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An Efficient Seven-Step Block Method for Numerical Solution of SIR and Growth Model

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Abstract- In this article, a new implicit continuous block method is developed using the interpolation and collocation techniques via Power series as the basis function. A constant step length within a seven-step interval of integration was adopted. The selected grid points were evaluated to get a continuous linear multistep method. The evaluation of the continuous method at the non-interpolation points produces the discrete schemes which form the block. The basic properties of the block method were investigated and found to be consistent, zero stable and hence convergent. The new method was tested on real life problems namely: SIR and Growth model. The results were found to compare favourably with the existing methods in terms of accuracy and efficiency.

Keywords- Block method, Growth Model, implicit, power series and SIR model.

1 Introduction

The direct numerical solution of first order initial value problem (IVP) of ordinary differential equations (ODEs) of the form in equation (1) is considered using linear multistep technique in this research. First order ODEs are important tools in solving real-life problems. Various natural phenomena are modeled using first order ODEs which are applied to many problems in physical sciences and engineering. Many problems in the form of (1) may not be easily solved analytically. Hence, numerical schemes are often developed to solve them.

$$y' = f(x, y), y(a) = y_0 \qquad (1)$$

Some authors have proposed linear multistep method to solve equation (1) (Awoyemi, 1992; Lambert, 1973). According to (Awoyemi, 1999), continuous linear multistep method has greater benefits over the discrete method in that the continuous linear multistep method gives better error estimation, provide a simplified coefficient for further analytical work at different points and guarantee easy appropriation of solution at all interior points of the integration interval. Predictor-corrector method for solving (1) was carried out by (Awoyemi, 2001; Onumanyi *et al*.1994), to mention a few. These authors individually implemented their methods with predictor-correct and adopted Taylor series expansion to supply starting values.

According to (Adesanya, 2012), the setback of the predictor-corrector method is that it is very costly as subroutines are very complicated to write because of the special techniques required to supply starting values and for varying the step size which leads to longer computer time and more human effort. Hence, it affects the accuracy of the method. Since the predictor-corrector method has several shortcomings, hence there is a need to develop other method to cater for the draw-backs. Therefore, scholars developed block method to take away the setback of predictor-corrector method (Jator and Li, 2009; Omole and Ogunware, 2018; Adeyefa and Fadaka, 2017).

In this research, the development of a seven point linear multistep block method for the numerical solution of SIR and Growth Model which are a class of first order IVPs of ODEs will be our focus.

2 Research Methodology

2.1 Method Derivation

In developing this method, power series of the form in equation (2) is considered as the approximate solution to (1), where k = 7 is the step length. Equation (3) is derived by differentiating (2).

$$y(x) = \sum_{j=0}^{k+1} a_j x^j \qquad (2)$$

$$y'(x) = \sum_{j=0}^{k+1} j a_j x^{j-1} = f(x, y) \qquad (3)$$

Interpolating (2) at $x_{n+j}, j = 4$ and collocating (3) at $x_{n+j}, j = 0, 1, \ldots, 7$. These equations are then combined to give a nonlinear system of equations of the form in equation (4).

Gaussian elimination technique is used in finding the values of a_j's in (4) which are then substituted into (2) to produce a continuous implicit scheme of the form

$$y(x) = \alpha_4(x) y_{n+4} + h\left(\sum_{j=0}^{k+1} \beta_j(x) f_{n+j}\right) \qquad (5)$$

Where y(x) is the numerical solution of the IVP, α_j and β_j are both constant.

