Systemic Risk, Interbank Market Contagion, and the Lender of Last Resort Function

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Abstract

We develop a theoretical model examining the financial stability policy of a central bank serving as both the lender of last resort and the regulator of the financial system. The model accommodates the possibility of financial contagion through interbank market linkages, and adverse feedback from the financial system to the real economy. It identifies the relative riskiness of the agents in the financial system, the probability of systemic distress, and the expected duration of credit supply reduction as the key factors influencing the design of financial stability policy. Model simulations indicate the existence of a substitution effect between reducing the expected scope of a central bank's assistance to an institution in distress and increasing bank reserve requirements.

JEL classification: E58, G01, G21, G28.

Keywords: Financial Stability, Central Bank Policy, Lender of Last Resort, Banking Crisis, Bank Regulation.

1 Introduction

The shortcomings of a financial stability policy based upon microprudential bank regulation and the provision of liquidity to solvent but illiquid institutions were revealed by the 2007 - 2009 financial crisis. The severity of this crisis initiated a fundamental re-evaluation of the role that central banks and bank capital regulation play in promoting the resilience of an increasingly complex and interconnected financial system. The regulatory response, initiated by the Third Basel Capital Accord, centres on increasing bank capital requirements and introducing new capital liquidity standards. However, the question of precisely identifying the targets and objectives of a new stability framework remains the subject of an ongoing academic and policy debate.

As bank failures may generate negative real economic externalities, and thus damage the fiscal position of the economy, recent approaches to designing financial stability frameworks are grounded in close coordination between the central bank and fiscal authorities (Allen, 2016). Freixas and Parigi (2014) conclude that the traditional view, namely that a central bank should only assist solvent but illiquid institutions often leads to a simplistic analysis of financial stability, and to an artificial separation between lending of last resort and bank bail-out interventions. Gorton and Huang (2004) point out that while the central bank should be committed to alleviating liquidity strains in the financial system, fiscal bailouts of insolvent banks provide an additional source of emergency liquidity. Reflecting this position, a distinctive feature of the analytical framework we develop is that it makes no distinction between central bank liquidity provision and government bail-outs, and treats both as a policymaker's decision to assist a bank in distress.

A second contribution of our modelling approach is to investigate the combined impact of bank regulatory reserve requirements and a central bank's commitment to act as the lender of last resort on the operation of an economic system in which financial sector dependencies are mediated through an interbank loan market. We consider different regulatory regimes, varying in the extent of policymaker's interventions. Specifically, the stylised economic system consists of a central bank serving as both lender of last resort and regulator of the financial system, two representative commercial banks interacting through an interbank market, and a non-financial, economic sector producing real output. We identify the appropriate design for central bank regulatory policy and market intervention when the objective is to maximise the difference between the expected economic output of the system and the costs associated with financial distress. In this context, we offer a holistic view of financial stability, which recognises the interdependencies between the financial system and the real economy.

Our central results can be briefly summarised. First, we determine key factors influencing the design of the optimal central bank financial stability policy to be the relative riskiness of financial institutions in the system, the probability of a financial crisis becoming systemic, and the expected duration of any ensuing credit supply reduction. Second, while central bank intervention which guarantees repayment of interbank loans is the preferred crisis response strategy in many instances, providing a bank in distress with direct balance sheet assistance becomes more desirable when either activity in the interbank market or the expected duration of bank failure-induced credit supply reductions increase. Third, our results establish the existence of a substitution effect between central bank policies designed to moderate the expectations of financial institutions with regard to receiving assistance, and regulations increasing bank reserve requirements. Finally, the model emphasises the paramount importance of a central bank's commitment to fostering the stable functioning of the interbank money market in times of economic stress.

2 Related literature

The various literatures analysing a central bank's lender of last resort function, financial contagion, and prudential bank regulation have been evolving largely independently until recently. Our paper contributes to synthesis of this research agenda by attempting to develop a more general, unified theory connecting financial stability and regulatory policy.

The original objectives of a lender of last resort are clarified in Thornton (1802) and Bagehot (1873), and suggest that "very large loans at very high interest rates are the best remedy for the worst malady of the money market [...] Any notion that money is not to be had, or that it may not be had at any price, only raises alarm to panic, and enhances panic to madness"(Bagehot, 1873: pp. 56 - 57). This claim finds support in Diamond and Dybvig's (1983) formulation, which shows that the efficient functioning of the banking system requires either a formal deposit insurance scheme or a central bank's commitment to lend to solvent but illiquid institutions. Such guarantees serve to eliminate the possibility that self-fulfilling prophecies of bank runs will destabilise the system.

During many crisis periods, however, illiquidity and insolvency often become indistinguishable. One consequence of this identification problem is that central banks may abandon the traditional principles associated with the lender of last resort function by extending loans or engaging in large-scale open-market purchases of the non-performing assets of any non-depository institution deemed to be systemically important (Buiter, 2008, Aglietta and Mojon, 2014). Receiving liquidity assistance, however, enables financial institutions to escape market discipline and promotes forbearance (Freixas and Parigi, 2014). Indeed, the empirical evidence in Dam and Koetter (2012) indicates that default probabilities in the financial system increase significantly following a policymaker's intervention. This trade-off between preventing financial contagion and creating moral hazard is formalised in Goodhart and Huang's (2005) lender of last resort model. The existence of a lender of last resort also affects financial institutions' financing decisions, particularly in terms of the liquidity of the reserves held, as they come to expect that the central bank's collateral quality policy may be loosened under crisis conditions (Wagner, 2007, Ratnovski, 2009, Koulischer and Struyven, 2014).

To address this problem, Goodhart and Huang (2005) propose an intervention strategy based around the notion of constructive ambiguity, the doctrine of not specifying in advance the precise goals and targets of financial stability policy and the formal mechanism of any central bank intervention. They argue that this encourages financial institutions to behave as if they are not protected, although the central bank may actually stand ready to intervene in a crisis. Nijskens (2014) highlights the fact that the credibility of such an approach is largely determined by the central bank's reputation for strictness. The recent financial crisis, however, has raised questions concerning the efficacy of a constructive ambiguity approach. Commentators argue that the lack of a clear understanding of how central banks are going to respond in a crisis leads to sudden significant changes in assistance expectations, with potentially devastating consequences for financial system stability (Vinogradov, 2012).[1]

Consequently, our model departs from the traditional analysis of constructive ambiguity. We examine a system in which the central bank's policy is common knowledge, thereby allowing it to efficiently anchor financial institutions' assistance expectations. This approach is closely akin to forward guidance on future interest rates, in which the central bank communicates the details of its monetary policy in an attempt to influence private expectations, aiming to reduce the resource misallocation that incorrectly held expectations may induce[2] (Cordella and Yeyati, 2002, Blinder, et al., 2008).

Many commentators focus on the interbank money market as the most prominent channel of financial contagion in a banking system. The interbank market interconnects banks' balance sheets, and serves as the channel to rapidly propagate one agent's distress throughout the financial system (Rochet and Tirole, 1996, Allen and Gale, 2000, Freixas, et al., 2000, Kiyotaki and Moore, 2002).[3] By intervention in the interbank market, the lender of last resort seeks to contain a crisis before it is transmitted any further (Goodhart and Huang, 2005). Further, interbank market interest rates transmit the effects of a change

[1] This point is illustrated by the changes in the level of the VIX index in late 2008, which increased from its long-term average of approximately 20 points to almost 90 points following the news that Lehman Brothers would be allowed to fail and yet AIG would be rescued a day later.

[2] This reflects the spirit of Friedman's (1962) analysis, who argues that following a well-publicised policy has favourable effects on people's attitudes and expectations, which in turn leads to better outcomes than if the same policy actions are implemented on a discretionary basis.

in the central bank's policy rate into the real economy, and are now commonly used as benchmark rates in pricing a variety of derivatives used by non-financial firms (International Monetary Fund, 2008, Brunnermeier, 2009). Thus, as a result of the consequent impact on the real sector of the economy, the costs of a sudden evaporation of confidence in this market can be substantial, and severely impede a central bank's ability to achieve its policy objectives (Nagel, 2013). Domanski, et al. (2014) argue that the growing importance of the interbank market calls for development of new operating frameworks that would allow for greater flexibility during times when banking sector liquidity is compromised. As our model explicitly accounts for the effects of financial contagion arising out of liquidity channel linkages intermediated through the interbank market, it contributes to identifying how a central bank may achieve this objective.

Existing theories of microprudential regulation are comprehensively surveyed in Dewatripont and Tirole (1994) and VanHoose (2007). This regulatory approach focuses on limiting financial distress in individual institutions by ensuring that they are well-capitalised and managed prudently (Borio, 2003). The regulatory paradigm shift towards macroprudential regulation seeks to limit the risk of system-wide financial distress resulting in a significant loss of real output (Borio, 2003). Galati and Moessner (2010) and Claessens (2014) provide an overview of macroprudential policy tools, detailing the distinctions between the two classes of regulatory regime. Repullo and Suarez (2013) argue that macroprudential regulations are less procyclical and result in an increased accumulation of capital reserves in economic expansions, thereby increasing the resilience of the financial system.

Recent regulatory changes, however, have been initiated without any solid theoretical foundations lending support to their efficacy. Examinations of their effectiveness primarily undertake qualitative assessments, or provide early estimates of the economic costs and resulting impact of the current transition towards macroprudentiality (e.g., Slovik and Cournède, 2011, or Angelini, et al., 2014). Arnold, et al. (2012) discuss the practical problems in accurately identifying systemic risk, a core concept around which the new framework is built. Giese, et al. (2013) survey the literature to identify the possible transmission channels of macroprudential policy and its effects on credit supply. They conclude that *ex ante* it is impossible to identify the effects with certainty. Mészáros (2013) emphasises the fact that the new framework suffers from a number of internal and technical difficulties, arguing that it may require an unprecedented level of intervention in order to achieve its new objectives. The theory of systemic risk outlined by Acharya (2009) suggests that any system relying on the use of risk-weighted assets is fundamentally unsuitable in a systemic context, and ought to be replaced by a framework based on

[3] Notwithstanding the fact that Acemoglu, et al. (2015) show that shocks of sufficiently small magnitude are more efficiently absorbed by a more densely interconnected financial network.

correlations between banks' asset returns. Van den Heuvel's (2008) model shows that bank capital requirements reduce banks' ability to create liquidity in the economy, creating a welfare cost equal to a 1% permanent reduction in consumption.

