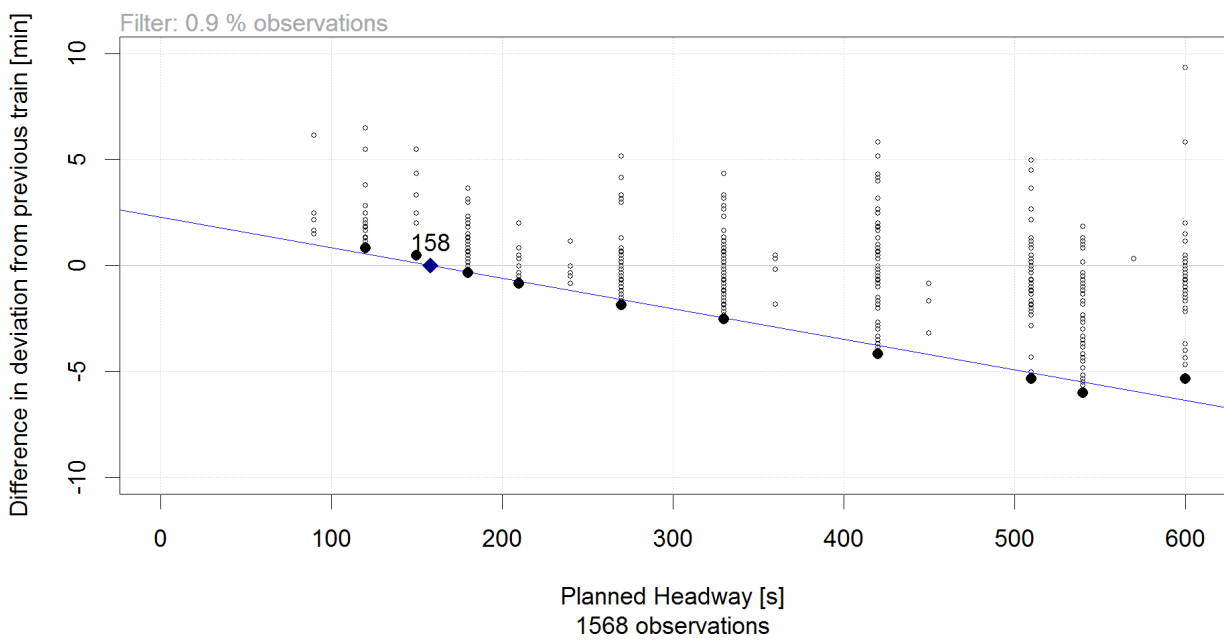


Figure 4: Differences in deviation in relation to the planned headway for sequences of exits and entrances at Høje Taastrup station, track 4. The bold dots are the minimum differences in deviation recorded for given planned headways. The blue line is the regression line of the headway buffer as a function of the planned headway. The diamond is the calculated minimum feasible headway.

Table 2: Results from historical data analysis compared to microsimulation.

Station	Registration pattern	Section ID 1	Section ID 2	Track no.	h_{\min} [s]		Difference [s]
					Historical data	Microsimulation	
KH	UU	261	261	5/6/7/8	96	94	-2
VAL	GI	2042	2033	2	118	113	-5
VAL	GU	2042	2042	2	142	139	-3
VAL	UG	2042	2042	2	101	116	15
VAL	UU	2042	2042	2	150	164	14
HIF	GG	452	452	2	64	82	18
HTÅ	UI	51	49	3	176	148	-28
HTÅ	UI	63	60	4	158	148	-10
HTÅ	UU	51	51	3	154	211	57
HTÅ	UU	63	63	4	234	211	-23
HTÅ	II	49	49	3	243	211	-32
HTÅ	II	60	60	4	236	211	-25
HTÅ	II	49	60	3-4	81	102	21
HTÅ	II	60	49	4-3	79	102	23