$$f_{n+j} = f\left(x_{n+j}, y_{n+j}, y'_{n+j}\right) \qquad (6)$$

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$$\begin{bmatrix} 1 & x_{n+4} & x_{n+4}^2 & x_{n+4}^3 & x_{n+4}^4 & x_{n+4}^5 & x_{n+4}^6 & x_{n+4}^7 \\ 0 & 1 & 2x_n & 3x_n^2 & 4x_n^3 & 5x_n^4 & 6x_n^5 & 7x_n^6 \\ 0 & 1 & 2x_{n+1} & 3x_{n+1}^2 & 4x_{n+1}^3 & 5x_{n+1}^4 & 6x_{n+1}^5 & 7x_{n+1}^6 \\ 0 & 1 & 2x_{n+2} & 3x_{n+2}^2 & 4x_{n+2}^3 & 5x_{n+2}^4 & 6x_{n+2}^5 & 7x_{n+2}^6 \\ 0 & 1 & 2x_{n+3} & 3x_{n+3}^2 & 4x_{n+3}^3 & 5x_{n+3}^4 & 6x_{n+3}^5 & 7x_{n+3}^6 \\ 0 & 1 & 2x_{n+4} & 3x_{n+4}^2 & 4x_{n+4}^3 & 5x_{n+4}^4 & 6x_{n+4}^5 & 7x_{n+4}^6 \\ 0 & 1 & 2x_{n+5} & 3x_{n+5}^2 & 4x_{n+5}^3 & 5x_{n+5}^4 & 6x_{n+5}^5 & 7x_{n+5}^6 \\ 0 & 1 & 2x_{n+6} & 3x_{n+6}^2 & 4x_{n+6}^3 & 5x_{n+6}^4 & 6x_{n+6}^5 & 7x_{n+6}^6 \\ 0 & 1 & 2x_{n+7} & 3x_{n+7}^2 & 4x_{n+7}^3 & 5x_{n+7}^4 & 6x_{n+7}^5 & 7x_{n+7}^6 \end{bmatrix} \begin{bmatrix} a_0 \\ a_1 \\ a_2 \\ a_3 \\ a_4 \\ a_5 \\ a_6 \\ a_7 \\ a_8 \end{bmatrix} = \begin{bmatrix} y_{n+4} \\ f_n \\ f_{n+1} \\ f_{n+2} \\ f_{n+3} \\ f_{n+4} \\ f_{n+5} \\ f_{n+6} \\ f_{n+7} \end{bmatrix} \qquad (4)$$

Using the transformation $t = \frac{x - x_{n+6}}{h}, \frac{dt}{dx} = \frac{1}{h}$, we obtain the continuous scheme below.

$$\beta_0(t) = \left(-\frac{3267}{70}t^2 + \frac{3024}{5}t^5 - \frac{278}{945} + \frac{938}{5}t^3 + \frac{7776}{35}t^7 - \frac{8703}{20}t^4 + 6t - \frac{2484}{5}t^6\right)$$

$$\beta_1(t) = \left(-\frac{17982}{5}t^5 + \frac{11484}{5}t^4 - \frac{4014}{5}t^3 - \frac{1448}{945} + 126t^2 - \frac{52488}{35}t^7 + 3186t^6\right)$$

$$\beta_2(t) = \left(-189t^2 + \frac{7911}{5}t^3 + \frac{151632}{35}t^7 - \frac{8}{35} - \frac{106083}{20}t^4 - 8748t^6 + \frac{46008}{5}t^5\right)$$

$$\beta_3(t) = \left(7002t^4 - \frac{65826}{5}t^5 + 13338t^6 - \frac{48600}{7}t^7 - \frac{1784}{945} - 1898 + 210t^2\right)$$

$$\beta_4(t) = \left(-12204t^6 + \frac{57024}{5}t^5 + \frac{46656}{7}t^7 - \frac{22905}{4}t^4 - \frac{315}{2}t^2 + 1476t^3 + \frac{106}{945}\right)$$

$$\beta_5(t) = \left(-53946t^5 + \frac{3402}{5}t^2 - \frac{1207224}{35}t^7 + \frac{130248}{5}t^4 + \frac{301806}{5}t^6 - \frac{32562}{5}t^3 - \frac{72}{35}\right)$$

$$\alpha_4(t) = 9$$

$$\beta_6(t) = \left(-18468t^6 + \frac{64}{105} + \frac{79704}{5}t^5 - \frac{149769}{20}t^4 - 189t^2 + \frac{384912}{35}t^7 + \frac{9171}{5}t^3\right)$$

$$\beta_7(t) = \left(-\frac{10206}{5}t^5 + 2430t^6 - \frac{52488}{35}t^7 + \frac{162}{7}t^2 - \frac{8}{105} - \frac{4698}{5}t^4\right)$$

Evaluating the above continuous method at the end point, gives the discrete scheme

$$y_{n+7} = y_{n+4} + \frac{h}{4480}\begin{bmatrix} 45f_n - 373f_{n+1} + 1377f_{n+2} - 3033f_{n+3} \\ +5297f_{n+4} + 1377f_{n+5} + 6795f_{n+6} + 1325f_{n+7} \end{bmatrix} \qquad (7)$$