One contribution of our model is that it accommodates this adverse relation between bank capital requirements, credit supply and economic output. A further contribution is that its formulation enables us to examine the joint effects of central bank regulation and lending of last resort on the efficient design of financial stability policy. Finally, we also contribute to the literature investigating the appropriate institutional allocation of the roles of lender of last resort and regulator of the banking system. Goodhart and Schoenmaker (1995) find no empirical evidence in support of either combining or separating these functions. At the same time, Kahn and Santos (2005) suggest that while centralising these roles may lead to excessive forbearance, a multi-institutional arrangement may instead create substantial costs arising from coordination failures. In contrast, our results suggest that combining the functions of the lender of last resort and the regulator of the banking system may improve the effectiveness of financial stability policy.

3 The model economy

The model economy consists of a central bank, two representative commercial banks connected through an interbank market, and firms operating in the real economy. The banks use liabilities and equity capital to provide loans to non-financial firms who invest in positive net present value projects. The central bank's policy objective is to appropriately deploy its mandate as lender of last resort and banking system regulator in order to maximise the difference between the real economic output generated in the system and the potential costs of economic distress. In this section we first detail the nature of the agents' behaviour in the model. We then proceed to discuss the structure of the interaction between them.

3.1 The commercial banks

Initially, at $t = 0$ the two commercial banks are of equal size, but have different exogenously-determined risk profiles, denoted prudent and risky. Both banks finance their operations through liabilities and equity capital, and maintain a certain level of central bank mandated highly liquid reserves dependent upon the amount of risky assets they hold. The time to maturity of the liabilities is sufficiently distant to make them irrelevant for decision-making within the time frames considered in the model. Thus, the levels of both liabilities and equity capital are assumed to be fixed.

The interbank money market is the conduit through which the banks interact to reallocate their initial endowment of risky assets. The lending bank is characterised as the prudent bank because it diversifies its portfolio of assets. The risky bank borrows in the interbank market in order to leverage its investment position, thereby taking on more risk. The risky bank does not commit the entire interbank loan to its risky investment, as it uses a fraction to increase the mandated amount of reserves it holds. Table 1 depicts the balance sheet structures of the two banks. Panel A captures their initial balance sheet position, Panels B and C show the changes following interbank market transaction.

Table 1: Balance sheet structures of the two commercial banks

Panel A: Initial balance sheet structure	
Assets	**Liabilities and Capital**
Risky Assets (a_t)	Liabilities (l)
Highly Liquid Reserves (k_t)	Equity Capital (c)
Panel B: Post-transaction balance sheet of the risky bank	
Assets	**Liabilities and Capital**
Risky Assets + Interbank Loan ($a_t + \frac{1}{1+\kappa} i_t$)	Interbank Loan (i_t)
Highly Liquid Reserves ($k_{u,t}$)	Liabilities (l)
	Equity Capital (c)
Panel C: Post-transaction balance sheet of the prudent bank	
Assets	**Liabilities and Capital**
Risky Assets - Interbank Loan ($a_t - i_t$)	Liabilities (l)
Interbank Loan (i_t)	Equity Capital (c)
Highly Liquid Reserves ($k_{s,t}$)	

This table outlines the initial, and the post-transaction balance sheet structures of the two commercial banks in the model.

The interbank loan pays a gross interest rate of ρ, which is proportional to the additional leverage the interbank loan generates, thereby accounting for the enhanced risk that the risky bank may default. Specifically,

$$\rho = 1 + \rho_0 \times \frac{i_t}{a_t} \tag{1}$$

where ρ_0 is a scaling factor.

The level of highly liquid reserves that the central bank requires the commercial banks to maintain corresponds to a given proportion of the value of risky assets that they hold. It is stipulated in the form of a total regulatory reserve ratio, κ.

Issuing an interbank loan is equivalent to reallocating endowment capital from risky assets to the interbank market, without changing the overall size of the prudent bank's balance sheet. Borrowing in that market increases the size of the risky bank's balance sheet. As a result, the levels of liquid reserves held by the two banks after the transaction will differ, as shown in equations (2) and (3) for the prudent and the risky bank respectively.

$$k_{s,t} = \kappa \times [(a_t - i_t) + i_t] = \kappa a_t \tag{2}$$

$$k_{u,t} = \kappa \times (a_t + \frac{1}{1+\kappa} i_t) \tag{3}$$

The investment payoffs to the banks differ and realise consecutively. The payoff to the risky bank realises first at $t = 1$, while that to the prudent one realises immediately afterwards at $t = 2$.

The payoffs of the risky bank ($\widetilde{R_H}$), and the prudent bank ($\widetilde{R_L}$) are given by:

$$\widetilde{R_H} = \begin{cases} R_H \text{ with probability } p(S_H) \\ 0 \text{ with probability } 1 - p(S_H) \end{cases} \tag{4}$$

$$\widetilde{R_L} = \begin{cases} R_L \text{ with probability } p(R_L|R_H) = 1 \\ R_L \text{ with probability } p(R_L|0) = p(S_L) \\ 0 \text{ with probability } 1 - p(S_L) \end{cases} \tag{5}$$

where R_H and R_L are the gross rates of return on the investments of the risky bank and the prudent bank respectively, $R_H > R_L$, $p(S_H)$ is the unconditional probability of the risky bank's investment project being a success, $p(R_L|R_H)$ is the probability of the prudent bank's success conditional on the risky bank's project succeeding, and $p(R_L|0) = p(S_L)$ is the probability of the prudent bank's success conditional on the risky bank's failure.

This structure leads to three possible states of the economy, according to the three combinations of the unconditional and conditional probabilities of the banks' investment projects being successful. (1) A good state, in which the investments of both banks are successful (unconditional probability of $p(S_H)$); (2) a neutral state, in which the investment of the risky bank fails but that of the prudent one is successful (unconditional probability of $[1 - p(S_H)] \times p(S_L)$); and (3) an adverse state, in which both investments fail (unconditional probability of $[1 - p(S_H)] \times [1 - p(S_L)]$).

Note that the information available at $t = 1$ is only partially revealing about the exact state of the economy if $\widetilde{R_H} = 0$. This in turn affects the central bank's policy.

We model the unconditional probability of a good state, $p(S_H)$, explicitly as a logistic function:

$$p(S_H) = \frac{1}{1 + e^{\alpha + \beta(a_t)}} \tag{6}$$

The initial equal size of the two banks is normalised to 1. Hence, a_t can be expressed in terms of κ as $a_t = \frac{1}{1+\kappa}$,[4] and Equation (6) is equivalent to:

$$p(S_H) = \frac{1}{1 + e^{\alpha + \beta(\frac{1}{1+\kappa})}} \tag{7}$$

where α is an independent factor which determines the probability that the economy will arrive in a good state, and β is a factor capturing the systemic sensitivity of that probability to changes in the level of investment in the risky asset or highly liquid reserves.

The function $p(S_H)$ is concave and decreasing in a_t (increasing in κ), thereby reflecting an important tenet of prudential regulation, namely that higher levels of reserves held by financial institutions enhance the stability and resilience of the financial system.

The terminal values of the two banks can be expressed as follows. For the risky bank:

$$V_{u,T} = max\Big([R_H(a_t + \frac{1}{1+\kappa} i_t) - \rho i_t + k_{u,t}], 0\Big) \tag{8}$$

which takes a value of 0 with unconditional probability $1 - p(S_H)$. For the prudent bank:

$$V_{s,T} = max\Big([R_L(a_t - i_t) + \rho i_t + k_{s,t}], 0\Big) \tag{9}$$

which takes a value of 0 with unconditional probability $[1 - p(S_H)] \times [1 - p(S_L)]$.

3.2 Non-financial firms

Non-financial firms in the model generate real economic output outside the financial system by investing funds they obtain from the banks in positive net present value investment projects. The non-financial sector consists of two types of perfectly competitive firms, higher and lesser creditworthy corporations. Higher (lower) creditworthy firms invest in less (more) risky projects as defined by the project's unconditional probability of success. Commercial banks invest in risky investments by channeling loans to the non-financial firms. A firm's creditworthiness is defined by the investment project it selects which in turn characterises the class of bank they borrow from, so the prudent bank channels funds to more creditworthy firms, and *vice versa*. All projects generate an average rate of return, r, net of the interest paid to the bank a firm borrows from.

The activities of the banks also add value to the economy, hence the net payoffs they earn are also accounted for in the economic output function. The total economic net output,

[4] Initially, the asset-side of a bank's balance sheet comprises only risky assets and highly liquid reserves, therefore $a_t + \kappa \times a_t = 1$ or $a_t = \frac{1}{1+\kappa}$.

Ω, is given by:

$$\Omega = (R_H - 1) \times (a_t + \frac{1}{1+\kappa} i_t) + (R_L - 1) \times (a_t - i_t) + r \times (2a_t - \frac{\kappa}{1+\kappa} i_t),^{5,6} \quad (10)$$

3.3 The central bank

The central bank's objective is to maximise the difference between the value of economic output generated in the system and the expected costs of economic distress. The value function of the central bank incorporates the economic output generated by the financial system and the non-financial firms, the additional costs generated by stress in the interbank market, the opportunity cost of requiring banks to hold highly liquid reserves, output losses resulting from bank failures, the direct costs of assisting a bank in distress, and the expected repayment of any emergency funding it may provide.

A default on a bank's interbank obligations would result in a significant impairment of the functioning of the interbank money market. Given this market's importance for an efficient transmission of monetary policy, as well as its importance in establishing benchmark interest rates, any impairment will lead to additional costs, denoted s, that the economy has to incur if the central bank does not intervene. In October 2008, during the financial crisis, the LIBOR-OIS spread, often considered to be a gauge of the health and efficient functioning of the interbank market, increased from a pre-crisis average of approximately 10 basis points to more than 350 basis points. Furthermore, the TED spread, measuring the perceived credit risk in the entire economy, increased from approximately 30 basis points to almost 460 basis points during the same period. The magnitude of these changes indicates how substantial the economic costs of a sudden evaporation of confidence in the interbank market can be. Indeed, Brunnermeier and Pedersen (2009) report that while the losses sustained by U.S. financial institutions alone between 2007 - 2008 were of the order of several hundred billion dollars, they were subsequently amplified to more than \$8 trillion in the overall stock market. The central bank's intervention to ensure smooth functioning of the interbank market can potentially mitigate these costs. Here, we assume such action simply involves guaranteeing the principal amount of all interbank market loans extended, so obliging lending banks to forego the interest they expect to earn on their funding position.

[5] This is a reduced form of the basic function of economic output specified as $\Omega = (R_H - 1) \times (a_t + \frac{1}{1+\kappa} i_t) - (\rho - 1) i_t + (R_L - 1) \times (a_t - i_t) + (\rho - 1) i_t + r(a_t + \frac{1}{1+\kappa} i_t) + r(a_t - i_t)$. The function accounts for the net returns the banks earn on their risky investments and the interbank position, and for the return of the non-financial sector based on the funds channeled by the two banks.