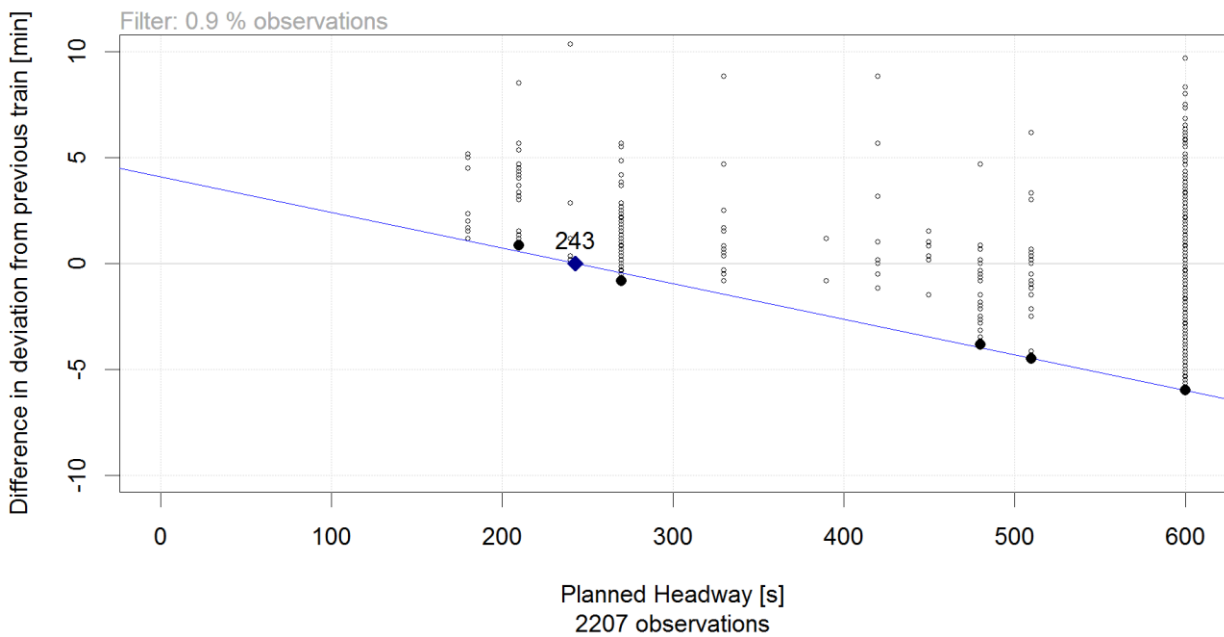
Discussion

Table 2 shows limited differences between the analysis of historical data and the microsimulation of minimum feasible headways. In general, the deviation between the two methods lies within a [-30, +30] s interval, apart from records at HTÅ, track 3. This specific case is affected by few outliers, possibly inaccurate time measures, shown in Figure 5. In particular, the estimated minimum feasible departure time at HTÅ track 3 seems infeasible, highlighting the necessity for further investigation.

In the other cases, the deviation between the two estimation methods finds partial explanation in the different granularity of the measuring systems. On the one hand, while it is possible to measure passing times with a second-precision in RailSys, the current time granularity for the trackside measurements on the Danish rail network is 10s. On the other hand, the microsimulation results depend on the quality of the modeling assumptions, including a deterministic minimum dwell time, and approximated driving behaviors. The presence of resulting negative buffers at HTÅ, visible in Figure 5, is noteworthy. At this station, a 4-tracked line section starts to fork into two lines at Roskilde, about 10 km beyond HTÅ. The minimum feasible headway between movements operated on the same track is clearly larger than movements occupying different tracks. The planned headway between trains originally scheduled on different tracks is smaller than the minimum feasible headway between movements operated on the same track. This is the case for points registered on the left side of the minimum feasible headway in Figure 5, left side. This results in a positive difference in deviation, namely a secondary delay.

Note that some of the influencing characteristics could not be measured. For example, the railway undertakings do not have to state the length and type of rolling stock used in operation, even though it might differ from the original plan. However, microsimulation tests suggested very limited differences in the liberation time of the blocking sections among different settings of rolling stock. The most relevant factor, the stopping pattern, is taken into account by means of the record type (I, U, or G).