2.2 DERIVATION OF THE BLOCK

The combination of the discrete methods obtained by evaluating the continuous scheme at all the non-interpolation points yield the block below in matrix form by means of matrix inversion.

$$\begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} y_{n+1} \\ y_{n+2} \\ y_{n+3} \\ y_{n+4} \\ y_{n+5} \\ y_{n+6} \\ y_{n+7} \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} y_{n-1} \\ y_{n-2} \\ y_{n-3} \\ y_{n-4} \\ y_{n-5} \\ y_{n-6} \\ y_n \end{bmatrix} + h \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & \frac{5257}{17280} \\ 0 & 0 & 0 & 0 & 0 & 0 & \frac{41}{140} \\ 0 & 0 & 0 & 0 & 0 & 0 & \frac{265}{896} \\ 0 & 0 & 0 & 0 & 0 & 0 & \frac{278}{945} \\ 0 & 0 & 0 & 0 & 0 & 0 & \frac{265}{896} \\ 0 & 0 & 0 & 0 & 0 & 0 & \frac{41}{140} \\ 0 & 0 & 0 & 0 & 0 & 0 & \frac{5257}{17280} \end{bmatrix} \begin{bmatrix} f_{n-1} \\ f_{n-2} \\ f_{n-3} \\ f_{n-4} \\ f_{n-5} \\ f_{n-6} \\ f_n \end{bmatrix}$$

$$+h \begin{bmatrix} \frac{139849}{120960} & -\frac{4511}{4480} & \frac{123133}{120960} & -\frac{88547}{120960} & \frac{1537}{4480} & -\frac{11351}{120960} & \frac{275}{24192} \\ \frac{1466}{945} & -\frac{71}{420} & \frac{68}{105} & -\frac{1927}{3780} & \frac{26}{105} & -\frac{29}{420} & \frac{8}{945} \\ \frac{1359}{896} & \frac{1377}{4480} & \frac{5927}{4480} & -\frac{3033}{4480} & \frac{1377}{4480} & -\frac{373}{4480} & \frac{9}{896} \\ \frac{1448}{945} & \frac{8}{35} & \frac{1784}{945} & -\frac{106}{945} & \frac{8}{35} & -\frac{64}{945} & \frac{8}{945} \\ \frac{36725}{24192} & \frac{775}{2688} & \frac{4625}{2688} & \frac{13625}{241192} & \frac{1895}{2688} & -\frac{275}{2688} & \frac{275}{24192} \\ \frac{54}{35} & \frac{27}{140} & \frac{68}{35} & \frac{27}{140} & \frac{54}{35} & \frac{41}{140} & 0 \\ \frac{25039}{17280} & \frac{343}{640} & \frac{20923}{17280} & \frac{20923}{17280} & \frac{343}{640} & \frac{25039}{17280} & \frac{5257}{17280} \end{bmatrix} \begin{bmatrix} f_{n+1} \\ f_{n+2} \\ f_{n+3} \\ f_{n+4} \\ f_{n+5} \\ f_{n+6} \\ f_{n+7} \end{bmatrix} \qquad (8)$$

Writing out the block explicitly gives the following

$$y_{n+1} = y_n + h\begin{bmatrix} \frac{5257}{17280}f_n + \frac{139849}{120960}f_{n+1} - \frac{4511}{4480}f_{n+2} + \frac{123133}{120960}f_{n+3} \\ -\frac{88547}{120960}f_{n+4} + \frac{1537}{4480}f_{n+5} - \frac{11351}{120960}f_{n+6} + \frac{275}{24192}f_{n+7} \end{bmatrix} \quad (9)$$

$$y_{n+2} = y_n + h\begin{bmatrix} \frac{41}{140}f_n + \frac{1466}{945}f_{n+1} - \frac{71}{420}f_{n+2} + \frac{68}{105}f_{n+3} \\ -\frac{1927}{3780}f_{n+4} + \frac{26}{105}f_{n+5} - \frac{29}{420}f_{n+6} + \frac{8}{945}f_{n+7} \end{bmatrix} \quad (10)$$