[6] This function incorporates the deadweight loss created by requiring commercial banks to maintain a certain level of reserves, which reduces credit supply, equal to $r \times (k_{s,t} + k_{u,t}) = r \times (2a_t + \frac{1}{1+\kappa} i_t)\kappa$.

Bank failures result in a sharp decline in credit supply[7] and diminish the amount of capital available for investment by non-financial firms, thus lowering total economic output. We assume that the loss of credit originating from a bank that has been resolved is not immediately replaced in the system. This loss of output, Γ, is modelled as a τ-period annuity paying $a_t \times (R_{H/L} - 1 + r)$ every period, equivalent to the return the banks and the non-financial firms could have generated had total credit supply not decreased.[8]

Finally, any central bank assistance to a bank in distress involves restoring that bank's balance sheet to its initial position. That is, the central bank provides the troubled bank with funds equal to the value of its liabilities and equity capital net of the highly liquid reserves it is required to hold, denoted by Λ. Since such funds are not a subsidy, the central bank's value function also accounts for the expected value of their repayment, $\Psi = \delta \times \Lambda$, where δ is a discount factor. As a result, the value function that the central bank maximises is given by:

$$V_{CB} = E[\Omega] - E[s] - E[\Gamma] - E[\Lambda] + E[\Psi] \tag{11}$$

corresponding to the difference between economic output and costs of economic distress.

3.4 Central bank's strategic policy objectives

In order to achieve its policy objective the central bank chooses between one of the following three strategies when responding to a crisis in the banking system. First, it may allow a bank in distress to be resolved. Second, it may intervene in the interbank market and guarantee the repayment of any interbank loans in order to prevent stress and contagion, while still allowing the bank in distress to be resolved.[9] Finally, it may attempt to avert a crisis altogether by immediately assisting the bank in distress through an injection of liquidity.

An important assumption is that the extent of the central bank's assistance capacity is restricted to just one bank in the system. This reflects the notion that accepting essentially worthless collateral in return for the support funds it provides leads to a significant deterioration of the overall quality of the assets held on the central bank's balance sheet (Buiter, 2008). Moreover, to fulfil its mandate the central bank always has an incentive to save at least one bank. A failure of both banks leads to a collapse of the entire financial

[7] As reported by the Bank of England, UK total credit supply decreased by almost 13% in 2008 - 2009.

[8] Using the average rate of return, r, as a discount rate, the present value of such an annuity is $\Gamma = ((R_{H/L} - 1 + r) \times a_t) \times (\frac{1-(1+r)^{-\tau}}{r})$

[9] The German bank rescue plan announced in October 2008 included a €400 billion guarantee of interbank loans with maturities of up to three years.

system, resulting in an infinitely large economic loss.

3.5 Structure and evolution of the game

At $t = 0$ the central bank selects from one of three potential strategies: no assistance provided at $t = 1$, an interbank loan repayment guarantee at $t = 1$, and providing direct balance sheet assistance to a bank in distress. This decision is reached subsequent to maximising the expected value of equation (11) for each strategy. The policy decision is then publicised and becomes common knowledge. The commercial banks determine the size of the interbank loan in order to maximise their expected terminal value given the announced central bank policy. The gross interest rate ρ is specified, the transactions in the interbank market take place, and the two banks undertake their project investments.

Between $t = 0$ and $t = 1$ a shock which determines the state of the economy occurs, and $\widetilde{R_H} \in (R_H, 0)$ realises at $t = 1$. If $\widetilde{R_H} = R_H$, then $\widetilde{R_L} = R_L$ in the subsequent period. In this case, both banks earn their expected returns, the risky bank repays the interbank loan together with any interest owing, and the central bank does not have to intervene, nor incurs any losses. If the realisation of $\widetilde{R_H} = 0$ at $t = 1$, the value of the risky bank's assets reduces to 0, and the path along which the subsequent events in the game evolve depends upon the announced central bank strategy. If the central bank does not respond to the failure of the risky bank's investment and allows it to be resolved, the central bank incurs the cost associated with the loss of future output, together with those arising from the ensuing stress in the interbank market, so that:

$$L_{CB,t=1} = \Gamma + s \tag{12}$$

The latter cost can be mitigated if central bank intervention is restricted to guaranteeing the interbank loan. Such a decision, however, implies that it incurs an additional cost equal to the value of the principal amount of the interbank loan, that is:

$$L_{CB,t=1} = \Gamma + i_t \tag{13}$$

Finally, the central bank can avoid incurring the cost of future output loss by assisting the risky bank through providing it with sufficient capital to restore its balance sheet to its initial position while still guaranteeing its interbank obligations. The cost of such intervention is equal to the value of the bank's liabilities and equity capital, net of the reserves held, so that:

$$L_{CB,t=1} = l + c + i_t - k_{u,t} \tag{14}$$

Using the balance sheet identity equation and substituting for ${k_{u,t}}$[10] the above cost becomes:

$$L_{CB,t=1} = a_t + \frac{1}{1+\kappa} i_t \tag{15}$$

The events at $t = 2$ depend on both the realisation of $\widetilde{R_L}$, and the actions of the central bank at $t = 1$. If the central bank makes no intervention at $t = 1$, the prudent bank is forced to write off its interbank position even if it earns the expected return on its investment project. If size of the interbank loan is sufficiently large, this creates financial distress for the prudent bank. A sufficient condition for distress to occur is that the value of the interbank loan exceeds the net return the bank earns on risky assets, namely:

$$\frac{R_L - 1}{R_L} a_t < i_t \tag{16}$$

If equation (16) holds,[11, 12] the central bank must provide the prudent bank with a capital injection equal to the value of the interbank loan less the net return the prudent bank earns to restore its solvency, so that:

$$L_{CB,t=2} = i_t - (R_L - 1) \times (a_t - i_t) \tag{17}$$

In the adverse state of the economy, when $R_L = 0$, the central bank provides the prudent bank with enough capital to restore its balance sheet to its initial position, that is:

$$L_{CB,t=2} = l + c - k_{s,t} = a_t \tag{18}$$

The central bank's decision to guarantee the risky bank's obligations in the interbank market means that a part of the emergency capital injection the prudent bank requires in an adverse state of the economy has already been provided at $t = 1$, thereby changing the cost of assistance to:

$$L_{CB,t=2} = a_t - i_t \tag{19}$$

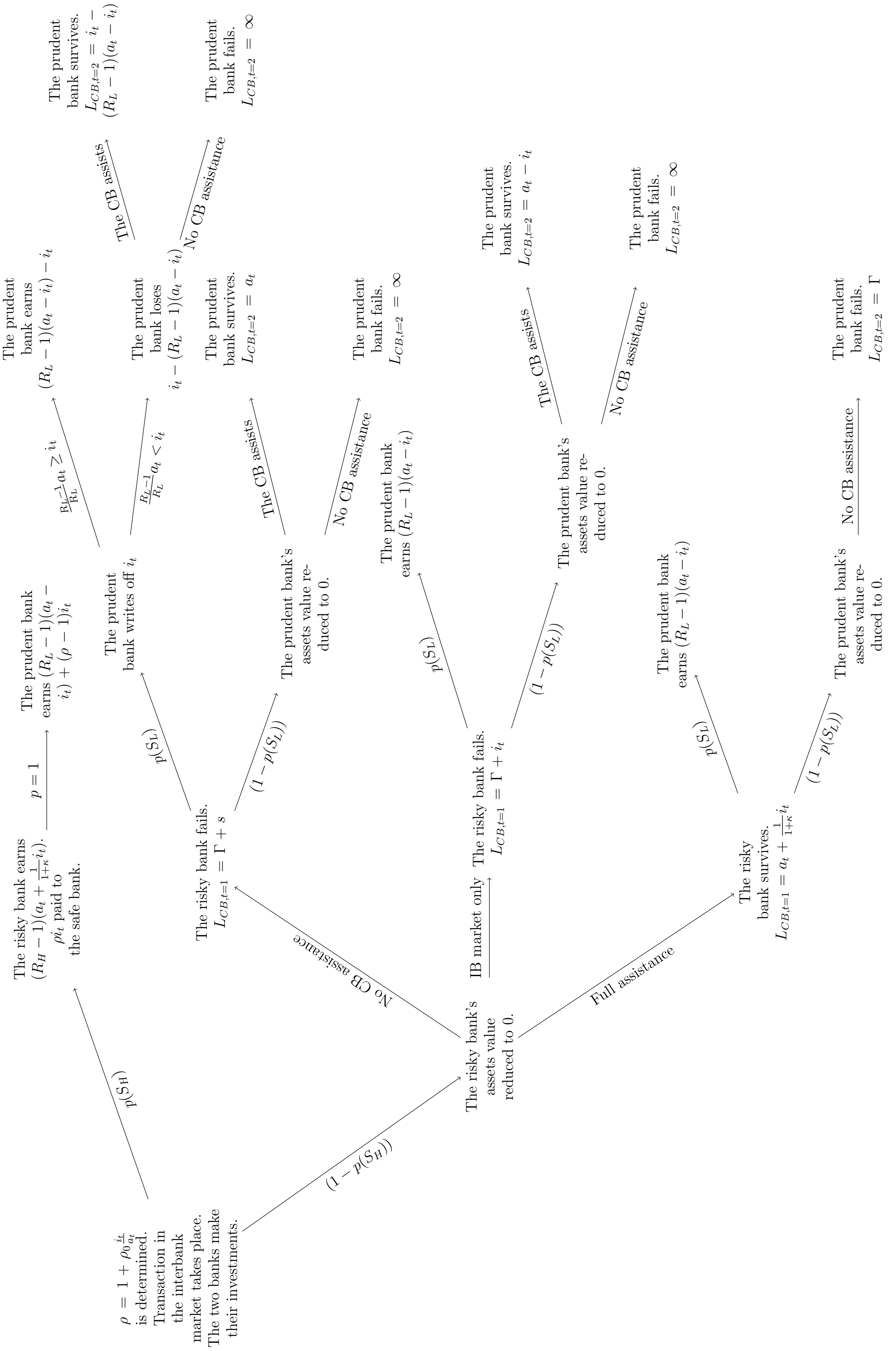
Finally, the prudent bank is resolved in a bad state of the economy if the central bank decides to fully assist the risky bank at $t = 1$. This situation arises as the central bank no longer has sufficient funds remaining to assist the prudent bank. Figure 1 depicts all the possible sequences of events, together with the costs that the central bank incurs in each state of the economy, as well as the payoffs to the two banks.

[10] Based on the initial balance sheet, $l + c = a_t + \kappa a_t$, and $k_{u,t} = \kappa \times (a_t + \frac{1}{1+\kappa} i_t)$.

[11] This is a reduced form of the inequality $(R_L - 1) \times (a_t - i_t) < i_t$.