Difference in deviation from previous train vs. Planned headway: HTÅ - II - 49 49



Difference in deviation from previous train vs. Planned headway: HTÅ - U U - 51 51

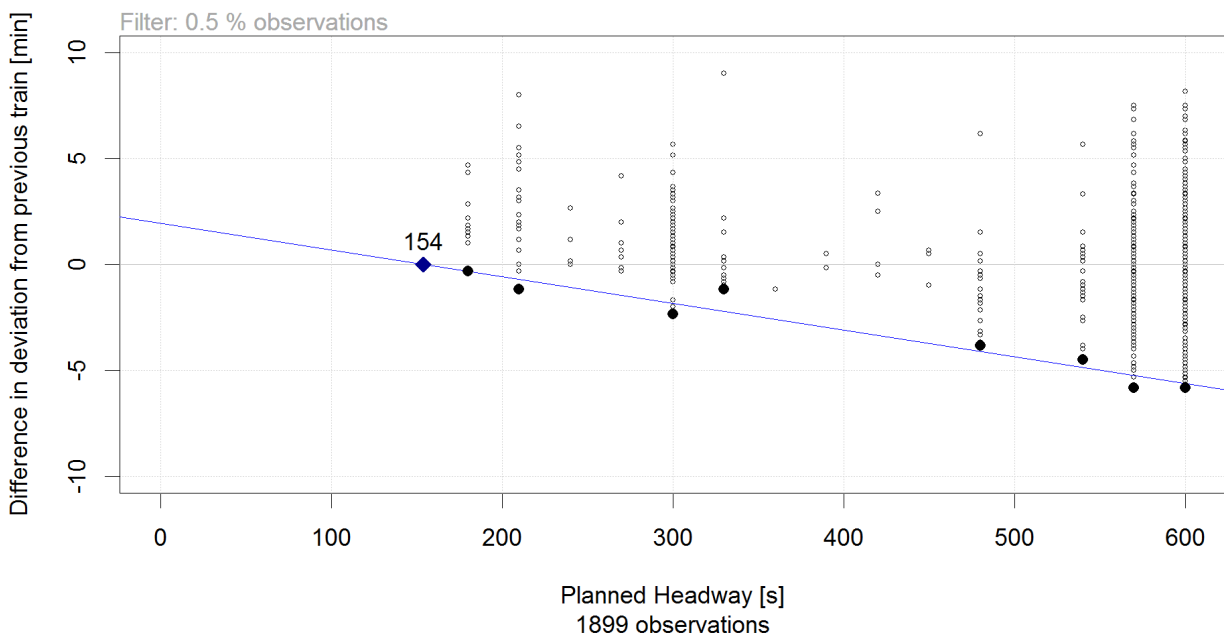


Figure 5: Minimum feasible headways at HTÅ, track 3. Arrival headways on the left, departure headways on the right.

Conclusions

This paper presents a historical data-based method to estimate the actual minimum feasible headway between conflicting movements in railway systems. The relationship between planned headways and recorded delays is investigated from the train timestamps automatically generated by the signaling system. The method is applied on the busiest railway line in Denmark and the results from recorded operations are validated through microsimulation.

The identified minimum feasible headways constitute the input data for multiple applications. Timetable optimization problems, simulation models at both mesoscopic and macroscopic level, and capacity and robustness assessment methods often require the minimum feasible headway times as input. The method

supports, thus, the improvement of railway schedules through a fact-based planning of the process times and buffers, as opposed to the current tradition of experience-based planning. Microsimulation models can also be calibrated and validated using the proposed method, through a systematic comparison of the minimum feasible headways measured from realized operation and from simulation. Further applications include the evaluation of the timetable reliability, as it is possible to extract the actual available headway buffer in the already planned schedules by subtracting the minimum feasible headways.

While previous methods described the relationship between headways and delay propagation from a theoretical perspective (Landex, 2008), this research presents a method based on the realized operation. Nevertheless, this method does not require detailed signal timestamps (Goverde & Meng, 2011; Daamen Winnie and Goverde, 2009; Richter T. , 2012), which simplifies the data acquisition process. The resulting minimum feasible headways clearly identify the potential conflicts in the timetables, whereas previous research based the identification of conflicts mainly on visual inspection of the delay and realized headway profiles (van Oort, Sparing, Brands, & Goverde, 2015). The found relationship between planned headway and realized recovery agrees with previous research (Yabuki, Ageishi, & Tomii, 2015; Corman & Kecman, 2018), even though this relationship had not been used to identify the minimum feasible headways. The Danish case study presented showed some weakness of the method against irregular data. In fact, a more sophisticated approach is under development to account for the recorded conditional distribution of differences in deviation for given values of planned headways. This will provide a method for assessing the probability that the minimum record value corresponds to the actual minimum possible difference in deviation, thus providing a better selection of the regression points and returning more accurate values of the minimum feasible headways.

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