$$y_{n+3} = y_n + h\begin{bmatrix} \frac{265}{896}f_n + \frac{1359}{896}f_{n+1} + \frac{1377}{4480}f_{n+2} + \frac{5927}{4480}f_{n+3} \\ -\frac{3033}{4480}f_{n+4} + \frac{1377}{4480}f_{n+5} - \frac{373}{4480}f_{n+6} + \frac{9}{896}f_{n+7} \end{bmatrix} \quad (11)$$

$$y_{n+4} = y_n + h\begin{bmatrix} \frac{278}{945}f_n + \frac{1448}{945}f_{n+1} + \frac{8}{35}f_{n+2} + \frac{1784}{945}f_{n+3} \\ -\frac{106}{945}f_{n+4} + \frac{8}{35}f_{n+5} - \frac{64}{945}f_{n+6} + \frac{8}{945}f_{n+7} \end{bmatrix} \quad (12)$$

$$y_{n+5} = y_n + h\begin{bmatrix} \frac{265}{896}f_n + \frac{36725}{24192}f_{n+1} + \frac{775}{2688}f_{n+2} + \frac{4625}{2688}f_{n+3} \\ +\frac{13625}{24192}f_{n+4} + \frac{1895}{2688}f_{n+5} - \frac{275}{2688}f_{n+6} + \frac{275}{24192}f_{n+7} \end{bmatrix} \quad (13)$$

$$y_{n+6} = y_n + h\begin{bmatrix} \frac{41}{140}f_n + \frac{54}{35}f_{n+1} + \frac{27}{140}f_{n+2} + \frac{68}{35}f_{n+3} \\ +\frac{27}{140}f_{n+4} + \frac{54}{35}f_{n+5} + \frac{41}{140}f_{n+6} \end{bmatrix} \quad (14)$$

$$y_{n+7} = y_n + h\begin{bmatrix} \frac{5257}{17280}f_n + \frac{25039}{17280}f_{n+1} + \frac{343}{640}f_{n+2} + \frac{20923}{17280}f_{n+3} \\ +\frac{20923}{17280}f_{n+4} + \frac{343}{640}f_{n+5} + \frac{25039}{17280}f_{n+6} + \frac{5257}{17280}f_{n+7} \end{bmatrix} \quad (15)$$

3 Analysis of Basic Properties of The Block

3.1 Order and Error Constant of the Block

Definition: A block linear multistep method of first order ODEs is said to be of order p if $\bar{c}_0 = \bar{c}_1 = \bar{c}_2 = \ldots = \bar{c}_p = 0$, $\bar{c}_{p+1} \neq 0$. Thus c's are the coefficients of h and y functions while $C_{P+1}$is the error constant (Omar and Abdelrahim, 2016) and (Omole and Ogunware, 2018). For our seven-step method, expanding the block in Taylor series expansion gives