[12] If this inequality does not hold, the prudent bank will be able to write off the interbank loan, and is not going to require any assistance from the central bank.

Figure 1: Event tree



3.6 Solving the model

To derive the optimal central bank policy the interactions in the model are simulated for plausible values of the specified parameters. We justify these parameter values below. This approach facilitates identification of the key decision-relevant factors for the central bank. Given the central bank's crisis response strategy announced at $t = 0$, we derive the expected terminal values of the two banks for each of the three strategies. Subsequently we evaluate the properties of the expected value functions in order to determine the size of the interbank loan that maximises the terminal values of each of the two banks.[13]

The optimal size of the interbank loan for the risky bank is constant regardless of the strategy selected by the central bank, and is given by:

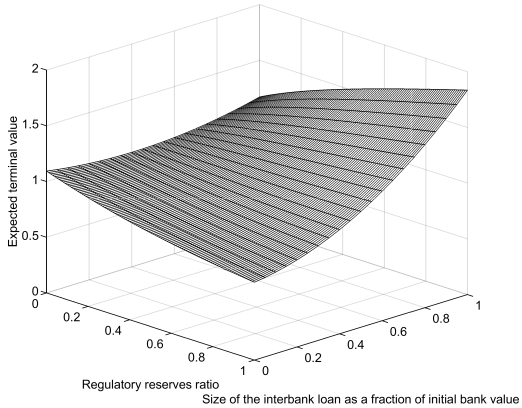
$$i_{u,t} = \frac{R_H - 1}{2\rho_0(1+\kappa)^2} \tag{20}$$

The optimal loan size depends positively on the level of returns the risky bank expects to earn, R_H, and is negatively related to both the cost of borrowing in the interbank market, ρ_0, and the level of highly liquid reserves, κ, the bank is required to maintain.

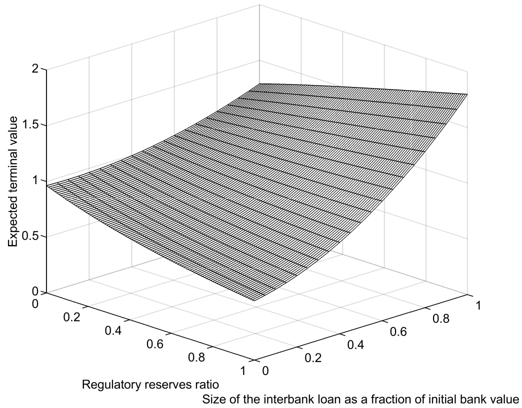
In case of the prudent bank, as shown in Figure 2, the expected terminal value is maximised at the highest possible volume of interbank lending for all levels of the regulatory reserves ratio, irrespective of which strategy the central bank selects.

[13] For details see Appendix A.

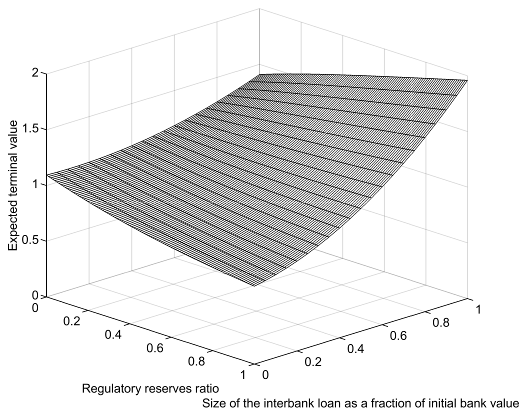
Figure 2: Expected terminal value of the prudent bank



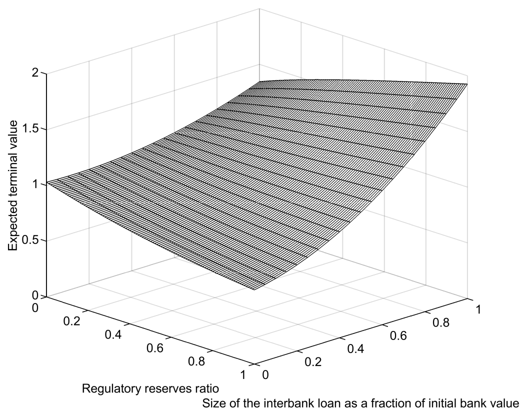
(a) No intervention at $t = 1$ and assistance to the prudent bank at $t = 2$



(b) No intervention at $t = 1$



(c) Interbank market intervention at $t = 1$



(d) Assisting the risky bank at $t = 1$

The surface plots demonstrate the expected terminal value of the prudent bank as a function of the regulatory reserves ratio and the size of the interbank loan under each of the policy strategies considered by the central bank. For all strategies, the expected terminal value of the bank is maximised at the highest possible volume of interbank lending for all levels of the regulatory reserves ratio.

The strategy selected by the central bank critically impacts the optimal size of the interbank loan for the prudent bank. If the central bank decides on a non-intervention strategy at $t = 1$, the consequences for the prudent bank vary depending on the relationship between the size of the interbank loan and the prudent bank's net earnings.

When the size of the interbank loan exceeds the net earnings of the prudent bank, $\frac{R_L-1}{R_L}a_t < i_t$, the prudent bank requires central bank's assistance at $t = 2$, as it is unable to absorb the loss associated with the write-off the interbank loan. As a consequence, we assume that due to its greater risk-aversion, the prudent bank limits the volume of interbank lending to $i_t = \frac{R_L-1}{R_L}a_t$ if the central bank adopts the non-intervention strategy.

As the repayment of the principal amount of the interbank loan in the adverse economic state is guaranteed under either of the two remaining strategies, the prudent bank is willing to issue an interbank loan of a size demanded by the risky bank, as shown in equation (20).

An important constraint is that the risky bank cannot borrow more than the prudent bank is willing to lend, and *vice versa*. Thus, the actual size of the interbank loan is given by the minimum of the two individual loan size functions:

$$i_t = min\{i_{s,t}, i_{u,t}\} \tag{21}$$

This mechanism also ensures that the expected terminal value of at least one of the two banks is maximised. The intersection of the two loan size functions corresponds to a simultaneous maximisation of their expected terminal values.

4 Simulation experiments

The value of the functions described in the preceding section depend upon a number of fixed parameters together with the reserve requirements ratio, κ, which can take any value between 0 and 1. When conducting the simulations below, the reserve requirements ratio is treated as the independent variable.[14]

4.1 Baseline simulation

Table 2 lists the selected parameter values for the baseline simulations. These values are chosen to constitute a reasonable reflection of those encountered in developed economies. Specifically, bank performance-related information available via Bloomberg Database and the FRED Database indicates that riskier banks have been generating a return on equity in excess of 20%, while more prudent banks earned approximately 10% over the last two decades. We believe our assumed parameter values for R_H and R_L are, therefore, reasonably conservative. As reported by Damodaran (2015), the average annual rate of return of S&P 500 listed firms between 1928 and 2014 is approximately 10%, justifying our chosen value for r. The choice of parameters α and β results in a probability of a successful investment by the risky bank of between 81% - 91%, reflecting the average cumulative five-year survival probability of a Ba-rated corporation (Hamilton and Cantor, 2006). Given the assumption about R_L, the likelihood of the prudent bank's success conditional on the risky one's failure, $p(S_L)$, is a reflection of its greater financial soundness and resilience. This difference in the banks' risk profiles is further reflected in the values assigned to their respective discount factors, δ_s and δ_u, which mirror a banks' cost of equity capital as

[14] All the simulations presented in this paper are undertaken using MatLab.

reported in Bloomberg. Our assumption about the duration of the credit supply decrease, τ, is motivated by the fact that following the crisis of 2007 - 2009, business conditions in most developed countries did not return to normal until at least early 2013. Finally, the choice of the value for s relates to Brunnermeier and Pedersen's (2009) analysis of the 2007 - 2008 liquidity crunch, in which the initial losses of the financial sector were reported to be approximately 5% of stock market capitalisation. Our assumption of $s = 0.1$ thus corresponds to 5% of the total initial value of the commercial banking system in the model.

Table 2: Fixed parameter values for the baseline simulations

Parameter	Assumed value
R_H	1.15
R_L	1.1
r	0.1
ρ_0	0.5
α	-3
β	1.5
$p(S_L)$	0.65
δ_s	0.75
δ_u	0.2
τ	5
s	0.1

This table reports the assumed fixed parameter values used in the baseline simulations.

4.2 The interbank loan market

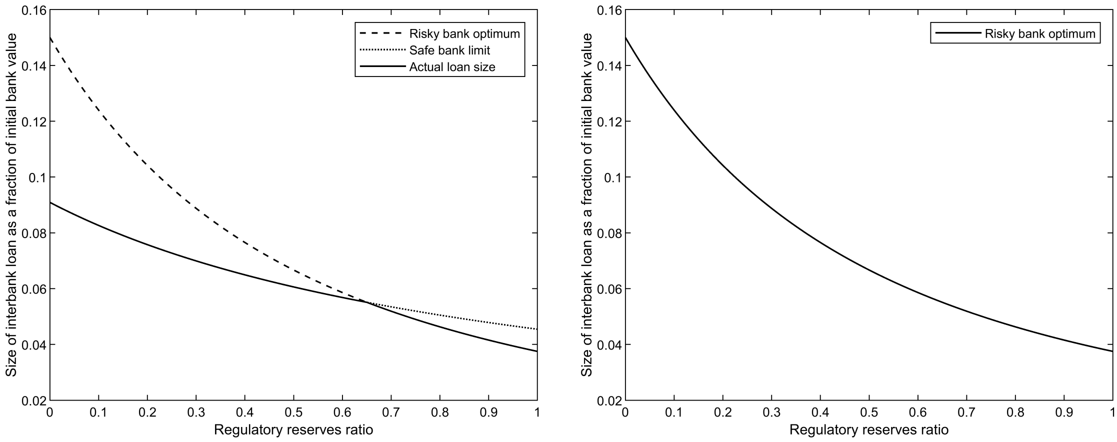
We determine the volume of interbank market transaction, $i_t = min\{i_{s,t}, i_{u,t}\}$, by comparing the risky bank's optimal loan size functions with the amount the prudent bank is willing to lend. These are depicted in Figure 3.

If the value of the interbank loan is lower than the net earnings of the prudent bank, $\frac{R_L-1}{R_L}a_t \geq i_t$, the prudent bank can absorb the losses suffered in the interbank market when the risky bank defaults on its loan obligations. Figure 3a depicts the simulated optimal loan size of the risky bank, as a fraction of initial bank value, defined as that which maximises a bank's expected terminal value for a given level of reserve requirements and the prudent bank's loan size limit implied by the inequality in (16).

The baseline results show that the difference between the risky bank's optimal loan size and the limit implied by prudent bank's net earnings diminishes as the regulatory reserves ratio is increased. We find that the size of the loan demanded by the risky bank exceeds the maximum amount the prudent bank is willing to lend for $\kappa < 0.65$, above which it becomes smaller than the prudent bank's loan size limit. Consequently, the actual volume of interbank lending is given by the minimum of the two loan sizes. This ensures that the prudent bank is always able to absorb the potential loss if the risky bank fails and the interbank loan has to be written off.

In case of the strategies based on intervening at $t = 1$, due to central bank's guarantees, the prudent bank is always willing to meet the risky bank's demand and issue a loan of a size allowing the latter to maximise its expected terminal value, shown in Figure 3b.

Figure 3: Interbank loan size, subject to no intervention at $t = 1$



(a) No intervention at $t = 1$ **(b)** Central bank intervention at $t = 1$

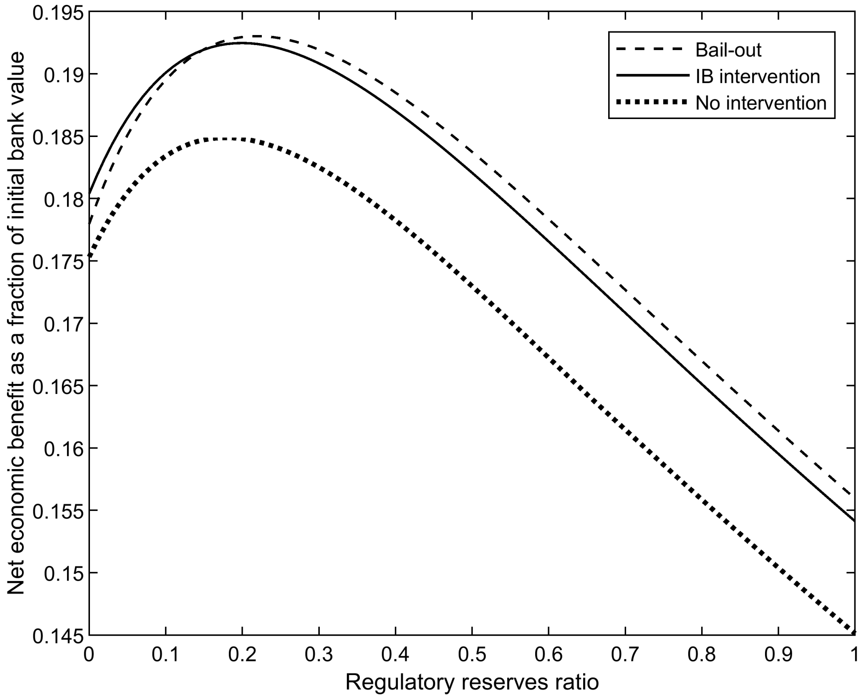
The dashed lines in Figure 3a depict the simulated risky bank's terminal value-maximising size of the interbank loan and the loan size below (above) which the prudent bank can (cannot) write it off without becoming financially distressed. The solid line in Figure 3a corresponds to the actual size of the interbank loan if the central bank does not intervene at $t = 1$, equivalent to the minimum of the risky bank's optimal loan size and the prudent bank's limit functions. Figure 3b plot the size of the interbank loan for either of the two remaining strategies, equivalent to risky bank's optimal loan size function.

Collectively, Figure 3 shows that for a plausible range of parameters, the size of the loan issued in the interbank market is equal either to that which reduces the prudent bank's risk of insolvency, or maximises the expected terminal value of the risky bank.

4.3 The central bank's value function

We now present the baseline results for the central bank's value function over the three defined strategies in Figure 4.

Figure 4: Baseline results for the central bank's strategies



The expected central bank's value function associated with the intervention in the interbank market at $t = 1$ is given by the solid line; that corresponding to direct balance sheet assistance at $t = 1$ by the dashed line, and the one resulting from no intervention at $t = 1$ by the dotted line. All expected value function are convex indicating there exists an optimal level of highly liquid reserve requirements, and that the benefits from increasing them are marginally diminishing.

The central bank's expected value function for all three strategies is convex, implying there exists an optimal level of highly liquid reserve requirements which maximises its objective function in all cases.

Figure 4 suggests that a strategy of no intervention at $t = 1$ generates the worst policy outcome for the central bank. Comparing the baseline results for the strategy of intervention in the interbank market at $t = 1$ to guarantee the interbank loans, with that of further assisting the risky bank by restoring its balance sheet, we find that the latter leads to a superior outcome for the central bank, as its expected value function is maximised at a higher level. However, this is achieved for a level of reserve requirements that is approximately 5 percentage points higher than in the situation where central bank intervention is restricted to guaranteeing loans in the interbank market. In effect, the liquid reserves of the risky bank must provide partial insurance against its riskiness in order for the central bank to find it optimal to provide further balance sheet assistance.

This preference for assisting the risky bank arises for two reasons. First, even though the central bank guarantees the repayment of the principal amount of the interbank loan under both strategies, it also expects the risky bank to eventually repay any assistance it receives.[15] If, however, the central bank only intervenes in the interbank market to guarantee loans, the amount of assistance provided becomes a deadweight loss. Second, the schedule of costs incurred by the central bank in each of the two strategies differs. The costs of assisting the risky bank incurred at $t = 1$ are lower than the associated costs of intervening in the interbank market.[16] Although interbank market intervention results in a smaller loss at $t = 2$, this loss is incurred only in the adverse state of the economy. Consequently, the degree to which the $t = 2$ loss affects the central bank's expected value function and its strategic preferences depends on the unconditional probability of the occurrence of an adverse state.

4.4 Determinants of central bank policy preferences

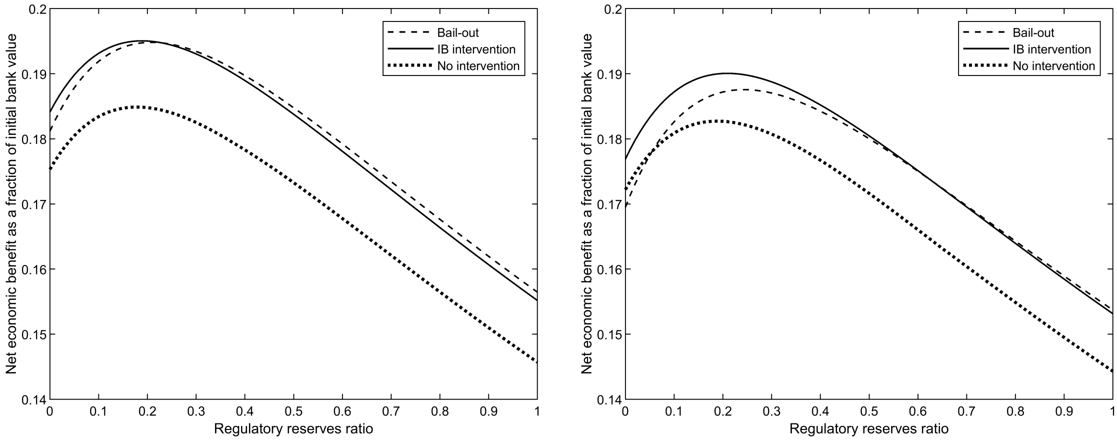
On the basis of the previous discussion, it is apparent that modifying the parameter values determining the size of the interbank loan (ρ_0), and the unconditional probability of the adverse state of the economy ($p(S_L)$) has implications for the central bank's strategic preferences. Figures 5a and 5b depict the effects of reducing the volume of interbank market activity by increasing ρ_0, and of increasing the unconditional probability of the adverse state, respectively. Both modifications enhance the likelihood that intervention in the interbank market to guarantee the loan repayments becomes the central bank's preferred strategy. A lower volume of interbank lending reduces the costs of assisting the bank in distress, as well as the pure loss the central bank incurs from guaranteeing the loan repayment, thereby reducing the optimal level of reserve requirements.

Conversely, a higher probability of an adverse state of the economy increases the expected costs to the central bank, leading to a higher optimal level of reserve requirements. Expected costs to the central bank are less if it confines its intervention to the interbank market, as the $t = 2$ loss associated with this strategy is lower than that which results from assisting the risky bank.

[15] The discount factors applied to the repayment of emergency funding provided to a commercial bank capture both the probability of repayment and the time a bank will require to return the funds.

[16] Intervention in the interbank market: $L_{CB,t=1} = \Gamma + i_t$ at $t = 1$, and $L_{CB,t=2} = a_t - i_t$ at $t = 2$; Assisting the risky bank: $L_{CB,t=1} = a_t + \frac{1}{1+\kappa} i_t$ at $t = 1$, and $L_{CB,t=2} = \Gamma$ at $t = 2$.

Figure 5: The impact of changes in ρ_0 and $p(S_L)$ on the order of central bank's strategic preferences

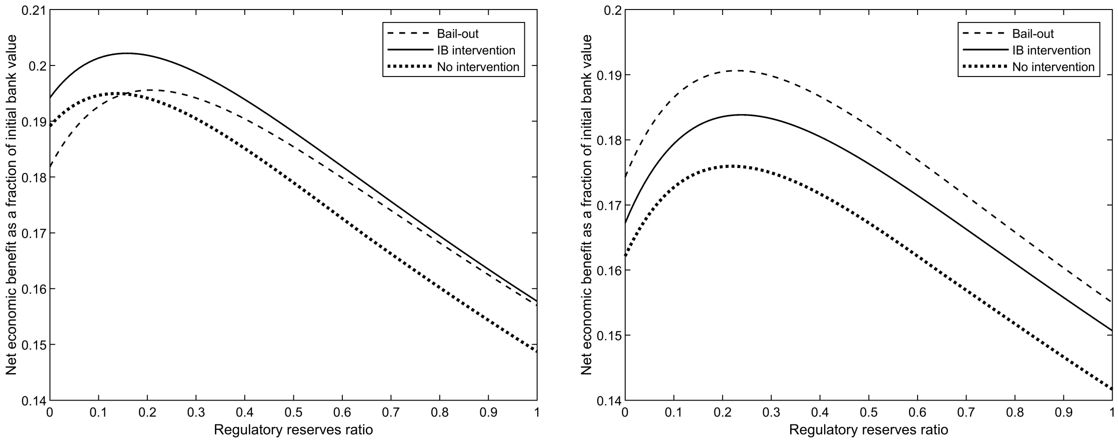


(a) $\rho_0 \uparrow$ $(\rho_0 = 0.6)$ **(b)** $p(S_L) \downarrow$ $(p(S_L) = 0.6)$

The strategy of interbank loan guarantee intervention (solid line) becomes the central bank's preferred strategy following increases in ρ_0, the sensitivity of the net interest rate on the interbank loan to the additional leverage the loan creates, resulting in lower volume of interbank activity. Further, reducing $p(S_L)$ the probability the prudent bank's investment project succeeds conditional on the failure of the risky bank, results in a higher probability of the adverse state of the economy. This makes the interbank loan market intervention more likely to be the central bank's preferred crisis response strategy.