$$\begin{bmatrix}
\sum_q^{\infty}\left(\frac{h^q}{q!}y^q\right) - \left(y_{n+1} - y_n - \frac{5257}{17280}hy'\right) - \sum_q^{\infty}\left(\frac{h^{q+1}}{q!}y^{q+1}\right)\begin{pmatrix}\left(\frac{139849}{120960}\right)(1)^q + \left(-\frac{451}{4480}\right)(2)^q + \left(\frac{123133}{120960}\right)(3)^q \\ +\left(-\frac{88547}{120960}\right)(4)^q + \left(\frac{1537}{4480}\right)(5)^q + \left(-\frac{11351}{120960}\right)(6)^q\left(\frac{275}{24192}\right)(7)^q\end{pmatrix} \\
\sum_q^{\infty}\left(\frac{(2h)^q}{q!}y^q\right) - \left(y_{n+2} - y_n - \frac{41}{140}hy'_{n n}\right) - \sum_q^{\infty}\left(\frac{h^{q+1}}{q!}y^{q+1}\right)\left(\begin{pmatrix}+\frac{1466}{945}(1)^q + \left(-\frac{71}{420}\right)(2)^q + \left(\frac{68}{105}\right)(3)^q \\ +\left(-\frac{1927}{3780}\right)(4)^q + \left(\frac{26}{105}\right)(5)^q + \left(-\frac{29}{420}\right)(6)^q + \left(\frac{8}{945}\right)(7)^q\end{pmatrix}\right) \\
\sum_q^{\infty}\left(\frac{(3h)^q}{q!}y^q\right) - \left(y_{n+3} - y_n - \frac{265}{896}hy'_n\right) - \sum_q^{\infty}\left(\frac{h^{q+1}}{q!}y^q\right)\begin{pmatrix}\left(\frac{1359}{896}\right)(1)^q + \left(\frac{1377}{4480}\right)(2)^q + \left(\frac{5927}{4480}\right)(3)^q \\ +\left(-\frac{3033}{4480}\right)(4)^q + \left(\frac{1377}{4480}\right)(5)^q + \left(-\frac{373}{4480}\right)(6)^q + \left(\frac{9}{896}\right)(7)^q\end{pmatrix} \\
\sum_q^{\infty}\left(\frac{(4h)^q}{q!}y^q\right) - \left(y_{n+4} - y_n - \frac{278}{945}hy'_{n n}\right) - \sum_q^{\infty}\left(\frac{h^{q+1}}{q!}y^q\right)\begin{pmatrix}\left(\frac{1448}{945}\right)(1)^q + \left(\frac{8}{35}\right)(2)^q + \left(\frac{1784}{945}\right)(3)^q \\ +\left(-\frac{106}{945}\right)(4)^q + \left(\frac{8}{35}\right)(5)^q + \left(-\frac{64}{945}\right)(6)^q + \left(\frac{8}{945}\right)(7)^q\end{pmatrix} \\
\sum_q^{\infty}\left(\frac{(5h)^q}{q!}y^q\right) - \left(y_{n+5} - y_n - \frac{265}{896}hy'_{n n}\right) - \sum_q^{\infty}\left(\frac{h^{q+1}}{q!}y^q\right)\begin{pmatrix}\left(\frac{36725}{24192}\right)(1)^q + \left(\frac{775}{2688}\right)(2)^q + \left(\frac{4625}{2688}\right)(3)^q \\ +\left(\frac{13625}{24192}\right)(4)^q + \left(\frac{1895}{2688}\right)(5)^q + \left(-\frac{275}{2688}\right)(6)^q + \left(\frac{275}{24192}\right)(7)^q\end{pmatrix} \\
\sum_q^{\infty}\left(\frac{(6h)^q}{q!}y^q\right) - \left(y_{n+6} - y_n - \frac{41}{140}hy'_{n n}\right) - \sum_q^{\infty}\left(\frac{h^{q+1}}{q!}y^q\right)\begin{pmatrix}\left(\frac{54}{35}\right)(1)^q + \left(\frac{27}{140}\right)(2)^q + \left(\frac{68}{35}\right)(3)^q \\ +\left(\frac{27}{140}\right)(4)^q + \left(\frac{54}{35}\right)(5)^q + \left(\frac{41}{140}\right)(6)^q\end{pmatrix} \\
\sum_q^{\infty}\left(\frac{(7h)^q}{q!}y^q\right) - \left(y_{n+7} - y_n - \frac{5257}{17280}hy'_{n n}\right) - \sum_q^{\infty}\left(\frac{h^{q+1}}{q!}y^q\right)\begin{pmatrix}\left(\frac{250}{17280}\right)(1)^q + \left(\frac{343}{640}\right)(2)^q + \left(\frac{20923}{17280}\right)(3)^q \\ +\left(\frac{20923}{17280}\right)(4)^q + \left(\frac{343}{640}\right)(5)^q + \left(\frac{25039}{17280}\right)(6)^q + \left(\frac{5257}{17280}\right)(7)^q\end{pmatrix}
\end{bmatrix} = \begin{bmatrix}0\\0\\0\\0\\0\\0\\0\end{bmatrix} \quad (16)$$

Hence, the block is of uniform order 8, with error constant
$[0.00133246803, 0.0060062, 0.0085425, 0.0214368, 0.0093566, 0.0083566, 0.0073545\ 0.0063565]^T$

3.2 Zero Stability of the Block Method

Given the general form of block method

$$A^{(0)}Y_m = A^{(i)}Y_{m-1} + h^{\mu}[B^{(i)}F_m + B^{(0)}F_{m-1}]$$

A block method is said to be zero stable, if the roots

$$det[\lambda A^{(0)} - A^{(i)}] = 0$$

of the first characteristic polynomial satisfy $|\lambda| \leq 1$ and for the roots with $|\lambda| \leq 1$, the multiplicity must not exceed the order of the differential equation (Omar and Kuboye, 2015). For our method,

$$A = \left[z\begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix} - \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix} \right] = 0$$

$$A = z^6(z-1) = 0, z = 0,0,0,0,0,0,0$$

Hence the block is zero stable

3.3 Consistency and Convergence

Our new block method is consistent since the order of each of the method is greater than 1 (Olanegan *et al*, 2015)