We find that this result is also sensitive to the assumptions concerning the duration of the reductions in credit supply resulting from bank failures, increasing the output gap. As shown in Figures 6a and 6b, the preferences of the central bank favour strategies in which the extent of its intervention is more limited for credit supply reductions of shorter duration. Conversely, the longer the expected period of reduced credit supply, the greater the central bank's incentive to provide full assistance to a bank in distress.

Figure 6: The impact of changes in τ on the order of central bank's strategic preferences



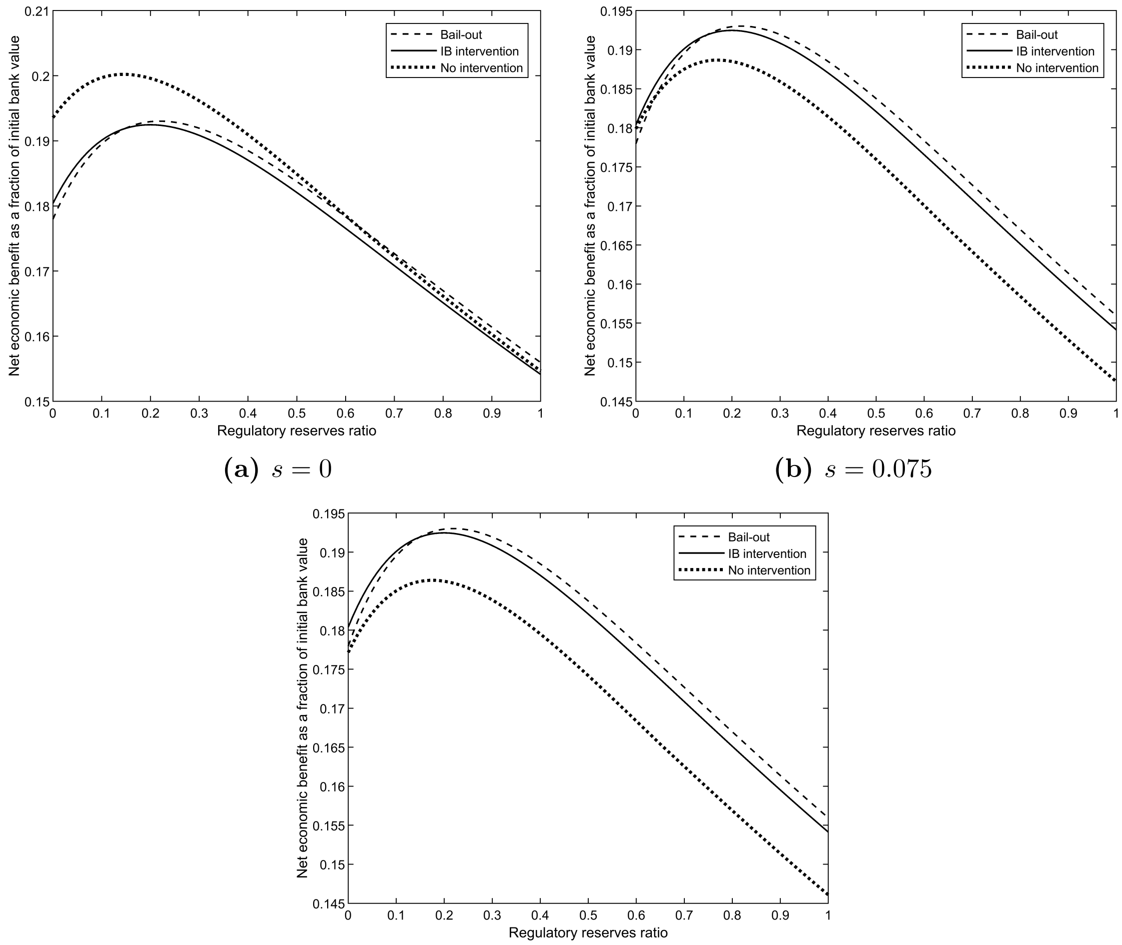
(a) $\tau \downarrow$ $(\tau = 4.5)$ **(b)** $\tau \uparrow$ $(\tau = 5.5)$

The ordering of central bank's strategic preferences is affected by the expected duration of credit supply reduction resulting from a bank failure. The shorter the expected duration of post-shock reduced credit availability, the more likely it is that an intervention restricted to interbank loan repayment guarantee is going to be the central bank's preferred strategy. Conversely, for longer expected credit supply reduction, the strategy of providing direct balance sheet assistance to a bank in distress becomes the preferred alternative.

These results demonstrate that the ordering of central bank's strategic preferences are influenced by trading-off the immediate direct costs incurred when responding to a crisis and the expected indirect costs incurred in subsequent periods when the financial system is relatively stable but not operating at its full capacity. Consequently, the central bank's response to shocks likely to result in short-term reductions of economic output ought to be directed towards preventing contagion via the interbank money market and ensuring that it remains stable. For longer expected credit supply reductions, the strategy of providing direct balance sheet assistance to a bank in distress becomes the preferred alternative. Importantly, irrespective of the expected duration of a reduction in supply of credit, not intervening at all at $t = 1$ is never the optimal crisis response strategy, as the central bank's preferences gradually shift from interbank loan repayment guarantee towards balance sheet assistance intervention only.

Note that the results of the baseline simulation and the simulations discussed above are based on the assumption that the economy incurs additional costs arising from stress in the interbank market. This stress may critically impair the efficiency of the monetary transmission mechanism. We find that once this cost is minimal ($s = 0$) the no intervention strategy actually becomes the most preferable option for the central bank, a result illustrated in Figure 7a. Furthermore, as shown in Figures 7b and 7c, we identify $s = 0.075$ and $s = 0.09$ as critical levels of that cost, changing the central bank's preference for one strategy over the remaining two. These results suggest that the introduction of an alternative mechanism supporting the efficient functioning of the interbank market in crisis conditions, other than an explicit guarantee of interbank loan repayments, would impact upon the central bank's strategic policy preferences. In this context, the regulatory liquidity standards introduced in the Third Basel Accord, requiring financial institutions to hold enough highly-liquid assets to meet their short-term financing needs in stressed market conditions, provide a recent example of a mechanism which could help to reduce the costs generated by an impairment of the interbank market's functioning during a crisis, and influence central bank strategy.

Figure 7: Expected central bank's value function for different levels of s



(a) $s = 0$

(b) $s = 0.075$

(c) $s = 0.09$

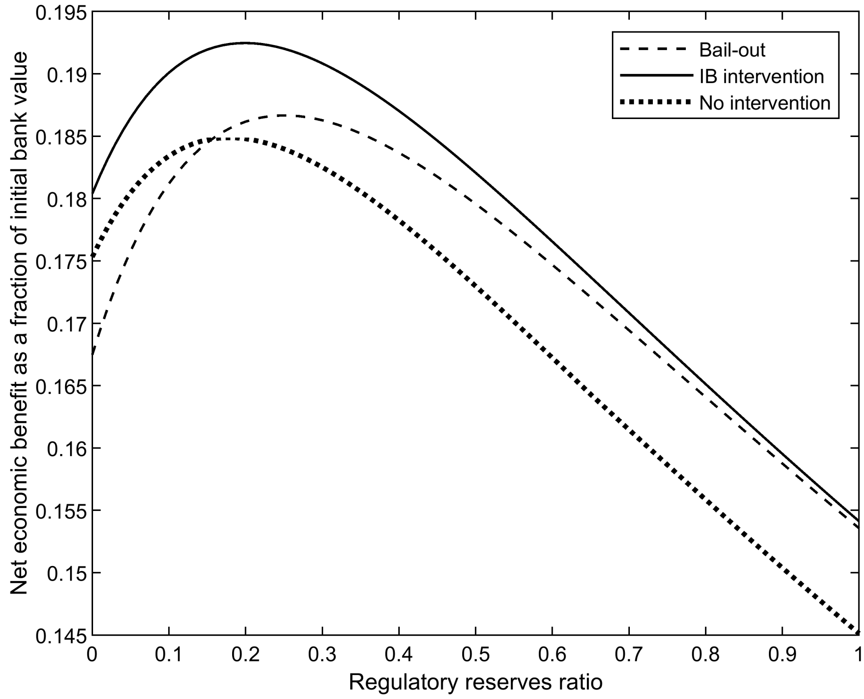
The expected cost of stress in the interbank market impacts the order of central bank's strategic preferences. As shown in Figure 7a, if a bank's default on its interbank obligations does not induce additional stress in the interbank money market, the strategy of no intervention at $t = 1$ (dotted line) results in the best outcome for the central bank. Once this cost is increased to 9% of the initial bank value ($s = 0.09$), this strategy becomes the central bank's least preferred option.

Figure 7c demonstrates that incurring an additional cost of $s = 0.09$ is sufficient to make the outcome of the strategy of no assistance at $t = 1$ the central bank's least preferred option for all values of the regulatory reserves ratio.

How the central bank perceives bank riskiness is also an important factor in determining its preferred strategy. This perception is captured by the discount factor applied to the repayment of any emergency assistance funding, which reflects both the time allocated for the repayment of central bank assistance as well as the default risk of the assisted institution.

We find that changing the discount factor's value from the baseline value of $\delta_u = 0.2$ to $\delta_u = 0.15$ (reflecting a higher default rate) is sufficient for the intervention in the interbank market to become the dominant strategy, a result we illustrate in Figure 8. The central bank will opt for the safer alternative of intervening in the interbank market and assisting the prudent bank if it believes that the risky bank is more likely to default in the future subsequent to receiving emergency funding, or alternatively that it is likely to require a very long time to repay the assistance funding.

Figure 8: Expected central bank value functions based on a lower risky bank's discount factor ($\delta_u \downarrow$, $\delta_u = 0.15$)



The central bank's assessment of the assisted institution's ability to repay the emergency funding it receives is an important factor determining the order of its strategic preferences. This future default risk and the time necessary to return the funds provided by the central bank are captured by the discount factor applied to compute the present value of expected liquidity repayment. Reducing the risky bank's discount factor makes the strategy of providing it with direct assistance at $t = 1$ (dashed line) a suboptimal strategy when compared to intervening in the interbank market only (solid line).

Table 3 produces comparative statics showing how both the central bank's expected value function, and the optimal level of the reserve requirements change when the levels of the fixed parameters are increased.

Table 3: Comparative statics

Parameter value	Optimal CB strategy	$E[V_{CB}]$	$\kappa_{maxE[V_{CB}]}$
Baseline R_H ($R_H = 1.15$)	Assist risky bank	0.193	0.22
$R_H \uparrow$ ($R_H = 1.175$)	Assist risky bank	0.211	0.19
$R_H \downarrow$ ($R_H = 1.125$)	Interbank market intervention	0.188	0.18
Baseline R_L ($R_L = 1.1$)	Assist risky bank	0.193	0.22
$R_L \uparrow$ ($R_L = 1.125$)	Interbank market intervention	0.209	0.17
$R_L \downarrow$ ($R_L = 1.075$)	Assist risky bank	0.179	0.23
Baseline r ($r = 0.1$)	Assist risky bank	0.193	0.22
$r \uparrow$ ($r = 0.125$)	Assist risky bank	0.228	0.16
$r \downarrow$ ($r = 0.075$)	Interbank market intervention	0.161	0.25
Baseline ρ_0 ($\rho_0 = 0.5$)	Assist risky bank	0.193	0.22
$\rho_0 \uparrow$ ($\rho_0 = 0.6$)	Interbank market intervention	0.195	0.19
$\rho_0 \downarrow$ ($\rho_0 = 0.4$)	Assist risky bank	0.190	0.23
Baseline α ($\alpha = -3$)	Assist risky bank	0.193	0.22
$\alpha \uparrow$ ($\alpha = -2.75$)	Assist risky bank	0.157	0.4
$\alpha \downarrow$ ($\alpha = -3.25$)	Interbank market intervention	0.234	0.05
Baseline β ($\beta = 1.5$)	Assist risky bank	0.193	0.22
$\beta \uparrow$ ($\beta = 1.6$)	Assist risky bank	0.182	0.3
$\beta \downarrow$ ($\beta = 1.4$)	Interbank market intervention	0.207	0.12
Baseline $p(S_L)$ ($p(S_L) = 0.65$)	Assist risky bank	0.193	0.22
$p(S_L) \uparrow$ ($p(S_L) = 0.7$)	Assist risky bank	0.199	0.2
$p(S_L) \downarrow$ ($p(S_L) = 0.6$)	Interbank market intervention	0.190	0.22
Baseline δ_s ($\delta_s = 0.75$)	Assist risky bank	0.193	0.22
$\delta_s \uparrow$ ($\delta_s = 0.8$)	Interbank market intervention	0.194	0.19
$\delta_s \downarrow$ ($\delta_s = 0.7$)	Assist risky bank	0.193	0.22
Baseline δ_u ($\delta_u = 0.2$)	Assist risky bank	0.193	0.22
$\delta_u \uparrow$ ($\delta_u = 0.25$)	Assist risky bank	0.199	0.18
$\delta_u \downarrow$ ($\delta_u = 0.15$)	Interbank market intervention	0.192	0.2
Baseline τ ($\tau = 5$)	Assist risky bank	0.193	0.22
$\tau \uparrow$ ($\tau = 5.5$)	Assist risky bank	0.191	0.23
$\tau \downarrow$ ($\tau = 4.5$)	Interbank market intervention	0.202	0.16
Baseline s ($s = 0.1$)	Assist risky bank	0.193	0.22
$s \uparrow$ ($s = 0.2$)	Assist risky bank	0.193	0.22
$s \downarrow$ ($s = 0$)	No intervention at $t = 1$	0.202	0.14

This table reports the changes in the expected value of the central bank's objective function and the level of regulatory reserves ratio which maximises it resulting from an increase in the values of the respective fixed parameters used in model simulations.

We discuss the effects of modifying parameters ρ_0, δ, τ, or s in detail in the preceding subsection. Increasing R_H, R_L, or r all influence the central bank's value function in a similar fashion, leading to higher expected economic output without changing the central bank's expected losses. These expected gains and losses, however, also depend on the probabilities determining the state of economy. Decreasing α and increasing $p(S_L)$ raises the unconditional probabilities of the economy being in a good or a neutral state, when the investments of the banks are successful. If the probability that the economy is in a good state is more sensitive (higher β) to the amount of reserves held by banks, then the optimal level of reserve requirements is also going to be higher. However, this results in a larger opportunity cost of regulation, thus reducing the maximum value of the central bank's objective function.

5 Discussion

The predictions arising from the model we outline have several important implications for the design of a central bank's financial stability policy. Importantly, our results emphasise the importance of a proactive central bank in achieving the objectives of a macroprudential financial policy. Positioned at an intersection of financial markets and the real economy, central banks are often best placed to identify adverse changes in the condition of the financial system (here reflected in interbank market conditions) faster than other market participants or policymakers. As such, they can gauge their impact on the real economy more accurately. This superior access to information plays a key role in determining appropriate policy design, which we now discuss.

First, our results imply there is a trade-off between the nature and scope of a central bank's response to a banking crisis, and the level of highly liquid reserves financial institutions are required to hold. A commercial bank's risk appetite is affected both by crisis assistance expectations and by the reserve requirements with which it has to comply. The absence of an implicit guarantee that the central bank will help in a crisis enhances market discipline, while high reserve requirements make riskier investment more costly.

We find that the optimal level of reserve requirements increases commensurately with the scope of the central bank's commitment to intervene at $t = 1$. The results indicate that the central bank's expected value function is maximised at a lower level of reserves when it only intervenes in the interbank market than if it assists the risky bank by restoring its balance sheet. This suggests that financial stability policy should be based on active management of assistance expectations, and setting reserve requirements accordingly. Our model shows that a strict commitment to limiting the scope of the central bank's intervention in response to a crisis and requiring financial institutions to hold high reserves may be economically wasteful.

Further, when comparing the effects of interbank loan guarantee intervention and of providing full assistance to the risky bank, a level of regulatory reserve requirements exists at which the value functions associated with the two strategies intersect. At this "switch level"of reserves, the central bank considers the two alternative strategies to be equally effective in achieving its objective. Importantly, this has further consequences for the central bank's policy decision if the regulatory bank reserve ratio is exogenously set, for example by Basel III requirements, at a different level to that which maximises its value function. Should the specified level of reserves be sufficiently greater (lower) than the one for which the two value functions intersect, the central bank can increase the expected value of its policy pre-commitment by a prior commitment to an alternative intervention policy instead. For example, the central bank can achieve a better outcome by announcing a commitment to providing full assistance to the risky bank rather than by limiting its intervention to interbank loan guarantee if the regulatory liquid reserves the commercial banks need to hold exceed the level for which it is indifferent between the two strategies, and *vice versa*. Since the individual sizes of the interbank loans that maximise the expected terminal values of the two commercial banks are the same for each of the two strategies, such a decision would not affect their investment policy but would result in a better expected outcome for the entire economic system. While an accurate identification of the preference switch level of regulatory capital may be difficult in practice due to the uncertainty in measuring or estimating variables determining the expected central bank value function, as long as the levels of reserve requirements that maximise the two expected value functions are of a similar order of magnitude, the potential welfare loss stemming from a commitment to a suboptimal intervention policy is not substantial. This uncertainty also implies that the central bank's preference for one strategy over the other may be affected by the reputation it wishes to uphold.

Next, the results support the use of countercyclical capital buffers. The optimal level of reserve requirements is an indicator of the relative importance of bank reserves given the set of conditions assumed in the simulation. The significance of liquid reserves in enhancing the central bank's value function increases with the unconditional probability of an adverse state of the economy, irrespective of how the central bank responds to a crisis. Since banks have no access to external financing, the higher reserve requirements in this adverse state can only be met through a fire sale of assets. A fire sale, however, leads to further depreciation of the value of a bank's remaining assets. One potential resolution to this issue is the adoption of a policy of countercyclical capital buffers.

The analysis identifies three key factors a central bank should consider when deciding whether to confine any intervention activity only to the interbank market, or to assist the risky bank. These factors are: the unconditional probability that the economy finds itself in an adverse state, the expected duration of an adverse credit supply shock, and an accurate assessment of the level of risky bank's distress probability. The simulation experiments suggest that an intervention limited to guaranteeing the repayment of interbank loans becomes more likely to be the central bank's preferred crisis response strategy when the following conditions hold: the volume of interbank lending is lower, the expected reduction in economic output is short-lived, and the probability of a system-wide crisis is enhanced.

Our baseline simulation results suggest that assisting the risky bank is the preferable strategy for the central bank. The optimal level of liquid reserves under that policy may be so high, however, that it would possibly impair the commercial banks' ability to perform their core function of liquidity creation and credit provision. This finding reflects Diamond and Dybvig's (1986) argument that financial stability policy motivated solely by macroeconomic concerns may undermine the primary reason for the existence of banks.

Finally, our paper emphasises the importance of a central bank's commitment to averting system-wide financial contagion through the interbank market. Our simulations suggest that additional economic costs created by stress in the interbank market equivalent to just 9% of the initial value of banking system assets is sufficient for the policy of no assistance to become the least preferable strategy for the central bank. The widespread use of interest rate derivatives by non-financial firms has created a new channel linking the stability of the financial system to that of the entire economy. In 2014, interest rate contracts accounted for more than 80% of OTC derivatives traded globally, with a notional amount outstanding of more than $500 trillion (Bank for International Settlements, 2015). Furthermore, a well-functioning interbank market is crucial for efficient transmission of monetary policy or of emergency liquidity injections. Severe stress in this market will impair a central bank's ability to achieve its policy objectives.

6 Conclusions

Following the financial crisis of 2007 - 2009, which discredited the notion of constructive ambiguity, defining the appropriate objectives for a central bank's financial stability policy has become less clear-cut. The subject continues to attract ongoing academic and policy debate.

An increasingly complex and interconnected economic system requires the central bank to play a multiplicity of roles; at the intersection of monetary and fiscal policies, as a lender of last resort, as both a supervisor and regulator of the banking sector, with additional responsibilities for monitoring the interbank money market (Freixas and Parigi, 2014).

In this context our model identifies the following as the key factors relevant for the design of a central bank's financial stability policy: the expected duration of a reduction in credit supply due to bank failures, the relative riskiness of the banks in the financial system, and the probability of a banking crisis becoming systemic. Furthermore, the model highlights the importance of ensuring the stability of the interbank money market for the security of the entire economic system.

Finally, our results establish the existence of a substitution effect between the scope of a central bank's assistance to an institution in distress, and the level of required reserve requirements. We find that active management of banks' assistance expectations yields similar results to changing reserve requirements. A poorly designed financial stability policy that does not strike the right balance between the two, however, may result in significant economic costs.