Theorem 1: Convergence

According to (Lambert, 1973), the necessary and sufficient condition for a linear multistep method to be convergent is for it to be consistent and zero stable. Also, (Akinfenwa et al, 2016), substantiated Lambert's stand on convergence as thus;

convergence = consistency + zero-stability.

Hence the new block method is convergent.

4 Numerical Experiments

Here, the performance of the new block method is examined on SIR and Growth model. The results obtained from the test examples are shown in tabular form. We used MATLAB codes for the computational purposes.

4.1 Test Problem 1 (SIR Model)

The SIR model is an epidemiological model that computes the theoretical number of people infected with a contagious illness in a closed population over time. The name of this class of models derives from the fact that they involve coupled equations relating the number of susceptible people$S(t)$, number of people infected$I(t)$, and the number of people who have recovered$R(t)$. This is a good and simple model for many infectious diseases including measles, mumps and rubella. It is given by the following three coupled equations:

$$\left\{\begin{aligned} \frac{dS}{dt} &= \mu(1-S) - \beta IS \\ \frac{dI}{dt} &= -\mu I - \gamma I + \beta IS \\ \frac{dR}{dt} &= -\mu R + \gamma I \end{aligned}\right\} \quad (17)$$

where μ, γ, β are positive parameters to be determined. Define y to be,

$$y = S + I + R$$

and adding the equations in (16) above, we obtain the following evolution equation for y,

$$y' = \mu(1-y)$$

Taking $\mu = 0.5$and attaching an initial condition $y(0) = 0.5$ (for a particular closed population). We obtain,

$$\frac{dy}{dt} = 0.5(1-y), y(0) = 0.5$$

whose exact solution is

$$y(t) = 1 - 0.5e^{-05t}$$

Source: (Sunday *et al*, 2013)

Table 1. Comparison of the result of test problem 1 with (Sunday *et al*, 2013) and (Omar and Adeyeye, 2016)

X	Exact solution	Computed solution	Error	Error (Omar and Adeyeye, 2016)	Error (Sunday et al, 2013)
0.1	0.524385287749642990	0.524385287749652100	9.103829E-015	4.956150E-06	5.574430E-12
0.2	0.547581290982020240	0.547581290982027350	7.105427E-015	4.725970E-06	3.946177E-12
0.3	0.569646011787471100	0.569646011787479980	8.881784E-015	8.979940E-06	8.183232E-12
0.4	0.590634623461009150	0.590634623461030350	2.120526E-014	8.552430E-06	3.436118E-11
0.5	0.610599608464297510	0.610599608464434280	1.367795E-013	1.219300E-05	1.929473E-10
0.6	0.629590889659141120	0.629590889659939370	7.982504E-013	1.160780E-05	1.879040E-10
0.7	0.647655955140643340	0.647655955144342150	3.698819E-012	1.471310E-05	1.776835E-10

4.2 Test Problem 2 (Growth Model)

A bacteria culture is known to grow at a rate proportional to the amount present. After one hour, 1000 strands of the bacteria are observed in the culture; and after four hours, 3000 strands. Find the number of strands of the bacteria present in the culture at time $t: 0 \leq t \leq 1$. Let $N(t)$, denote the number of bacteria strands in the culture at time t, the initial value problem modeling this problem is given by,

$$\frac{dN}{dt} = 0.366N, N(0) = 694$$

The exact solution is given by

$$N(t) = 694e^{0366t}$$

Source: (Sunday *et al*, 2013)

Table 2. Comparison of the result of test problem 2 with (Sunday *et al*, 2013)