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Appendix A: Derivation of the interbank loan size functions

The risky bank

a) No central bank assistance:

If the central bank provides no assistance to the risky bank at $t = 1$ it earns its expected returns with probability $p(S_H)$, and 0 otherwise. As a result, its expected terminal value is given by

$$E[V_{u,T}] = p(S_H) \times \big[R_H(a_t + \frac{1}{1+\kappa}i_t) - \rho i_t + k_{u,t}\big] \tag{A.1}$$

b) Central bank assistance:

Should the central bank decide to assist the risky bank at $t = 1$ and provide sufficient funding to restore the bank's balance sheet to its initial position, the risky bank's terminal value is equal to $a_t + \kappa a_t$, and its expected terminal value of the risky bank is given by

$$E[V_{u,T}] = p(S_H) \times \big[R_H(a_t + \frac{1}{1+\kappa}i_t) - \rho i_t + k_{u,t}\big] + \big[1 - p(S_H)\big](a_t + \kappa a_t) \tag{A.2}$$

Comparing (A.1) and (A.2), the additional term in equation (A.2) is independent of i_t, so both (A.1) and (A.2) provide the same value for the size of the interbank loan which maximises the expected terminal value of the risky bank.

Therefore:

$$E[V_{u,T}] = p(S_H) \times \big[R_H(a_t + \frac{1}{1+\kappa}i_t) - \rho i_t + k_{u,t}\big] \tag{A.3}$$

Since $\rho = 1 + \rho_0 \frac{i_t}{a_t}$,

$$E[V_{u,T}] = p(S_H) \times \big[R_H(a_t + \frac{1}{1+\kappa}i_t) - (1 + \rho_0 \frac{i_t}{a_t})i_t + \kappa(a_t + \frac{1}{1+\kappa}i_t)\big] \tag{A.4}$$

Taking the first derivative with respect to i_t and setting it equal to zero:

$$\frac{\partial E[V_{u,T}]}{\partial i_t} = p(S_H)\big[R_H \frac{1}{1+\kappa} - 1 - 2\rho_0 \frac{i_t}{a_t} + \frac{\kappa}{1+\kappa}\big] \tag{A.5}$$

$$0 = p(S_H)\big[R_H \frac{1}{1+\kappa} - 1 - 2\rho_0 \frac{i_t}{a_t} + \frac{\kappa}{1+\kappa}\big] \tag{A.6}$$

Since $1 = \frac{1+\kappa}{1+\kappa}$,

$$0 = p(S_H)\big[\frac{R_H - 1 - \kappa + \kappa}{1+\kappa} - 2\rho_0 \frac{i_t}{a_t}\big] \tag{A.7}$$

$$i_t = \frac{(R_H - 1)a_t}{2\rho_0(1+\kappa)} \tag{A.8}$$

Since $a_t = \frac{1}{1+\kappa}$,

$$i_t = \frac{R_H - 1}{2\rho_0(1+\kappa)^2} \tag{A.9}$$

Taking the second derivative of (A.4) with respect to i_t produces

$$\frac{\partial^2 E[V_{u,T}]}{\partial i_t^2} = -2p(S_H)\rho_0(1+\kappa) \tag{A.10}$$

which is strictly negative.

This indicates the size of the interbank loan that maximises the expected terminal value of the risky bank independently of the strategy the central bank selects.

The prudent bank

a) No central bank assistance:

If the central bank decides not to intervene at $t = 1$ the effect on the prudent bank's terminal value depend on whether the inequality $\frac{R_L-1}{R_L}a_t < i_t$ holds. Two cases are relevant.

For $\frac{R_L-1}{R_L}a_t \geq i_t$, the net earnings of the prudent bank exceed the value of the interbank loan to be written off, resulting in the following expected terminal value:

$$\begin{aligned} E[V_{s,T}] = {} & p(S_H)\big[R_L(a_t - i_t) + \rho i_t + k_{s,t}\big] + \big[1 - p(S_H)\big]p(S_L)\big[R_L(a_t - i_t) - i_t + k_{s,t}\big] \\ & + \big[1 - p(S_H)\big]\big[1 - p(S_L)\big](a_t + \kappa a_t) \end{aligned} \tag{A.11}$$

As the final term in equation (A.9) is independent of i_t, the optimal size of the interbank loan for the prudent bank can be computed as:

$$E[V_{s,T}] = p(S_H)\big[R_L(a_t - i_t) + \rho i_t + \kappa a_t\big] + \big[1 - p(S_H)\big]p(S_L)\big[R_L(a_t - i_t) - i_t + \kappa a_t\big] \tag{A.12}$$

which is equivalent to:

$$\begin{aligned} E[V_{s,T}] = {} & p(S_H)\Big[R_L a_t - R_L i_t + i_t + \rho_0 \frac{i_t^2}{a_t} + \kappa a_t - p(S_L)[R_L a_t - R_L i_t - i_t + \kappa a_t]\Big] \\ & + p(S_L)\big[R_L a_t - R_L i_t - i_t + \kappa a_t\big] \end{aligned} \tag{A.13}$$

It follows, by taking the first derivative with respect to i_t and setting it equal to zero:

$$\frac{\partial E[V_{s,T}]}{\partial i_t} = R_L\big[p(S_H)p(S_L) - p(S_H) - p(S_L)\big] + \big[p(S_H)p(S_L) + p(S_H) - p(S_L)\big] + 2p(S_H)\rho_0\frac{i_t}{a_t} \tag{A.14}$$

$$0 = R_L\big[p(S_H)p(S_L) - p(S_H) - p(S_L)\big] + \big[p(S_H)p(S_L) + p(S_H) - p(S_L)\big] + 2p(S_H)\rho_0\frac{i_t}{a_t} \tag{A.15}$$

$$i_t = \frac{\Big[R_L\big[p(S_H) + p(S_L) - p(S_H)p(S_L)\big] + \big[p(S_L) - p(S_H) - p(S_H)p(S_L)\big]\Big]a_t}{2p(S_H)\rho_0} \tag{A.16}$$

Since $a_t = \frac{1}{1+\kappa}$,

$$i_t = \frac{R_L\big[p(S_H) + p(S_L) - p(S_H)p(S_L)\big] + \big[p(S_L) - p(S_H) - p(S_H)p(S_L)\big]}{2p(S_H)\rho_0(1+\kappa)} \tag{A.17}$$

Equation (A.17) denotes the size of the interbank loan resulting in a zero first derivative for the prudent bank if the central bank decides not to intervene at $t = 1$, and the prudent bank's net earnings are sufficient to enable it to write off the interbank loan.

For $\frac{R_L - 1}{R_L}a_t < i_t$, the prudent bank's net earnings are not sufficient to absorb the loss associated with writing off the interbank loan. Therefore it requires the central bank's assistance in both the neutral and adverse state of the economy. Consequently, its terminal value function is given by the following expression:

$$E[V_{s,T}] = p(S_H)\big[R_L(a_t - i_t) + \rho i_t + k_{s,t}\big] + \big[1 - p(S_H)\big](a_t + \kappa a_t) \tag{A.18}$$

Taking the first derivative with respect to i_t and setting it equal to zero:

$$\frac{\partial E[V_{s,T}]}{\partial i_t} = p(S_H)\big[-R_L + 1 + 2\rho_0\frac{i_t}{a_t}\big] \tag{A.19}$$

$$0 = p(S_H)\big[-R_L + 1 + 2\rho_0\frac{i_t}{a_t}\big] \tag{A.20}$$

$$i_t = \frac{(R_L - 1)a_t}{2\rho_0} \tag{A.21}$$

Since $a_t = \frac{1}{1+\kappa}$,

$$i_t = \frac{R_L - 1}{2\rho_0(1+\kappa)} \tag{A.22}$$

Equation (A.22) gives the size of the interbank loan resulting in a zero first derivative for the prudent bank if the central bank decides not to intervene at $t = 1$, and the prudent bank's net earnings are insufficient to enable it to write off the interbank loan.

b) Central bank intervenes in the interbank market:

If the central bank decides to intervene in the interbank market it provides the prudent bank with the principal amount of the interbank loan in the neutral state, and assists it in the adverse state, resulting in the following expected terminal value function:

$$E[V_{s,T}] = p(S_H)\big[R_L(a_t - i_t) + \rho i_t + k_{s,t}\big] + \big[1 - p(S_H)\big]p(S_L)\big[R_L(a_t - i_t) + i_t + k_{s,t}\big] + \big[1 - p(S_H)\big]\big[1 - p(S_L)\big](a_t + \kappa a_t) \tag{A.23}$$

Taking the first derivative with respect to i_t and setting it equal to zero:

$$\frac{\partial E[V_{s,T}]}{\partial i_t} = \big[p(S_H) + p(S_L) - p(S_H)p(S_L)\big](-R_L + 1) + 2p(S_H)\rho_0 \frac{i_t}{a_t} \tag{A.24}$$

$$0 = \big[p(S_H) + p(S_L) - p(S_H)p(S_L)\big](-R_L + 1) + 2p(S_H)\rho_0 \frac{i_t}{a_t} \tag{A.25}$$

$$i_t = \frac{\big[p(S_H) + p(S_L) - p(S_H)p(S_L)\big](R_L - 1)a_t}{2p(S_H)\rho_0} \tag{A.26}$$

Since $a_t = \frac{1}{1+\kappa}$,

$$i_t = \frac{(R_L - 1)\big[p(S_H) + p(S_L) - p(S_H)p(S_L)\big]}{2p(S_H)\rho_0(1 + \kappa)} \tag{A.27}$$

Equation (A.27) depicts the size of the interbank loan resulting in a zero first derivative for the prudent bank if the central bank decides to intervene in the interbank market at $t = 1$.

c) Central bank assists risky bank at $t = 1$:

Finally, if the central bank assists the risky bank at $t = 1$, the prudent bank will receive the principal amount of the interbank loan in the neutral state, and will be resolved in the adverse state, as the central bank can only assist one commercial bank. Consequently, its expected terminal value is given by the following function:

$$E[V_{s,T}] = p(S_H)\big[R_L(a_t - i_t) + \rho i_t + \kappa a_t\big] + \big[1 - p(S_H)\big]p(S_L)\big[R_L(a_t - i_t) + i_t + \kappa a_t\big] \tag{A.28}$$

This function's first derivative with respect to i_t is equivalent to (A.24). As a result, the size of the loan if the central bank assists the risky bank at $t = 1$ is given by equation

(A.27).

The second derivatives of (A.12), (A.18), (A.23) and (A.28) with respect to i_t are given by

$$\frac{\partial^2 E[V_{s,T}]}{\partial i_t^2} = 2p(S_H)\rho_0(1+\kappa) \tag{A.29}$$

which is strictly positive, indicating that the expected terminal value function of the safe bank has a global minimum.

As shown in the main body of the paper (see: Figure 2), the maximum expected terminal value of the prudent bank is achieved at the highest possible volume of interbank lending.