X	Exact solution	Computed solution	Error	Error (Sunday et al, 2013)
0.1	719.870950484131980000	719.870950484132660000	6.821210E-013	1.830358E−011
0.2	746.706318949463250000	746.706318949463930000	6.821210E-013	1.250555E−011
0.3	774.542056995183660000	774.542056995184340000	6.821210E-013	1.227818E−011
0.4	803.415456425155070000	803.415456425154730000	3.410605E-013	3.137757E−011
0.5	833.365199208096560000	833.365199208089050000	7.503331E-012	2.216893E−010
0.6	864.431409300187850000	864.431409300137600000	5.024958E-011	2.060005E−010
0.7	896.655706399515910000	896.655706399278530000	2.373781E-010	2.171419E−010

4.3 Discussion of Result

From the numerical solution of the two test problems (SIR and Growth model) solved by the new method, the result displayed in table (1) showed the superiority of the new block method over that of (Sunday *et al*, 2013) and (Omar and Adeyeye, 2016). In table (2), it is clearly observed that the new block method outperforms the method of (Sunday *et al*, 2013) in terms of accuracy for the solution of test problem 2.

5 Conclusion

This article has proposed a new seven-step scheme for the computational solution of SIR and Growth model which are a class of first order IVPs of ODEs. From the two test problems solved by the new scheme, the new method has been proven to be effective in handling first order ODEs initial value problems directly. This fact is evidently seen from the accuracy of the numerical results presented. Hence the new method is accurate, efficient and consistent.

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References

Adesanya, A.O. (2011): Block Methods for the solution of higher order initial value problems in ordinary differential equations. Unpublished Ph.D Thesis, Federal University of Technology, Akure, Nigeria.

Adeyefa, E. O. & Fadaka, O., (2017): A Step Block Algorithm for Solving First Order Initial Value Problems in Ordinary Differential Equations, FUOYE Journal of Engineering and Technology, 2(1), 110-113.

Akinfenwa, O. A., Ogunseye, H. A. & Okunuga, S. A. (2016). Block Hybrid Method for Solution of Fourth Order Ordinary Differential equations, Nigerian Journal of Mathematics and Applications, 25, 140 – 150.

Awoyemi, D. O., (1992). On some continuous linear multistep methods for initial value problems. Unpublished Ph. D Thesis. University of Ilorin, Nigeria.

Awoyemi, D. O. (1999). A Class of Continuous Methods for General Second Order Initial Value Problem in Ordinary Differential Equation. Inter. J. Computer math, 72, 29-37.

Awoyemi, D.O. (2001). A new sixth order algorithm for general second order ordinary differential equations. International Journal of Computer Math.77, 117-124.

Jator, S.N. & Li, J. (2009). A self-stationary linear multistep method for a direct solution of general second order IVPs; International Journal of Computer Math. 86(5), 817-836.

Lambert, J. D. (1973). Computational methods in ordinary differential equations. John Wiley and Sons, New York.

Olanegan, O. O., Ogunware, B. G., Omole E. O. Oyinloye, T. S. & Enoch, B. T. (2015). Some Variable Hybrids Linear Multistep Methods for Solving First Order Ordinary Differential Equations Using Taylor's Series; IOSR Journal of Mathematics 11, 08-13.

Omar, Z. & Abeldrahim, R. (2016). New Uniform Order Single Step Hybrid Block Method for Solving Second Order Ordinary Differential Equations; International Journal of Applied Engineering Research 11(4), 2402-2406.

Omar, Z. & Adeyeye, O. (2016). Numerical Solution of First Order Initial Value Problems Using a Self-Starting Implicit Two-Step Obrechkoff-Type Block Method. Journal of Mathematics and Statistics 12(2):127-134.

Omar, Z. & Kuboye, J. O. (2015). Computation of an Accurate Implicit Block Method for Solving Third Order Ordinary Differential Equations Directly; Global Journal of Pure and Applied Mathematics. 11,177-186.

Omole, E. O. & Ogunware, B. G. (2018). 3-Point Single Hybrid Block Method (3PSHBM) for Direct Solution of General Second Order Initial Value Problem of Ordinary Differential Equations. Journal of Scientific Research & Reports 20(3): 1-11; ISSN: 2320-0227.

Onumanyi, P., Awoyemi, D.O., Jator, S.N & Sirisena, U.W. (1994). New Linear Multistep Methods with Continuous Coefficients for First Order Initial Value Problem. J. Nig., Math. SOC. 13; 37-51.

Sunday, J., Odekunle M.R. & Adesanya, A.O. (2013). Order six block integrator for the solution of first order ordinary differential equations. Int. J. Math. Soft Comput., 3: 87-96.